$D^* \equiv N$ bound state in strange three-body systems

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The recent update of the strangeness -2 ESC08c Nijmegen potential incorporating the NAGARA and KISO events predicts a ΞN bound state, D^* , in the ${}^{3}S_{1}(I = 1)$ channel. We study if the existence of this two-body bound state could give rise to stable three-body systems. For this purpose we solve the bound state problem of three-body systems where the ΞN state is merged with N's, Λ 's, Σ 's, or Ξ 's, making use of the most recent updates of the two-body ESC08c Nijmegen potentials. We found that there appear stable states in the ΞNN and $\Xi \Xi N$ systems, the $\Xi \Lambda N$ and $\Xi \Sigma N$ systems being unbound.

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I. INTRODUCTION

The hyperon-nucleon (YN) and hyperon-hyperon (YY)interactions are not only of interest by themselves but they constitute also the input for microscopic calculations of few- and many-body systems involving strangeness, such as exotic neutron star matter [1–5] or hypernuclei [6–8]. Recently the so-called KISO event was reported, the first clear evidence of a deeply bound state of Ξ^{-} -¹⁴N [9]. Although microscopic calculations are impossible in this case and, consequently, their interpretation will be always afflicted by large uncertainties, the ESC08c Nijmegen potential has been recently updated to account for the most recent experimental information of the strangeness -2 sector, the KISO [9] and the NAGARA [10] events, concluding the existence of a bound state, D^* , in the ${}^{3}S_{1}(I = 1)\Xi N$ channel with a binding energy of 1.56 MeV [11,12].

In a recent series of papers [13,14] we have studied the consequences of the existence of this ΞN bound state in fewbaryon systems with nucleons, specifically because for some quantum numbers such states could be stable, which can be easily tested against future data. In Ref. [13] we analyzed the possible existence of ΞNN bound states in isospin 3/2 channels, motivated by the decoupling from the lowest $\Lambda\Lambda N$ channel, due to isospin conservation, which would make a possible bound state stable. We found a $\Xi NN J^P = \frac{1}{2}^+$ bound state with a binding energy of about 2.5 MeV.¹ In Ref. [14] we found a ΞNN deeply bound state with quantum numbers $(I)J^P = (\frac{1}{2})\frac{3}{2}^+$, lying 13.5 MeV below the Ξd threshold, due to the coherent effect of the deuteron, a *NN* bound state, and the D^* , a ΞN bound state.

In a manner similar to the existence of the deuteron, a *NN* bound state which is responsible for the existence of the

triton, NNN, and the hypertriton, ΛNN , stable three-body bound states; in this paper we study if the existence of the $D^* \equiv N$ bound state could give rise to other stable few-body systems when it is merged with N's, Λ 's, Σ 's or Ξ 's. The possible existence of stable few-body states containing a ΞN two-body subsystem is suggested by the attractive character of the $\Lambda \Xi$, $\Sigma \Xi$, and $\Xi \Xi$ interactions for some partial waves [15-21]. There are also preliminary studies of the $\Xi \Xi N$ system [22] indicating that lattice QCD calculations of multibaryon systems are now within sight. To carry out our objectives, we will study the ΞNN , $\Xi \Lambda N$, $\Xi \Sigma N$, and $\Xi \Xi N$ three-body systems. We will make use of the most recent updates of the ESC08c Nijmegen potentials in the strangeness -1, -2, -3, and -4 sectors [11,20,21], accounting for the recent KISO [9] and NAGARA [10] events in the strangeness -2 sector.

Recent preliminary results from lattice QCD suggest an overall attractive ΞN interaction [23] which may be relevant for the first Ξ hypernucleus reported in Ref. [9]. Besides the recent update of the ESC08c Nijmegen model [11,12], there are other models predicting bound states in the ΞN system prior to the KISO event, such as the chiral constituent quark model of Ref. [24]. However, one should keep in mind that there are other models for the ΞN interaction, such as the hybrid quark-model-based analysis of Ref. [25], the effective field theory approach of Ref. [26], and even some of the earlier models of the Nijmegen group [16] that do not present ΞN bound states and in which, in general, the interactions are weakly attractive or repulsive. Thus, one does not expect that these models will give rise to ΞNN or ΞYN bound states. On the other hand, current Ξ hypernuclei studies [6–8] have been performed by means of ΞN interactions derived from the Nijmegen models, and thus our study complements such previous works for the simplest systems that could be studied exactly.

The paper is organized as follows. We will use Sec. II to describe all technical details to solve the three-body boundstate Faddeev equations. In Sec. III we will construct the two-body amplitudes needed for the solution of the bound state three-body problem. Our results will be presented and discussed in Sec. IV. Finally, in Sec. V we summarize our main conclusions.

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¹This binding energy was recalculated in Ref. [14] including, in addition to the ΞN isospin-spin (i, j) = (1, 1) channel, also the ΞN (1,0) channel, which is mainly repulsive, obtaining a bound state with a binding energy slightly smaller, 1.33 MeV below threshold.

II. THE THREE-BODY BOUND-STATE FADDEEV EQUATIONS

We will restrict ourselves to the configurations where all three particles are in S-wave states so that the Faddeev equations for the bound-state problem in the case of three baryons with total isospin I and total spin J are

$$T_{i;IJ}^{i_ij_i}(p_iq_i) = \sum_{j \neq i} \sum_{i_jj_j} h_{ij;IJ}^{i_ij_i;i_jj_j} \frac{1}{2} \int_0^\infty q_j^2 dq_j$$

$$\times \int_{-1}^1 d\cos\theta \, t_{i;i_ij_i} (p_i, p_i'; E - q_i^2/2\nu_i)$$

$$\times \frac{1}{E - p_j^2/2\mu_j - q_j^2/2\nu_j} \, T_{j;IJ}^{i_jj_j}(p_jq_j), \quad (1)$$

where $t_{i;i_ij_i}$ stands for the two-body amplitudes with isospin i_i and spin j_i . p_i is the momentum of the pair jk (with ijk an even permutation of 123) and q_i the momentum of particle iwith respect to the pair jk. μ_i and ν_i are the corresponding reduced masses,

$$\mu_i = \frac{m_j m_k}{m_j + m_k},$$

$$\nu_i = \frac{m_i (m_j + m_k)}{m_i + m_j + m_k},$$
(2)

and the momenta p'_i and p_j in Eq. (1) are given by

$$p'_{i} = \sqrt{q_{j}^{2} + \frac{\mu_{i}^{2}}{m_{k}^{2}}q_{i}^{2} + 2\frac{\mu_{i}}{m_{k}}q_{i}q_{j}\cos\theta},$$

$$p_{j} = \sqrt{q_{i}^{2} + \frac{\mu_{j}^{2}}{m_{k}^{2}}q_{j}^{2} + 2\frac{\mu_{j}}{m_{k}}q_{i}q_{j}\cos\theta}.$$
(3)

 $h_{ij;IJ}^{i_i j_i ; i_j j_j}$ are the spin–isospin coefficients,

$$\begin{aligned} h_{ij;IJ}^{i_{i}j_{i};i_{j}j_{j}} \\ &= (-)^{i_{j}+\tau_{j}-I}\sqrt{(2i_{i}+1)(2i_{j}+1)}W(\tau_{j}\tau_{k}I\tau_{i};i_{i}i_{j}) \\ &\times (-)^{j_{j}+\sigma_{j}-J}\sqrt{(2j_{i}+1)(2j_{j}+1)}W(\sigma_{j}\sigma_{k}J\sigma_{i};j_{i}j_{j}), \end{aligned}$$

$$(4)$$

where *W* is the Racah coefficient and τ_i , i_i , and *I* (σ_i , j_i , and *J*) are the isospins (spins) of particle *i*, of the pair *jk*, and of the three-body system.

Since the variable p_i in Eq. (1) runs from 0 to ∞ , it is convenient to make the transformation

$$x_i = \frac{p_i - b}{p_i + b},\tag{5}$$

where the new variable x_i runs from -1 to 1 and *b* is a scale parameter that has no effect on the solution. With this transformation, Eq. (1) takes the form

$$T_{i;IJ}^{i_i,j_i}(x_iq_i) = \sum_{j \neq i} \sum_{i_j,j_j} h_{i_j;IJ}^{i_i,j_i,j_j} \frac{1}{2} \int_0^\infty q_j^2 dq_j$$

$$\times \int_{-1}^1 d\cos\theta \, t_{i;i_i,j_i} \big(x_i, x_i'; E - q_i^2 / 2\nu_i \big)$$

$$\times \frac{1}{E - p_j^2 / 2\mu_j - q_j^2 / 2\nu_j} \, T_{j;IJ}^{i_j,j_j}(x_jq_j). \tag{6}$$

Since in the amplitude $t_{i;i_i,j_i}(x_i, x'_i; e)$ the variables x_i and x'_i run from -1 to 1, one can expand this amplitude in terms of Legendre polynomials as

$$t_{i;i_i j_i}(x_i, x_i'; e) = \sum_{nr} P_n(x_i) \tau_{i;i_i j_i}^{nr}(e) P_r(x_i'),$$
(7)

TABLE I. Low-energy parameters of the most recent updates of the ESC08c Nijmegen interactions for the ΛN [11], ΣN [11], ΞN [11], $\Lambda \Xi$ [21], $\Sigma \Xi$ [21], and $\Xi \Xi$ [21] systems, and the parameters of the corresponding local potentials given by Eq. (19).

	(i, j)	<i>a</i> (fm)	r_0 (fm)	A (MeV fm)	$\mu_A (\mathrm{fm}^{-1})$	B (MeV fm)	$\mu_B (\mathrm{fm}^{-1})$
ΛN	(1/2,0)	-2.62	3.17	280	2.00	655	3.55
	(1/2, 1)	-1.72	3.50	170	1.95	670	4.60
ΣN	(3/2,0)	-3.91	3.41	122	1.47	388	3.55
	(3/2, 1)	0.61	-2.35	329	4.12	124	1.71
ΞN	$(0,0)^{a}$			120	1.30	510	2.30
	(0,1)	-5.357	1.434	377	2.68	980	6.61
	(1,0)	0.579	-2.521	290	3.05	155	1.60
	(1,1)	4.911	0.527	568	4.56	425	6.73
ΛΞ	(1/2,0)	-9.83	2.38	370	2.20	970	3.90
	(1/2, 1)	-12.9	2.00	130	1.90	340	4.50
$\Sigma \Xi$	(3/2,0)	-2.80	2.45	111	2.00	315	4.73
	(3/2, 1)	-10.9	1.92	147	2.07	790	6.33
	(0,1)	0.53	1.63	210	1.60	560	2.05
ΞΞ	(1,0)	-7.25	2.00	155	1.75	490	5.60

^aThis channel is discussed in Sec. III.

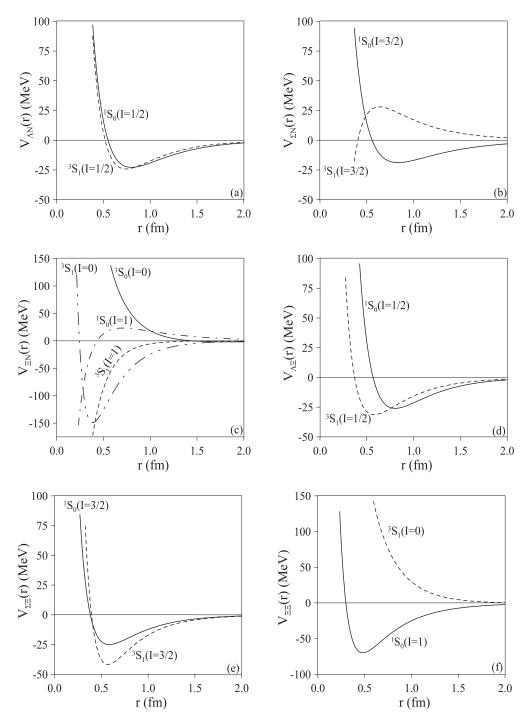


FIG. 1. (a) $V_{\Delta N}(r)$ potential as given by Eq. (19) with the parameters of Table I. (b) Same as (a) for the $V_{\Sigma N}(r)$ potential. (c) Same as (a) for the $V_{\Xi N}(r)$ potential. (d) Same as (a) for the $V_{\Delta \Xi}(r)$ potential. (e) Same as (a) for the $V_{\Sigma \Xi}(r)$ potential. (f) Same as (a) for the $V_{\Xi \Xi}(r)$ potential.

where the expansion coefficients are given by

Applying expansion (7) in Eq. (6), one gets

$$T_{i;IJ}^{i_i j_i}(x_i q_i) = \sum_n P_n(x_i) T_{i;IJ}^{n i_i j_i}(q_i),$$
(9)

$$\tau_{i;i_ij_i}^{nr}(e) = \frac{2n+1}{2} \frac{2r+1}{2} \int_{-1}^{1} dx_i \\ \times \int_{-1}^{1} dx_i' P_n(x_i) t_{i;i_ij_i}(x_i, x_i'; e) P_r(x_i').$$
(8)

where $T_{i;IJ}^{ni_i j_i}(q_i)$ satisfies the one-dimensional integral equation

$$T_{i;IJ}^{ni_ij_i}(q_i) = \sum_{j \neq i} \sum_{m_ij_j} \int_0^\infty dq_j A_{ij;IJ}^{ni_ij_j;m_ij_j}(q_i,q_j;E) T_{j;IJ}^{mi_jj_j}(q_j),$$
(10)

with

$$A_{ij;IJ}^{ni_{i}j_{i};mi_{j}j_{j}}(q_{i},q_{j};E) = h_{ij;IJ}^{i_{i}j_{i};i_{j}j_{j}} \sum_{r} \tau_{i;i_{i}j_{i}}^{nr} \left(E - q_{i}^{2}/2\nu_{i}\right) \frac{q_{j}^{2}}{2} \\ \times \int_{-1}^{1} d\cos\theta \, \frac{P_{r}(x_{i}')P_{m}(x_{j})}{E - p_{j}^{2}/2\mu_{j} - q_{j}^{2}/2\nu_{j}}$$
(11)

The three amplitudes $T_{1;IJ}^{ri_1j_1}(q_1)$, $T_{2;IJ}^{mi_2j_2}(q_2)$, and $T_{3;IJ}^{ni_3j_3}(q_3)$ in Eq. (10) are coupled together. The number of coupled equations can be reduced, however, when two of the particles are identical. The reduction procedure for the case where one has two identical fermions has been described before [27,28] and will not be repeated here. With the assumption that particles 2 and 3 are identical and particle 1 is the different one, only the amplitudes $T_{1;IJ}^{ri_1j_1}(q_1)$ and $T_{2;IJ}^{mi_2j_2}(q_2)$ are independent from each other and they satisfy the coupled integral equations

$$T_{1;IJ}^{ri_{1}j_{1}}(q_{1}) = 2\sum_{mi_{2}j_{2}} \int_{0}^{\infty} dq_{3} A_{13;IJ}^{ri_{1}j_{1};mi_{2}j_{2}}(q_{1},q_{3};E) T_{2;IJ}^{mi_{2}j_{2}}(q_{3}),$$
(12)

$$T_{2;IJ}^{ni_{2}j_{2}}(q_{2}) = \sum_{mi_{3}j_{3}} g \int_{0}^{\infty} dq_{3} A_{23;IJ}^{ni_{2}j_{2};mi_{3}j_{3}}(q_{2},q_{3};E) T_{2;IJ}^{mi_{3}j_{3}}(q_{3}) + \sum_{ri_{1}j_{1}} \int_{0}^{\infty} dq_{1} A_{31;IJ}^{ni_{2}j_{2};ri_{1}j_{1}}(q_{2},q_{1};E) T_{1;IJ}^{ri_{1}j_{1}}(q_{1}),$$
(13)

with the identical-particle factor

$$g = (-)^{1+\sigma_1+\sigma_3-j_2+\tau_1+\tau_3-i_2},$$
(14)

where σ_1 (τ_1) stand for the spin (isospin) of the different particle and σ_3 (τ_3) for those of the identical ones.

Substitution of Eq. (12) into Eq. (13) yields an equation with only the amplitude T_2 ,

$$T_{2;IJ}^{ni_2j_2}(q_2) = \sum_{mi_3j_3} \int_0^\infty dq_3 K_{IJ}^{ni_2j_2;mi_3j_3}(q_2,q_3;E) T_{2;IJ}^{mi_3j_3}(q_3),$$
(15)

where

$$K_{IJ}^{ni_{2}j_{2};mi_{3}j_{3}}(q_{2},q_{3};E) = gA_{23;IJ}^{ni_{2}j_{2};mi_{3}j_{3}}(q_{2},q_{3};E) + 2\sum_{r_{1}j_{1}}\int_{0}^{\infty} dq_{1}A_{31;IJ}^{ni_{2}j_{2};ri_{1}j_{1}}(q_{2},q_{1};E)A_{13;IJ}^{ri_{1}j_{1};mi_{3}j_{3}}(q_{1},q_{3};E).$$
(16)

III. TWO-BODY AMPLITUDES

We have constructed the two-body amplitudes for all subsystems entering the three-body problems studied by solving the Lippmann–Schwinger equation of each (i, j) channel,

$$t^{ij}(p,p';e) = V^{ij}(p,p') + \int_0^\infty p''^2 dp'' V^{ij}(p,p'') \\ \times \frac{1}{e - p''^2/2\mu} t^{ij}(p'',p';e),$$
(17)

where

$$V^{ij}(p,p') = \frac{2}{\pi} \int_0^\infty r^2 dr \ j_0(pr) V^{ij}(r) j_0(p'r), \qquad (18)$$

and the two-body potentials consist of an attractive and a repulsive Yukawa term, i.e.,

$$V^{ij}(r) = -A \frac{e^{-\mu_A r}}{r} + B \frac{e^{-\mu_B r}}{r}.$$
 (19)

The parameters of all ΛN , ΣN , ΞN , $\Lambda \Xi$, $\Sigma \Xi$, and $\Xi \Xi$ channels were obtained by fitting the low-energy data of each

TABLE II. Two-body *NN*, *YN*, and *YY* isospin-spin (i, j) channels that contribute to a given three-body state with total isospin *I* and total spin *J*. The last column indicates the corresponding threshold for each state, given by $\sum_{i=1}^{3} M_i - E$, where M_i are the masses of the baryons of each channel; B_1 stands for the binding energy of the deuteron and B_2 for the binding energy of the $D^* \Xi N$ state.

	(I,J)	ΛN	ΞN	$\Lambda \Xi$	$\Sigma \Xi(\Sigma N)$	$\Xi\Xi(NN)$	Ε
ΞNN	(1/2,1/2) (1/2,3/2) (3/2,1/2) (3/2,3/2)		(0,0),(0,1),(1,0),(1,1) (0,1),(1,1) (1,0),(1,1) (1,1) (1,1)			(0,1),(1,0) (0,1) (1,0)	$B_1 \\ B_1 \\ B_2 \\ B_2 \\ B_2$
$\Xi\Lambda N$	(0,1/2) (0,3/2) (1,1/2) (1,3/2)	$(1/2,0),(1/2,1) \\ (1/2,1) \\ (1/2,0),(1/2,1) \\ (1/2,1)$	(0,0),(0,1) (0,1) (1,0),(1,1) (1,1)	$(1/2,0),(1/2,1) \\ (1/2,1) \\ (1/2,0),(1/2,1) \\ (1/2,1)$			$0\\0\\B_2\\B_2$
$\Xi\Sigma N$	(2,1/2) (2,3/2)		(1,0),(1,1) (1,1)		(3/2,0),(3/2,1) (3/2,1)		B_2 B_2
ΞΞN	(1/2, 1/2) (1/2, 3/2) (3/2, 1/2) (3/2, 3/2)		$(0,0),(0,1),(1,0),(1,1) \\ (0,1),(1,1) \\ (1,0),(1,1) \\ (1,1)$			(0,1),(1,0) (0,1) (1,0)	$B_2 \\ B_2 \\ B_2 \\ B_2 \\ B_2$

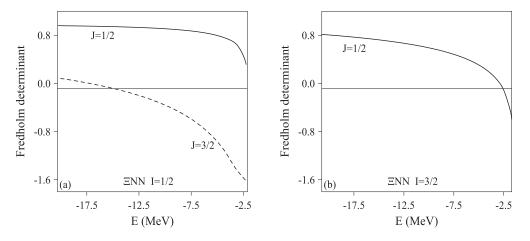


FIG. 2. (a) Fredholm determinant for the J = 1/2 and J = 3/2 $I = 1/2 \equiv NN$ channels. (b) Fredholm determinant for the J = 1/2 $I = 3/2 \equiv NN$ channel.

channel as given in the most recent update of the strangeness -1 and -2 [11] and strangeness -3 and -4 [21] ESC08c Nijmegen potentials. The low-energy data and the parameters of these models are given in Table I. The $\Xi N^{-1}S_0$ (I = 0) potential was fitted to the ΞN phase shifts given in Fig. 14 of Ref. [11] without taking into account the inelasticity, i.e., assuming $\rho = 0$ (this two-body channel does not contribute to the three-body bound states found in this work). For the ΣN system we only consider the I = 3/2 channels, because the I = 1/2 channels would decay strongly to ΛN states. Analogously, for the $\Sigma \Xi$ system we only consider the I = 3/2 channels, because the I = 1/2 channels would decay strongly to $\Lambda \Xi$ states. In the case of the NN (0,1) and (1,0) channels we use the Malfliet-Tjon models [29] with the parameters given in Ref. [30].

The potentials obtained are shown in Fig. 1. In Fig. 1(a) we show the $V_{\Lambda N}(r)$ potential that it is constrained by the existence of experimental data. The interaction is attractive at intermediate range and strongly repulsive at short range, but without having bound states. The same could be said about the $I = 3/2 V_{\Sigma N}(r)$ potentials shown in Fig. 1(b). The existence of $\Sigma^{\pm} p$ cross sections tightly constrains the

interaction. As can be seen the ${}^{3}S_{1}(I = 3/2)$ potential is strongly repulsive at intermediate range, which makes rather unlikely the existence of three-body bound states containing this ΣN channel. In Fig. 1(c) we show the $V_{\Xi N}(r)$ potential, where one notes the attractive character of the ${}^{3}S_{1}(I = 1) \Xi N$ partial wave, giving rise to the D^* bound state with a binding energy of 1.67 MeV. We also confirm how all the $J = 1 \Xi N$ interactions are attractive [21]. The $V_{\Lambda\Xi}(r)$ potentials shown in Fig. 1(d) are rather similar to the $V_{\Lambda N}(r)$ case, the intermediate range attraction not being enough to generate two-body bound states. The $I = 3/2 V_{\Sigma\Xi}(r)$ potentials are shown in Fig. 1(e); analogously to the $\Lambda \Xi$ case, being attractive they do not present two-body bound states. Regarding the $\Xi\Xi$ interaction, Fig. 1(f), we observe the attractive character of the ${}^{1}S_{0}(I = 1)$ potential; although having bound states in earlier versions of the ESC08c Nijmegen potential [16], in the most recent update of the strangeness -4 sector it does not present a bound state [21]. The existence of bound states in the $\Xi\Xi$ system has been predicted by different calculations in the literature [15,17,18]. It can be definitively stated that all models agree on the fairly important attractive character of this channel, either with or without a bound state [19].

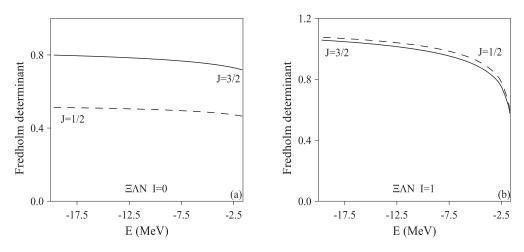


FIG. 3. (a) Fredholm determinant for the J = 1/2 and J = 3/2 $I = 0 \Xi \Lambda N$ channels. (b) Fredholm determinant for the J = 1/2 and J = 3/2 $I = 1 \Xi \Lambda N$ channels.

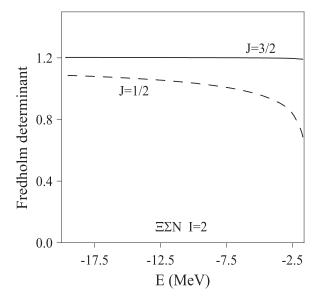


FIG. 4. Fredholm determinant for the J = 1/2 and $J = 3/2 I = 2 \Sigma N$ channels.

IV. RESULTS AND DISCUSSION

We show in Table II the channels of the different twobody subsystems contributing to each (I, J) three-body state. For the $\Xi \Sigma N$ system we only consider the I = 2 channels, because the I = 0 and 1 would decay strongly to $\Xi \Lambda N$ states. The three-body problem is solved by means of the ESC08c Nijmegen interactions described in Sec. III and given in Table I. The binding energies are measured with respect to the lowest threshold, indicated in Table II for each particular state.

We show in Fig. 2 the Fredholm determinant of all ΞNN channels that had been previously studied in Refs. [13,14]. As we can see in Fig. 2(b), a bound state is found for the $(I)J^P = (\frac{3}{2})\frac{1}{2}^+ \Xi NN$ state, 1.33 MeV below the corresponding threshold, $2m_N + m_{\Xi} - B_2$, where B_2 is the binding energy of the $D^* \Xi N$ state. However, the most interesting result of the ΞNN system is shown in Fig. 2(a): the very large binding energy of the $(\frac{1}{2})\frac{3}{2}^+$ state, which would make it easy

to identify experimentally as a sharp resonance lying some 15.7 MeV below the ΞNN threshold. The $\Lambda\Lambda-\Xi N$ (i,j) = (0,0) transition channel, which is responsible for the decay $\Xi NN \rightarrow \Lambda\Lambda N$, does not contribute to the $(I)J^P = (\frac{1}{2})\frac{3^+}{2}$ state in a pure *S*-wave configuration [14]. One would need at least the spectator nucleon to be in a *D* wave or that the $\Lambda\Lambda-\Xi N$ transition channel to be in one of the negative parity *P*-wave channels, with the nucleon spectator also in a *P* wave. Thus, due to the angular momentum barriers, the resulting decay width of the $(\frac{1}{2})\frac{3^+}{2}$ state is expected to be very small.

For the ΞNN three-baryon system with (I, J) = (3/2, 3/2), only the $(i, j) = (1, 1) \Xi N$ channel contributes (see Table II), and the corresponding Faddeev equations with two identical fermions can be written as [31]

$$T = -t_N^{N\Xi} G_0 T. ag{20}$$

Thus, due to the negative sign in the right-hand side, the ΞN interaction is effectively repulsive and, therefore, no bound state is possible in spite of the attraction of the ΞN subsystem. The minus sign in Eq. (20) is a consequence of the identity of the two nucleons, since the first term of the right-hand side of Eq. (20) proceeds through Ξ exchange, and it corresponds to a diagram where the initial and final states differ only in that the two identical fermions have been interchanged, which brings the minus sign. This effect has been pointed out before [32]. This is the reason why the Fredholm determinant for the $(I,J) = (3/2,3/2) \Xi N N$ channel is not shown in Fig. 2(b).

We show in Fig. 3 the Fredholm determinant of all $\Xi \Lambda N$ channels. As can be seen, although the ΛN interaction is attractive [see Fig. 1(a)], it is not enough to generate bound states in the three-body system. The channels with I = 1 are more attractive than those with I = 0, where the Fredholm determinant is rather flat, but they are far from being bound. Note that whereas in the ΞNN and $\Xi \Xi N$ systems the ΞN interaction in the bound-state channel appears twice, in the $\Xi \Lambda N$ system this interaction appears only once, which is the reason why this last system has no bound states.

We present in Fig. 4 the Fredholm determinant of the $I = 2\Xi\Sigma N$ channels. As expected, due to the contribution of the strongly repulsive ${}^{3}S_{1}(I = 3/2) \Sigma N$ channel in all

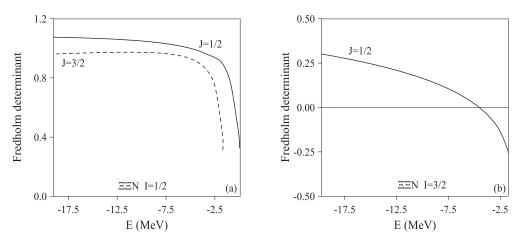


FIG. 5. (a) Fredholm determinant for the J = 1/2 and J = 3/2 $I = 1/2 \Xi \Xi N$ channels. (b) Fredholm determinant for the J = 1/2 $I = 3/2 \Xi \Xi N$ channel.

TABLE III. Separation energy, in MeV, of the different $(I)J^P$ three-body states containing ΞN subsystems.

$(I)J^{P}$	$(\frac{1}{2})\frac{3}{2}^+$	$(\frac{3}{2})\frac{1}{2}^+$
$\Xi N N$	13.54	1.33
$\Xi \Xi N$		2.85

 $I = 2 \ \Xi \Sigma N$ three-body systems, there do not appear any bound states.

Finally, we show in Fig. 5 the Fredholm determinant of all $\Xi \Xi N$ channels. The Fredholm determinant for the $(I)J^P = (3/2)3/2^+$ channel is not shown in Fig. 5(b) for the same reason explained above for the ΞNN system: it is strongly repulsive. In the $\Xi \Xi N$ system there appears a bound state with quantum numbers $(I)J^P = (\frac{3}{2})\frac{1}{2}^+$, 2.85 MeV below the lowest threshold, $2m_{\Xi} + m_N - B_2$, where B_2 stands for the binding energy of the $D^* \Xi N$ subsystem. Since this $\Xi \Xi N$ state has isospin 3/2 it can not decay into $\Xi \Lambda \Lambda$ due to isospin conservation, so it would be stable. This stable state appears in spite of the fact that the last update of the ESC08c Nijmegen $\Xi\Xi {}^{1}S_{0}(I = 1)$ potential has no bound states, as is predicted, however, by several models in the literature. If bound states would exist for the $\Xi\Xi$ system, the three-body state would become deeply bound, as happens for the ΞNN system. The I = 1/2 channels are also attractive but they are not bound.

We summarize in Table III the stable bound states of the different three-body systems containing a ΞN subsystem.

V. SUMMARY

Recent results in the strangeness -2 sector, the so-called KISO event, reported clear evidence of a deeply bound state of Ξ^{-14} N. which could point out that the average ΞN interaction might be attractive. We have made use of the most recent updates of the ESC08c Nijmegen potential in the different strangeness sectors, accounting for the recent experimental information, to study the bound state problem of three-body systems containing a ΞN subsystem: ΞNN , $\Xi \Lambda N$, $\Xi \Sigma N$, and $\Xi \Xi N$. We have found that the ΞNN system presents bound states with quantum numbers $(I)J^P = (3/2)1/2^+$ and $(1/2)3/2^+$, the last one being a deeply bound state lying 13.54 MeV below the Ξd threshold. The $\Xi \Lambda N$ system is unbound for all possible quantum numbers due to a reduced

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contribution of the ΞN interaction in the bound-state channel. We found the same for the $\Xi \Sigma N$ system; in this case the negative results are even reinforced by the contribution of the repulsive ${}^{3}S_{1}(I = 3/2) \Sigma N$ interaction. The $\Xi \Xi N$ system presents a bound state with quantum numbers $(I)J^{P} =$ $(3/2)1/2^{+}$. The states with isospin 3/2 would be stable due to isospin conservation. The state with isospin 1/2 is expected to present a very small decay width due to angular momentum barriers. The $\Xi \Xi N$ bound state does exist in spite of the fact that we have used the most recent update of the ESC08c Nijmegen potential that does not predict $\Xi \Xi$ bound states. If bound states would exist for the $\Xi \Xi$ system, as predicted by several models in the literature, the state would become deeply bound, as happens for the ΞNN system.

As stated in the Introduction, the hyperon-nucleon and hyperon-hyperon interactions are basic inputs for microscopic calculations of few- and many-body systems involving strangeness, such as hypernuclei or exotic neutron star matter. It is expected that the recently approved hybrid experiment E07 at J-PARC, could shed light on the uncertainties of our knowledge of the hadron-hadron interaction in the baryon octet. Meanwhile the scarce experimental information, together with the impossibility of microscopic calculations to study observations like the ones reported in Ref. [9], means its interpretation will be always afflicted by large uncertainties, giving rise to ample room for speculation. The detailed theoretical investigation presented in our recent works about the possible existence of bound states based on realistic models is a basic tool to advance knowledge of the details of the hyperon-nucleon and hyperon-hyperon interactions. First, it could help to raise the awareness of the experimentalist that it is worthwhile to investigate few-baryon systems, specifically because for some quantum numbers such states could be stable. Second, it makes clear that strong and attractive YN and YYinteractions have consequences for the few-body sector and can be easily tested against future data.

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