Nuclear correlations and neutrino emissivity from the neutron branch of the modified Urca process

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The neutrino emissivity from the neutron branch of the modified Urca process is calculated. The nuclear correlation effects are taken into account by employing the correlation functions extracted from the lowest-order constrained variational (LOCV) method applied to asymmetric nuclear matter. Two-body nucleon interaction is modeled by a realistic Argonne AV18 potential. In order to get consistency with semiempirical saturation parameters of nuclear matter and the existence of $2M_{\odot}$ pulsars, we add a phenomenological Urbana UIX three-body potential to the nucleon Hamiltonian and apply a newly formulated version of the LOCV method that allows for three-body nucleon interactions. We find that at fixed temperature neutrino emissivity is a (weakly) decreasing function of density, due to quenching of the contribution from tensor correlations with increasing density. This is in variance with all previous works. We also find that three-body forces allow for the opening of the direct Urca process at nucleon density 0.3 fm^{-3}.

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I. INTRODUCTION

Nuclear matter is of great relevance in the study of the physics of different systems, such as heavy ion collisions, supernova explosions, and neutron stars [1–4]. Different many body theories have been developed to study nuclear matter properties based on microscopic calculations. Among these methods one can recall the variational methods [5,6], Brueckner-type calculations in both relativistic and nonrelativistic versions [7,8], and Monte Carlo methods in various versions [9,10]. One of the main tasks of the mentioned approaches is to find the equation of state (EOS) of the system under investigation. Regarding this task, neutron stars are good candidates for exploring the superdense nuclear matter EOS. Properties such as the mass-radius relation of these dense stars have been calculated using different EOSs (e.g., [6,11]) and the Tolman-Openheimer-Volkoff (TOV) equation of hydrostatic equilibrium [12] and compared with observational data [13]. But apart from the EOS, the correlation properties of nuclear matter (effect of interactions on the nucleon motion) also play an important role in understanding some features of dense nuclear matter. In a neutron star, interacting nucleons manifest many-body effects which are directly related to the nucleonic interactions. These effects can influence many properties of the system. Important phenomena such as dissipation due to viscosity and neutrino transport (emission, scattering, and absorption) are related to the many-body effects. Some works have been devoted already to the study of many-body effects on properties such as viscosity, etc. through correlation functions [14,15], but such effects have not been systematically investigated within the variational approach as far as the neutrino transport properties of dense matter are concerned. If we restrict ourselves to a domain in which the stellar matter is transparent to neutrinos, i.e., below 10^9 K [16], it is worthwhile to investigate how the neutrino emissivity in main neutrino producing processes and, as a result, the cooling behavior of the star depend on the realistic correlation functions and whether such dependence predicts noticeable differences compared to simple models that ignored some important many body effects. In fact, such studies can provide a tool for probing many-body methods as well as the input realistic nucleon-nucleon (N-N) potentials.

The modified Urca process (hereafter Murca; for the origin of the names Urca and Murca, see Sec. 3.3.5 of Ref. [17]) was introduced by Chiu and Salpeter [18]. They wrote that their estimate of the neutrino energy loss was "essentially the result of a dimensional analysis and could be in error by several orders of magnitude." Then followed calculations by other authors [19–22]. The series of pioneer articles of 1964–1965 was concluded with a detailed paper by Bahcall and Wolf [16] who also critically reviewed previous calculations of the Murca cooling rate.

Friman and Maxwell in their classical paper calculated the neutrino emissivity of different neutrino producing mechanisms, including Murca, taking the medium- and long-range strong interaction into account through the one-pion-exchange model and the short-range component by a phenomenological Fermi-liquid description [23].

Blaschke and others studied the nuclear in-medium effects on the emissivity of neutrinos in Murca processes within a thermodynamic T-matrix approach [24]. In order to facilitate numerical calculation, they assumed a bare nucleon-nucleon interaction of nonlocal-separable type. Under such an assumption, the equation for the in-medium T matrix was reduced to a system of algebraic equations. The obtained Murca emissivity was about half (one-fifth) of the Friman and Maxwell result [23] at baryon density $0.5n_0$ $(1.5n_0)$, where n_0 is the nuclear saturation density. Throughout the present paper $n_0 = 0.16\ \mathrm{fm}^{-3}$, which corresponds to the mass-energy density $\rho_0 = 2.7 \times 10^{14}\ \mathrm{g\,cm}^{-3}$.

Another approach to the calculation of the Murca emissivity was based on the Landau-Migdal theory of Fermi liquids (results reviewed in Ref. [25]). This approach treats systematically various strong-interaction effects associated with neutrino emission generated by the charged-current weak-interaction processes. A series of calculations started with a quasiparticle approximation for nucleons [26,27]. The contribution resulting from possible softening of the pion mode in dense nuclear matter with increasing density was also evaluated [28]. Generally, Murca emissivities obtained in the Landau-Migdal approach (called "medium modified Urca" in Ref. [29]) were much larger than those obtained by Friman and Maxwell [23]; moreover, they increased strongly with increasing density, and were plagued by huge uncertainties.

Applying the quasiparticle approximation for nucleons, Sawyer and Soni proposed an approach based on pair correlation functions for calculating the nuclear matrix element [30]. Within this approach they could extract an analytical expression for the neutrinos' opacity due to their absorption by neutrons in the presence of an additional neutron, i.e., $\nu_e nn \rightarrow npe$. But they applied an unrealistic pure hard-core N-N repulsion when modeling the two-body correlation function. Following the formalism of Ref. [30], Haensel and Jerzak obtained neutrino opacity due to $\nu_e nn \rightarrow npe$ using realistic correlation functions, i.e., ones including tensor correlation and calculated using a realistic N-N potential. This showed the crucial importance of tensor forces [31]. They used, unfortunately, correlation functions obtained by other authors for symmetric nuclear matter at n_0, instead of correlation functions calculated at each density in beta-stable nuclear matter. In the present paper we will overcome these shortcomings of Ref. [31]

One of the many body techniques which casts many body properties through correlation functions directly from a systematic approach is the so-called variational method. The lowest-order constrained variational (LOCV) theory is a fully self-consistent variational method that has successfully been used in describing various properties of baryonic systems in the last three decades [32–34]. In the LOCV framework the state-dependent correlation functions can directly be extracted by solving the Euler-Lagrange equations in a variational way [32].

The purpose of the present paper is to calculate and investigate the emissivity of neutrinos in the neutron branch (n-branch) Murca process, which is one of the main neutrino producing processes in the neutron star interior [35], using LOCV state-dependent correlation functions.

The paper proceeds as follows. The independent pair approximation for calculation of neutrino emissivity using the Fermi Golden rule is justified in Sec. II. Then the calculation of emissivity based on realistic N-N correlation functions is presented in Sec. III. Section IV is devoted to our results that are obtained using the correlation functions arising from the LOCV method. Our discussions and conclusions are presented in Sec. V. A brief review of the LOCV method is given in the Appendix.

II. INDEPENDENT PAIR APPROXIMATION FOR NUCLEON QUASIPARTICLES

We consider a neutron star core composed mainly of neutrons of density n_n, with an admixture of protons of density n_p, and electrons and muons. We assume $T < 10^9$ K, so that nucleons and leptons are strongly degenerate Fermi systems and the neutron star is transparent to neutrinos [16]. The plasma of nucleons and leptons is homogeneous and electrically neutral, and all matter constituents are in equilibrium with respect to weak-interaction processes (beta equilibrium). Leptons can be treated as free Fermi gases. Nucleons interact via strong (nuclear) forces and form a strongly asymmetric ($n_n \gg n_p$) nuclear matter. Throughout this paper, we limit our selves to nonsuperfluid (normal) nuclear matter.

A strongly interacting system can be described using the Landau theory of Fermi liquids [36]. In this theory, real nuclear matter is replaced by a system of nucleon uasi-particles of the same number density n_n and n_p. The Fermi momenta for quasiparticles coincide with those for a free Fermi gas of nucleons, $p_n = \hbar(3\pi^2 n_n)^{1/3}$ and $p_p = \hbar(3\pi^2 n_p)^{1/3}$. There is a correspondence between the ground state and low-lying excited states in a noninteracting Fermi gas of nucleons and a strongly interacting Fermi liquid treated in terms of nucleon quasiparticles. Such low-lying excited states are associated with deviation of the occupation numbers of states in the vicinity of Fermi surfaces from the ground-state one. Namely, the ground state corresponds to occupation of all momentum states within a Fermi sphere, while a low-lying excited state corresponds to filling a number of states above the Fermi surface and emptying the same number of states within this surface. In what follows we will use the term "states" exclusively for the quasiparticle states. They can be labeled by momentum and spin, similarly to the states of nucleons in a free Fermi gas model. Because of the Pauli blocking of the final states, only transitions between the states close to the corresponding Fermi surfaces for neutrons and protons can contribute to neutrino emission. The same picture is valid for free Fermi gases of electrons and muons, for which, however, quasiparticles coincide with particles.

For strongly degenerate nuclear matter, those excited momentum states which can be involved in the neutrino emission processes constitute a dilute gas of elementary excitations (Landau quasiparticles). The wave function of the dilute gas of Landau quasiparticles can be constructed using the independent pair approximation. The many-body initial and final state in the emission process will be expressed in terms of the correlated two-particle (pair) wave function in nuclear matter. In the case of the Brueckner-Hartree-Fock (BHF) theory of nuclear matter, the correlated two-body wave functions can be calculated using the Bethe-Goldstone equation. The pair correlation function in LOCV is calculated directly from Euler-Lagrange equations resulting from minimization of the energy functional and normalization condition.

Recently, the pair correlation functions for nuclear matter were calculated using both LOCV and BHF approaches with realistic nucleon interactions in the same conditions. It has been found that both methods give similar pair correlation functions [37]. Having constructed the many-body wave function in the independent pair approximation, and using the properties of the Landau quasiparticle distribution function at temperature T (it coincides with that of the free Fermi gas, but involves quasiparticle energies), we can calculate the Murca neutrino emissivity using the Fermi Golden Rule.

As is shown in the next section, in the leading order in T the calculation reduces to the radial integrals of expressions involving pair correlation functions. The pre-factors in the final expression involve effective masses of Landau quasiparticles, resulting from the statistical factors (density of initial and final momentum states per unit energy). In the present paper we use pair correlation functions calculated using the LOCV method. The effective mass of nucleons has been calculated in the framework of LOCV method as well, but only for the symmetric nuclear matter using single particle potential energy at zero temperature [38]. Although it is in principle possible to calculate them for asymmetric and beta-stable matter, since these data are not available yet, we employ nucleon effective masses in the $T = 0$ system calculated in BHF method for the same realistic bare nucleon interactions as in our LOCV calculation [39]. We are allowed to do so because, as was mentioned, it is shown in Ref. [37] that the defect functions arising from LOCV and BHF methods are almost the same at zero temperature and they are in good agreement regarding the behavior of correlation functions.

III. NEUTRINO EMISSIVITY FROM THE *n* BRANCH OF THE MODIFIED URCA PROCESS

After the temperature of neutron star core interior has fallen below 10^9 K, the star is transparent to neutrinos and matter constituents are degenerate [16]. Below muon threshold, the direct Urca (Durca) process is blocked in neutron star matter with proton abundance less than $1/9 \simeq 0.11$ because of the energy and momentum conservation for nucleon quasiparticles [12]. Above muon threshold, the constraint on n_p/n is stronger [40]. In the absence of the Durca reactions which would lead to a fast cooling of a neutron star, the Murca processes remain main mechanisms that control the cooling rate. Detailed description of neutrino emission processes can be found in Refs. [17,41]. Similarly as Durca, Murca involves charged-current weak interaction between nucleons and leptons. However, in Murca process, the nucleon undergoing the charged-current process is correlated via two-body nuclear force with another nucleon which acts as "active spectator." Nuclear force allows there for the transfer of momentum between the nucleons involved in charged-current process and the "active spectators" allow therefore for the energy-momentum balance in the Murca reactions. The basic Murca process (neutron beta decay) is

$$\begin{aligned} n + n &\to n + p + e + \overline{\nu}_e, \\ \boldsymbol{p}_1 s_1 \quad \boldsymbol{p}_2 s_2 &\to \boldsymbol{p}_{1'} s_{1'} \quad \boldsymbol{p}_p s_p \quad \boldsymbol{p}_e s_e \quad \boldsymbol{p}_{\overline{\nu}}, \end{aligned} \tag{1}$$

where the asymptotic momenta and spins in the initial and final states are indicated to fix the notation. An inverse process is

$$n + p + e \to n + n + \nu_e. \tag{2}$$

In the proton branch of Murca the active spectator is a proton,

$$n + p \to p + p + e + \overline{\nu}_e, \tag{3}$$

$$p + p + e \to p + n + \nu_e. \tag{4}$$

Reactions analogous to (1)–(4) are possible if muons are present in neutron star matter, with electrons replaced by muons.

The neutron branch of the Murca process does not have a density threshold: it can happen at all densities. The proton branch has a threshold. In npe matter it is open if $p_{\mathrm{F}p} > \frac{1}{4} p_{\mathrm{F}n}$, where $p_{\mathrm{F}n}$ ($p_{\mathrm{F}p}$) is the neutron (proton) Fermi momentum. This condition implies that the proton fraction $Y_p > Y_{\mathrm{c}p} = \frac{1}{65} = 0.0154$, satisfied almost everywhere in the neutron star core, except for EOSs with very low symmetry energy at $\rho \leqslant \rho_0$ [42]. In this work we focus on the neutron branch of the Murca process, which yields higher neutrino emissivity than the proton branch, Eqs. (3) and (4) [17].

In neutron star matter in beta equilibrium (an assumption valid in a cooling neutron star) the rates of processes (1) and (2) (neutron branch of Murca) are equal. So it is sufficient to calculate the rate of any of them and double the result. We first obtain an expression for the electron neutrinos' emissivity, then we modify it to get the muon neutrinos' emissivity.

A. The emissivity carried by $\nu_e, \overline{\nu}_e$

The rate at which the neutrinos with energy E_ν are emitted in any of reactions Eqs. (1)–(4) can be computed using the Fermi golden rule. The Murca process represented by Eq. (1) is generated by the weak charged-current interaction with the Hamiltonian operator density, which in the approximation of nonrelativistic nucleons is

$$\hat{H}^{\mathrm{cc}}_{\mathrm{int}}(\boldsymbol{x}) = \frac{G}{\sqrt{2}} (\hat{\psi}^{(p)\dagger} \hat{\psi}^{(n)} \hat{\overline{\psi}}_e \gamma_0 (1 - \gamma^5) \psi_\nu - c_A \hat{\psi}^{(p)\dagger} \sigma^i \hat{\psi}^{(n)} \hat{\overline{\psi}}_e \gamma_i (1 - \gamma^5) \hat{\psi}_\nu), \tag{5}$$

where G is the Fermi weak coupling constant, and summation over repeated indices is assumed. In these equations $\hat{\psi}$'s are the particle's field operators, c_A is the axial vector renormalization, and σ's and γ's are Pauli and Dirac matrices, respectively.

The neutrino emissivity [energy carried away by neutrinos from processes (1) and (2) from 1 cm^3 in 1 s] is obtained by multiplying the rate by E_ν and integrating over dE_ν. For the sum of contributions from reactions Eqs. (1) and (2) we get

$$\begin{aligned} Q_\nu^{\mathrm{Mn}(e)} = {}& \frac{2}{(2\pi\hbar)^{18}} \int d^3p_1 d^3p_2 d^3p_{1'} d^3p_p d^3p_e d^3p_{\overline{\nu}} E_{\overline{\nu}} \frac{1}{2} \langle |\mathcal{M}_{\mathrm{if}}|^2 \rangle_{\mathrm{spin}} (2\pi)^4 \delta(E_1 + E_2 - E_{1'} - E_p - E_e - E_{\overline{\nu}_e}) \\ & \times \delta^{(3)}(\boldsymbol{p}_1 + \boldsymbol{p}_2 - \boldsymbol{p}_{1'} - \boldsymbol{p}_p - \boldsymbol{p}_e - \boldsymbol{p}_{\overline{\nu}_e}) \mathfrak{n}_n(E_1) \mathfrak{n}_n(E_2) [1 - \mathfrak{n}_n(E_{1'})][1 - \mathfrak{n}_p(E_p)][1 - \mathfrak{n}_e(E_e)], \end{aligned} \tag{6}$$

where we use notation of Eq. (1), $E_n(\boldsymbol{p}_1)$, $E_n(\boldsymbol{p}_2)$, $E_n(\boldsymbol{p}_{1'})$, and $E_p(\boldsymbol{p}_p)$ are nucleon (quasiparticle) energies, $E_{\overline{\nu}_e} = cp_{\overline{\nu}_e}$, and electrons are treated as a free Fermi gas. The factor $\frac{1}{2}$ is introduced to avoid the double counting of the nn states. The functions

$\mathfrak{n}_j[E_j(\boldsymbol{p}_j)]$ are the Fermi-Dirac distributions. The quantity $\langle|\mathcal{M}_{\text{if}}|^2\rangle_{\text{spin}}$ is the spin averaged squared matrix element of the weak-interaction Hamiltonian.

Due to the high degeneracy of the neutron star matter constituents, the available phase space is determined by the temperature and, as was discussed in Sec. II, only momentum states close to the corresponding Fermi surface contribute to Q_ν. Consequently, one can replace in the integrand of Eq. (6)

$$d^3p_i = m_i^* p_{\text{F}i}\, dE_i\, d\Omega_i, \tag{7}$$

where $d\Omega_i$ is the solid angle in the direction of $\boldsymbol{p}_i$ and the effective mass m_i^* $(i = n,p)$ is calculated at the corresponding Fermi surface,

$$m_i^* = \frac{p_{\text{F}i}}{v_{\text{F}i}}, \tag{8}$$

where $v_{\text{F}i}$ is the quasiparticle velocity at the Fermi surface,

$$v_{\text{F}i} = \left(\frac{\partial E_i}{\partial p}\right)_{p=p_{\text{F}i}}. \tag{9}$$

For electrons

$$m_e^* = \frac{p_{\text{F}e}}{c}. \tag{10}$$

To evaluate the mean squared matrix element we construct the initial and final states of nucleons in the independent pair approximation. We follow the notations of Ref. [31] and construct the correlated two-body state vectors as

$$\left|\Psi^{(nn)}_{s_1s_2}\right\rangle = \frac{1}{V}\int \mathrm{d}^3x_1 d^3x_2\, e^{\frac{i}{\hbar}(\boldsymbol{p}_1\cdot\boldsymbol{x}_1+\boldsymbol{p}_2\cdot\boldsymbol{x}_2)} \times \sum_{s_as_b} f^{nn}_{s_as_1,s_bs_2}(\boldsymbol{x}_1-\boldsymbol{x}_2)\,\hat{\psi}^{(n)}_{s_a}(\boldsymbol{x}_1)\,\hat{\psi}^{(n)}_{s_b}(\boldsymbol{x}_2)|0\rangle, \tag{11}$$

$$\left|\Psi^{(np)}_{s_{1'}s_p}\right\rangle = \frac{1}{V}\int d^3x' d^3y'\, e^{\frac{i}{\hbar}(\boldsymbol{p}_{1'}\cdot\boldsymbol{x}'+\boldsymbol{p}_p\cdot\boldsymbol{y}')} \times \sum_{s_as_b} f^{np}_{s_as'_1,s_bs_p}(\boldsymbol{x}'-\boldsymbol{y}')\,\hat{\psi}^{(n)}_{s_a}(\boldsymbol{x}')\,\hat{\psi}^{(p)}_{s_b}(\boldsymbol{y}')|0\rangle. \tag{12}$$

The correlation functions f are in general 4×4 matrices in the spin space, and become unit matrices for a pair of free nucleons.

For the nn pair, f is assumed to be purely central, because the tensor force is weak in this isospin channel,

$$f^{nn}_{s_as_1,s_bs_2}(\boldsymbol{r}) = f^{nn}(r)\,\delta_{s_as_1}\delta_{s_bs_2}, \tag{13}$$

where $r = |\boldsymbol{r}|$. For the np pair, f contains, in addition to a central component, also a tensor component, because of a strong tensor force in the two-body state ${}^3S_1 - {}^3D_1$:

$$f^{(np)}_{s_as'_1,s_bs_p}(\boldsymbol{r}) = f^{(np)}_c(r)\,\delta_{s_as'_1}\delta_{s_bs_p} + f^{(np)}_t(r)[S^{(np)}(\boldsymbol{n})]_{s_as'_1,s_bs_p}, \tag{14}$$

where $\boldsymbol{n}$ is a unit vector in the direction of $\boldsymbol{r}$, and

$$S^{(np)}(\boldsymbol{n}) = 3\,\sigma^i_n n_i\,\sigma^j_p n_j - \sigma^k_n\sigma^k_p \tag{15}$$

is the usual tensor operator.

Several additional steps can be made which allow for getting an analytic expression for $Q_\nu^{\text{Mn}(e)}$ [12]. The energy of a neutrino is of the order of $k_{\text{B}}T$, where k_{B} is the Boltzmann constant, and therefore much smaller than the relevant energies of n, p, and e. Therefore, we can remove $E_{\overline{\nu}_e}$ and $\boldsymbol{p}_{\overline{\nu}_e}$ from the arguments of the delta function in Eq. (6). Then, using the "triangle approximation" $\boldsymbol{p}_1 \cong \boldsymbol{p}_{1'} + \boldsymbol{p}_2$, i.e., neglecting proton and electron Fermi momenta compared to the neutron one, we finally get

$$Q_\nu^{\text{Mn}(e)} = \frac{11513}{945}\frac{G^2\left(1+3c_A^2\right)}{c^4\pi\hbar^{13}2^{10}} m_n^{*3} m_p^* \frac{k_{F_p}k_{F_e}^2}{k_{F_n}^6}\mathcal{R}(k_{\text{F}n})(k_{\text{B}}T)^8, \tag{16}$$

where $k_{\text{F}j} = p_{\text{F}j}/\hbar$. The dimensionless factor $\mathcal{R}$ contains the effects of N-N interaction through correlation functions [31]:

$$\mathcal{R}(k_{\text{F}n}) = F_c^2 + 12.7F_t^2, \tag{17}$$

where

$$F_c = 4\pi k_{\text{F}n}^3\int_0^\infty dr\, r^2 j_0(k_{\text{F}n}r)\left[f_c^{(nn)}(r) f_c^{(np)}(r) - 1\right], \tag{18}$$

$$F_t = 4\pi k_{\text{F}n}^3\int_0^\infty dr\, r^2 j_2(k_{\text{F}n}r) f_c^{(nn)}(r) f_t^{(np)}(r). \tag{19}$$

Here, $j_l(x)$ are the spherical Bessel functions of order l. Notice that for $f = f_c = 1$ (no correlations, no tensor correlations) one gets $\mathcal{R} = 0$, as expected.

By substituting appropriate values of constants in Eq. (16), we obtain an expression useful for numerical calculations:

$$Q_\nu^{\text{Mn}(e)} = 5.92\times10^{19}\frac{n_0}{n_n}{Y_e}^{2/3}{Y_p}^{1/3} \times \left(\frac{m_n^*}{m_n}\right)^3\frac{m_p^*}{m_p}\mathcal{R}(k_{F_n}){T_9}^8\ \text{erg}\,\text{cm}^{-3}\text{s}^{-1}, \tag{20}$$

where $n_0 = 0.16\ \text{fm}^{-3}$ is the saturation density of symmetric nuclear matter, $T_9 = T/10^9$, and $Y_e = n_e/n$ and $Y_p = n_p/n$ are the electron and proton fractions, respectively. For a simple model of npe matter, i.e., neglecting the presence of muons, we have $Y_e = Y_p$.

B. The emissivity carried by $\nu_\mu, \overline{\nu}_\mu$

When the Fermi energy of electrons exceeds the rest energy of muons, $E_{\text{F}e} = c(m_e^2c^2 + p_{\text{F}e}^2)^{1/2} \simeq p_{\text{F}e}c > m_\mu c^2 = 105.56$ MeV, muons are present in neutron star matter. As the muon Fermi momentum is smaller than the electron one, blocking of the electron Durca process implies *a fortiori* blocking of the muon Durca one. Then, only Murca reactions involving muons can proceed:

$$n + n \to p + n + \mu + \overline{\nu}_\mu, \quad n + p + \mu \to n + n + \nu_\mu. \tag{21}$$

Muons can be treated as a free Fermi gas at $T = 0$, so that $E_\mu(p) = c(m_\mu^2c^2 + p^2)^{1/2}$ and therefore

$$v_{\text{F}\mu} = \left(\frac{\partial E_\mu}{\partial p}\right)_{p=p_{\text{F}\mu}} = c^2\frac{p_{\text{F}\mu}}{E_{\text{F}\mu}} = c\frac{p_{\text{F}\mu}}{p_{\text{F}e}}, \tag{22}$$

where the beta equilibrium condition $E_{\text{F}\mu} = cp_{\text{F}e}$ was used. Therefore, while the muon mass $m_\mu \simeq 207m_e$, in $npe\mu$ matter in beta equilibrium

$$m_\mu^* = m_e^* = \frac{p_{\mathrm{F}e}}{c}. \tag{23}$$

To obtain a complete expression for the emissivity of muon neutrinos we should replace one of the $k_{\mathrm{F}e}$ in Eq. (16) by $k_{\mathrm{F}\mu}$ and include the muon threshold factor. Applying these modifications, we get following expression for the emissivity of muon neutrinos from the neutron branch of the Murca process:

$$Q_\nu^{\mathrm{Mn}(\mu)} = 5.92 \times 10^{19} \frac{n_0}{n_n} Y_\mu \left(\frac{Y_p}{Y_e}\right)^{1/3} \left(\frac{m_n^*}{m_n}\right)^3 \frac{m_p^*}{m_p} \times \mathcal{R}(k_{\mathrm{F}n}) T_9{}^8 \Theta(k_{\mathrm{F}e} - k_{c\mu})\ \mathrm{erg\,cm^{-3}s^{-1}}, \tag{24}$$

where $k_{c\mu} = m_\mu c^2/\hbar c = 0.517\ \mathrm{fm}^{-1}$.

The total emissivity of the neutron branch of the Murca process is obtained by adding expressions in Eqs. (20) and (24).

IV. RESULTS

As is apparent from Eqs. (16), (20), and (24), the main parameters involved in the calculation of neutrino emissivity at a given baryon density, which are directly related to the nuclear interactions in neutron star matter, are proton and electron fractions Y_p and Y_e, from which one obtains $Y_\mu = Y_p - Y_e$ and $Y_n = 1 - Y_p$, effective masses m_n^* and m_p^*, and correlation factor $\mathcal{R}$. We need a reliable many-body technique to evaluate these parameters. As mentioned in the Introduction, the method we have adopted is LOCV. A brief description of the LOCV method is presented in the Appendix. As an input N-N interaction we have used the Argonne V-18 (AV18) potential [43]. However, it is well known that the two-body force (2BF) cannot reproduce the saturation properties of nuclear matter, and addition of a three-body force (3BF) is required. In this work we have employed the Urbana IX model for 3BF in an extended LOCV method [44]. So we also examine the effect of 3BF on the neutrino emissivity by comparing the results obtained using 2BF only and those obtained using the 2BF+3BF Hamiltonian.

A. Proton and electron fraction

The nucleon component of neutron star matter is strongly asymmetric nuclear matter with $Y_p \ll 1$. Electrical neutrality requires $Y_p = Y_e + Y_\mu$. In beta-stable $npe\mu$ matter (BSM), leptons and nucleons are in beta equilibrium, which implies a relation between the chemical potentials $\mu_n = \mu_p + \mu_e$ and $\mu_e = \mu_\mu$, where μ_i and n_i are chemical potential and number density of particle i. Electrical neutrality and beta equilibrium determine actually the values of Y_i at a given baryon density n. Neglecting nuclear interactions, i.e., assuming a free Fermi gas model (FFG), we get

$$Y_e^{\mathrm{FFG}} \cong 0.00547 \frac{n}{n_0}. \tag{25}$$

Interaction between nucleons strongly increases Y_e due to an attractive potential well for protons moving in neutron matter. Friman and Maxwell [23] proposed to include the effect of

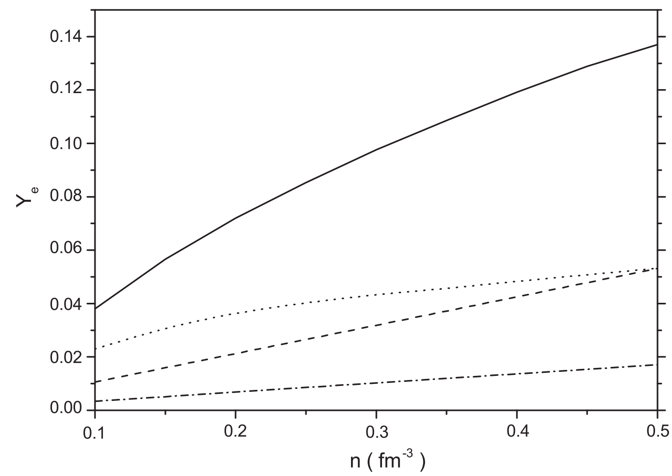

FIG. 1. Electron fractions predicted by the noninteracting Fermi gas model (dash-dotted line), the Friman and Maxwell modified version [23] (dashed line), LOCV with two-body force only (short-dashed line), and LOCV with two-body plus three-body forces (solid line).

strong interaction by modifying the numerical prefactor in Eq. (25):

$$Y_e \cong 0.0168 \frac{n}{n_0}. \tag{26}$$

The values of Y_e resulting from these approximations and those obtained from the LOCV method with and without 3BF are shown in Fig. 1. At $n = n_0$ the electron fraction obtained using the LOCV method without 3BF is about twice that proposed in Ref. [23]. We have also presented the LOCV method's predicted values of Y_p in Fig. 2 with and without 3BF.

Inclusion of 3BF strongly increases electron and proton fractions. Already at n_0, we obtain $Y_p(\mathrm{2BF}) = 0.03$ and $Y_p(\mathrm{2BF+3BF}) = 0.07$. The contribution from 3BF grows

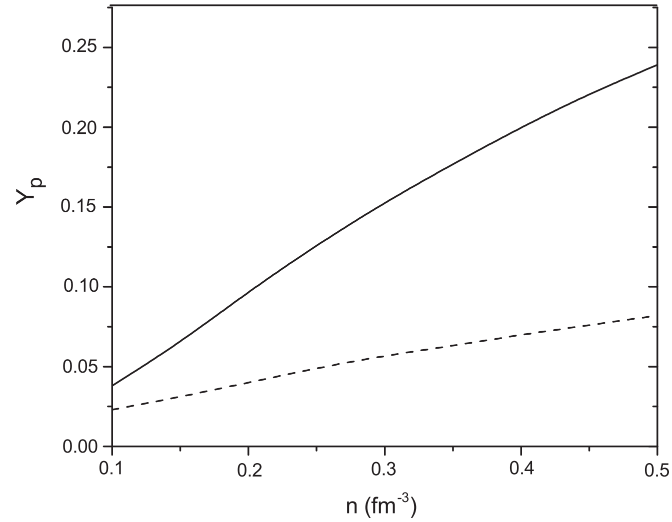

FIG. 2. LOCV proton fraction for two-body force only (dotted line) and the two-body plus three-body forces case (solid line).

rapidly with density, and at $2n_0$ we get $Y_p(\mathrm{2BF}) = 0.06$ and $Y_p(\mathrm{2BF} + \mathrm{3BF}) = 0.16$.

All fractions increase with baryon density. Including 3BF in the LOCV calculation results in a dramatic increase of the value of proton, electron, and muon fractions with density (the value of Y_μ can be deduced from Figs. 1 and 2).

An important consequence is that including 3BF into the LOCV calculations in the manner worked out in Ref. [44] allows for opening the Durca process above $n_{\mathrm{Du}} = 0.3\ \mathrm{fm}^{-3}$. This means that for $n > 0.3\ \mathrm{fm}^{-3}$ the Murca process is irrelevant for neutron star cooling, being a negligible addition to the Durca emissivity. Notice that the Durca process is not allowed even at $0.5\ \mathrm{fm}^{-3}$, the largest density considered in the present paper, when only 2BF is considered.

B. Correlation factor

The effects of nucleon correlations in nuclear matter are encapsulated in the correlation factor $\mathcal{R}$. In Eqs. (18) and (19) we approximated f^{nn} by the most important LOCV correlation function in the $^1S_0$ channel and f^{np}s with those in the $^3S_1$–$^3D_1$ channel. Recall that, in the numerical evaluation of the electron neutrinos' absorption rate, the authors of Ref. [31] used symmetric nuclear matter (SNM) correlation functions calculated using the FHNC many-body method [5] computed with Reid soft-core 2BF [45] at saturation density. The dependence of correlation functions on density was neglected in Ref. [31]. In Fig. 3 we compared $\mathcal{R}$ of Ref. [31] and the one calculated using our f's of SNM at n_0, with the AV18 2BF instead of the Reid soft-core one of [31]. We have also presented there a curve obtained using density-dependent correlation functions in SNM. Notice that using density-dependent correlation functions, apart from a dramatic reduction in the value of $\mathcal{R}$, has qualitatively changed the shape of $\mathcal{R}(n)$. When one uses the correlation functions calculated at n_0, the $\mathcal{R}$ factor grows monotonically with increasing n. In the case when one uses f's calculated consistently at each n, $\mathcal{R}(n)$ for SNM first grows, reaching a broad maximum at $n \sim 2n_0$, and then slowly decreases with increasing n.

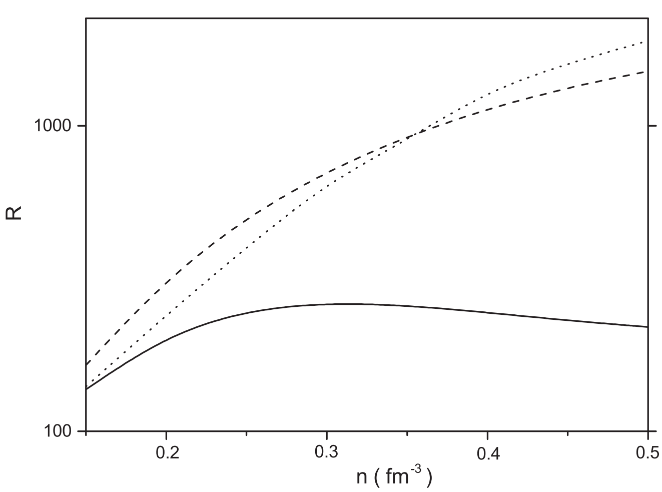

FIG. 3. Correlation factors $\mathcal{R}$ obtained using symmetric nuclear matter (SNM) input: density-dependent LOCV correlation functions (solid line), LOCV correlation functions at saturation density (dashed line), and FHNC correlation functions at saturation density (dotted line).

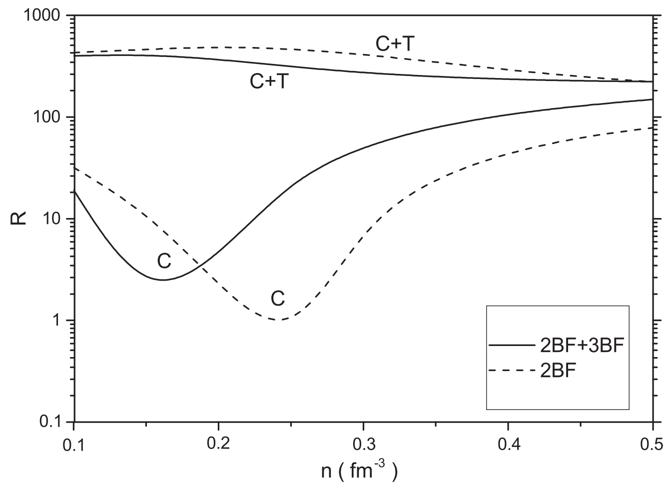

FIG. 4. Different components of the correlation $\mathcal{R}$ factor calculated with LOCV density-dependent correlation functions in beta-stable matter with and without three-body force. Solid lines are the results obtained including three-body force.

In Fig. 4, the central and central-plus-tensor part of the $\mathcal{R}$ factor with and without 3BF were compared. We assumed beta-stable matter (BSM) and we used density dependent correlation functions. One sees that including 3BF decreased the value of the central part of $\mathcal{R}$ at $n < 0.2\ \mathrm{fm}^{-3}$ and increased it for $n > 0.2\ \mathrm{fm}^{-3}$. The tensor contribution to $\mathcal{R}$ is much larger than the central one. The total $\mathcal{R}$ decreases when 3BF is switched on. However, at highest densities the central component of $\mathcal{R}$ gets closer to the total $\mathcal{R}$ both for 2BF and 2BF+3BF calculations. This means that the effect of the tensor character weakens as density increases. This is due to the cancellations in the integrand involving $j_2(k_{\mathrm{F}n} r)$ in the expression for F_t. These cancellations become more and more effective with an increasing $k_{\mathrm{F}n}$.

In Fig. 5 we compared the $\mathcal{R}$ factors, calculated using LOCV density-dependent correlation functions, for three cases of composition of nuclear matter: pure neutron matter (PNM), symmetric nuclear matter (SNM), and beta-stable matter (BSM) with and without 3BF. It is seen that, in all three cases, adding 3BF has led to a decrease in the value of $\mathcal{R}$. For SNM the relative effect of adding 3BF is the smallest. On the other hand, significant difference is seen between the SNM's and BSM's $\mathcal{R}$ factors. This implies that applying the correct approximation for the composition of nucleon sector of stellar matter as well as including 3BF are both important for correct treatment of strong interaction effects in neutrino emissivity calculations.

C. Impact of different factors on Murca emissivity

In Fig. 6 we plotted electron neutrino emissivity at $T_0 = 3 \times 10^8$ K vs n for BSM using density-dependent correlation functions, calculated within the LOCV method in the 2BF and 2BF+3BF cases. In Fig. 7 the corresponding curves including

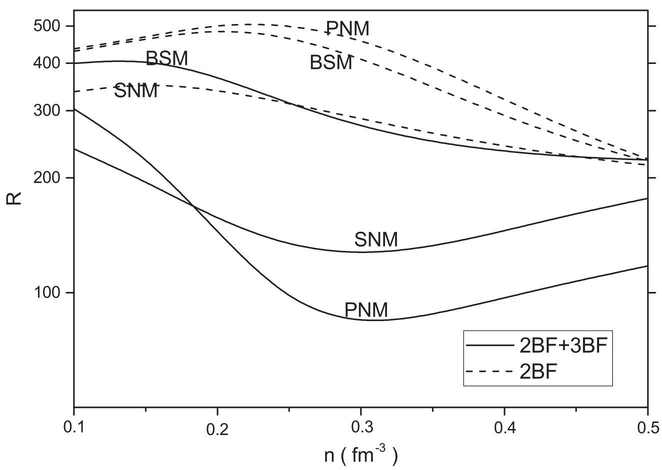

FIG. 5. Comparison of correlation factors calculated with LOCV density-dependent correlation functions in three different scenarios: symmetric nuclear matter (SNM), beta-stable $npe\mu$ matter (BSM), and pure neutron matter (PNM) with and without three-body forces. Solid lines refer to results including three-body forces.

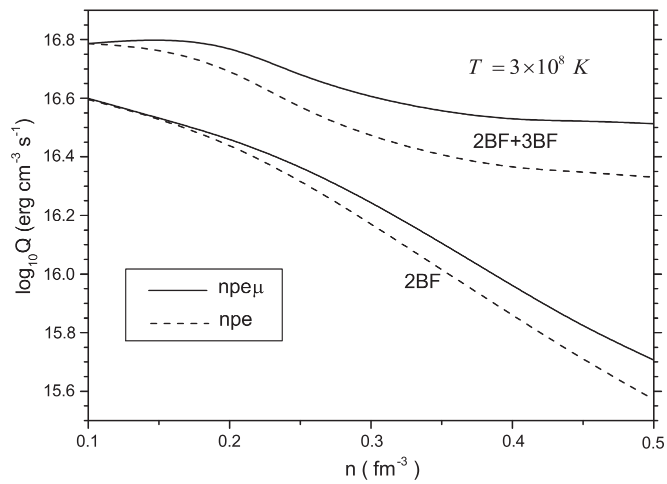

FIG. 7. Electron plus muon neutrino emissivity of beta-stable matter (solid line) and electron neutrino emissivity of muonless beta-stable matter (dotted line) with and without three-body force.

electron and muon neutrinos were shown. The $Q^{\mathrm{Mn}(n)}$ curves at any T can be obtained using the scaling relation

$$Q^{\mathrm{Mn}}(T) = Q^{\mathrm{Mn}}(T_0)\left(\frac{T}{T_0}\right)^8. \tag{27}$$

In all cases emissivity is a monotonically decreasing function of density. Inclusion of 3BF while decreasing the $\mathcal{R}$ factor has had an overall effect of shifting the curve of emissivity to higher values.

V. DISCUSSION AND CONCLUSION

We have calculated the emissivity of electron and muon neutrinos in the n-branch Murca process in an $npe\mu$ model of a neutron star. We treated the rate of the Murca using the approach of Ref. [31] to account for the effect of strong interaction and nuclear many-body properties through realistic two-body correlation functions. We employed the correlation functions calculated by a variational microscopic many-body method, the so-called LOCV method, assuming the AV18 two-body potential, and then we studied the effect of including a phenomenological three-body UIX potential.

In Fig. 8 we have presented the outcome of comparison between our results for electron neutrino emissivity and those from other works. Since the nuclear matrix element in other works has been evaluated near the nuclear matter saturation density only, we have also used the LOCV beta-stable matter correlation functions at this specific density.

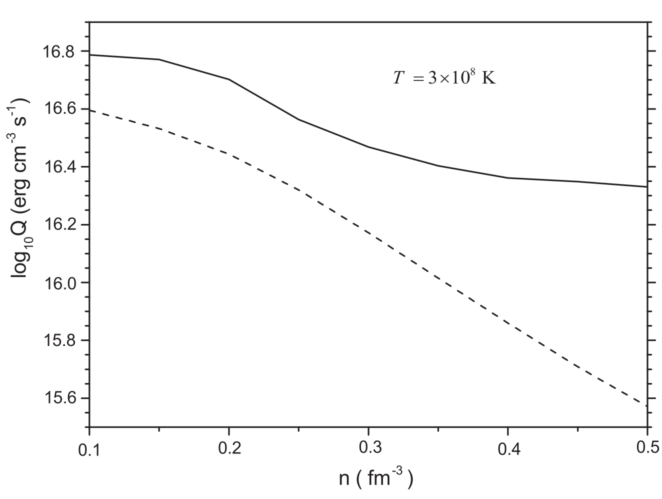

FIG. 6. Electron neutrino emissivity of beta-stable matter at $T = 3 \times 10^8$ K, obtained using two-body force only (2BF, dashed line) and two-body plus three-body forces (2BF+3BF, solid line).

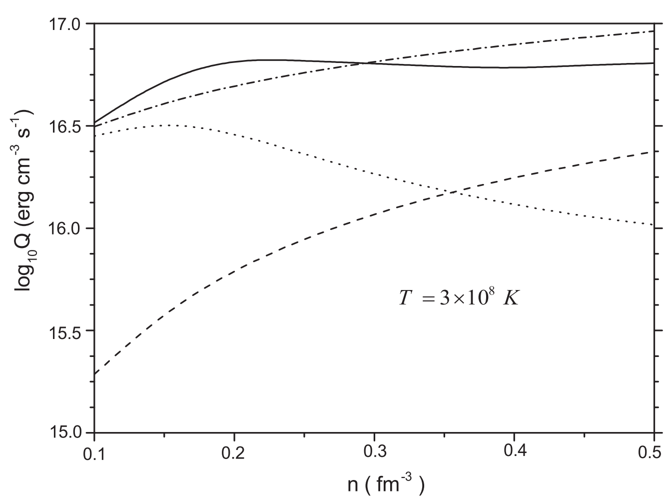

FIG. 8. Comparison of electron neutrino emissivity from the Murca n-branch process calculated at $T = 3 \times 10^8$ K using different approaches. Solid line: LOCV with three-body force included; dotted line: LOCV without three-body force; dash-dotted line: Friman and Maxwell [23] approach; dashed line: Sawyer and Soni [30] hard-core interaction with core radius 0.9 fm.

The calculation of Friman and Maxwell [23] (hereafter FM) is based on the one-pion-exchange description for the long-range tensor part of the strong interaction and a Fermi liquid parametrization [36] for the short-range part of it. Their result is shown with the dash-dotted line. As is seen in Fig. 8 our density-independent results for 2BF are close to those of FM at low density. However, as in our 2BF case the relative effect of tensor correlations decreases with increasing n (see Fig. 4), and our curve diverges from the FM one. However, after adding 3BF, whose effects grows rapidly with increasing density, our emissivity becomes closer to that of FM (somewhat larger below 0.2 fm^{-3}, reaching a broad maximum at $\approx$0.2 fm^{-3}, and somewhat smaller at higher density).

As pointed out in Sec. I, treating the effect of strong interaction in the nuclear matrix element through correlation functions was already introduced in the work of Sawyer and Soni in Ref. [30] (hereafter SS). This was actually the starting idea for the authors of Ref. [31] (hereafter HJ). But SS used a pure hard-core correlation function and so their estimated correlation factor was dramatically sensitive to the assumed value of the hard-core radius. For the sake of comparison, results of SS for an unrealistically large hard-core radius 0.9 fm are shown as the dashed line in Fig. 8.

Figure 7 shows our prediction of the electron and muon neutrinos' emissivity curves from the neutron branch of the Murca process. In contrast to the Q^{Mn} of FM and SS, our values are monotonically decreasing with increasing density. This property is valid in both 2BF and 2BF+3BF cases. Moreover, one sees steeper decrease compared with the LOCV results in Fig. 8 obtained using the n_0 correlation functions.

The calculation reported in Ref. [24] was based on the thermodynamic T-matrix "in-medium" technique. The quasiparticle approximation was used, as in our calculation, which implied T^8 dependence of neutrino emissivity. A nonlocal-separable N-N potential of Mongan [46] was assumed to make the calculation feasible. Medium effects were basically dispersive (a momentum-dependent potential in which a nucleon pair moves), and the exclusion principle effect from the background nucleons constrained the scattering of nucleon pair. Generally, the Q^{Mn} of Blaschke *et al.* [24] while close to FM's and to our results at subnuclear density (small medium effects), become some five times smaller than the FM one at 0.25 fm^{-3}, the highest n considered in Ref. [24].

Calculations of Murca neutrino emissivity reported in Refs. [26,27] and reviewed in Refs. [25] have been performed within the Fermi liquid theory of nuclear matter. They went beyond the quasiparticle approximation, and included additional many-body effects in strongly interacting nucleonic medium (neutrino emission from intermediate particle-hole states, renormalization of the weak charged-current vertices, and collective effects due to possible softening of the pion-like modes). With conservative assumptions about relevant parameters, the authors obtained Q^{Mn} similar to FM at $n = n_0$, and an order of magnitude larger than FM at $n = 2n_0$. Assuming a strong softening of the pion-like mode (a possibility more popular in 1980s than today), an increase by factors 10^3 and 10^4 at n_0 and $2n_0$, respectively, was found. Increase of Q^{Mn} with density is very strong, but dramatic cancellation of various large contributions could change these results. While the calculations in Ref. [26,27] show (potential) importance of various strong interaction effects, neglected in a more standard approach like that in the present paper, a lack of knowledge of crucial constants entering with high power (e.g., 6 or even 8) into formulas for Q^{Mn} in Refs. [26,27] does not allow making controllable numerical predictions and definitive comparisons.

Concluding, employing a realistic description of strong interaction in dense nuclear matter and accounting for the density and composition dependence of nucleon correlations in neutron star matter are essential for getting Murca emissivities from neutron star cores. Still, due to lack of knowledge of strong interactions in nucleon matter at supranuclear density and deficiencies of the many-body theories, the uncertainty in Q^{Mn} is disappointingly large.

In the LOCV calculation of Ref. [44] adding the three-body force increased the maximum allowable mass for neutron stars from $1.68M_\odot$ to $2.33M_\odot$. Our calculations shows that inclusion of 3BF into the LOCV calculation is also crucial for neutrino emissivity. In particular, inclusion of 3BF allows opening of the Durca process at $2n_0$, while with only 2BF Durca is blocked at all densities considered. Qualitatively, a similar situation was found for A18 and A18+UIX interactions by Akmal *et al.* [6] who used variational chain summation (VCS) techniques: Durca blocked at any density for the A18 two-body force, and Durca allowed above $5n_0$ when three-body UIX interaction is included. However, at $n < 3n_0$ considered by us, they get a rather small increase of x_p due to inclusion of the three-body force, compared to our doubling of the proton fraction. As already can be seen from Table III in Ref. [44], there are important differences in the symmetry energy values at saturation density: 37.4 MeV [44] compared with 33.3 MeV [6], which is strongly correlated with the difference in proton fraction, $x_p \propto E_{\mathrm{sym}}^3$ (see, e.g., [47]). We expect that a very rapid growth of x_p with density is related to the large value of symmetry-energy slope parameter L. This would probably affect our reported results regarding the effect of three-body correlations on neutrino emissivity and the Durca threshold.

In this work we adopted the nucleon effective masses obtained using a different many-body theory (Brueckner-Hartree-Fock [39]) because of the lack of the appropriate data in the LOCV approach. We hope to calculate in the future the important effective-mass factor, appearing in the power 4 in the emissivity expression, in the LOCV framework for BSM and consider its effect on our results. Furthermore, since in the absence of the Durca process different neutrino producing processes should compete to determine the thermal evolution of the star, calculation of the emissivity of other processes such as the proton branch Murca process and N-N bremsstrahlung [17,41] is also important. We hope to address these issues in our future work devoted to application of the LOCV method to physics of neutron star cores.

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APPEENDIX

The LOCV method is a fully self-consistent variational method that has achieved many successes in predicting nuclear matter properties during recent decades. In this method we introduce a trial many-body wave function in the form

$$\Psi(12\ldots A)=\hat{\mathcal{F}}(12\ldots A)\Phi(12\ldots A), \tag{A1}$$

where $\hat{\mathcal{F}}$ is an A-body correlation operator and is considered as a symmetrized product of pair correlation operators (the Jastrow form of $\hat{\mathcal{F}}$ [48]), i.e.,

$$\hat{\mathcal{F}}(12\ldots A)=\hat{\mathcal{S}}\prod_{i<j}\hat{f}(ij), \tag{A2}$$

where each $\hat{f}(ij)$ is assumed to be built out of a set of operators $\hat{\mathcal{Q}}_k$ that appear in two-particle interaction. These two-body operators acting in the spaces of the ij nucleon pairs are multiplied by functions $f_k(r)$ where $r=|\boldsymbol{r}_i-\boldsymbol{r}_j|$. Then we construct a cluster expansion of the expectation value of our nuclear hamiltonian, i.e.,

$$\hat{\mathcal{H}}=\sum_i\frac{\hat{p}_i^2}{2m_i}+\sum_{i<j}\hat{V}_{ij}+\sum_{i<j<k}\hat{V}_{ijk}+\cdots. \tag{A3}$$

For the energy per nucleon we obtain

$$E(A)=\frac{1}{A}\frac{\langle\Psi|\hat{\mathcal{H}}|\Psi\rangle}{\langle\Psi|\Psi\rangle}=E_1+E_2+\cdots. \tag{A4}$$

Here, E_1 is the A-body kinetic energy, independent of f_k, and E_2 is the two-body energy contribution,

$$E_2=\frac{1}{2A}\sum_i\langle ij|\hat{\mathcal{W}}(12)|ij\rangle_a, \tag{A5}$$

where $\hat{\mathcal{W}}(12)$ is an effective two-body potential defined as

$$\hat{\mathcal{W}}(12)=[f(12),[\hat{T}_1+\hat{T}_2,\hat{f}(12)]]+\hat{f}(12)\hat{V}\hat{f}(12), \tag{A6}$$

in which $\hat{T}_1+\hat{T}_2$ is the kinetic energy operator of a noninteracting (1,2) pair and $\hat{V}(1,2)$ is the interparticle potential operator. We can write the two-body energy as a functional of correlation functions while E_1 is independent of f_i:

$$E_2[f]=\int\mathfrak{L}(f',f,r)dr, \tag{A7}$$

where $\mathfrak{L}$ depends on correlation functions f_k, their radial derivatives, and the N-N potential parameters as an input. We can now minimize E_2 with respect to the variations of the f_k's in each channel of interaction [33]. We impose a natural normalization constraint in the form of [34]

$$\frac{1}{A}\sum_{ij}\langle ij|f_{\rm P}^2(12)-f_k^2(12)|ij\rangle_a=1. \tag{A8}$$

The function $f_{\rm P}$ is the modified Pauli correlation function which for nn pair has the form

$$f_{\rm P}=\left(1-\frac{9}{2}\left(\frac{j_1(k_{{\rm F}n}r_{12})}{k_{{\rm F}n}r_{12}}\right)^2\right)^{-\frac{1}{2}}, \tag{A9}$$

where $j_1(x)$ is spherical Bessel function of order 1. The function $f_{\rm P}$ for the pp pair is obtained by putting $k_{{\rm F}p}$ instead of $k_{{\rm F}n}$. For the np pair $f_{\rm P}=1$. Constraint (A9) introduces a Lagrange multiplier through which all f's are coupled. So we end up with a set of Euler-Lagrange equations for correlation functions subject to the normalization constraint. It is noticeable that $\chi=\langle\psi_{12}|\psi_{12}\rangle-1$ has the role of a smallness parameter in the cluster expansion, and by choosing an appropriate correlation function we can truncate the expression (28) up to two-body terms and keep the higher terms as small as possible [49]. Finally, the correlation functions are extracted through solving the arising coupled differential equations. More details can be found in Ref. [50] and the references therein.

It is well known that two-body forces alone cannot reproduce the saturation properties of symmetric nuclear matter. Also the three-body forces play an important role at supernormal densities of nuclear systems. So, in order to include the three-body forces in the LOCV method, we have employed an Urbana type three-body interaction in the form [51]

$$V_{123}=V_{123}^{2\pi}+V_{123}^R, \tag{A10}$$

where $V_{123}^{2\pi}$ and V_{123}^R are the two-pion exchange and the shorter-range phenomenological parts respectively [51]. Dealing with the full three-body problem is difficult, and to avoid such difficulties we have introduced a density dependent effective two-body interaction $\bar{V}_{12}(r)$ (DDETI) weighted by the LOCV two-body correlation functions $f(r)$ at each density and averaged over the third particle coordinates [44]:

$$\bar{V}_{12}(r)=n\int d^3r_3\sum_{\sigma_3,\tau_3}f^2(r_{13})f^2(r_{23})V_{123}. \tag{A11}$$

By inserting Eq. (37) in the above equation and doing some algebra, one can achieve the operator structure of DDETI as

$$\bar{V}_{12}(r)=(\tau_1.\tau_2)(\sigma_1.\sigma_2)V_{\sigma\tau}^{2\pi}(r)+S_{12}(\hat{\mathbf{r}})(\tau_1.\tau_2)V_t^{2\pi}(r)+V_c^R(r), \tag{A12}$$

where

$$V_{\sigma\tau}^{2\pi}(r)=\frac{2\pi}{r}n\int_0^\infty|\mathbf{r}_{13}|d|\mathbf{r}_{13}|\int_{|\mathbf{r}_{12}-\mathbf{r}_{13}|}^{|\mathbf{r}_{12}+\mathbf{r}_{13}|}|\mathbf{r}_{32}|d|\mathbf{r}_{32}|f^2(|\mathbf{r}_{13}|)f^2(|\mathbf{r}_{32}|)\sum_{cyc}\sum_{\sigma_3\tau_3}4A\times[Y(m_\pi|\mathbf{r}_{13}|)Y(m_\pi|\mathbf{r}_{32}|)+2P_2(\hat{\mathbf{r}}_{13}.\hat{\mathbf{r}}_{23})T(m_\pi|\mathbf{r}_{13}|)T(m_\pi|\mathbf{r}_{32}|)], \tag{A13a}$$

$$V_t^{2\pi}(r) = \frac{2\pi}{r} n \int_0^\infty |\mathbf{r}_{13}| d|\mathbf{r}_{13}| \int_{|\mathbf{r}_{12}-\mathbf{r}_{13}|}^{|\mathbf{r}_{12}+\mathbf{r}_{13}|} |\mathbf{r}_{32}| d|\mathbf{r}_{32}| f^2(|\mathbf{r}_{13}|) f^2(|\mathbf{r}_{32}|) \sum_{cyc} \sum_{\sigma_3\tau_3} 4A \times [Y(m_\pi|\mathbf{r}_{13}|)T(m_\pi|\mathbf{r}_{32}|)P_2(\hat{\mathbf{r}}_{12}.\hat{\mathbf{r}}_{23})$$
$$+T(m_\pi|\mathbf{r}_{13}|)Y(m_\pi|\mathbf{r}_{32}|)P_2(\hat{\mathbf{r}}_{12}\cdot\hat{\mathbf{r}}_{13}) + T(m_\pi|\mathbf{r}_{13}|)T(m_\pi|\mathbf{r}_{32}|)P], \tag{A13b}$$
$$V_c^R(r) = \frac{2\pi}{r} n \int_0^\infty |\mathbf{r}_{13}| d|\mathbf{r}_{13}| \int_{|\mathbf{r}_{12}-\mathbf{r}_{13}|}^{|\mathbf{r}_{12}+\mathbf{r}_{13}|} |\mathbf{r}_{32}| d|\mathbf{r}_{32}| f^2(|\mathbf{r}_{13}|) f^2(|\mathbf{r}_{32}|) \sum_{cyc} \sum_{\sigma_3\tau_3} U \times [T(m_\pi|\mathbf{r}_{13}|)T(m_\pi|\mathbf{r}_{32}|)]^2. \tag{A13c}$$

In the above equations $P_2(x)$'s are the usual Legendre polynomials and P is equal to $-\frac{3}{2}(\hat{\mathbf{r}}_{13}\cdot\hat{\mathbf{r}}_{23})(\hat{\mathbf{r}}_{13}\cdot\hat{\mathbf{r}}_{23}) + 3(\hat{\mathbf{r}}_{12}\cdot\hat{\mathbf{r}}_{23})(\hat{\mathbf{r}}_{12}\cdot\hat{\mathbf{r}}_{13}) - P_2(\hat{\mathbf{r}}_{12}\cdot\hat{\mathbf{r}}_{13}) - P_2(\hat{\mathbf{r}}_{12}\cdot\hat{\mathbf{r}}_{23})$. The z axis is taken along the vector $\mathbf{r}_{12}$ and $r = |\mathbf{r}_{12}|$. m_π indicates the average pion mass and $Y(m_\pi r)$ and $T(m_\pi r)$ are the Yukawa and tensor functions respectively. The factors A and U in the above equations are determined in such a way that we can reproduce the correct saturation properties of symmetric nuclear matter at zero temperature.

First we extract the two-body correlation functions by using the two-body interactions at each channel and insert them in the above equation to obtain the DDETI. Then by adding this DDETI to the bare two-body interaction we achieve an effective two-body potential. Now we use this effective two-body interaction in the LOCV procedure. As an outcome of this process, a so-called effective two-body correlation function is obtained. Then, again we insert the mentioned effective two-body correlation function in Eq. (38) and repeat this iteration process till convergence is reached. The final effective two-body correlation function and the corresponding effective two-body potential are used to find the equation of state.

In the case of AV18 as the bare two-body interaction, we have found saturation energy -23.37 MeV at $n_0 = 0.32$ fm^{-3} and symmetry energy $E_{\rm sym}(n_0) = 39.13$ MeV and by adding the three-body interaction the saturation energy is reduced to -15.58 MeV at $n_0 = 0.17$ fm^{-3} and symmetry energy $E_{\rm sym}(n_0) = 37.51$ MeV [44]. So it is seen that including 3BF in the LOCV method via above approximation has improved the saturation energy and density; however, the symmetry energy is still out of the standard range. More details can be found in Ref. [44].

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