## **Spin of the proton in chiral effective field theory**

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Proton spin is investigated in chiral effective field theory through an examination of the singlet axial charge,  $a_0$ , and the two nonsinglet axial charges,  $a_3$  and  $a_8$ . Finite-range regularization is considered as it provides an effective model for estimating the role of disconnected sea-quark loop contributions to baryon observables. Baryon octet and decuplet intermediate states are included to enrich the spin and flavor structure of the nucleon, redistributing spin under the constraints of chiral symmetry. In this context, the proton spin puzzle is well understood with the calculation describing all three of the axial charges reasonably well. The strange quark contribution to the proton spin is negative with magnitude 0.01. With appropriate  $Q^2$  evolution, we find the singlet axial charge at the experimental scale to be  $\hat{a}_0 = 0.31^{+0.04}_{-0.05}$ , consistent with the range of current experimental values.

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In 1988 the European Muon Collaboration (EMC) published their polarized deep inelastic measurement of the proton's spin-dependent structure function  $g_1$ . Their result suggested that the quark spins summed over the up, down, and strange quark flavors contribute only a small fraction of the proton's spin [\[1\]](#page-5-0). The EMC data shocked the particle physics community, because it was thought to be contradictory to the apparently successful, naive quark model descriptions of proton structure where the constituent quarks carry the total proton spin. It inspired a vigorous global program of experimental and theoretical developments to understand the internal spin structure of the proton extending for nearly three decades. For reviews of the spin structure of the proton, see, for example, Refs. [\[2–10\]](#page-5-0).

The experimental efforts at CERN [\[1,11–13\]](#page-5-0), DESY [\[14\]](#page-5-0), Thomas Jefferson National Accelerator Facility [\[15\]](#page-5-0), BNL Relativistic Heavy Ion Collider [\[16,17\]](#page-5-0), and Stanford Linear Accelerator Center [\[18\]](#page-5-0) have been impressive. A summary of the status and recent experimental results on the spin structure of the nucleon can be found in Ref. [\[2\]](#page-5-0). Unlike the early EMC result, which suggested that the quark spin contribution,  $\Sigma$ , might be consistent with zero (14%  $\pm$  9%  $\pm$ 21% [\[1\]](#page-5-0)), today the experimental measurements indicate that the nucleon's flavor-singlet axial charge measured in polarized deep inelastic scattering is  $0.35 \pm 0.03$ (stat.)  $\pm 0.05$ (syst.) at  $Q^2 = 3 \text{ GeV}^2$ . This tends to about one-third of the total spin  $0.33 \pm 0.03$ (stat.)  $\pm 0.05$ (syst.) as  $Q^2 \rightarrow \infty$  [\[13,14,19\]](#page-5-0).

The matrix elements of the nonsinglet axial current  $J_{5\mu}^k$  and the singlet axial current  $J_{5\mu}$  are defined as

$$
\langle p, s | \overline{\psi} \gamma^{\mu} \gamma_5 \frac{\lambda^k}{2} \psi | p, s \rangle = M s^{\mu} a_k, \quad k = 1, 2, \dots, 8, \quad (1)
$$

$$
\langle p, s | \overline{\psi} \gamma^{\mu} \gamma_5 \psi | p, s \rangle = 2Ms^{\mu} a_0 = 2Ms^{\mu} \Sigma, \tag{2}
$$

where  $\lambda^k$  are generators of the flavor group and  $\psi =$  $(u,d,s,\ldots)$  is a vector in flavor space. The singlet axial current is not conserved owing to the Adler-Bell-Jackiw anomaly. As a result, the flavor-singlet matrix element can receive an additional contribution from gluon polarization [\[20–23\]](#page-5-0). This led to the early idea that the measured singlet component  $a_0$ receives an important contribution from the gluon polarization  $\Delta G$ ; i.e.,

$$
a_0 = \Sigma - N_f \frac{\alpha_s}{2\pi} \Delta G.
$$
 (3)

The polarized gluon distribution function  $\Delta G$  was estimated to be less than  $0.3$  at a scale of 1 GeV<sup>2</sup> in the MIT bag model [\[24\]](#page-5-0). From the extensive experimental studies one finds that the absolute value is of the order  $|\Delta G| \simeq 0.2-0.3$  for  $Q^2 =$ 3 GeV<sup>2</sup> [\[25,26\]](#page-5-0). This amount of gluon polarization, by itself, is far too small to resolve the problem of the small value of  $\Sigma$ through the axial anomaly.

Another explanation for the small value of  $\Sigma$  draws on the strange quark contribution to the proton spin. The nonsinglet axial charge  $a_8$  extracted from hyperon  $\beta$  decays under the assumption of SU(3) flavor symmetry is  $a_8 = \Delta u + \Delta d$  –  $2\Delta s = 0.58 \pm 0.03$  [\[27\]](#page-5-0). If the strange quark contribution to the proton spin were around  $-0.08$ , the proton spin,  $\Sigma$ , expressed as  $a_8 + 3\Delta s \simeq -0.34$ , would be close to the experimental data. However, the uncertainty of  $a_8$  could be as large as 20% [\[28,29\]](#page-5-0). A recent reevaluation of the nucleon's axial charges in the cloudy-bag model, taking into account the effect of the one-gluon-exchange hyperfine interaction and the meson cloud, led to the value  $a_8 = 0.46 \pm 0.05$  [\[30\]](#page-5-0). In this case,  $\Delta s$  was found to be of order 0.01 in magnitude (and negative), with the small value of  $a_8$  a consequence of SU(3) breaking.

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<span id="page-1-0"></span>Soon after the release of the EMC data it was realized that the effect of the pion cloud of the nucleon, associated with chiral symmetry breaking, would be to lower the quark spin content of the nucleon [\[31\]](#page-5-0). This is because pion emission tends to flip the nucleon spin and hence the spins of the quarks in it, while the quarks in the pion necessarily carry orbital angular momentum but no spin. This effect was calculated in the cloudy bag model and the effect of the pion cloud together with the relativistic motion of the light quarks in the bag [\[32\]](#page-5-0) reduced  $\Sigma$  to around 0.5. An alternative approach to the problem recognized that, given the standard spin-dependent one-gluon-exchange correction to the energy of the nucleon, there must be a corresponding exchange current correction to the proton spin [\[33\]](#page-5-0). This too reduces the proton spin by around 0.15 below the naive bag model result of 0.65. It is only recently that studies of the  $\Delta$  nucleon mass splitting in lattice QCD [\[34\]](#page-5-0) provided the justification for combining the pion cloud and one-gluon exchange effects [\[3\]](#page-5-0). This led to a theoretical result in the range 0.35 to 0.40, which is compatible with the aforementioned experimental value and, after QCD evolution, with the results of lattice QCD for the angular momentum carried by the quarks in the proton [\[35\]](#page-5-0).

The scale dependence of  $a_0$  presents another consideration in understanding the fraction of the proton spin carried by quarks [\[36\]](#page-5-0). Consideration of the general features of QCD evolution long ago led to the conclusion that the natural scale at which to match a quark model to QCD is quite low, so that most of the momentum of the proton is carried by valence quarks and one can think of the gluons as having been integrated out of the theory. In Jaffe's scenario, the small value of the experimental proton spin is attributable to differences in the energy scale of the experimental result and the quark model results. Because the anomalous dimension of the singlet axial current is nontrivial, its matrix element  $a_0$  is scale dependent. With the  $Q^2$  evolution, it is possible that the large proton spin at low  $Q^2$  will be reduced through  $Q^2$  evolution to the large  $Q^2$  of the experimental result. As mentioned in Ref. [\[36\]](#page-5-0), it is difficult to get a reliable evolution at low  $Q^2$  because perturbative QCD is not applicable. More specifically, one cannot determine which evolution line presented in Ref. [\[36\]](#page-5-0) is correct. One needs a direct calculation of  $\Sigma$  at the low energy scale.

In this paper, we investigate the proton spin carried by the quarks in the framework of effective field theory, assuming that at the corresponding low scale the gluons have been integrated out, with the only residue being a spin-dependent effective interaction between quarks. The idea of applying chiral symmetry constraints to the proton spin problem is crucial for this paper. This idea appeared for the first time in the paper by Brodsky, Ellis, and Karliner [\[37\]](#page-5-0).  $\Sigma$  and the nonsinglet axial charges  $a_3$  and  $a_8$  are calculated simultaneously in the chiral effective field theory. In this approach the proton structure is enhanced through the dressing of the proton by octet-meson and both octet and decuplet baryon intermediate states. These processes enrich the spin and flavor structure of the nucleon, redistributing spin under the constraints of chiral symmetry. As we will see, this formalism is able to describe all three axial charges in a reasonable manner.

We consider heavy baryon chiral perturbation theory and include octet and decuplet intermediate-state baryons. The



FIG. 1. The one-loop Feynman diagrams for calculating the quark contribution to the proton spin. The thin and thick solid lines are for the octet and decuplet baryons, respectively.

lowest-order chiral Lagrangian used in the calculation of the nucleon spin distribution function is expressed as

$$
L_v = i Tr \overline{B}_v (v \cdot \mathcal{D}) B_v + 2DTr \overline{B}_v S_v^{\mu} \{A_{\mu}, B_v\} + 2FTr \overline{B}_v S_v^{\mu} [A_{\mu}, B_v] - i \overline{T}_v^{\mu} (v \cdot \mathcal{D}) T_{v\mu} + \mathcal{C}(\overline{T}_v^{\mu} A_{\mu} B_v + \overline{B}_v A_{\mu} T_v^{\mu}),
$$
(4)

where  $S_v^{\mu}$  is the covariant spin operator defined as

$$
S_v^{\mu} = \frac{i}{2} \gamma^5 \sigma^{\mu \nu} v_{\nu}.
$$
 (5)

Here  $v^{\nu}$  is the nucleon four velocity. In the rest frame, we have  $v^{\nu} = (1,0,0,0)$ . D, F, and C are the standard SU(3)-flavor coupling constants.

According to the Lagrangian, the one-loop Feynman diagrams, which contribute to the quark spin fraction of the proton, are plotted in Fig. 1. Working with the chiral coefficients of full QCD [\[38,39\]](#page-5-0), the contribution of the doubly represented  $u$ -quark sector of the proton to the proton spin, described by diagram (a) of Fig. 1, is expressed as

$$
\Delta u^a = \left[ C_{N\pi} I_{2\pi}^{NN} + C_{\Sigma K} I_{2K}^{N\Sigma} + C_{\Lambda \Sigma K} I_{5K}^{N\Omega} \right. \left. + C_{N\eta} I_{2\eta}^{NN} \right] s_u,
$$
\n(6)

where the first through fourth terms in the bracket are the contributions from the  $\pi N$ ,  $K \Sigma$ , the  $K(\Lambda - \Sigma)$  transition, and the  $\eta N$  intermediate states, respectively. The u-quark contribution with a  $\Lambda$  intermediate state vanishes. The coefficients,  $C$ , of the integrals,  $I$ , are expressed as

$$
C_{N\pi} = -\frac{(D+F)^2}{288\pi^3 f_\pi^2},\tag{7}
$$

$$
C_{\Sigma K} = -\frac{5(D - F)^2}{288\pi^3 f_\pi^2},\tag{8}
$$

$$
C_{\Lambda \Sigma K} = \frac{(D - F)(D + 3F)}{288\pi^3 f_\pi^2},\tag{9}
$$

$$
C_{N\eta} = -\frac{2}{3} \frac{(3F - D)^2}{288\pi^3 f_\pi^2}.
$$
 (10)

These coefficients reflect the SU(3)-flavor symmetry considered in obtaining the meson-baryon couplings (proportional to  $F$  and  $D$ ), the angular-momentum composition of the intermediate meson-baryon intermediate states, and the SU(6) spin-flavor wave function of the intermediate-state baryon <span id="page-2-0"></span>considered in assigning a quark-sector spin contribution. The latter is discussed in further detail below.

With the above coefficients, one can write the  $d$  quarksector contribution to the proton spin of Fig.  $1(a)$  as

$$
\Delta d^{a} = \left[\frac{7}{2}C_{N\pi}I_{2\pi}^{NN} + \frac{1}{5}C_{\Sigma K}I_{2K}^{N\Sigma} - C_{\Lambda \Sigma K}I_{5K}^{N\Lambda \Sigma}\right] - \frac{1}{4}C_{N\eta}I_{2\eta}^{NN}\right]s_{d}.
$$
 (11)

Similarly, the strange quark contribution to the proton spin from diagram (a) of Fig.  $1$  is written as

$$
\Delta s^a = \left[ -\frac{3}{10} C_{\Sigma K} I_{2K}^{N\Sigma} + C_{\Lambda K} I_{2K}^{N\Lambda} \right] s_s, \tag{12}
$$

where

$$
C_{\Lambda K} = -\frac{1}{2} \frac{(D + 3F)^2}{288\pi^3 f_\pi^2}.
$$
 (13)

In the above equations, the low-energy coefficients  $s_q$  ( $q =$  $u,d,s$ ) describe the tree-level quark contribution to the baryon spin. For example, for the intermediate proton and neutron, their spins are expressed as

$$
s_p = \frac{4}{3}s_u - \frac{1}{3}s_d, \quad s_n = \frac{4}{3}s_d - \frac{1}{3}s_u. \tag{14}
$$

In the naive quark model, the value of  $s_q$  is 1. However, it is smaller than 1 owing to the relativistic and confinement effects [\[32\]](#page-5-0).

Diagram (b) of Fig. [1](#page-1-0) illustrates decuplet baryon intermediate states. The u-sector contribution to the proton spin from this diagram is

$$
\Delta u^b = \left[ C_{\Delta \pi} I_{2\pi}^{N\Delta} + C_{\Sigma^* K} I_{2K}^{N\Sigma^*} \right] s_u, \tag{15}
$$

where the coefficients  $C_{\Delta \pi}$  and  $C_{\Sigma^* K}$  are

$$
C_{\Delta \pi} = \frac{35C^2}{648\pi^3, f_\pi^2},\tag{16}
$$

$$
C_{\Sigma^* K} = \frac{5}{28} C_{\Delta \pi}.
$$
 (17)

The d and s quark-sector contributions are

$$
\Delta d^b = \left[\frac{2}{7}C_{\Delta\pi}I_{2\pi}^{N\Delta} + \frac{1}{5}C_{\Sigma^*K}I_{2K}^{N\Sigma^*}\right]s_d\tag{18}
$$

and

$$
\Delta s^b = \frac{3}{5} C_{\Sigma^* K} I_{2K}^{N\Sigma^*} s_s. \tag{19}
$$

In deriving these equations, the tree-level quark contributions to the spin of decuplet baryons are used. For example,

$$
s_{\Delta^+} = 2s_u + s_d, \quad s_{\Sigma^{*-}} = 2s_d + s_s. \tag{20}
$$

These contributions will also be reduced upon taking relativistic and confinement effects into account.

Diagrams (c) and (d) of Fig. [1](#page-1-0) provide contributions from intermediate states involving an octet-decuplet transition. The  $u$  quark-sector contribution to the proton spin from these diagrams is expressed as

$$
\Delta u^{c+d} = \left[C_{N\Delta\pi}I_{3\pi}^{N\Delta} + C_{\Sigma\Sigma^*K}I_{5K}^{N\Sigma\Sigma^*} + C_{\Lambda\Sigma^*K}I_{5K}^{N\Lambda\Sigma^*}\right]s_u,
$$
\n(21)

where

$$
C_{N\Delta\pi} = -\frac{(D+F)\mathcal{C}}{27\pi^3 f_\pi^2},\tag{22}
$$

$$
C_{\Sigma\Sigma^*K} = -\frac{5}{8} \frac{(D-F)\mathcal{C}}{27\pi^3 f_\pi^2},\tag{23}
$$

$$
C_{\Lambda\Sigma^*K} = -\frac{1}{8} \frac{(D+3F)\mathcal{C}}{27\pi^3 f_\pi^2}.
$$
 (24)

The d and s quark-sector contributions are

 $\Delta d^c$ 

$$
^{+d} = \left[ -C_{N\Delta\pi} I_{3\pi}^{N\Delta} + \frac{1}{5} C_{\Sigma\Sigma^* K} I_{5K}^{N\Sigma\Sigma^*} \right. \\
\left. -C_{\Lambda\Sigma^* K} I_{5K}^{N\Lambda\Sigma^*} \right] s_d,\tag{25}
$$

$$
\Delta s^{c+d} = -\frac{6}{5} C_{\Sigma \Sigma^* K} I_{5K}^{N \Sigma \Sigma^*} s_s.
$$
 (26)

The integrals in the above equations,  $I_{2j}^{\alpha\beta}$ ,  $I_{5j}^{\alpha\beta\gamma}$ , and  $I_{3j}^{\alpha\beta}$  are defined in Ref. [\[39\]](#page-5-0).

Including the tree-level contribution, the total  $u$ -,  $d$ -, and s-quark sector contributions to the spin of the proton are

$$
\Delta u = \frac{4}{3}Zs_u + \Delta u^a + \Delta u^b + \Delta u^{c+d},
$$
  
\n
$$
\Delta d = -\frac{1}{3}Zs_d + \Delta d^a + \Delta d^b + \Delta d^{c+d},
$$
  
\n
$$
\Delta s = \Delta s^a + \Delta s^b + \Delta s^{c+d}.
$$
\n(27)

Here Z is the wave-function renormalization constant calculated from the standard diagrams corresponding to those of Fig. [1.](#page-1-0) Values are listed in Table I.

In the numerical calculations, the SU(3)-flavor couplings are  $D = 0.8, F = 0.46$ . The decuplet coupling  $C = -1.2$  [\[40\]](#page-5-0). The regulator in the integrals is chosen to be of a dipole form,

$$
u(k) = \frac{1}{(1 + k^2/\Lambda^2)^2},
$$
\n(28)

with  $\Lambda = 0.8 \pm 0.2$  GeV. This prescription is known to model the contributions of disconnected sea-quark loop contributions well [\[41–43\]](#page-5-0).

The final quark spin contributions are related to the lowenergy coefficients  $s_u$ ,  $s_d$ , and  $s_s$ . These are the tree-level values of the quark spin and are unity in the naive constituent-quark model. Relativistic and confinement effects associated with

TABLE I. The predictions of the meson-cloud model presented herein for proton spin structure as a function of the regulator parameter,  $\Lambda = 0.8 \pm 0.2$ , governing the size of the meson-cloud dressings of the proton.

$\Lambda$ (GeV)	∼	$S_a$	$\Delta u$	$\Delta d$	$\Delta s$	$g_A$	$a_8$	2	$\hat{a}_0$ (3 GeV <sup>2</sup> )
0.6	0.84	0.83	0.93	$-0.35$	$-0.003$	1.27	0.59	0.58	0.35
0.8	0.71	0.82	0.90	$-0.38$	$-0.007$	1.27	0.53	0.51	0.31
1.0	0.58	0.76	0.86	$-0.41$	$-0.014$	l 77 .	0.47	0.43	0.26

light quarks suppress this value. We begin by assuming  $s_u$  =  $s_d = s_s = s_q$  and treat  $s_q$  as a parameter constrained by the axial charge  $a_3 = 1.27$ . With  $\Lambda = 0.8$  GeV, the central value of  $s_q$  that we find is  $s_q = 0.79$ , less than 1, as expected. Because the strange quark is expected to be less relativistic, this value may be an overestimate of the spin suppression in that case. However, the strange quark contribution to the proton spin is small and so the approximation is adequate for this purpose.

With  $s_q = 0.79$ , the u, d, and s quark contributions to the proton spin are

$$
\Delta u = 0.94
$$
,  $\Delta d = -0.33$ ,  $\Delta s = -0.01$ . (29)

The axial charge  $a_8 = 0.63$  and  $\Sigma = 0.61$ . Before considering the necessary  $Q^2$  evolution to the value  $Q^2 = 3 \text{ GeV}^2$  relevant to the experimental data, it is interesting to consider other improvements to our use of SU(6)-spin-flavor wave functions in attributing quark spin to intermediate meson-baryon states.

Although it lies outside the framework of chiral effective field theory, the effect of one-gluon-exchange (OGE) is particularly important for spin-dependent quantities. Hogaason and Myhrer [\[44\]](#page-5-0) showed that the incorporation of the exchange current correction arising from the effective OGE force shifts the tree-level nonsinglet charge,  $a_3$ , from  $\frac{5}{3}s_q$  to  $\frac{5}{3}s_q - G$ , where  $G$  is about 0.05. Thus, if one were to include the OGE correction,  $s_q$  would be somewhat larger at 0.82 if one chose it to reproduce the axial charge  $a_3 = 1.27$ . For the charges,  $a_0$  and  $a_8$ , the OGE correction shifts their tree-level values from  $s_q$  to  $s_q - 3G$  [\[3\]](#page-5-0). In this case,  $a_0 = 0.51$  and  $a_8 = 0.53$ . Correspondingly, the quark contributions to the proton spin are

$$
\Delta u = 0.90
$$
,  $\Delta d = -0.38$ ,  $\Delta s = -0.01$ . (30)

The results show that the strange quark contribution to the proton spin is very small relative to the u and d contributions. The axial charge,  $a_8 = 0.53$ , is intermediate between the value extracted under the assumption of SU(3) symmetry from hyperon  $\beta$  decay,  $0.58 \pm 0.03$  [\[27\]](#page-5-0), and that obtained in the cloudy-bag model,  $0.46 \pm 0.05$  [\[30\]](#page-5-0).

To provide an estimate of the uncertainty in these results, we vary the regulator parameter,  $\Lambda$ , governing the size of meson cloud contributions to proton structure. Considering  $\Lambda =$  $0.8 \pm 0.2$  GeV, the uncertainties in the quark contributions to proton spin are

$$
\Delta u = +0.90^{+0.03}_{-0.04},\tag{31}
$$

$$
\Delta d = -0.38^{+0.03}_{-0.03},\tag{32}
$$

$$
\Delta s = -0.007^{+0.004}_{-0.007}.\tag{33}
$$

The axial charges with the corresponding error bars are

$$
a_0 = \Sigma = 0.51^{+0.07}_{-0.08}
$$
 and  $a_8 = 0.53^{+0.06}_{-0.06}$ . (34)

The nonsinglet axial current is conserved in the limit of massless quarks and the anomalous dimension for the nonsinglet axial current vanishes. Therefore, the nonsinglet matrix elements  $a_3$  and  $a_8$  are scale independent. However, the anomalous dimension of the singlet axial current is nontrivial, and  $a_0$  is a scale-dependent quantity. Consistent with the idea that at a sufficiently low scale the valence quarks

dominate and the gluons have been effectively integrated out of the theory, we set  $a_0 = \Sigma$  at that scale. Then, to compare the result for  $a_0$  calculated within chiral effective field theory to experiment,  $\hat{a}_0(Q^2)$  is obtained through next-to-next-to-leading order (NNLO) QCD evolution to  $Q^2 = 3 \text{ GeV}^2$ .

The  $Q^2$  evolution equation has the form [\[45\]](#page-5-0)

$$
\frac{d}{dt}\hat{a}_0(t) = -N_f \frac{\alpha_s}{2\pi} \gamma_{gq} \hat{a}_0(t),\tag{35}
$$

where  $t = \ln Q^2/\mu^2$ . After integrating in  $\alpha_s$  from a normalization scale of  $\mu^2$  to  $Q^2$ , one obtains [\[45\]](#page-5-0)

$$
\ln \frac{\hat{a}_0(Q^2)}{\hat{a}_0(\mu^2)} = \frac{6N_f}{33 - 2N_f} \frac{\alpha_s(Q^2) - \alpha_s(\mu^2)}{\pi} \times \left\{ 1 + \left[ \frac{83}{24} + \frac{N_f}{36} - \frac{33 - 2N_f}{8(153 - 19N_f)} \right] \times \frac{\alpha_s(Q^2) + \alpha_s(\mu^2)}{\pi} \right\},
$$
(36)

with the NNLO calculation of the anomalous dimension,  $\gamma_{gq}$ , taken from Ref. [\[46\]](#page-5-0).

In Fig. 2 we illustrate the  $Q^2$  evolution of  $\hat{a}_0(Q^2)$  commencing with our result of Eq. (34) attributed to the scale  $\mu^2 =$ 0.5 GeV<sup>2</sup> as in Ref. [\[36\]](#page-5-0). Initially,  $\hat{a}_0(Q^2)$  decreases rapidly with increasing  $Q^2$ , raising concerns about the application of an NNLO calculation for  $Q^2 < 1$  GeV<sup>2</sup>. However, in the context of the model uncertainty presented in Fig. 2 the present  $Q^2$  evolution will suffice.

At  $Q^2 = 3 \text{ GeV}^2$ , our calculation of the proton spin can be compared with experiment. Our model provides

$$
\hat{a}_0(3 \text{ GeV}^2) = 0.31_{-0.05}^{+0.04},\tag{37}
$$

which agrees with the experimental measurement of  $0.35 \pm$ 0.03(stat.)  $\pm$  0.05(syst.) at  $Q^2 = 3 \text{ GeV}^2$ .

Finally, we discuss how the results depend on the choice of the regulator and the coupling constants. In the above



FIG. 2.  $Q^2$  evolution of the singlet axial charge of Eq. (34),  $a_0 =$  $\hat{a}_0$  ( $\mu = 0.5 \text{ GeV}^2$ ) for the proton. The upper, middle, and lower lines are for the values  $\Lambda = 0.6, 0.8,$  and 1 GeV, respectively, and provide insight into the role of the meson cloud and the sensitivity of our result at  $\Lambda = 0.8$  to variations in the size of the meson-cloud dressing of the proton.

Regulator	$\Lambda$ (GeV)		$S_{\alpha}$	$\Delta u$	$\Delta d$	$\Delta s$	$g_A$	$a_8$	∠	$\hat{a}_0$ (3 GeV <sup>2</sup> )
Dipole	$0.8\,$	0.71	0.82	0.90	$-0.38$	$-0.007$	1.27	0.53	0.51	0.31
Monopole	0.496	0.65	0.79	0.88	$-0.39$	$-0.012$	1.27	0.52	0.49	0.30
Gaussian	0.616	0.75	0.828	0.90	$-0.37$	$-0.005$	1.27	0.55	0.53	0.32
Sharp cutoff	0.418	0.79	0.837	0.91	$-0.36$	$-0.002$	1.27	0.56	0.55	0.34

TABLE II. The predictions of proton spin structure for different regulators with the corresponding  $\Lambda$ 's same as Ref. [\[47\]](#page-5-0).

calculations, the regulator was chosen to be a dipole form. The other types of functions with the corresponding  $\Lambda$ 's were discussed in Ref. [\[47\]](#page-5-0). It was shown that the physical results were close to each other for different regulators with proper  $\Lambda$ 's. Here we also apply the same monopole and Gaussian functions, as well as the sharp cutoff and the numerical results are listed in Table II. From the table, one can see that the values for each regulator only differ a little. In our error bar estimation,  $\Lambda = 0.8 \pm 0.2$  GeV for dipole form is very generous. If  $\Lambda$  is changed by  $\pm 10\%$  for all the above regulators, the values are still in the range of our error bar.

We also do the numerical calculation with the coupling constants  $D = 0.76$ ,  $F = 0.5$ , and  $C = -1.5$ . The obtained values are listed in Table III and show no big difference for different parameter sets. In addition, because the contribution to the proton spin from strange quark is very small, which is the order of 0.01, the changes of the coupling constants in the strange quark sector owing to the SU(3) symmetry breaking have negligible effect on the proton spin.

In summary, we have examined the proton spin fractions carried by quarks using a model in which the meson-cloud dressings of the proton are characterized by chiral effective field theory, regularized through a regulator characterizing the nontrivial size of the source of the meson cloud. Finite-range regularization provides an effective model for estimating the role of disconnected sea-quark loop contributions to baryon observables [\[41–43,48–50\]](#page-5-0). Both baryon octet and decuplet intermediate states are included to enrich the spin and flavor structure of the nucleon, redistributing spin under the constraints of chiral symmetry. Drawing on extensive experience [\[39,41–43,47,48,51–55\]](#page-5-0), the preferred regulator parameter is  $\Lambda = 0.8$  GeV. To gain insight into the role of the meson cloud and uncertainties associated in determining the size of the meson-cloud contributions, we have varied  $\Lambda$ from 0.6 to 1 GeV.

The coefficient  $s_q$ , which takes relativistic and confinement effects into account is constrained by the experimental axial charge  $a_3 = g_A = 1.27$ . The one-gluon-exchange correction to the axial charges is also taken into consideration. Because each quark-sector contribution is calculated separately, the nonsinglet charges,  $a_3$  and  $a_8$ , and the singlet charge  $a_0$  are obtained simultaneously. The results are summarized in Table [I.](#page-2-0)

Our model provides significant insight into the the proton spin puzzle. The main conclusions are as follows:

- (1) At low energy scales the total quark spin contribution to the proton spin,  $\Sigma = 0.51^{+0.07}_{-0.08}$ , is only of order one-half in the valence region.
- (2) As indicated in Table [I,](#page-2-0) all three of the quark spin contributions  $\Delta u$ ,  $\Delta d$ , and  $\Delta s$  decrease in value as one increases the size of the meson-cloud contribution by increasing  $\Lambda$ . As a result the net spin carried by the quarks,  $\Sigma$ , diminishes with increasing meson-cloud contributions. This is in accord with the increased role of orbital angular momentum [\[35\]](#page-5-0) between the odd-parity mesons and the even-parity baryons of the proton's meson cloud considered herein.
- (3) The parameter  $s_a$  reflecting the role of relativistic and confinement effects and constrained by  $a_3$  is around 0.82, smaller than 1, as expected, but larger than the typical "ultrarelativistic" value of 0.65. Again, increasing the size of the meson-cloud contributions diminishes this value. For example, at  $\Lambda = 1$  GeV,  $s_q = 0.76$ .
- (4) The nonsinglet charge  $a_8 = 0.53^{+0.06}_{-0.06}$  lies between the value extracted from the hyperon  $\beta$  decays under the assumption of SU(3) symmetry,  $0.58 \pm 0.03$ , and the value  $0.46 \pm 0.05$  obtained in the cloudy-bag model [\[30\]](#page-5-0). Because the experimental value of  $a_0$  extracted from deep inelastic scattering data depends on this quantity, further work to pin down the extent of SU(3) breaking would be valuable.
- (5) The strange quark contribution to the proton spin is negative and its absolute value is of the order 0.01. Larger  $\Lambda$  values admit stronger hyperon contributions, which act to increase this magnitude.
- (6) The experimental value of the  $a_0$  at 3 GeV<sup>2</sup> is reproduced through a combination of the chiral correction and  $Q^2$  evolution of  $\Sigma$  from a scale of

TABLE III. The predictions of proton spin structure for different sets of coupling constants  $D$ ,  $F$ , and  $C$  with dipole regulator.

D		$F$ c z		$S_a$	$\Delta u$	$\Delta d$	$\Delta s$	$g_A$		$a_8 \qquad \qquad \Sigma$	$\hat{a}_0$ (3 GeV <sup>2</sup> )
0.8	0.46	$-1.2$	0.71	0.82	0.90	$-0.38$	$-0.007$	1.27	0.53	0.51	0.31
0.8	0.46	$-1.5$	0.68	0.78	0.90	$-0.37$	$-0.006$	1.27	0.55	0.53	0.32
0.76	0.5	$-1.2$	0.71	0.82	0.89	$-0.38$	$-0.007$	1.27	0.53	0.51	0.31
0.76	0.5	$-1.5$	0.68	0.78	0.90	$-0.37$	$-0.005$	1.27	0.54	0.53	0.32

<span id="page-5-0"></span>0.5 GeV<sup>2</sup> [36]. We find that  $\hat{a}_0$  (3 GeV<sup>2</sup>) is 0.31<sup>+0.04</sup>,05, which agrees with the experimental measurement of  $0.35 \pm 0.03$ (stat.)  $\pm 0.05$ (syst.).

Future work should explore the role of higher-order terms in the  $Q^2$  evolution of  $a_0$  and explore nonperturbative treatments that can provide further insight into the connection between models of hadron structure and modern experimental results.

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