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Consistency between the monopole strength of the Hoyle state determined by structural calculation and that extracted from reaction observables

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We analyze the α-^{12}C inelastic scattering to the 0_2^+ state of ^{12}C, the Hoyle state, in a fully microscopic framework. With no free adjustable parameter, the inelastic cross sections at forward angles are well reproduced by the microscopic reaction calculation using the transition density of ^{12}C obtained by the resonating group method and the nucleon-nucleon g matrix interaction developed by the Melbourne group. It is thus shown that the monopole transition strength obtained by the structural calculation is consistent with that extracted from the reaction observable, suggesting no missing monopole strength of the Hoyle state.

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The 0_2^+ state of ^{12}C, the so-called Hoyle state, has intensively been studied theoretically and experimentally [1–13]. Despite a rather clear understanding of its three-α structure, the description of the Hoyle state appeared in reaction observables, the (α,α') inelastic scattering cross section in particular, has not been achieved. It was reported in many studies [14–17] that the (α,α') cross section theoretically obtained with using the transition density of ^{12}C from the ground state to the 0_2^+ state significantly overshot the observed cross section. This puzzle is called *the missing monopole strength of the Hoyle state* [15].

In these preceding studies, however, a semimicroscopic treatment of the distorting potential between α and ^{12}C as well as the coupling potentials for the excitation of ^{12}C was adopted. This suggests some ambiguities in the distorting and coupling potentials that connect the structural information and the reaction observable. In the present study we apply a fully microscopic framework to the (α,α') inelastic scattering to the 0_2^+ state of ^{12}C and show that the calculated result agrees with the experimental cross section and that essentially there is no room for the missing monopole strength.

In this study we adopt the g-matrix folding model with the target-density approximation (TDA) [18–20]; the local density of the target nucleus is used as an input density for the g matrix. This TDA g-matrix approach has been derived from the nucleus-nucleus multiple scattering theory [21] in Ref. [19] and shown to work well for describing the elastic scattering of ^{3}He [19] and α [18,20] off several moderately heavy and heavy target nuclei. We do not include the chiral three-nucleon force (3NF) modification to the g matrix because its effect on α elastic [20] and inelastic [22] scattering was shown to be very small except at backward angles.

We consider α as a projectile (P) and ^{12}C as a target nucleus (T). The α particle is assumed to stay in the ground state, whereas the transitions of ^{12}C between the 0_1^+,2_1^+,0_2^+,3_1^-,2_2^+, and 0_3^+ states are explicitly taken into account. The coupled-channel (CC) equation to be solved is given by

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dR^2}+\frac{\hbar^2}{2\mu}\frac{L(L+1)}{R^2}+F^J_{\gamma L,\gamma L}(R)+U^{\rm Coul}(R)-E_\gamma\right]\chi^J_{\gamma L}(R)=-\sum_{\gamma' L'\neq\gamma L}F^J_{\gamma L,\gamma' L'}(R)\chi^J_{\gamma' L'}(R), \tag{1}$$

$$\begin{aligned}F^J_{\gamma L,\gamma' L'}(R)=\frac{1}{4\pi}\sum_\lambda i^{L'-L+\lambda}(-1)^{L'-L+I-J+\lambda}\sqrt{(2L+1)(2L'+1)(2I+1)(2\lambda+1)}W(LIL'I'|J\lambda)(L'0L0|\lambda0)\\ \times\left\{\int d\hat{\boldsymbol{R}}d\boldsymbol{r}_{\rm P}d\boldsymbol{r}_{\rm T}\rho_{\rm P}(r_{\rm P})\rho^\lambda_{\gamma\gamma'}(r_{\rm T})g^{\rm (dr)}(s;\rho)[Y_\lambda(\hat{\boldsymbol{R}})\otimes Y_\lambda(\hat{\boldsymbol{r}}_{\rm T})]_{00}\right.\\ \left.+\int d\hat{\boldsymbol{R}}d\boldsymbol{p}d\boldsymbol{s}\rho_{\rm P}(p)\hat{j}_1(k_{\rm F}(p)s)\rho^\lambda_{\gamma\gamma'}(t)\hat{j}_1(k_{\rm F}(t)s)g^{\rm (ex)}(s;\rho)j_0(k(R)s/M)[Y_\lambda(\hat{\boldsymbol{R}})\otimes Y_\lambda(\hat{\boldsymbol{t}})]_{00}\right\},\end{aligned} \tag{2}$$

where $\chi^J_{\gamma L}(R)$ is the radial part of the P-T scattering wave function in the (γL) channel; γ specifies the state of T; and L is the orbital angular momentum between P and T. The total spin of T in the γ state is denoted by I. The definition of the coordinates is given in Fig. 1. E_γ is defined by $E_\gamma = E-\varepsilon_\gamma$ with E being the incident energy of P in the center-of-mass system and ε_γ being the excitation energy of T. M is defined by $M = A_{\rm P}A_{\rm T}/(A_{\rm P}+A_{\rm T})$, where $A_{\rm P(T)}$ is the mass number of P (T). The Coulomb potential between P and T is denoted by $U^{\rm Coul}(R)$. Y_λ is the spherical harmonics, $\hat{j}_1(k_{\rm F}s)$ is defined by $3/(k_{\rm F}s)j_1(k_{\rm F}s)$,

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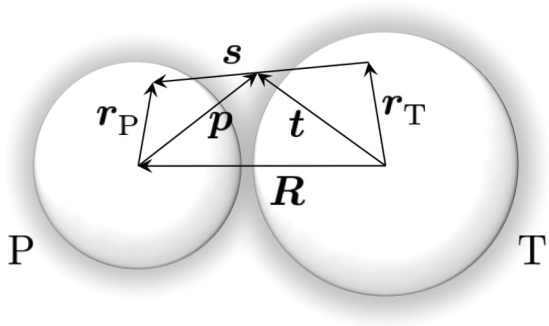

FIG. 1. Definition of the coordinates.

j_n is the spherical Bessel function, $(L'0L0\lambda 0)$ is the Clebsch-Gordan coefficient, and $W(LSL'S'|J\lambda)$ is the Racah coefficient.

This coupled-channel approach with a g matrix has widely been adopted so far [22–25]; the possible double counting for the coupling to nonelastic channels is expected to be negligible, as discussed in Ref. [22]. Equation (2) contains two key ingredients. One is the nuclear transition density $\rho^{\lambda}_{\gamma\gamma'}$ and the other is the g matrix $g^{(\mathrm{dr/ex})}$; the superscript dr (ex) indicates the direct (exchange) part of g, an explicit form of which is shown in, e.g., Ref. [26].

We adopt the transition density of ^{12}C obtained by the resonating group method (RGM) based on a three-α model [2]; the 0_1^+,2_1^+ (4.44 MeV), 0_2^+ (7.65 MeV), 3_1^- (9.64 MeV), 2_2^+ (9.84 MeV) [16], and 0_3^+ (10.3 MeV) states are considered as mentioned above. These densities are shown to reproduce the elastic and inelastic form factors for electron scattering and are thus highly reliable. In the CC calculation of the (α,α') process, we include all the six states listed above; we use the experimental values of the excitation energies. For the ground state density of α, ρ_{P}, we take the phenomenological one determined from electron scattering [27] in which the finite-size effect of proton charge is unfolded with a standard procedure [28].

As for g, we use the Melbourne g-matrix interaction [29], which has been highly successful, with no free parameters, in describing various nucleon-nucleus elastic and inelastic cross sections in a wide range of incident energies. The use of a g-matrix interaction having a predictive power is one of the most essential features of the present study. As mentioned, we use the TDA for evaluating the argument ρ in $g^{(\mathrm{dr/ex})}$, i.e., $\rho = \rho^{\lambda=0}_{\gamma\gamma}(r_{\mathrm{m}})$, where r_{m} denotes the midpoint of the interacting two nucleons. For the nondiagonal potentials, we take the average of the densities in the initial and final states, i.e., $\rho_{\mathrm{T}} = [\rho^{\lambda=0}_{\gamma\gamma}(r_{\mathrm{m}}) + \rho^{\lambda=0}_{\gamma'\gamma'}(r_{\mathrm{m}})]/2$.

It should be noted that this framework is not fully microscopic in the sense that we do not use the same interaction in the structure and reaction parts. Following the multiple scattering theory, we use the effective interaction based on a bare nucleon-nucleon force at positive energies; in practice the Melbourne g-matrix interaction is adopted. In principle, in the structure part a similar prescription can be used at negative energies. Although an attempt in this direction is ongoing in the nuclear physics field, it has not been completed yet; the description of the 0_2^+ state of ^{12}C is known to be extremely difficult in particular. In this situation, it will be reasonable to use a structure wave function that reproduces the properties of the nucleus, whatever the interaction for the structure part is.

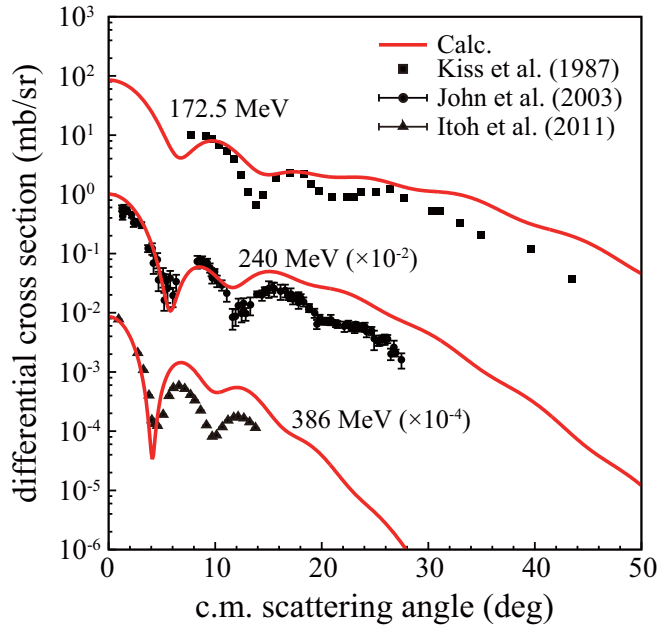

FIG. 2. Differential cross sections of α-^{12}C inelastic scattering to the 0_2^+ state at 172.5, 240, and 386 MeV, as a function of the scattering angle in the center-of-mass system. The experimental data are taken from Refs. [16,30,31].

Figure 2 shows the differential cross sections of $^{12}\mathrm{C}(\alpha,\alpha')^{12}\mathrm{C}(0_2^+)$ at 172.5, 240, and 386 MeV, compared with the experimental data [16,30,31]. A relativistic correction for the kinematics is included in the present calculations, although its effect is found to be negligibly small. One sees that with no free adjustable parameters the inelastic cross section to the 0_2^+ state of ^{12}C is reproduced well at forward angles at these three energies. At larger angles the calculation slightly overshoots the experimental data. As a possible reason for this discrepancy, we consider that the 3NF effects will slightly decrease the cross section at backward angles, as found in Ref. [22]. In addition, couplings with higher excited states, which are not included in this work, may change the cross sections at those angles. At any rate, however, there seems to be no room for the missing monopole strength; if it could exist, the inelastic cross section would decrease at all angles and the good agreement at forward angles would be lost, though it is not so clear at 172.5 MeV because of the lack of data at very forward angles. We show the results for the elastic scattering in Fig. 3 in the same way as in Fig. 2; a very good agreement between the calculated cross section and the experimental data is obtained, which confirms the reliability of the microscopic reaction calculation adopted.

In a recent work [33] it was shown that the monopole transition strength, $4.5 \pm 0.5\ e\mathrm{fm}^2$, to the 0_2^+ state determined by comparing the theoretical result with the experimental data is consistent with that deduced from electron scattering. However, the antisymmetrized molecular dynamics calculation adopted in the reaction analysis gives a larger value ($6.6\ e\mathrm{fm}^2$)

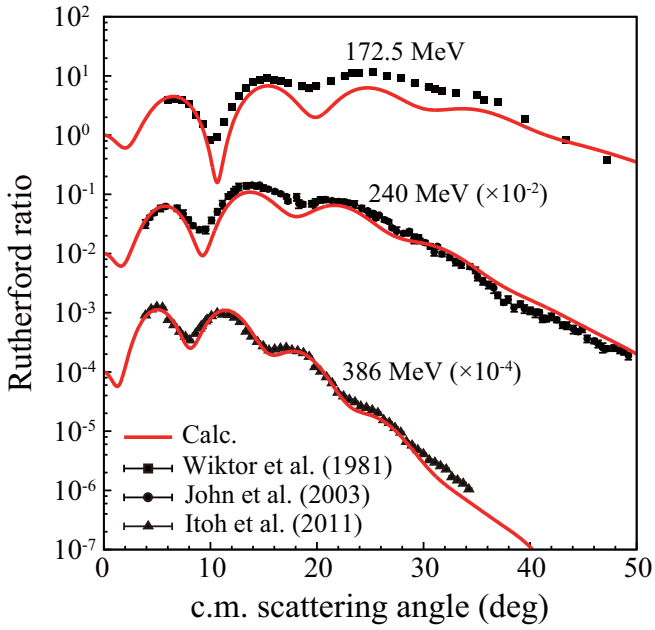

FIG. 3. Same as in Fig. 2 but for the elastic scattering (the ratio to the Rutherford cross section). The experimental data are taken from Refs. [16,30,32].

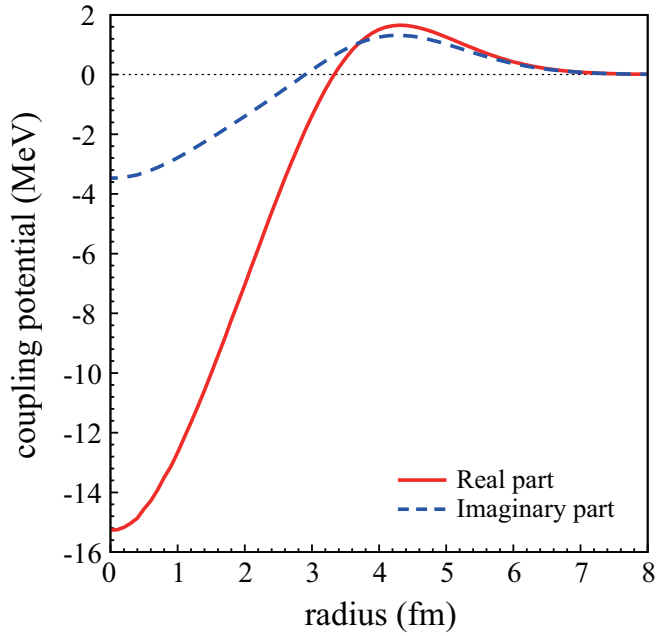

FIG. 4. Coupling potential between the 0_1^+ and 0_2^+ states of ^{12}C for the (α,α') scattering at 172.5 MeV. The solid (dashed) line corresponds to the real (imaginary) part of the coupling potential.

of the transition strength. Therefore in Ref. [33] the structural input and the reaction observables still have a gap of about 30%. Furthermore, the interaction strength was adjusted so as to reproduce the elastic scattering data; the renormalization factors for the real (imaginary) part are 1.05 (1.27) and 1.24 (1.38) for the analyses at 240 and 386 MeV, respectively.

Finally, we show by the solid (dashed) line in Fig. 4 the real (imaginary) part of the coupling potential between the 0_1^+ and 0_2^+ states of ^{12}C for the α inelastic scattering at 172.5 MeV. Important characteristics of the coupling potential are that it has a peak at the origin and is well concentrated in the nuclear interior region. In Ref. [17] it was shown phenomenologically that this behavior of the coupling potential is essential to reproduce the absolute value of the ^{12}C$(\alpha,\alpha')^{12}$C(0_2^+) cross section. It should be noted, however, that the origin of this behavior in the present study is completely different from that in Ref. [17].

In summary, we have calculated the α-^{12}C inelastic cross section to the 0_2^+ of ^{12}C with a microscopic coupled-channels method using the RGM transition density of ^{12}C and the Melbourne g-matrix interaction. We have obtained a good agreement between the calculated and measured values of the inelastic cross section at forward angles, as well as that of the elastic cross section. It suggests that the monopole transition strength obtained by the RGM calculation is consistent with the value extracted from the reaction observable. Thus, it is concluded that there is no missing monopole strength of the Hoyle state.

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