Numerical values of the f^F, f^D, and f^S coupling constants in the SU(3) invariant interaction Lagrangian of the vector-meson nonet with $1/2^+$ octet baryons

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It is demonstrated how the use of all existing experimental information on electric and magnetic nucleon form factors, described by the unitary and analytic (U&A) nucleon electromagnetic structure model in spacelike and timelike regions simultaneously, can provide numerical values of f^F, f^D, and f^S coupling constants in SU(3) invariant interaction Lagrangian of the vector-meson nonet with $1/2^+$ octet baryons. The latter, together with universal vector-meson coupling constants f_V, play an essential role in a prediction of $1/2^+$ octet hyperon electromagnetic form factor behaviors.

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I. INTRODUCTION

Recently the BESIII Collaboration has published [1] new results on proton electromagnetic (EM) form factors (FFs) in the timelike region by measuring the process $e^+e^- \to p\bar{p}$ with very high precision at 12 different center-of-mass energies from 4.9836 to 13.4762 GeV^2. In such a way, the obtained results have substantially enriched a set of already existing experimental points on proton EM FFs in the spacelike and timelike regions. Moreover, the BESIII Collaboration and earlier also the BaBar Collaboration [2], measuring the polar angular distribution $F(\cos\theta_p)$ of produced protons at a few different energies, have been able to determine for the first time separate information on the proton electric and proton magnetic FFs in the timelike region.

On the other hand, a very successful microscopic model for the proton EM FFs in the near-threshold timelike region has been developed [3] based on the assumption that the behavior of the EM FFs as a function of energy is given by the initial- or final-state interaction between proton and antiproton in the processes $e^+e^- \leftrightarrow p\bar{p}$. In this approach the antinucleon-nucleon potential is constructed in the framework of chiral effective field theory [4] and fitted to results of partial-wave analysis of existing $\bar{p}p$ scattering data. As a result, predictions of the proton EM FFs $G_E^p(t)$ and $G_M^p(t)$ and their ratios are in good agreement with existing data up to $t = 3.986$ GeV^2, i.e., within the region of validity of the model under consideration.

Future plans of the BESIII Collaboration are to extend such data measurements for $e^+e^- \to Y\bar{Y}$ processes, where Y are hyperons from the $1/2^+$ octet baryons $p, n, \Lambda, \Sigma^+, \Sigma^0, \Sigma^-, \Xi^0$, and Ξ^-.

In a preparation of such a program, it can be useful to have some model predictions for hyperon EM FF behaviors and subsequently for the corresponding differential and total cross sections.

However, the model presented in Ref. [3] is not applicable for such model predictions of hyperon EM FFs as it is based on the existence of baryon-antibaryon scattering data, which is not possible for the case of hyperons.

In this paper we demonstrate a scheme within which one can do such predictions. As one can see, an important role in the prediction of hyperon EM FFs is played by the advanced unitary and analytic (U&A) nucleon EM structure model, respecting the SU(3) symmetry and OZI rule [5–7] violation. A similarly important role is played by knowledge of the universal vector-meson coupling constants f_V to be extracted from the experimental values of vector-meson lepton widths

$$\Gamma(V \to e^+e^-) = \frac{\alpha^2 m_V}{3}\left(\frac{f_V^2}{4\pi}\right)^{-1}, \tag{1}$$

with $\alpha = \frac{1}{137}$ being the QED fine structure coupling constant and m_V being the vector-meson mass, and knowledge of numerical values of f^F, f^D, and f^S coupling constants in the SU(3) invariant Lagrangian

$$\begin{aligned} L_{VB\bar{B}} = {} & \frac{i}{\sqrt{2}} f^F\left[\bar{B}^\alpha_\beta \gamma_\mu B^\beta_\gamma - \bar{B}^\beta_\gamma \gamma_\mu B^\alpha_\beta\right](V_\mu)^\gamma_\alpha \\ & + \frac{i}{\sqrt{2}} f^D\left[\bar{B}^\beta_\gamma \gamma_\mu B^\alpha_\beta + \bar{B}^\alpha_\gamma \gamma_\mu B^\beta_\gamma\right](V_\mu)^\gamma_\alpha \\ & + \frac{i}{\sqrt{2}} f^S \bar{B}^\alpha_\beta \gamma_\mu B^\beta_\alpha \omega^0_\mu, \end{aligned} \tag{2}$$

describing strong interactions of the nonet of vector-mesons with $1/2^+$ octet baryons, where $B, \bar{B}$, and V are baryon, antibaryon, and vector-meson octuplet matrices and ω^0_μ is the ω-meson singlet.

The advanced U&A nucleon EM structure model represents a harmonious unification of the vector-meson pole contributions and cut structure in the complex plane of EM FFs. The cuts represent the so-called continua contributions in nucleon EM FFs.

The nucleon EM FFs are analytic functions in the whole complex plane of their variable t, besides the cuts on the positive real axis from the lowest branch point t_0 to $+\infty$.

The shape of nucleon EM FFs is directly related to the existence of complex conjugate pairs of poles on unphysical sheets of the Riemann surface in t variable, corresponding to unstable true neutral vector-mesons [8] $\rho(770), \omega(782)$, $\phi(1020)$; $\rho'(1450), \omega'(1420), \phi'(1680)$; and $\rho''(1700), \omega''(1650), \phi''(2170)$ with the quantum numbers of the photon to be found explicitly in the process of electron-positron annihilation into hadrons.

As a result the complex nature of nucleon EM FFs for $t > t_0$ in the timelike region is secured by imaginary parts of the vector-meson poles on unphysical sheets of the Riemann surface in the t variable and the cuts on the positive real axis of the first so-called physical sheet of the Riemann surface, whereby imaginary parts of the EM FFs are given by a difference of the FF values on the upper boundary of the cuts and the FF values on the lower boundary of the cuts.

Every nucleon electric FF is canonically normalized to the nucleon electric charge and every nucleon magnetic FF is normalized to the nucleon magnetic moment.

All these nucleon EM FFs show asymptotic behaviors as predicted by the quark model of hadrons to be proven also in the framework of the QCD [9].

The advanced U&A nucleon EM structure model depends on the coupling constants ratios (f_{VNN}/f_V) to be defined for every one of the nine above-mentioned unstable true neutral vector-mesons in the vector meson dominance (VMD) model. However, not all nine of them appear explicitly in the advanced U&A nucleon EM structure model as free parameters. In the process of a construction of the VMD model, which is automatically normalized and has the correct asymptotic behavior, some of these coupling constant ratios can be expressed through the Particle Data Group (PDG) mass values of the true neutral vector-mesons under consideration and other coupling constant ratios. At this point there is freedom in the choice as to which of the coupling constant ratios (f_{VNN}/f_V) will be left as free parameters of the model.

The problem is that not for all nine true neutral vector-mesons under consideration an experimental information on the lepton width $\Gamma(V \to e^+e^-)$ exists. The latter is known [8] only for ground-state vector-mesons $\rho(770)$, $\omega(782)$, and $\phi(1020)$. For all the first and second excited states Ref. [8] declares, their lepton decays have been seen but up to now they have not been precisely measured experimentally.

It is for this reason that in the present advanced nucleon EM structure model we keep the coupling constant ratios $(f_{\rho NN}/f_\rho)$,$(f_{\omega NN}/f_\omega)$, and $(f_{\phi NN}/f_\phi)$ [and $(f_{\omega' NN}/f_{\omega'})$ and $(f_{\phi' NN}/f_{\phi'})$] for which some model estimations of $f_{\omega'}$ and $f_{\phi'}$ exist [10], as free parameters to be determined from comparison of the present advanced nucleon EM structure model with all existing data on nucleon EM FFs in spacelike and timelike regions simultaneously.

The estimation of $f_{\omega'}$ and $f_{\phi'}$ [and also $f_{\rho'}$ for a determination of $f_{\rho' NN}$ from $(f_{\rho' NN}/f_{\rho'})$ to be completely given by the PDG mass values of vector-mesons under consideration] in Ref. [10] will only be a model admixture in subsequent determination of the $1/2^+$ octet hyperon EM FF behaviors.

II. EXISTING EXPERIMENTAL INFORMATION ON NUCLEON EM FFS

The EM structure of the nucleons (the proton and neutron iso-doublets), as revealed experimentally for the first time in elastic unpolarized electron-proton scattering in the middle of the past century, is completely described by four independent scalar functions, the electric $G_{Ep}(t)$,$G_{En}(t)$ and the magnetic $G_{Mp}(t)$,$G_{Mn}(t)$ FFs, dependent on a single variable which is chosen as the momentum transfer squared $t = -Q^2$ of the virtual photon γ^*. The experimental information on these functions consists of the following 11 different sets of data:

(1) the ratio $\mu_p G_E^p(t)/G_M^p(t)$ in the spacelike $(t < 0)$ region from polarization experiments [11–15]
(2) $G_E^p(t)$ in the spacelike $(t < 0)$ region [16]
(3) $|G_E^p(t)|$ in the timelike $t > 0$ region only from experiments assuming $|G_E^p(t)| = |G_M^p(t)|$
(4) $G_M^p t)$ in the spacelike $(t < 0)$ region [16–23]
(5) $|G_M^p(t)|$ in the timelike $t > 0$ region [1,2,24–32]
(6) $|G_E^p(t)/G_M^p(t)|$ in the timelike $t > 0$ region [1,2]
(7) $G_E^n(t)$ in the spacelike $(t < 0)$ region [33–39]
(8) $|G_E^n(t)|$ in the timelike $t > 0$ region only from experiments assuming $|G_E^n(t)| = |G_M^n(t)|$
(9) $G_M^n(t)$ in the spacelike $(t < 0)$ region [33,40–46]
(10) $|G_M^n(t)|$ in the timelike $t > 0$ region [28]
(11) the ratio $\mu_n G_E^n(t)/G_M^n(t)$ in the spacelike $(t < 0)$ region from polarization experiments with the light nuclei [47,48],

which are not all equally trustworthy.

The most reliable, of all the above-mentioned data, are the experimental points for $\mu_p G_E^p(t)/G_M^p(t)$ [11–15] in the spacelike region, which have been extracted from simultaneous measurement of the transverse component,

$$P_t = \frac{h}{I_0}(-2)\sqrt{\tau(1+\tau)}G_E^p G_M^p \tan\theta/2, \qquad (3)$$

and the longitudinal component,

$$P_l = \frac{h(E_e + E_{e'})}{I_0 m_p}\sqrt{\tau(1+\tau)}{G_M^p}^2 \tan^2\theta/2, \qquad (4)$$

of the recoil proton's polarization in the polarized electron scattering plane of the polarization transfer process $\overrightarrow{e}p \to e\overrightarrow{p}$, where h is the electron beam helicity, E_e is the beam energy, $E_{e'}$ is the scattered e' energy, θ is the e^- scattering angle, I_0 is the unpolarized cross section excluding σ_{Mott}, and $\tau = -\frac{t}{4m_p^2}$, by means of the relation

$$\mu_p \frac{G_{Ep}}{G_{Mp}} = -\frac{P_t}{P_l}\frac{(E_e + E_{e'})}{2m_p}\tan\theta/2. \qquad (5)$$

The data [11–15] clearly demonstrate that a general belief in the dipole behavior of the proton electric FF $G_E^p(t)$ in the spacelike region obtained by the Rosenbluth method from the process of the elastic scattering of unpolarized electrons on unpolarized protons $e^-p \to e^-p$ described by the differential cross section in the laboratory system

$$\frac{d\sigma^{\text{lab}}(e^-p \to e^-p)}{d\Omega} = \frac{\alpha^2}{4E^2}\frac{\cos^2(\theta/2)}{\sin^4(\theta/2)}\frac{1}{1+\left(\frac{2E}{m_p}\right)\sin^2(\theta/2)} \times \left[\frac{{G_E^p}^2(t) - \frac{t}{4m_p^2}{G_M^p}^2(t)}{1-\frac{t}{4m_p^2}} - 2\frac{t}{4m_p^2}{G_M^p}^2(t)\tan^2(\theta/2)\right] \qquad (6)$$

with the QED fine structure constant $\alpha = 1/137$, the incident electron energy E, and the scattering angle θ, before the polarization experiments [11–15] have been carried out, is no more valid.

This is a reason why we further ignore all older data on $G_E^p(t)$ in the spacelike region and we take into account only the newest data from MAMI [16] at the region $-0.55242 < t < -0.0152$ GeV^2, where the application of the Rosenbluth method is still justified.

Second in reliability are the data on the proton magnetic FF $G_M^p(t)$ [16–23] in the spacelike region, which have been obtained by the Rosenbluth method from experimental information on the differential cross section describing elastic scattering of unpolarized electrons on unpolarized protons. As one can see from the expression of the differential cross section (6), with increased negative values of t, the proton magnetic FF is dominant in comparison to the proton electric FF and so the data on $G_M^p(t)$ extracted by the Rosenbluth method are reliable.

Less reliable are data [1,2,24–32] on the absolute value of the magnetic FF $|G_M^p(t)|$ in the timelike $t > 0$ region (the same is true of the neutron magnetic FF $|G_M^n(t)|$ in the timelike $t > 0$ region). A reason is that the most of them are extracted from the total cross section of the electron-positron annihilation process into proton-antiproton pair

$$\sigma_{\rm tot}^{c.m.}(e^+e^- \to p\bar{p}) = \frac{4\pi\alpha^2\beta_p}{3t}\left[\left|G_M^p(t)\right|^2 + \frac{2m_p^2}{t}\left|G_E^p(t)\right|^2\right] \quad (7)$$

with $\beta_p = \sqrt{1-\frac{4m_p^2}{t}}$, or from the total cross section of the antiproton-proton annihilation into electron-positron pair

$$\sigma_{\rm tot}^{c.m.}(\bar{p}p \to e^+e^-) = \frac{2\pi\alpha^2}{3p_{c.m.}\sqrt{t}}\left[\left|G_M^p(t)\right|^2 + \frac{2m_p^2}{t}\left|G_E^p(t)\right|^2\right] \quad (8)$$

with $p_{c.m.}$ being the antiproton momentum in the c.m. system. And in both cross sections is assumed for high energy values, either equality $|G_E^p(t)| = |G_M^p(t)|$, which is exactly valid only at the proton-antiproton threshold following directly from the definition of both FFs, or identity $|G_E^p(t)| = 0$. As a result the total cross sections under consideration give then information only on $|G_M^p(t)|$. Such approach is not well formed after all.

A very promising method for determination of $|G_E^p(t)|$ and $|G_M^p(t)|$ is demonstrated in Refs. [1,2] by a measurement of the distribution of θ_p, the angle between the proton momentum in the $p\bar{p}$ rest frame, and the momentum of the $p\bar{p}$ system in the e^+e^- c.m. frame and by a subsequent fitting of the obtained experimental points by the expression

$$F(\cos\theta_p) = N_{\rm norm}\left[1+\cos^2\theta_p + \frac{4m_p^2}{t}R^2(1-\cos^2\theta_p)\right], \quad (9)$$

obtained from

$$\frac{d\sigma(t)}{d\Omega} = \frac{\alpha^2\beta_p C}{4t}\left[\left|G_M^p(t)\right|^2(1+\cos^2\theta_p) + \frac{4m_p^2}{t}\left|G_E^p(t)\right|^2\sin^2\theta_p\right] \quad (10)$$

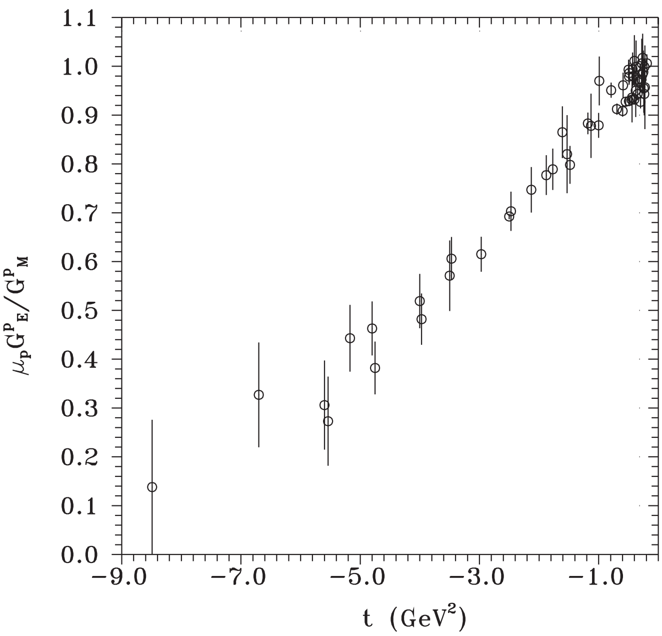

FIG. 1. Data on the ratio $\mu_p G_E^p(t)/G_M^p(t)$ in the spacelike ($t < 0$) region from polarization experiments [11–15].

with the Coulomb correction factor C, where

$$R = \frac{|G_E^p(t)|}{|G_M^p(t)|} \quad (11)$$

and

$$N_{\rm norm} = \frac{2\pi\alpha^2\beta_p L}{4t}\left[1.94 + 5.04\frac{m_p^2}{t}R^2\left|G_M^p(t)\right|^2\right] \quad (12)$$

is the overall normalization factor and L is the integrated luminosity.

However, the latter method is under development and the ratios R with $|G_M^p(t)|$ at several values of the energy t have up to now been determined [1,2].

The existing data on $G_E^n(t)$,$G_M^n(t)$ in the spacelike region and the data on $\mu_n G_E^n(t)/G_M^n(t)$ are even more model dependent as they have been extracted from the scattering processes of electrons on light nuclei using various theoretical model ingredients.

All these data are presented in Figs. 1–11.

Despite of all their shortcomings they will be further analyzed by one advanced U&A nucleon EM structure model simultaneously.

III. ADVANCED UNITARY AND ANALYTIC NUCLEON EM STRUCTURE MODEL

First we shall demonstrate how it is possible in the advanced nucleon EM structure model to keep the coupling constant ratios $(f_{\rho NN}/f_\rho)$,$(f_{\omega NN}/f_\omega)$, and $(f_{\phi NN}/f_\phi)$ [and also $(f_{\omega' NN}/f_{\omega'})$ and $(f_{\phi' NN}/f_{\phi'})$] as free parameters of the model, to be then subsequently determined numerically in comparison with the advanced U&A model to all existing data simultaneously.

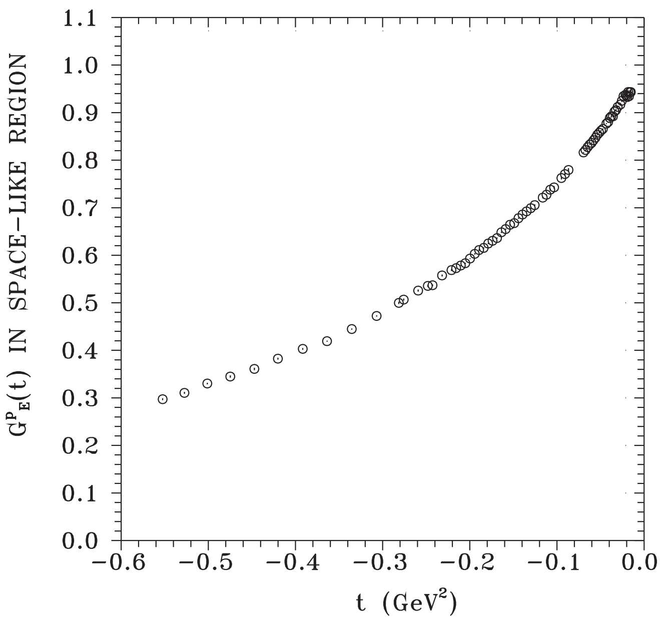

FIG. 2. Proton electric FF data in the spacelike ($t < 0$) region from MAMI [16].

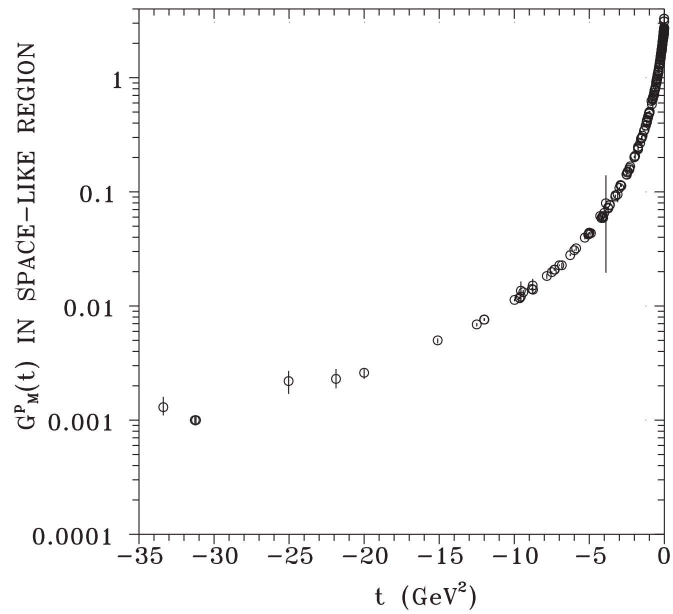

FIG. 4. Proton magnetic FF data in the spacelike ($t < 0$) region [16–23].

Nevertheless, before that we would like to note that the nucleon EM FFs $G_E^p(t), G_M^p(t), G_E^n(t)$, and $G_M^n(t)$ are very suitable for extraction of experimental information on the nucleon EM structure from the earlier mentioned physical quantities (3)–(9). For a construction of various nucleon EM structure models the flavor-independent isoscalar and isovector parts $F_{1s}^N(t), F_{1v}^N(t), F_{2s}^N(t)$, and $F_{2v}^N(t)$ of the Dirac and Pauli FFs to be defined by a parametrization of the matrix element of the nucleon EM current

$$\langle N|J_\mu^{EM}|N\rangle = e\bar{u}(p')\left\{\gamma_\mu F_1^N(t) + \frac{i}{2m_N}\sigma_{\mu\nu}(p'-p)_\nu F_2^N(t)\right\}u(p) \tag{13}$$

are more suitable.

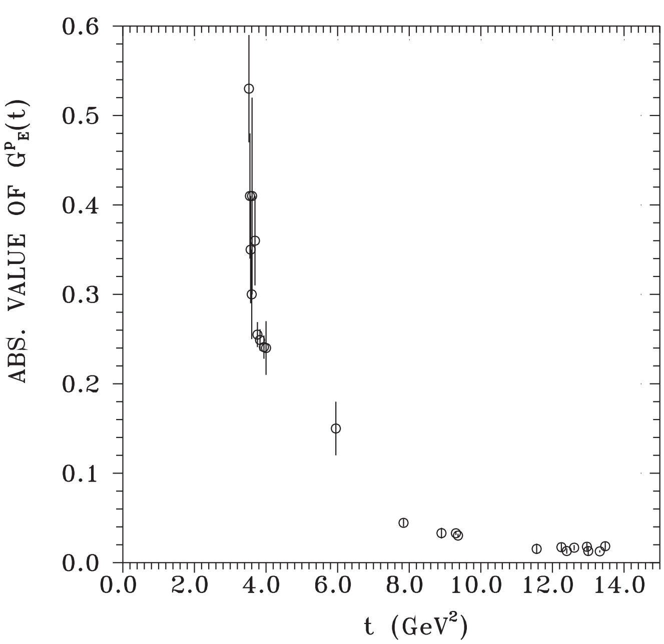

FIG. 3. Proton electric FF data in the timelike ($t > 0$) region from experiments in which $|G_E^p(t)| = |G_M^p(t)|$ has been assumed.

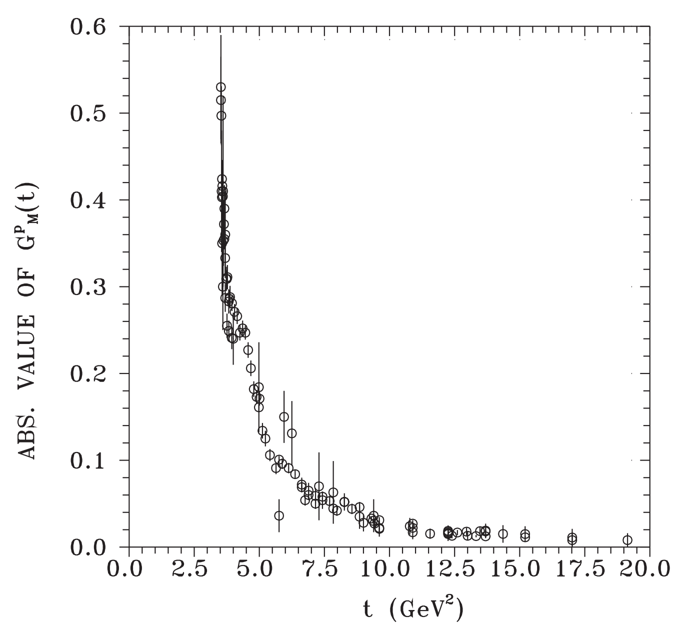

FIG. 5. Proton magnetic FF data in the timelike ($t > 0$) region [1,2,24–32].

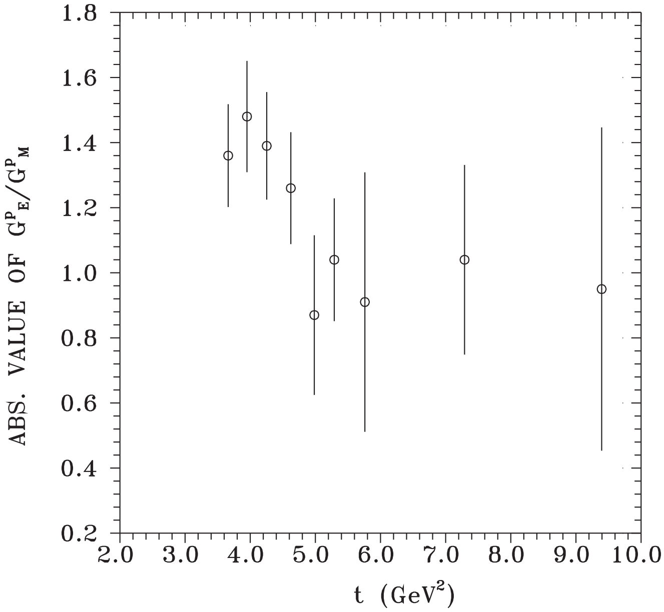

FIG. 6. Data on the ratio $|G_E^p(t)/G_M^p(t)|$ in the timelike ($t > 0$) region [1,2].

Both sets of these FFs are related as follows:

$$\begin{aligned}
G_E^p(t) &= [F_{1s}^N(t) + F_{1v}^N(t)] + \frac{t}{4m_p^2}[F_{2s}^N(t) + F_{2v}^N(t)],\\
G_M^p(t) &= [F_{1s}^N(t) + F_{1v}^N(t)] + [F_{2s}^N(t) + F_{2v}^N(t)],\\
G_E^n(t) &= [F_{1s}^N(t) - F_{1v}^N(t)] + \frac{t}{4m_n^2}[F_{2s}^N(t) - F_{2v}^N(t)],\\
G_M^n(t) &= [F_{1s}^N(t) - F_{1v}^N(t)] + [F_{2s}^N(t) - F_{2v}^N(t)],
\end{aligned} \tag{14}$$

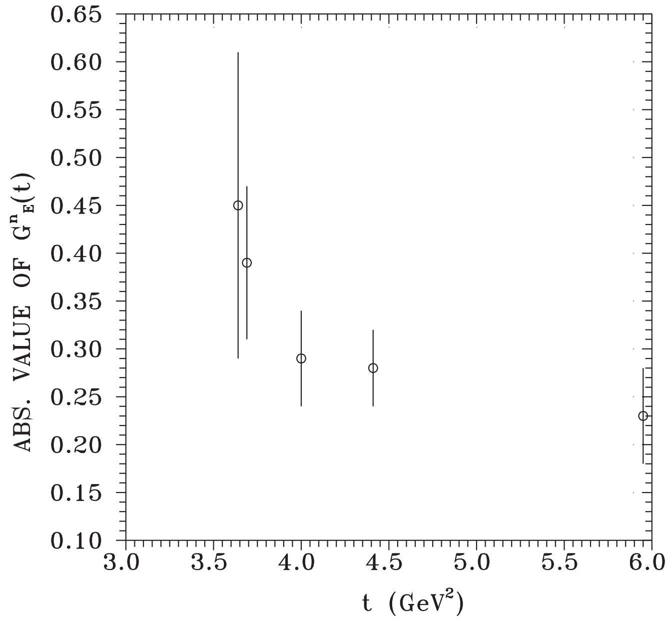

FIG. 8. Neutron electric FF data in the timelike ($t > 0$) region from experiments in which $|G_E^n(t)| = |G_M^n(t)|$ has been assumed.

whereby experimental fact of a production of true neutral vector-meson resonances with quantum numbers of the photon in $e^+e^- \to$ hadrons is in the first approximation taken into account by saturation of $F_{1s}^N(t)$ and $F_{2s}^N(t)$ with neutral isoscalar vector mesons $\omega(782),\phi(1020)$, $\omega'(1420),\phi'(1680),\omega''(1650)$, and $\phi''(2170)$ [and $F_{1v}^N(t)$ and $F_{2v}^N(t)$] with neutral isovector vector-meson resonances $\rho(770),\rho'(1450)$, and $\rho''(1700)$ in the corresponding VMD FF parametrization in the zero-width approximation.

For the sake of generality let us consider FF $F(t)$ with a normalization $F(0) = F_0$, the asymptotic behavior

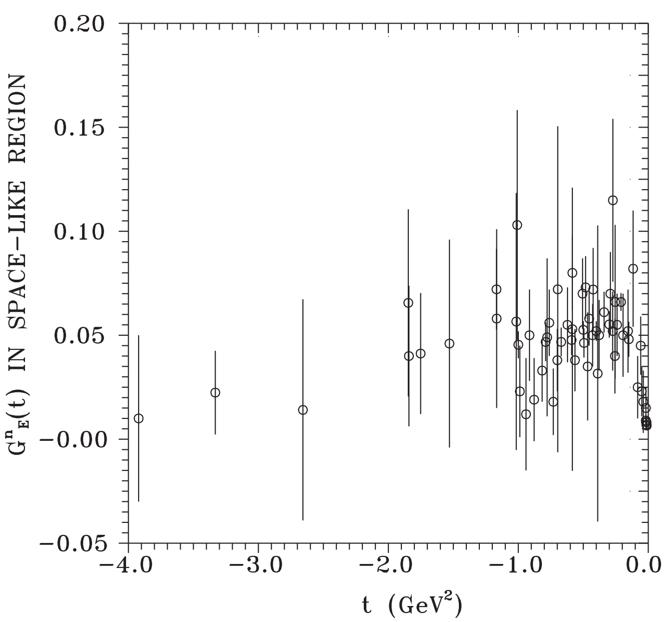

FIG. 7. Neutron electric FF data in the spacelike ($t < 0$) region [33–39].

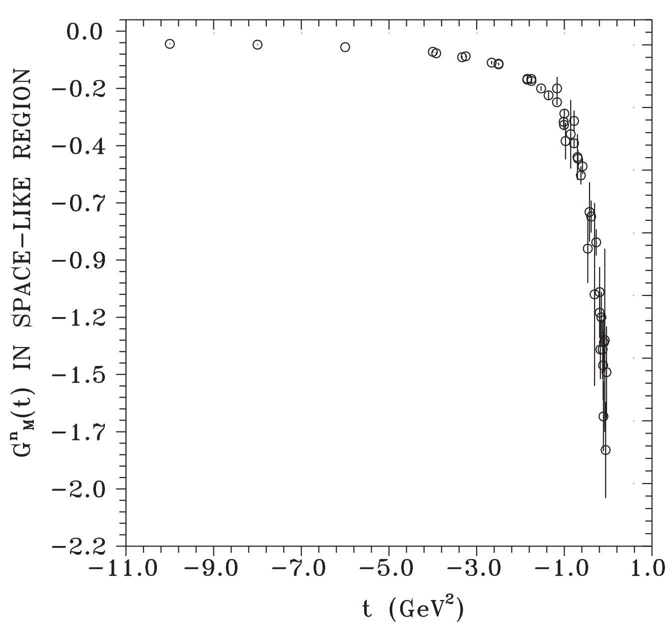

FIG. 9. Neutron magnetic negative FF data in the spacelike ($t < 0$) region [33,40–46].

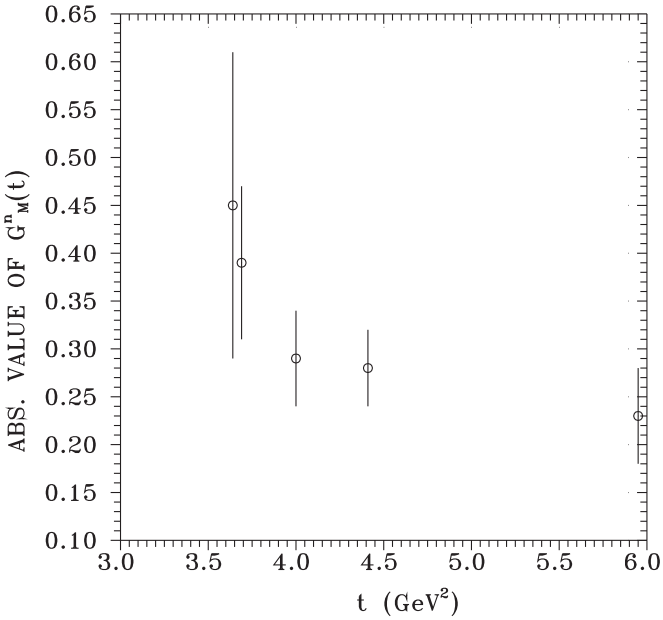

FIG. 10. Neutron magnetic FF data in the timelike ($t > 0$) region [28].

$F(t)_{t\to\infty} \sim 1/t^m$, and saturated with n-true neutral vector mesons V_n. Then in the framework of the standard VMD model in the zero-width approximation one can write

$$F(t) = \frac{m_1^2}{m_1^2 - t}a_1 + \frac{m_2^2}{m_2^2 - t}a_2 + \cdots + \frac{m_n^2}{m_n^2 - t}a_n, \quad (15)$$

where $a_n = (f_{V_nNN}/f_{V_n}), f_{V_nNN}$ is the coupling constant of an interaction of the nth vector-meson with nucleons, and f_{V_n} is the universal vector-meson coupling constant to be determined numerically from the lepton width of the nth vector-meson (1).

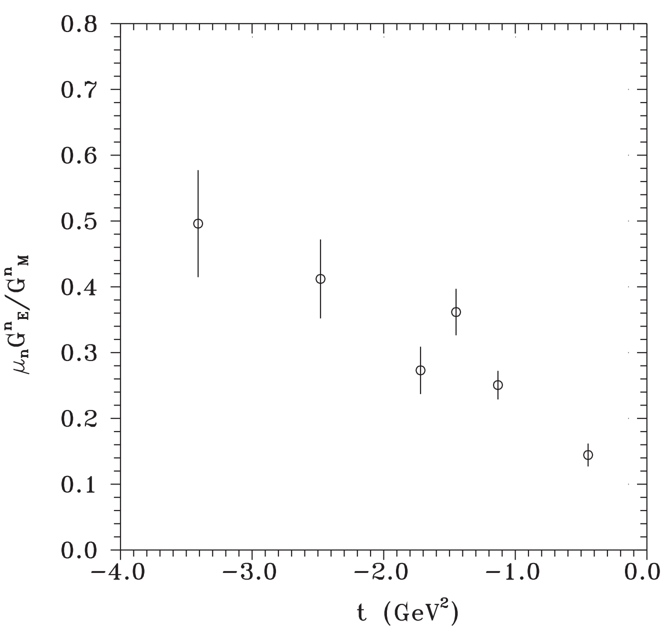

FIG. 11. Data on the ratio $\mu_n G_E^n(t)/G_M^n(t)$ in the spacelike ($t < 0$) region from polarization experiments on the light nuclei [47,48].

Now requiring the normalization of (15) at $t = 0$, one gets the following equation for coupling constant ratios (f_{iNN}/f_i):

$$\sum_{i=1}^{n}(f_{iNN}/f_i) = F_0. \quad (16)$$

Then by transforming (15) into a common denominator one gets in the numerator a polynomial of t^{n-1} degree. In order to achieve the required asymptotic behavior of $F(t)$, one has to put to zero some coefficients in the polynomial in the numerator, starting from the highest degree t^{n-1}, and one comes to another $m - 1$ equation:

$$\sum_{i=1}^{n} m_i^{2r}(f_{iNN}/f_i) = 0; r = 1,2,\ldots,m-1 \quad (17)$$

for n coupling constant ratios. As a result, a solution to the system of equations (16) and (17) will give m coupling constant ratios through the PDG masses of vector mesons and all additional coupling constant ratios $(f_{m+1}NN/f_{m+1}),\ldots,(f_nNN/f_n)$, which will be considered as free parameters of the model.

The general solution to the system of Eqs. (16) and (17) for $n > m$ leads [49] to the FF to be saturated by n vector-meson resonances in the form suitable for the unitarization

$$\begin{aligned} F(t) = {} & F_0\frac{\prod_{j=1}^{m} m_j^2}{\prod_{j=1}^{m}\left(m_j^2 - t\right)} \\ & + \sum_{k=m+1}^{n}\left\{\sum_{j=1}^{m}\frac{m_k^2}{\left(m_k^2 - t\right)}\frac{\prod_{j\neq i,j=1}^{m} m_j^2}{\prod_{j\neq k,j=1}^{m}\left(m_j^2 - t\right)}\right. \\ & \times\frac{\prod_{j\neq i,j=1}^{m}\left(m_j^2 - m_k^2\right)}{\prod_{j\neq i,j=1}^{m}\left(m_j^2 - m_i^2\right)} \\ & \left. - \frac{\prod_{j=1}^{m} m_j^2}{\prod_{j=1}^{m}\left(m_j^2 - t\right)}\right\}(f_{kNN}/f_k) \end{aligned} \quad (18)$$

and for $n = m$

$$F(t) = F_0\frac{\prod_{j=1}^{m} m_j^2}{\prod_{j=1}^{m}\left(m_j^2 - t\right)}, \quad (19)$$

for which the required asymptotic behavior $F(t)_{t\to\infty} \sim 1/t^m$ and for $t = 0$ the normalization $F(0) = F_0$ are fulfilled automatically.

Now these general results will be applied to the flavor-independent isoscalar and isovector parts of the Dirac and Pauli nucleon FFs $F_{1s}^N(t), F_{1v}^N(t), F_{2s}^N(t)$, and $F_{2v}^N(t)$ by means of which all required properties of the nucleon $U\&A$ electromagnetic structure model, including

(1) experimental fact of a production of unstable vector-meson resonances in e^+e^- annihilation processes into hadrons
(2) the analytic properties
(3) the reality conditions
(4) the unitarity conditions
(5) normalizations
(6) asymptotic behaviors

are achieved.

However, before it is achieved we have to specify normalizations and asymptotic behaviors of flavor-independent isoscalar and isovector parts of the Dirac and Pauli nucleon FFs.

The nucleon EM FFs are normalized as follows:

$$G_E^p(0)=1; G_M^p(0)=\mu_p; G_E^n(0)=0; G_M^n(0)=\mu_n \quad (20)$$

with μ_p and μ_n being proton and neutron magnetic moments, respectively.

The asymptotic behaviors of nucleon EM FFs are

$$G_E^p(t)_{|_{t\to\infty}}=G_M^p(t)_{|_{t\to\infty}}=G_E^n(t)_{|_{t\to\infty}}=G_M^n(t)_{|_{t\to\infty}}\sim\frac{1}{t^2}. \quad (21)$$

Then from (14) one obtains the normalizations of isoscalar and isovector parts of the Dirac and Pauli nucleon FFs

$$F_{1s}^N(0)=F_{1v}^N(0)=\tfrac{1}{2};\quad F_{2s}^N(0)=\tfrac{1}{2}(\mu_p+\mu_n-1); \quad F_{2v}^N(0)=\tfrac{1}{2}(\mu_p-\mu_n-1) \quad (22)$$

and their asymptotic behaviors are

$$F_{1s}^N(t)_{|_{t\to\infty}}=F_{1v}^N(t)_{|_{t\to\infty}}\sim\frac{1}{t^2}; \quad F_{2s}^N(t)_{|_{t\to\infty}}=F_{2v}^N(t)_{|_{t\to\infty}}\sim\frac{1}{t^3}. \quad (23)$$

If we apply (18) to $F_{1s}^N(t)$ with $m=2$ and $n=\omega'',\phi'',\omega',\phi',\omega$, and ϕ we obtain

$$\begin{aligned}F_{1s}^N(t)=&\frac{1}{2}\frac{m_{\omega''}^2m_{\phi''}^2}{(m_{\omega''}^2-t)(m_{\phi''}^2-t)}+\Bigg\{\frac{m_{\phi''}^2m_{\omega'}^2}{(m_{\phi''}^2-t)(m_{\omega'}^2-t)}\frac{(m_{\phi''}^2-m_{\omega'}^2)}{(m_{\phi''}^2-m_{\omega''}^2)}+\frac{m_{\omega''}^2m_{\omega'}^2}{(m_{\omega'}^2-t)(m_{\omega''}^2-t)}\frac{(m_{\omega''}^2-m_{\omega'}^2)}{(m_{\omega''}^2-m_{\phi''}^2)}\\&-\frac{m_{\omega''}^2m_{\phi''}^2}{(m_{\omega''}^2-t)(m_{\phi''}^2-t)}\Bigg\}(f_{\omega'NN}^{(1)}/f_{\omega'})\\&+\Bigg\{\frac{m_{\phi'}^2m_{\phi''}^2}{(m_{\phi'}^2-t)(m_{\phi''}^2-t)}\frac{(m_{\phi''}^2-m_{\phi'}^2)}{(m_{\phi''}^2-m_{\omega''}^2)}+\frac{m_{\omega''}^2m_{\phi'}^2}{(m_{\omega''}^2-t)(m_{\phi'}^2-t)}\frac{(m_{\omega''}^2-m_{\phi'}^2)}{(m_{\omega''}^2-m_{\phi''}^2)}-\frac{m_{\omega''}^2m_{\phi''}^2}{(m_{\omega''}^2-t)(m_{\phi''}^2-t)}\Bigg\}(f_{\phi'NN}^{(1)}/f_{\phi'})\\&+\Bigg\{\frac{m_{\phi''}^2m_{\omega}^2}{(m_{\phi''}^2-t)(m_{\omega}^2-t)}\frac{(m_{\phi''}^2-m_{\omega}^2)}{(m_{\phi''}^2-m_{\omega''}^2)}+\frac{m_{\omega}^2m_{\omega''}^2}{(m_{\omega}^2-t)(m_{\omega''}^2-t)}\frac{(m_{\omega''}^2-m_{\omega}^2)}{(m_{\omega''}^2-m_{\phi''}^2)}-\frac{m_{\omega''}^2m_{\phi''}^2}{(m_{\omega''}^2-t)(m_{\phi''}^2-t)}\Bigg\}(f_{\omega NN}^{(1)}/f_{\omega})\\&+\Bigg\{\frac{m_{\phi}^2m_{\phi''}^2}{(m_{\phi}^2-t)(m_{\phi''}^2-t)}\frac{(m_{\phi''}^2-m_{\phi}^2)}{(m_{\phi''}^2-m_{\omega''}^2)}+\frac{m_{\omega''}^2m_{\phi}^2}{(m_{\omega''}^2-t)(m_{\phi}^2-t)}\frac{(m_{\omega''}^2-m_{\phi}^2)}{(m_{\omega''}^2-m_{\phi''}^2)}-\frac{m_{\omega''}^2m_{\phi''}^2}{(m_{\omega''}^2-t)(m_{\phi''}^2-t)}\Bigg\}(f_{\phi NN}^{(1)}/f_{\phi}).\end{aligned} \quad (24)$$

If we apply it to $F_{1v}^N(t)$ with $m=2$ and $n=\rho'',\rho',\rho$ we obtain

$$\begin{aligned}F_{1v}^N(t)=&\frac{1}{2}\frac{m_{\rho''}^2m_{\rho'}^2}{(m_{\rho''}^2-t)(m_{\rho'}^2-t)}+\Bigg\{\frac{m_{\rho}^2m_{\rho'}^2}{(m_{\rho}^2-t)(m_{\rho'}^2-t)}\frac{(m_{\rho'}^2-m_{\rho}^2)}{(m_{\rho'}^2-m_{\rho''}^2)}+\frac{m_{\rho}^2m_{\rho''}^2}{(m_{\rho}^2-t)(m_{\rho''}^2-t)}\frac{(m_{\rho''}^2-m_{\rho}^2)}{(m_{\rho''}^2-m_{\rho'}^2)}\\&-\frac{m_{\rho''}^2m_{\rho'}^2}{(m_{\rho''}^2-t)(m_{\rho'}^2-t)}\Bigg\}(f_{\rho NN}^{(1)}/f_{\rho}).\end{aligned} \quad (25)$$

If we apply it to $F_{2s}^N(t)$ with $m=3$ and $n=\omega'',\phi'',\omega',\phi',\omega,\phi$ we obtain

$$\begin{aligned}F_{2s}^N(t)=&\frac{1}{2}(\mu_p+\mu_n-1)\frac{m_{\omega''}^2m_{\phi''}^2m_{\omega'}^2}{(m_{\omega''}^2-t)(m_{\phi''}^2-t)(m_{\omega'}^2-t)}+\Bigg\{\frac{m_{\phi''}^2m_{\phi'}^2m_{\omega'}^2}{(m_{\phi''}^2-t)(m_{\phi'}^2-t)(m_{\omega'}^2-t)}\frac{(m_{\phi''}^2-m_{\phi'}^2)(m_{\omega'}^2-m_{\phi'}^2)}{(m_{\phi''}^2-m_{\omega''}^2)(m_{\omega'}^2-m_{\omega''}^2)}\\&+\frac{m_{\omega''}^2m_{\omega'}^2m_{\phi'}^2}{(m_{\omega''}^2-t)(m_{\omega'}^2-t)(m_{\phi'}^2-t)}\frac{(m_{\omega''}^2-m_{\phi'}^2)(m_{\omega'}^2-m_{\phi'}^2)}{(m_{\omega''}^2-m_{\phi''}^2)(m_{\omega'}^2-m_{\phi''}^2)}+\frac{m_{\omega''}^2m_{\phi''}^2m_{\phi'}^2}{(m_{\omega''}^2-t)(m_{\phi''}^2-t)(m_{\phi'}^2-t)}\frac{(m_{\omega''}^2-m_{\phi'}^2)(m_{\phi''}^2-m_{\phi'}^2)}{(m_{\omega''}^2-m_{\omega'}^2)(m_{\phi''}^2-m_{\omega'}^2)}\\&-\frac{m_{\omega''}^2m_{\phi''}^2m_{\omega'}^2}{(m_{\omega''}^2-t)(m_{\phi''}^2-t)(m_{\omega'}^2-t)}\Bigg\}(f_{\phi'NN}^{(2)}/f_{\phi'})+\Bigg\{\frac{m_{\phi''}^2m_{\omega'}^2m_{\omega}^2}{(m_{\phi''}^2-t)(m_{\omega'}^2-t)(m_{\omega}^2-t)}\frac{(m_{\phi''}^2-m_{\omega}^2)(m_{\omega'}^2-m_{\omega}^2)}{(m_{\phi''}^2-m_{\omega''}^2)(m_{\omega'}^2-m_{\omega''}^2)}\\&+\frac{m_{\omega''}^2m_{\omega'}^2m_{\omega}^2}{(m_{\omega''}^2-t)(m_{\omega'}^2-t)(m_{\omega}^2-t)}\frac{(m_{\omega''}^2-m_{\omega}^2)(m_{\omega'}^2-m_{\omega}^2)}{(m_{\omega''}^2-m_{\phi''}^2)(m_{\omega'}^2-m_{\phi''}^2)}+\frac{m_{\omega''}^2m_{\phi''}^2m_{\omega}^2}{(m_{\omega''}^2-t)(m_{\phi''}^2-t)(m_{\omega}^2-t)}\frac{(m_{\omega''}^2-m_{\omega}^2)(m_{\phi''}^2-m_{\omega}^2)}{(m_{\omega''}^2-m_{\omega'}^2)(m_{\phi''}^2-m_{\omega'}^2)}\\&-\frac{m_{\omega''}^2m_{\phi''}^2m_{\omega'}^2}{(m_{\omega''}^2-t)(m_{\phi''}^2-t)(m_{\omega'}^2-t)}\Bigg\}(f_{\omega NN}^{(2)}/f_{\omega})+\Bigg\{\frac{m_{\phi''}^2m_{\omega'}^2m_{\phi}^2}{(m_{\phi''}^2-t)(m_{\omega'}^2-t)(m_{\phi}^2-t)}\frac{(m_{\phi''}^2-m_{\phi}^2)(m_{\omega'}^2-m_{\phi}^2)}{(m_{\phi''}^2-m_{\omega''}^2)(m_{\omega'}^2-m_{\omega''}^2)}\end{aligned}$$

$$+\frac{m_{\omega''}^2 m_{\omega'}^2 m_\phi^2}{(m_{\omega''}^2-t)(m_{\omega'}^2-t)(m_\phi^2-t)}\frac{(m_{\omega''}^2-m_\phi^2)(m_{\omega'}^2-m_\phi^2)}{(m_{\omega''}^2-m_{\phi''}^2)(m_{\omega'}^2-m_{\phi''}^2)}+\frac{m_{\omega''}^2 m_{\phi''}^2 m_\phi^2}{(m_{\omega''}^2-t)(m_{\phi''}^2-t)(m_\phi^2-t)}\frac{(m_{\omega''}^2-m_\phi^2)(m_{\phi''}^2-m_\phi^2)}{(m_{\omega''}^2-m_{\omega'}^2)(m_{\phi''}^2-m_{\omega'}^2)}$$
$$-\frac{m_{\omega''}^2 m_{\phi''}^2 m_{\omega'}^2}{(m_{\omega''}^2-t)(m_{\phi''}^2-t)(m_{\omega'}^2-t)}\bigg\}(f_{\phi NN}^{(2)}/f_\phi). \tag{26}$$

Application of (19) to $F_{2v}^N(t)$ with $m=3$ and $n=\rho'',\rho',\rho$ leads to

$$F_{2v}^N(t)=\frac{1}{2}(\mu_p-\mu_n-1)\frac{m_{\rho''}^2 m_{\rho'}^2 m_\rho^2}{(m_{\rho''}^2-t)(m_{\rho'}^2-t)(m_\rho^2-t)}. \tag{27}$$

The expressions (24)–(27) are automatically normalized and they have the asymptotic behaviors as predicted by the quark model of hadrons.

Unitarization of the model, i.e., an incorporation of the correct analytic properties of the nucleon EM FFs, is achieved by the nonlinear transformations

$$t=t_0^s+\frac{4(t_{\rm in}^{1s}-t_0^s)}{[1/V(t)-V(t)]^2};\quad t=t_0^v+\frac{4(t_{\rm in}^{1v}-t_0^v)}{[1/W(t)-W(t)]^2};$$
$$t=t_0^s+\frac{4(t_{\rm in}^{2s}-t_0^s)}{[1/U(t)-U(t)]^2};\quad t=t_0^v+\frac{4(t_{\rm in}^{2v}-t_0^v)}{[1/X(t)-X(t)]^2}, \tag{28}$$

respectively, and a subsequent inclusion of the nonzero values of vector-meson widths.

In nonlinear transformations $t_0^s=9m_\pi^2$, $t_0^v=4m_\pi^2$, $t_{\rm in}^{1s}, t_{\rm in}^{1v}, t_{\rm in}^{2s}, t_{\rm in}^{2v}$ are the square-root branch points, as is transparent from the inverse transformations

$$V(t)=i\frac{\sqrt{\left(\frac{t_{\rm in}^{1s}-t_0^s}{t_0^s}\right)^{1/2}+\left(\frac{t-t_0^s}{t_0^s}\right)^{1/2}}-\sqrt{\left(\frac{t_{\rm in}^{1s}-t_0^s}{t_0^s}\right)^{1/2}-\left(\frac{t-t_0^s}{t_0^s}\right)^{1/2}}}{\sqrt{\left(\frac{t_{\rm in}^{1s}-t_0^s}{t_0^s}\right)^{1/2}+\left(\frac{t-t_0^s}{t_0^s}\right)^{1/2}}+\sqrt{\left(\frac{t_{\rm in}^{1s}-t_0^s}{t_0^s}\right)^{1/2}-\left(\frac{t-t_0^s}{t_0^s}\right)^{1/2}}} \tag{29}$$

and similarly for $W(t),U(t)$, and $X(t)$, which map the corresponding four-sheeted Riemann surfaces always into one $V-,W-,U-,X-$ plane.

Practically let us demonstrate the unitarization in the case of the Dirac isoscalar FF (24). The nonlinear transformation $t=t_0^s+\frac{4(t_{\rm in}^{1s}-t_0^s)}{[1/V(t)-V(t)]^2}$ implies also the relations $m_r^2=t_0^s+\frac{4(t_{\rm in}^{1s}-t_0^s)}{[1/V_{r0}-V_{r0}]^2}$ and $0=t_0^s+\frac{4(t_{\rm in}^{1s}-t_0^s)}{[1/V_N-V_N]^2}$.

Then every term

$$\frac{m_r^2}{m_r^2-t}\equiv\frac{m_r^2-0}{m_r^2-t}=\left(\frac{1-V^2}{1-V_N^2}\right)^2\frac{(V_N-V_{r0})(V_N+V_{r0})(V_N-1/V_{r0})(V_N+1/V_{r0})}{(V-V_{r0})(V+V_{r0})(V-1/V_{r0})(V+1/V_{r0})} \tag{30}$$

in (24) is factorized into asymptotic term and the so-called finite-energy term (for $|t|\to\infty$ it turns out to be a real constant) gives a resonant behavior around $t=m_r^2$.

One can prove

(1) if $m_r^2-\Gamma_r^2/4<t_{\rm in}\Rightarrow V_{r0}=-V_{r0}^*$
(2) if $m_r^2-\Gamma_r^2/4>t_{\rm in}\Rightarrow V_{r0}=1/V_{r0}^*$

which in case 1 leads to the expression

$$\frac{m_r^2}{m_r^2-t}=\left(\frac{1-V^2}{1-V_N^2}\right)^2\frac{(V_N-V_{r0})(V_N-V_{r0}^*)(V_N-1/V_{r0})(V_N-1/V_{r0}^*)}{(V-V_{r0})(V-V_{r0}^*)(V-1/V_{r0})(V-1/V_{r0}^*)} \tag{31}$$

and in case 2 leads to the following expression:

$$\frac{m_r^2}{m_r^2-t}=\left(\frac{1-V^2}{1-V_N^2}\right)^2\frac{(V_N-V_{r0})(V_N-V_{r0}^*)(V_N+V_{r0})(V_N+V_{r0}^*)}{(V-V_{r0})(V-V_{r0}^*)(V+V_{r0})(V+V_{r0}^*)}. \tag{32}$$

Lastly, introducing the nonzero width of the resonance by a substitution

$$m_r^2\to(m_r-i\Gamma_r/2)^2 \tag{33}$$

(i.e., simply one has to get rid of 0 in subindices of the previous two expressions), one gets in case 1

$$\frac{m_r^2}{m_r^2-t}\to\left(\frac{1-V^2}{1-V_N^2}\right)^2\frac{(V_N-V_r)(V_N-V_r^*)(V_N-1/V_r)(V_N-1/V_r^*)}{(V-V_r)(V-V_r^*)(V-1/V_r)(V-1/V_r^*)}=\left(\frac{1-V^2}{1-V_N^2}\right)^2 L_r(V) \tag{34}$$

and in case 2

$$\frac{m_r^2}{m_r^2-t} \to \left(\frac{1-V^2}{1-V_N^2}\right)^2 \frac{(V_N-V_r)(V_N-V_r^*)(V_N+V_r)(V_N+V_r^*)}{(V-V_r)(V-V_r^*)(V+V_r)(V+V_r^*)} = \left(\frac{1-V^2}{1-V_N^2}\right)^2 H_r(V). \tag{35}$$

Then for every isoscalar and isovector Dirac and Pauli FF one obtains just one analytic and smooth function with the domain from $-\infty$ to $+\infty$ in the forms

$$\begin{aligned}F_{1s}^N[V(t)] = \left(\frac{1-V^2}{1-V_N^2}\right)^4 \Bigg\{ &\frac{1}{2}H_{\omega''}(V)H_{\phi''}(V) + \Bigg[H_{\phi''}(V)H_{\omega'}(V)\frac{(C^{1s}_{\phi''}-C^{1s}_{\omega'})}{(C^{1s}_{\phi''}-C^{1s}_{\omega''})} + H_{\omega''}(V)H_{\omega'}(V)\frac{(C^{1s}_{\omega''}-C^{1s}_{\omega'})}{(C^{1s}_{\omega''}-C^{1s}_{\phi''})}\\
&- H_{\omega''}(V)H_{\phi''}(V)\Bigg]\big(f^{(1)}_{\omega' NN}/f_{\omega'}\big) + \Bigg[H_{\phi''}(V)H_{\phi'}(V)\frac{(C^{1s}_{\phi''}-C^{1s}_{\phi'})}{(C^{1s}_{\phi''}-C^{1s}_{\omega''})} + H_{\omega''}(V)H_{\phi'}(V)\frac{(C^{1s}_{\omega''}-C^{1s}_{\phi'})}{(C^{1s}_{\omega''}-C^{1s}_{\phi''})}\\
&- H_{\omega''}(V)H_{\phi''}(V)\Bigg]\big(f^{(1)}_{\phi' NN}/f_{\phi'}\big) + \Bigg[H_{\phi''}(V)L_{\omega}(V)\frac{(C^{1s}_{\phi''}-C^{1s}_{\omega})}{(C^{1s}_{\phi''}-C^{1s}_{\omega''})}\\
&+ H_{\omega''}(V)L_{\omega}(V)\frac{(C^{1s}_{\omega''}-C^{1s}_{\omega})}{(C^{1s}_{\omega''}-C^{1s}_{\phi''})} - H_{\omega''}(V)H_{\phi''}(V)\Bigg]\big(f^{(1)}_{\omega NN}/f_{\omega}\big)\\
&+ \Bigg[H_{\phi''}(V)L_{\phi}(V)\frac{(C^{1s}_{\phi''}-C^{1s}_{\phi})}{(C^{1s}_{\phi''}-C^{1s}_{\omega''})} + H_{\omega''}(V)L_{\phi}(V)\frac{(C^{1s}_{\omega''}-C^{1s}_{\phi})}{(C^{1s}_{\omega''}-C^{1s}_{\phi''})} - H_{\omega''}(V)H_{\phi''}(V)\Bigg]\big(f^{(1)}_{\phi NN}/f_{\phi}\big)\Bigg\}\end{aligned} \tag{36}$$

dependent on 5 free physically interpretable parameters $(f^{(1)}_{\omega' NN}/f_{\omega'})$,$(f^{(1)}_{\phi' NN}/f_{\phi'})$, $(f^{(1)}_{\omega NN}/f_{\omega})$,$(f^{(1)}_{\phi NN}/f_{\phi})$, and t^{1s}_{in},

$$\begin{aligned}F_{1v}^N[W(t)] = \left(\frac{1-W^2}{1-W_N^2}\right)^4 \Bigg\{ &\frac{1}{2}L_{\rho}(W)L_{\rho'}(W) + \Bigg[L_{\rho'}(W)L_{\rho''}(W)\frac{(C^{1v}_{\rho'}-C^{1v}_{\rho''})}{(C^{1v}_{\rho'}-C^{1v}_{\rho})} + L_{\rho}(W)L_{\rho''}(W)\frac{(C^{1v}_{\rho}-C^{1v}_{\rho''})}{(C^{1v}_{\rho}-C^{1v}_{\rho'})} - L_{\rho}(W)L_{\rho'}(W)\Bigg]\\
&\times \big(f^{(1)}_{\rho NN}/f_{\rho}\big)\Bigg\}\end{aligned} \tag{37}$$

dependent on 2 free physically interpretable parameters $(f^{(1)}_{\rho NN}/f_{\rho})$ and t^{1v}_{in},

$$\begin{aligned}F_{2s}^N[U(t)] = \left(\frac{1-U^2}{1-U_N^2}\right)^6 \Bigg\{ &\frac{1}{2}(\mu_p+\mu_n-1)H_{\omega''}(U)H_{\phi''}(U)H_{\omega'}(U) + \Bigg[H_{\phi''}(U)H_{\omega'}(U)H_{\phi'}(U)\frac{(C^{2s}_{\phi''}-C^{2s}_{\phi'})(C^{2s}_{\omega'}-C^{2s}_{\phi'})}{(C^{2s}_{\phi''}-C^{2s}_{\omega''})(C^{2s}_{\omega'}-C^{2s}_{\omega''})}\\
&+ H_{\omega''}(U)H_{\omega'}(U)H_{\phi'}(U)\frac{(C^{2s}_{\omega''}-C^{2s}_{\phi'})(C^{2s}_{\omega'}-C^{2s}_{\phi'})}{(C^{2s}_{\omega''}-C^{2s}_{\phi''})(C^{2s}_{\omega'}-C^{2s}_{\phi''})} + H_{\omega''}(U)H_{\phi''}(U)H_{\phi'}(U)\frac{(C^{2s}_{\omega''}-C^{2s}_{\phi'})(C^{2s}_{\phi''}-C^{2s}_{\phi'})}{(C^{2s}_{\omega''}-C^{2s}_{\omega'})(C^{2s}_{\phi''}-C^{2s}_{\omega'})}\\
&- H_{\omega''}(U)H_{\phi''}(U)H_{\omega'}(U)\Bigg]\big(f^{(2)}_{\phi' NN}/f_{\phi'}\big) + \Bigg[H_{\phi''}(U)H_{\omega'}(U)L_{\omega}(U)\frac{(C^{2s}_{\phi''}-C^{2s}_{\omega})(C^{2s}_{\omega'}-C^{2s}_{\omega})}{(C^{2s}_{\phi''}-C^{2s}_{\omega''})(C^{2s}_{\omega'}-C^{2s}_{\omega''})}\\
&+ H_{\omega''}(U)H_{\omega'}(U)L_{\omega}(U)\frac{(C^{2s}_{\omega''}-C^{2s}_{\omega})(C^{2s}_{\omega'}-C^{2s}_{\omega})}{(C^{2s}_{\omega''}-C^{2s}_{\phi''})(C^{2s}_{\omega'}-C^{2s}_{\phi''})}\\
&+ H_{\omega''}(U)H_{\phi''}(U)L_{\omega}(U)\frac{(C^{2s}_{\omega''}-C^{2s}_{\omega})(C^{2s}_{\phi'}-C^{2s}_{\omega})}{(C^{2s}_{\omega''}-C^{2s}_{\omega'})(C^{2s}_{\phi''}-C^{2s}_{\omega'})} - H_{\omega''}(U)H_{\phi''}(U)H_{\omega'}(U)\Bigg]\big(f^{(2)}_{\omega NN}/f_{\omega}\big)\\
&+ \Bigg[H_{\phi''}(U)H_{\omega'}(U)L_{\phi}(U)\frac{(C^{2s}_{\phi''}-C^{2s}_{\phi})(C^{2s}_{\omega'}-C^{2s}_{\phi})}{(C^{2s}_{\phi''}-C^{2s}_{\omega''})(C^{2s}_{\omega'}-C^{2s}_{\omega''})} + H_{\omega''}(U)H_{\omega'}(U)L_{\phi}(U)\frac{(C^{2s}_{\omega''}-C^{2s}_{\phi})(C^{2s}_{\omega'}-C^{2s}_{\phi})}{(C^{2s}_{\omega''}-C^{2s}_{\phi''})(C^{2s}_{\omega'}-C^{2s}_{\phi''})}\\
&+ H_{\omega''}(U)H_{\phi''}(U)L_{\phi}(U)\frac{(C^{2s}_{\omega''}-C^{2s}_{\phi})(C^{2s}_{\phi''}-C^{2s}_{\phi})}{(C^{2s}_{\omega''}-C^{2s}_{\omega'})(C^{2s}_{\phi''}-C^{2s}_{\omega'})} - H_{\omega''}(U)H_{\phi''}(U)H_{\omega'}(U)\Bigg]\big(f^{(2)}_{\phi NN}/f_{\phi}\big)\Bigg\}\end{aligned} \tag{38}$$

dependent on 4 free physically interpretable parameters $(f^{(2)}_{\phi' NN}/f_{\phi'})$,$(f^{(2)}_{\omega NN}/f_{\omega})$, $(f^{(2)}_{\phi NN}/f_{\phi})$,$t^{2s}_{\text{in}}$,

$$F_{2v}^N[X(t)] = \left(\frac{1-X^2}{1-X_N^2}\right)^6 \left\{\frac{1}{2}(\mu_p-\mu_n-1)L_{\rho}(U)L_{\rho'}(U)H_{\rho''}(U)\right\} \tag{39}$$

dependent on 1 free physically interpretable parameter $t_{\rm in}^{2v}$, where

$$
\begin{aligned}
L_r(V) &= \frac{(V_N - V_r)(V_N - V_r^*)(V_N - 1/V_r)(V_N - 1/V_r^*)}{(V - V_r)(V - V_r^*)(V - 1/V_r)(V - 1/V_r^*)},\\
C_r^{1s} &= \frac{(V_N - V_r)(V_N - V_r^*)(V_N - 1/V_r)(V_N - 1/V_r^*)}{-(V_r - 1/V_r)(V_r - 1/V_r^*)}, \quad r = \omega,\phi
\end{aligned} \tag{40}
$$

$$
\begin{aligned}
H_l(V) &= \frac{(V_N - V_l)(V_N - V_l^*)(V_N + V_l)(V_N + V_l^*)}{(V - V_l)(V - V_l^*)(V + V_l)(V + V_l^*)},\\
C_l^{1s} &= \frac{(V_N - V_l)(V_N - V_l^*)(V_N + V_l)(V_N + V_l^*)}{-(V_l - 1/V_l)(V_l - 1/V_l^*)}, \quad l = \omega'',\phi'',\omega',\phi',
\end{aligned} \tag{41}
$$

$$
\begin{aligned}
L_k(W) &= \frac{(W_N - W_k)(W_N - W_k^*)(W_N - 1/W_k)(W_N - 1/W_k^*)}{(W - W_k)(W - W_k^*)(W - 1/W_k)(W - 1/W_k^*)},\\
C_k^{1v} &= \frac{(W_N - W_k)(W_N - W_k^*)(W_N - 1/W_k)(W_N - 1/W_k^*)}{-(W_k - 1/W_k)(W_k - 1/W_k^*)}, \quad k = \rho'',\rho',\rho,
\end{aligned} \tag{42}
$$

$$
\begin{aligned}
L_r(U) &= \frac{(U_N - U_r)(U_N - U_r^*)(U_N - 1/U_r)(U_N - 1/U_r^*)}{(U - U_r)(U - U_r^*)(U - 1/U_r)(U - 1/U_r^*)},\\
C_r^{2s} &= \frac{(U_N - U_r)(U_N - U_r^*)(U_N - 1/U_r)(U_N - 1/U_r^*)}{-(U_r - 1/U_r)(U_r - 1/U_r^*)}, \quad r = \omega,\phi,
\end{aligned} \tag{43}
$$

$$
\begin{aligned}
H_l(U) &= \frac{(U_N - U_l)(U_N - U_l^*)(U_N + U_l)(U_N + U_l^*)}{(U - U_l)(U - U_l^*)(U + U_l)(U + U_l^*)},\\
C_l^{2s} &= \frac{(U_N - U_l)(U_N - U_l^*)(U_N + U_l)(U_N + U_l^*)}{-(U_l - 1/U_l)(U_l - 1/U_l^*)}, \quad l = \omega'',\phi'',\omega',\phi',
\end{aligned} \tag{44}
$$

$$
\begin{aligned}
L_k(X) &= \frac{(X_N - X_k)(X_N - X_k^*)(X_N - 1/X_k)(X_N - 1/X_k^*)}{(X - X_k)(X - X_k^*)(X - 1/X_k)(X - 1/X_k^*)},\\
C_k^{2v} &= \frac{(X_N - X_k)(X_N - X_k^*)(X_N - 1/X_k)(X_N - 1/X_k^*)}{-(X_k - 1/X_k)(X_k - 1/X_k^*)}, \quad k = \rho',\rho,
\end{aligned} \tag{45}
$$

$$
\begin{aligned}
H_{\rho''}(X) &= \frac{(X_N - X_{\rho''})(X_N - X_{\rho''}^*)(X_N + X_{\rho''})(X_N + X_{\rho''}^*)}{(X - X_{\rho''})(X - X_{\rho''}^*)(X + X_{\rho''})(X + X_{\rho''}^*)},\\
C_{\rho''}^{2v} &= \frac{(X_N - X_{\rho''})(X_N - X_{\rho''}^*)(X_N + X_{\rho''})(W_X + X_{\rho''}^*)}{-(X_{\rho''} - 1/X_{\rho''})(X_{\rho''} - 1/X_{\rho''}^*)},
\end{aligned} \tag{46}
$$

since in a fitting procedure of all existing data by means of this version of the U&A nucleon EM structure model simultaneously one finds

$$
\begin{aligned}
&\left(m_\omega^2 - \Gamma_\omega^2/4\right) < t_{\rm in}^{1s}; \quad \left(m_\phi^2 - \Gamma_\phi^2/4\right) < t_{\rm in}^{1s}; \quad \left(m_{\omega'}^2 - \Gamma_{\omega'}^2/4\right) > t_{\rm in}^{1s}; \quad \left(m_{\phi'}^2 - \Gamma_{\phi'}^2/4\right) > t_{\rm in}^{1s};\\
&\left(m_{\omega''}^2 - \Gamma_{\omega''}^2/4\right) > t_{\rm in}^{1s}; \quad \left(m_{\phi''}^2 - \Gamma_{\phi''}^2/4\right) > t_{\rm in}^{1s};
\end{aligned} \tag{47}
$$

$$
\left(m_\rho^2 - \Gamma_\rho^2/4\right) < t_{\rm in}^{1v}; \quad \left(m_{\rho'}^2 - \Gamma_{\rho'}^2/4\right) < t_{\rm in}^{1v}; \quad \left(m_{\rho''}^2 - \Gamma_{\rho''}^2/4\right) < t_{\rm in}^{1v}; \tag{48}
$$

$$
\begin{aligned}
&\left(m_\omega^2 - \Gamma_\omega^2/4\right) < t_{\rm in}^{2s}; \quad \left(m_\phi^2 - \Gamma_\phi^2/4\right) < t_{\rm in}^{2s}; \quad \left(m_{\omega'}^2 - \Gamma_{\omega'}^2/4\right) > t_{\rm in}^{2s}; \quad \left(m_{\phi'}^2 - \Gamma_{\phi'}^2/4\right) > t_{\rm in}^{2s};\\
&\left(m_{\omega''}^2 - \Gamma_{\omega''}^2/4\right) > t_{\rm in}^{2s}; \quad \left(m_{\phi''}^2 - \Gamma_{\phi''}^2/4\right) > t_{\rm in}^{2s};
\end{aligned} \tag{49}
$$

$$
\left(m_\rho^2 - \Gamma_\rho^2/4\right) < t_{\rm in}^{2v}; \quad \left(m_{\rho'}^2 - \Gamma_{\rho'}^2/4\right) < t_{\rm in}^{2v}; \quad \left(m_{\rho''}^2 - \Gamma_{\rho''}^2/4\right) > t_{\rm in}^{2v}. \tag{50}
$$

IV. RESULTS OF THE ANALYSIS OF EXISTING NUCLEON EM FF DATA

We have collected 534 reliable experimental points on the nucleon EM structure from more than 40 independent experiments, as shown in Figs. 1–11. They have been analyzed simultaneously by means of the 9-resonance U&A model of the nucleon EM structure as formulated in the previous section. The minimum of the $\chi^2 = 2214$ has been achieved with the values of 12 free parameters of the model with a clear physical meaning, presented in Table I,

TABLE I. The numerical values of the free parameters of the nucleon U&A EM structure model, respecting SU(3) symmetry, formulated in the previous section.

$t_{\rm in}^{1s} = (1.0442 \pm 0.0200)\ \mathrm{GeV}^2$
$t_{\rm in}^{2s} = (1.0460 \pm 0.1399)\ \mathrm{GeV}^2$
$t_{\rm in}^{1v} = (2.9506 \pm 0.5326)\ \mathrm{GeV}^2$
$t_{\rm in}^{2v} = (2.3449 \pm 0.7656)\ \mathrm{GeV}^2$
$(f^{(1)}_{\omega NN}/f_\omega) = (1.5717 \pm 0.0022)$
$(f^{(1)}_{\phi NN}/f_\phi) = (-1.1247 \pm 0.0011)$
$(f^{(1)}_{\omega' NN}/f_{\omega'}) = (0.0418 \pm 0.0065)$
$(f^{(1)}_{\phi' NN}/f_{\phi'}) = (0.1879 \pm 0.0010)$
$(f^{(2)}_{\omega NN}/f_\omega) = (-0.2096 \pm 0.0067)$
$(f^{(2)}_{\phi NN}/f_\phi) = (0.2657 \pm 0.0067)$
$(f^{(2)}_{\phi' NN}/f_{\phi'}) = (0.1781 \pm 0.0029)$
$(f^{(1)}_{\rho NN}/f_\rho) = (0.3747 \pm 0.0022)$

whereby the results are not very sensitive to the position of the effective inelastic thresholds $t_{\rm in}^{1s}$,$t_{\rm in}^{2s}$, $t_{\rm in}^{1v}$, and $t_{\rm in}^{2v}$.

The corresponding description of the data by means of this U&A model with numerical values of parameters given in Table I is shown in Figs. 12–19.

Of course one could not expect that a description of such gigantic set of data to be obtained from so many independent experiments, every one of them charged with corresponding statistical and systematical errors, will be consistent with rules of standard statistics. We have been able to reduce the total χ^2 on 522 deg of freedom only to the value 4.24.

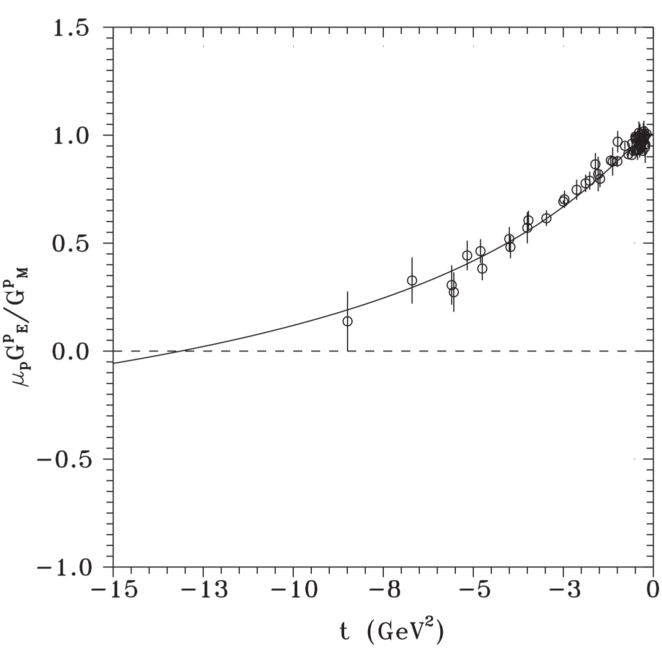

FIG. 12. Prediction of proton electric to magnetic FFs ratio behavior in spacelike region by U&A model respecting SU(3) symmetry and its comparison with existing data.

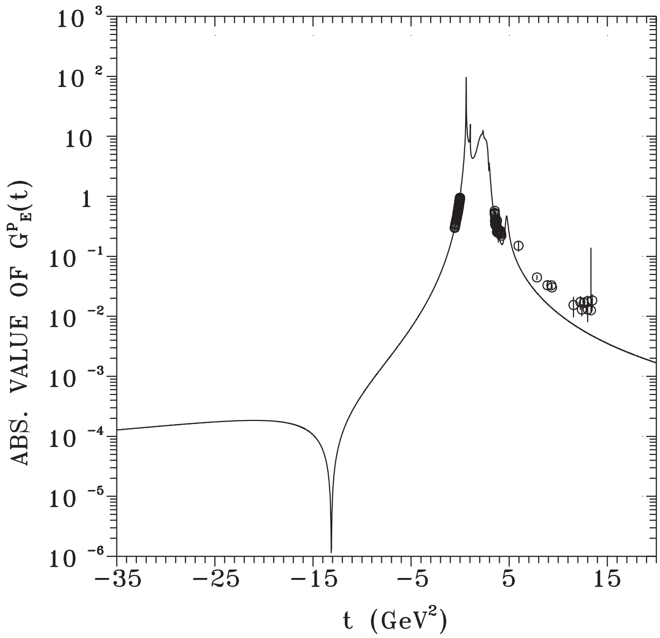

FIG. 13. Prediction of proton electric FF behavior by U&A model respecting SU(3) symmetry and its comparison with existing data.

The results of the analysis can be summarized as follows:

(1) A reasonable description (see Fig. 12) of the most reliable nucleon EM structure data, i.e., the data on the ratio $\mu_p G_E^p(t)/G_M^p(t)$ in the spacelike region to be obtained in polarization experiments, is achieved.

(2) Description of all other existing data (see Figs. 13–19) is quite reasonable too, besides an inconsistency of the data on neutron EM FFs in the timelike region with all

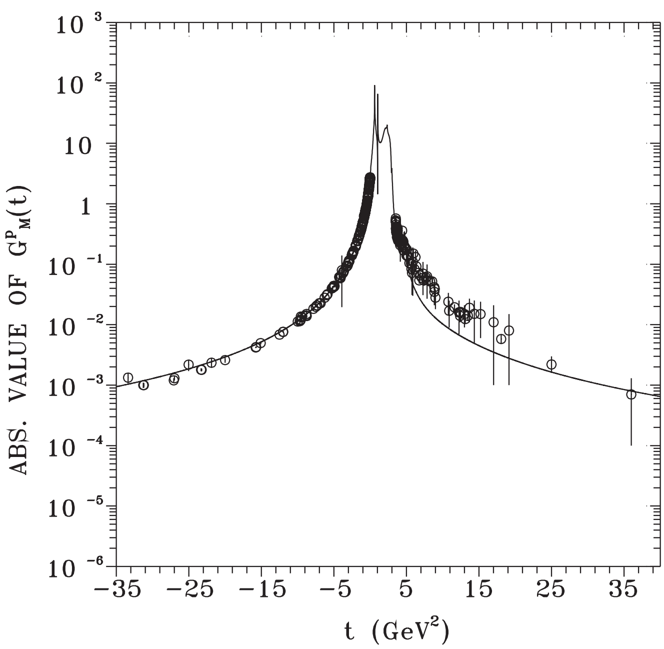

FIG. 14. Prediction of proton magnetic FF behavior by U&A model respecting SU(3) symmetry and its comparison with existing data.

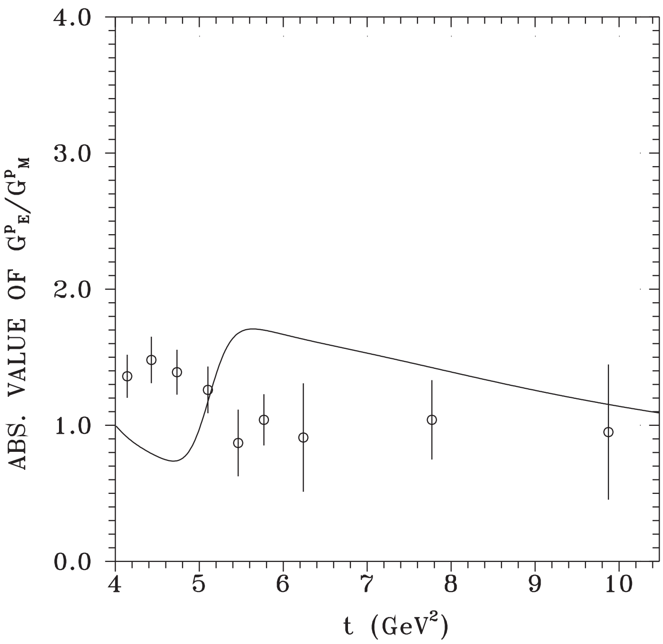

FIG. 15. Prediction of the absolute value of proton electric to magnetic FFs ratio behavior in the timelike region by the U&A model with respect to SU(3) symmetry and its comparison with existing data.

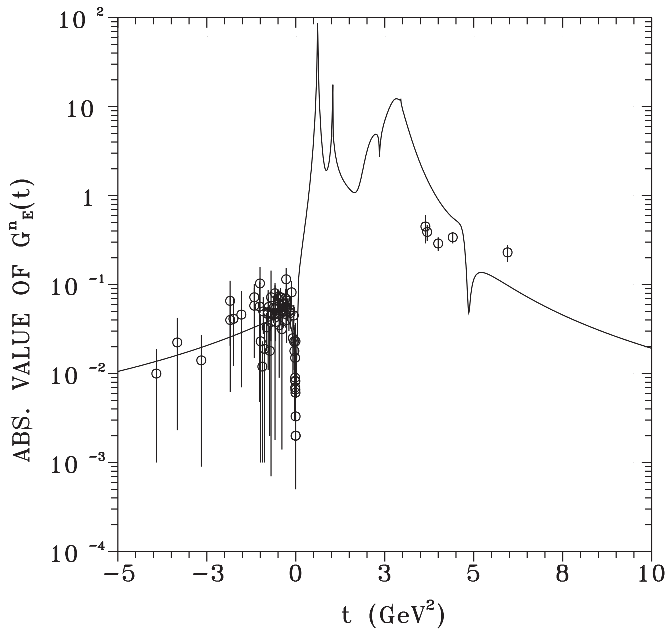

FIG. 17. Prediction of neutron electric FF behavior by the U&A model with respect to SU(3) symmetry and its comparison with existing the data.

other data on nucleon EM FFs, indicating that the total cross section of $e^+e^- \to n\bar{n}$ is considerably larger than has been found in FENICE experiment [28]. So, we are coming to the same conclusions as pointed out by one of the authors in Refs. [50,51] published more than 25 years ago.

(3) Again the existence of the zero of the proton electric FF $G_E^p(t)$ approximately at $t_z = -13$ GeV^2 is confirmed, which has been predicted in Ref. [52] for the first time.

(4) Electric and magnetic mean square charge radii of the nucleons are determined to be $\langle r_{Ep}^2\rangle = (0.7182 \pm 0.0369)$ fm^2, $\langle r_{Mp}^2\rangle = (0.7573 \pm 0.0133)$ fm^2, $\langle r_{En}^2\rangle = (-0.1162 \pm 0.0369)$ fm^2,

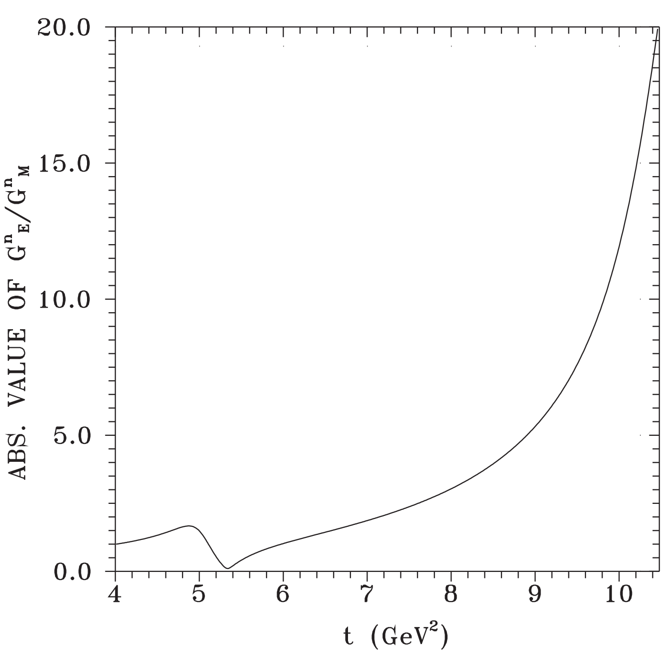

FIG. 16. Prediction of the absolute value of neutron electric to magnetic FFs ratio behavior in the timelike region by the U&A model with respect to SU(3) symmetry.

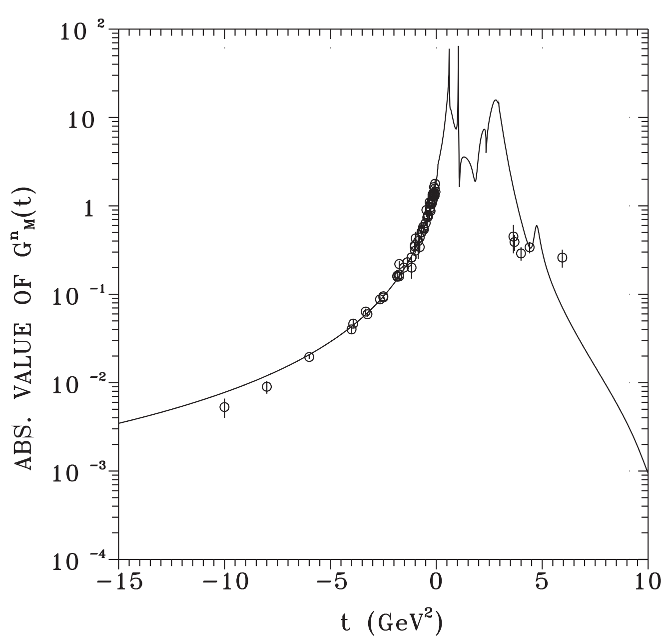

FIG. 18. Prediction of neutron magnetic FF behavior by the U&A model respecting SU(3) symmetry and its comparison with existing data.

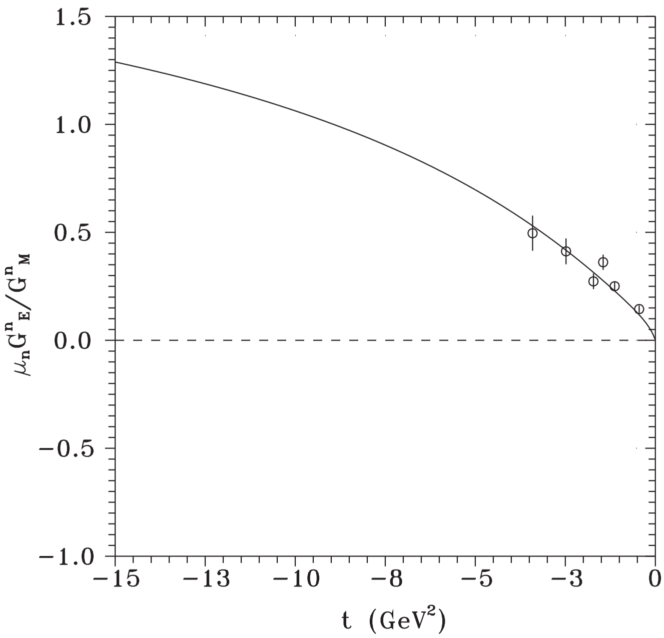

FIG. 19. Prediction of neutron electric to magnetic FFs ratio behavior in the spacelike region by the U&A model respecting SU(3) symmetry and its comparison with existing data.

$\langle r^2_{Mn}\rangle = (0.8312 \pm 0.0195)$ fm^2. Since the electric root mean square charge radius of the proton has been obtained by the U&A model simultaneously describing all existing 11 data sets on nucleon EM structure and its value is consistent with the value $\langle r_{Ep}\rangle = 0.84184 \pm 0.00067$ fm determined in the muon hydrogen atom spectroscopy [53], it seems to us that the electric proton charge radius puzzle is solved. Recently, researchers [54] also analyzing the nucleon EM structure data using by another approach came to the same conclusions..

(5) Electric mean square charge radius of the neutron is found to be almost identical with the value given in Ref. [8].

In Figs. 20–25 it is clearly demonstrated that the nucleon EM FFs represented by the U&A model indeed fulfill the reality condition $G^*(t) = G(t^*)$, i.e., they all are real functions from $-\infty$ up to the lowest branch point $t_0 = 4m_\pi^2$ on the positive real axis.

The imaginary parts of all nucleon EM FFs are different from zero only from the lowest branch point at $t = 0.0784$ GeV^2 to $+\infty$ and their behaviors are given by the unitarity conditions of the corresponding FFs.

V. NUMERICAL VALUES OF THE f^F, f^D, f^S AND $f^{F'}, f^{D'}, f^{S'}$ COUPLING CONSTANTS

The SU(3) invariant Lagrangian (2) of vector-meson nonet $\rho^-,\rho^0,\rho^+,K^{*-},K^{*0},\bar{K}^{*0},K^{*+}$, and ω,ϕ with $1/2^+$ octet baryons $p,n,\Lambda,\Sigma^+,\Sigma^0$, Σ^-,Ξ^0, and Ξ^- provides the following expressions:

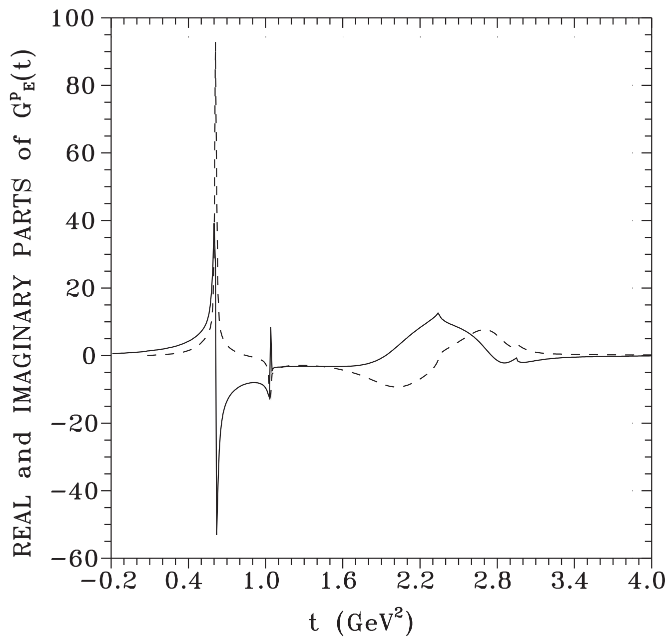

FIG. 20. Prediction of real (solid line) and imaginary (dashed line) parts of the proton electric FF by the U&A model respecting SU(3) symmetry.

$$f^{(1)}_{\rho NN} = \frac{1}{2}\left(f^D_1 + f^F_1\right),$$
$$f^{(1)}_{\omega NN} = \frac{1}{\sqrt{2}} f^S_1 \cos\theta - \frac{1}{2\sqrt{3}}\left(3f^F_1 - f^D_1\right)\sin\theta, \quad (51)$$
$$f^{(1)}_{\phi NN} = \frac{1}{\sqrt{2}} f^S_1 \sin\theta + \frac{1}{2\sqrt{3}}\left(3f^F_1 - f^D_1\right)\cos\theta,$$

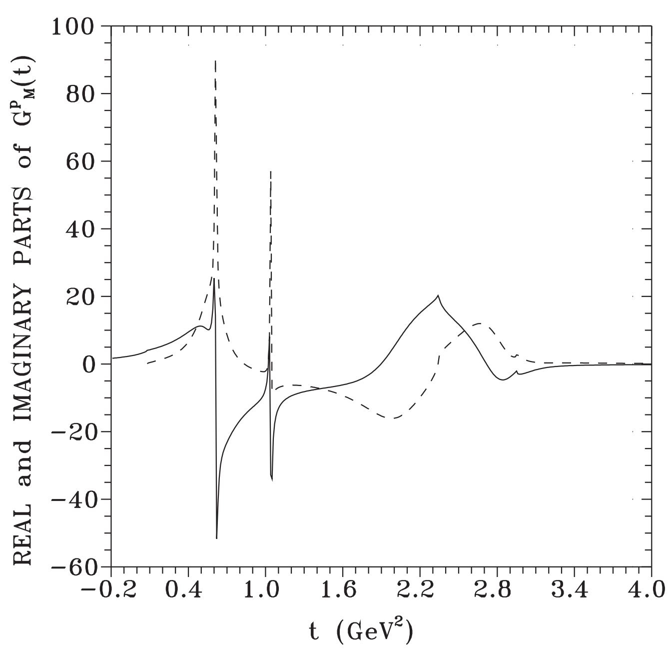

FIG. 21. Prediction of real (solid line) and imaginary (dashed line) parts of the proton magnetic FF by the U&A model respecting SU(3) symmetry.

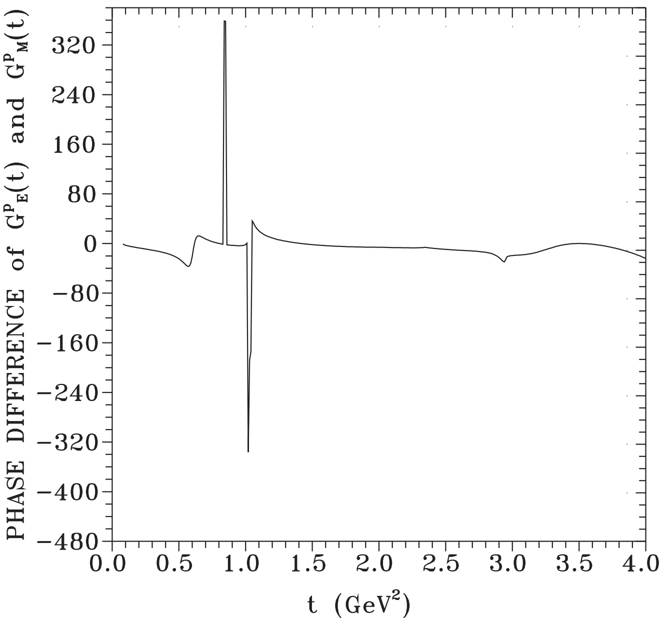

FIG. 22. Prediction of a phase difference of the proton electric and magnetic FFs by the U&A model with respect to SU(3) symmetry.

and

$$
\begin{aligned}
f^{(2)}_{\rho NN} &= \frac{1}{2}\left(f^D_2 + f^F_2\right),\\
f^{(2)}_{\omega NN} &= \frac{1}{\sqrt{2}} f^S_2 \cos\theta - \frac{1}{2\sqrt{3}}\left(3f^F_2 - f^D_2\right)\sin\theta, \quad (52)\\
f^{(2)}_{\phi NN} &= \frac{1}{\sqrt{2}} f^S_2 \sin\theta + \frac{1}{2\sqrt{3}}\left(3f^F_2 - f^D_2\right)\cos\theta,
\end{aligned}
$$

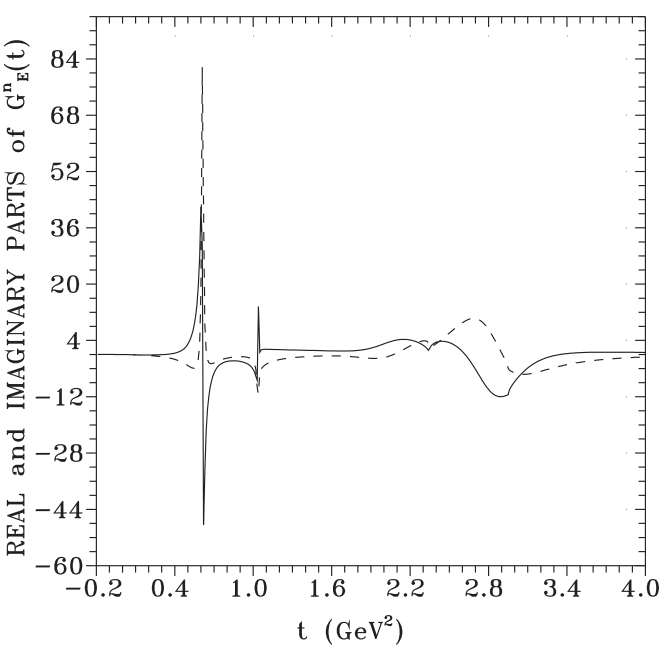

FIG. 23. Prediction of real (solid line) and imaginary (dashed line) parts of the neutron electric FF by the U&A model with respect to SU(3) symmetry.

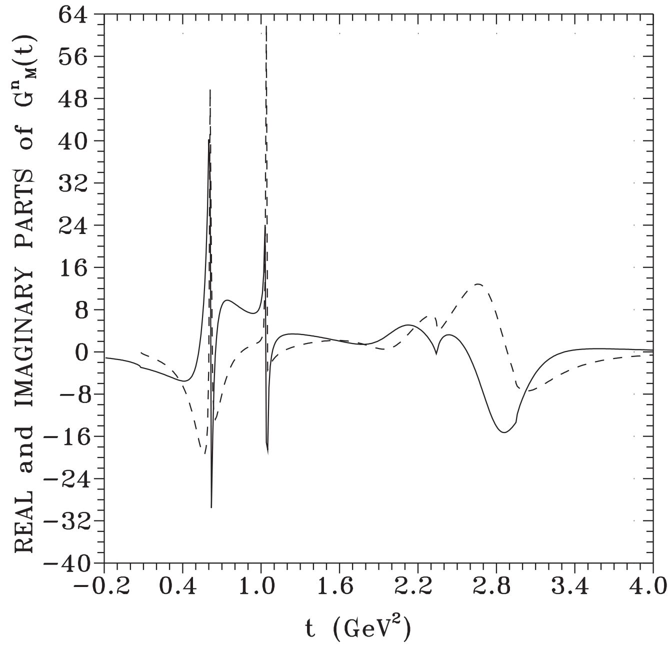

FIG. 24. Prediction of real (solid line) and imaginary (dashed line) parts of the neutron magnetic FF by the U&A model with respect to SU(3) symmetry.

where angle $\theta = 43.8^\circ$ and it is given by the Gell-Mann-Okubo quadratic mass formula:

$$m^2_{\phi(1020)} \cos^2\theta + m^2_{\omega(782)} \sin^2\theta = \frac{4m^2_{K^*(892)} - m^2_{\rho(770)}}{3}. \quad (53)$$

From the SU(3) invariant Lagrangian of the first excited vector-meson nonet $\rho^{-'}, \rho^{0'}, \rho^{+'}, K^{*-'}, K^{*0'}, \bar{K}^{*0'}, K^{*+'}$, ω', and ϕ' with $1/2^+$ octet baryons $p, n, \Lambda, \Sigma^+, \Sigma^0$,

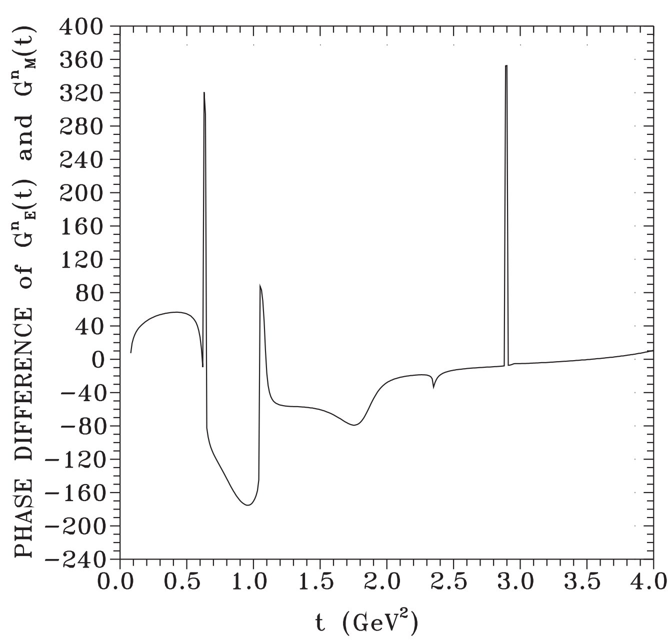

FIG. 25. Prediction of a phase difference of the neutron electric and magnetic FFs by the U&A model with respect to SU(3) symmetry.

Σ^-, Ξ^0, and Ξ^-

$$L_{V'B\bar{B}} = \frac{i}{\sqrt{2}} f^{F'} [\bar{B}^\alpha_\beta \gamma_\mu B^\beta_\gamma - \bar{B}^\beta_\gamma \gamma_\mu B^\alpha_\beta](V'_\mu)^\gamma_\alpha + \frac{i}{\sqrt{2}} f^{D'} [\bar{B}^\beta_\gamma \gamma_\mu B^\alpha_\beta + \bar{B}^\alpha_\gamma \gamma_\mu B^\beta_\gamma](V'_\mu)^\gamma_\alpha + \frac{i}{\sqrt{2}} f^{S'} \bar{B}^\alpha_\beta \gamma_\mu B^\beta_\alpha \omega^{0'}_\mu \quad (54)$$

another two systems of expressions

$$f^{(1)}_{\rho'NN} = \frac{1}{2}\left(f_1^{D'} + f_1^{F'}\right),$$
$$f^{(1)}_{\omega'NN} = \frac{1}{\sqrt{2}} f_1^{S'} \cos\theta' - \frac{1}{2\sqrt{3}}\left(3f_1^{F'} - f_1^{D'}\right)\sin\theta', \quad (55)$$
$$f^{(1)}_{\phi'NN} = \frac{1}{\sqrt{2}} f_1^{S'} \sin\theta' + \frac{1}{2\sqrt{3}}\left(3f_1^{F'} - f_1^{D'}\right)\cos\theta',$$

and

$$f^{(2)}_{\rho'NN} = \frac{1}{2}\left(f_2^{D'} + f_2^{F'}\right),$$
$$f^{(2)}_{\omega'NN} = \frac{1}{\sqrt{2}} f_2^{S'} \cos\theta' - \frac{1}{2\sqrt{3}}\left(3f_2^{F'} - f_2^{D'}\right)\sin\theta', \quad (56)$$
$$f^{(2)}_{\phi'NN} = \frac{1}{\sqrt{2}} f_2^{S'} \sin\theta' + \frac{1}{2\sqrt{3}}\left(3f_2^{F'} - f_2^{D'}\right)\cos\theta'$$

are obtained, where angle $\theta' = 50.3^\circ$ and it is again given by the Gell-Mann-Okubo quadratic mass formula

$$m^2_{\phi'(1680)}\cos^2\theta' + m^2_{\omega'(1420)}\sin^2\theta' = \frac{4m^2_{K^{*'}(1680)} - m^2_{\rho'(1450)}}{3}, \quad (57)$$

but with the first excited states of the corresponding particles.

So, if one knows numerical values of the coupling constants on the left-hand side of the expressions (51), (52), (55), and (56) one can find numerical values of all f^F, f^D, and f^S and $f^{F'}, f^{D'}$, and $f^{S'}$ coupling constants in both SU(3) invariant Lagrangians (3) and (54), which are needed for a prediction of EM FFs behaviors of all 1/2 octet hyperons $\Lambda, \Sigma^+, \Sigma^0, \Sigma^-$, Ξ^0, and Ξ^-.

It is straightforward to determine (51) and (52) from the numerical values in Table I, besides $(f^{(2)}_{\rho NN}/f_\rho)$, which is calculated by means of the relation

$$\left(f^{(2)}_{\rho NN}/f_\rho\right) = \frac{1}{2}(\mu_p - \mu_n - 1)\frac{C^{2v}_{\rho''}C^{2v}_{\rho'}}{\left(C^{2v}_{\rho'} - C^{2v}_{\rho}\right)\left(C^{2v}_{\rho''} - C^{2v}_{\rho}\right)} = 2.8956, \quad (58)$$

and the values $f_\rho = 4.9582, f_\omega = 17.0620$, and $f_\phi = -13.4428$ are calculated from existing data [8] on lepton width $\Gamma(V \to e^+e^-)$ by means of the relation $\Gamma(V \to e^+e^-) = \frac{\alpha^2 m_V}{3}\left(\frac{f_V^2}{4\pi}\right)^{-1}$.

The results are

$$f^{(1)}_{\rho NN} = 1.8578, \quad f^{(1)}_{\omega NN} = 26.8163, \quad f^{(1)}_{\phi NN} = 15.1191 \quad (59)$$

and

$$f^{(2)}_{\rho NN} = 14.3570, \quad f^{(2)}_{\omega NN} = -3.5762, \quad f^{(2)}_{\phi NN} = -3.5718. \quad (60)$$

However, for excited vector meson coupling constants in (55) and (56), a big problem appeared. There are no data (see Ref. [8]) on $\Gamma(V \to e^+e^-)$ for $\omega'(1420), \rho'(1450)$, and $\phi'(1680)$ in order to determine $f_{\rho'}, f_{\omega'}$, and $f_{\phi'}$. The first two are calculated from lepton widths estimated by Donnachie and Clegg [10] $f_{\rho'} = 13.6491$ and $f_{\omega'} = 47.6022$ and the third one for ϕ' is determined to be $f_{\phi'} = -33.6598$ from the relations $f^2_{\rho'} : f^2_{\omega'} : f^2_{\phi'} = \frac{1}{9} : 1 : \frac{1}{2}$ following (see [55]) from the quark structure of the corresponding vector mesons and the electric charges of the three constituent quarks from which these vector mesons are compounded.

Having numerical values of $f_{\rho'}, f_{\omega'}$, and $f_{\phi'}$ and calculating the coupling constant ratios for excited vector mesons that are missing in Table I by the relations

$$\left(f^{(1)}_{\rho'NN}/f_{\rho'}\right) = \frac{1}{2}\frac{C^{1v}_{\rho''}}{\left(C^{1v}_{\rho''} - C^{1v}_{\rho'}\right)} - \frac{\left(C^{1v}_{\rho''} - C^{1v}_{\rho}\right)}{\left(C^{1v}_{\rho''} - C^{1v}_{\rho'}\right)}\left(f^{(1)}_{\rho NN}/f_\rho\right) = 0.7635, \quad (61)$$

$$\left(f^{(2)}_{\rho'NN}/f_{\rho'}\right) = -\frac{1}{2}(\mu_p - \mu_n - 1)\frac{C^{2v}_{\rho''}C^{2v}_{\rho}}{\left(C^{2v}_{\rho''} - C^{2v}_{\rho'}\right)\left(C^{2v}_{\rho'} - C^{2v}_{\rho}\right)} = -1.3086, \quad (62)$$

$$\left(f^{(2)}_{\omega'NN}/f_{\omega'}\right) = \frac{1}{2}(\mu_p + \mu_n - 1)\frac{C^{2s}_{\omega''}C^{2s}_{\phi''}}{\left(C^{2s}_{\phi''} - C^{2s}_{\omega'}\right)\left(C^{2s}_{\omega''} - C^{2s}_{\omega'}\right)} - \frac{\left(C^{2s}_{\phi''} - C^{2s}_{\omega}\right)\left(C^{2s}_{\omega''} - C^{2s}_{\omega}\right)}{\left(C^{2s}_{\phi''} - C^{2s}_{\omega'}\right)\left(C^{2s}_{\omega''} - C^{2s}_{\omega'}\right)}\left(f^{(2)}_{\omega NN}/f_\omega\right) - \frac{\left(C^{2s}_{\phi''} - C^{2s}_{\phi}\right)\left(C^{2s}_{\omega''} - C^{2s}_{\phi}\right)}{\left(C^{2s}_{\phi''} - C^{2s}_{\omega'}\right)\left(C^{2s}_{\omega''} - C^{2s}_{\omega'}\right)}\left(f^{(2)}_{\phi NN}/f_\phi\right) - \frac{\left(C^{2s}_{\phi''} - C^{2s}_{\phi'}\right)\left(C^{2s}_{\omega''} - C^{2s}_{\phi'}\right)}{\left(C^{2s}_{\phi''} - C^{2s}_{\omega'}\right)\left(C^{2s}_{\omega''} - C^{2s}_{\omega'}\right)}\left(f^{(2)}_{\phi'NN}/f_{\phi'}\right) = -0.5771, \quad (63)$$

one obtains the results

$$f^{(1)}_{\rho'NN} = 10.4211, \quad f^{(1)}_{\omega'NN} = 1.9900, \quad f^{(1)}_{\phi'NN} = -6.3247 \quad (64)$$

and

$$f^{(2)}_{\rho'NN} = -17.8612, \quad f^{(2)}_{\omega'NN} = -27.4712, \quad f^{(2)}_{\phi'NN} = -5.9948. \quad (65)$$

Now, by a solution of the systems of algebraic equations (51), (52), (55), and (56) according to three unknown constants f^F, f^D, and f^S one comes to the following expressions:

$$f_1^F = \tfrac{1}{2}\left[\sqrt{3}\left(f^{(1)}_{\phi NN}\cos\theta - f^{(1)}_{\omega NN}\sin\theta\right) + f^{(1)}_{\rho NN}\right],$$
$$f_1^D = \tfrac{1}{2}\left[3f^{(1)}_{\rho NN} - \sqrt{3}\left(f^{(1)}_{\phi NN}\cos\theta - f^{(1)}_{\omega NN}\sin\theta\right)\right], \quad (66)$$
$$f_1^S = \sqrt{2}\left(f^{(1)}_{\omega NN}\cos\theta + f^{(1)}_{\phi NN}\sin\theta\right),$$

$$f_2^F = \tfrac{1}{2}\left[\sqrt{3}\left(f^{(2)}_{\phi NN}\cos\theta - f^{(2)}_{\omega NN}\sin\theta\right) + f^{(2)}_{\rho NN}\right],$$
$$f_2^D = \tfrac{1}{2}\left[3f^{(2)}_{\rho NN} - \sqrt{3}\left(f^{(2)}_{\phi NN}\cos\theta - f^{(2)}_{\omega NN}\sin\theta\right)\right], \quad (67)$$
$$f_2^S = \sqrt{2}\left(f^{(2)}_{\omega NN}\cos\theta + f^{(2)}_{\phi NN}\sin\theta\right),$$

$$f_1^{F'} = \tfrac{1}{2}\left[\sqrt{3}\left(f^{(1)}_{\phi' NN}\cos\theta' - f^{(1)}_{\omega' NN}\sin\theta'\right) + f^{(1)}_{\rho' NN}\right],$$
$$f_1^{D'} = \tfrac{1}{2}\left[3f^{(1)}_{\rho' NN} - \sqrt{3}\left(f^{(1)}_{\phi' NN}\cos\theta' - f^{(1)}_{\omega' NN}\sin\theta'\right)\right], \quad (68)$$
$$f_1^{S'} = \sqrt{2}\left(f^{(1)}_{\omega' NN}\cos\theta' + f^{(1)}_{\phi' NN}\sin\theta'\right),$$

$$f_2^{F'} = \tfrac{1}{2}\left[\sqrt{3}\left(f^{(2)}_{\phi' NN}\cos\theta' - f^{(2)}_{\omega' NN}\sin\theta'\right) + f^{(2)}_{\rho' NN}\right],$$
$$f_2^{D'} = \tfrac{1}{2}\left[3f^{(2)}_{\rho' NN} - \sqrt{3}\left(f^{(2)}_{\phi' NN}\cos\theta' - f^{(2)}_{\omega' NN}\sin\theta'\right)\right], \quad (69)$$
$$f_2^{S'} = \sqrt{2}\left(f^{(2)}_{\omega' NN}\cos\theta' + f^{(2)}_{\phi' NN}\sin\theta'\right),$$

and by a substitution of numerical values (59), (60), (64), and (65), respectively, one finds the numerical values

$$f_1^F = -5.69470, \quad f_1^D = 9.4103, \quad f_1^S = 42.1706, \quad (70)$$
$$f_2^F = 7.08952, \quad f_2^D = 21.6245, \quad f_2^S = -7.1465, \quad (71)$$
$$f_1^{F'} = 0.38580, \quad f_1^{D'} = 20.45639, \quad f_1^{S'} = -5.0842, \quad (72)$$
$$f_2^{F'} = 6.0577, \quad f_2^{D'} = -41.7801, \quad f_2^{S'} = -31.3390 \quad (73)$$

of all scrutinized coupling constants in SU(3) invariant interaction Lagrangians of vector-meson nonets with $1/2^+$ octet baryons. This allows us to predict behaviors of EM FFs of all $1/2^+$ octet hyperons in spacelike and timelike regions, including their real and imaginary parts, phase differences of electric and magnetic FFs, and corresponding differential and total cross sections, in which the EM structure of hyperons is planned to be measured. Predictions on all above mentioned quantities will be given in future papers, which will be published elsewhere.

VI. CONCLUSIONS

All existing 11 sets of data on nucleon EM FFs in spacelike and timelike regions from more than 40 different experiments have been analyzed, from which 534 reliable experimental points are here reasonably described by 9 resonance U&A models of nucleon EM structure, which respects the SU(3) symmetry and OZI rule violation. The analysis revealed the following results:

(1) An existence of the zero of $G_{Ep}(t)$ around $t_z = 13\ \mathrm{GeV}^2$ is again confirmed.
(2) The value of the proton charge rms radius coincides with the value obtained in the muon hydrogen atom spectroscopy experiment and in this way the existing puzzle seems to be solved.
(3) The value of the neutron charge mean squared radius is identical to the value given earlier.
(4) The coupling constants in the SU(3) invariant Lagrangian of the vector meson nonet interaction with $1/2^+$ octet baryons f^F, f^D, and f^S are determined numerically, which allows us to predict all quantities describing the EM structure of $1/2^+$ octet hyperons.

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