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Transverse-momentum–flow correlations in relativistic heavy-ion collisions

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The correlation between the transverse momentum and the azimuthal asymmetry of the flow is studied. A correlation coefficient is defined between the average transverse momentum of hadrons emitted in an event and the square of the elliptic or triangular flow coefficient. The hydrodynamic model predicts a positive correlation of the transverse momentum with the elliptic flow, and almost no correlation with the triangular flow in Pb-Pb collisions at LHC energies. In p-Pb collisions the new correlation observable is very sensitive to the mechanism of energy deposition in the first stage of the collision.

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I. INTRODUCTION

Collective expansion of the fireball in relativistic heavy-ion collisions generates an azimuthally asymmetric transverse flow. To a first approximation the collective expansion transforms the azimuthal asymmetry of the fireball into the elliptic or triangular flow in the final spectra [1]. An essential issue in the analysis of the hydrodynamic response is the identification of the relevant parameters of the initial state governing the final response [2,3]. Another important topic recently studied concerns nonlinearities in the hydrodynamics response [4–6].

One way to study nonlinearities in the hydrodynamic response is to measure higher order moments between flow coefficients [5]. Another possibility is to use event shape engineering [7]. This technique has been used in a number of experimental analyses [8,9] and theoretical studies [10]. Experimental results indicate that for a subsample of events with higher elliptic flow the transverse momentum spectra get harder [9].

Transverse-momentum fluctuations from event to event are caused by fluctuations in the initial size of the fireball [11,12]. Correlations between the average transverse flow and the coefficients of azimuthally asymmetric flow could reveal interesting information both on the correlation in the initial state between the size and the eccentricities and on the correlations of the strength of the hydrodynamic response with the flow coefficients. In the following, the correlation coefficient between the average transverse flow and the square of the elliptic or triangular flow coefficient is proposed as a robust observable to study such effects.

II. TRANSVERSE-MOMENTUM–FLOW CORRELATIONS

The covariance of any observable $\mathcal{O}$ with the square of the flow coefficient can be defined as

$$\mathrm{cov}(v_n\{2\}^2,\mathcal{O}) = \left\langle \frac{1}{N_{\rm pairs}} \sum_{i\neq k} e^{in\phi_i} e^{-in\phi_k} (\mathcal{O} - \langle \mathcal{O} \rangle) \right\rangle, \quad (1)$$

where the sum is over pairs of particles not used in the calculation of the observable $\mathcal{O}$. The simplest way to achieve it is to use separate pseudorapidity intervals for the calculation of the flow coefficient and $\mathcal{O}$. The Pearson coefficient for the correlation between $\mathcal{O}$ and the flow coefficient is

$$R(v_n\{2\}^2,\mathcal{O}) = \frac{\mathrm{cov}(v_n\{2\}^2,\mathcal{O})}{\sqrt{\mathrm{Var}(v_n\{2\}^2)\mathrm{Var}(\mathcal{O})}}. \quad (2)$$

By definition the Pearson coefficient is in the range $[-1,1]$.

Specifically, for the average transverse momentum in the event $\mathcal{O} = [p_\perp] = \frac{1}{N}\sum_i p_\perp^i$ one gets

$$\mathrm{cov}(v_n\{2\}^2,[p_\perp]) = \left\langle \frac{1}{N_{\rm pairs} N} \sum_{i\neq k\neq j} e^{in\phi_i} e^{-in\phi_k} (p_j - \langle [p_\perp] \rangle) \right\rangle. \quad (3)$$

In the following, particles in the sums are taken from three different pseudorapidity intervals A, B, C, $\eta_i \in [-2.5,-0.75]$, $\eta_k \in [0.75,2.5]$, and $\eta_j \in [-0.5,0.5]$. I have checked that similar results can be obtained using one large interval, but excluding self-correlations. The main reason to use three separate pseudorapidity intervals is to reduce nonflow effects. The Pearson correlation coefficient is

$$R(v_n\{2\}^2,[p_\perp]) = \frac{\left\langle \frac{1}{N_A N_B} \sum_{i\in A,k\in B} e^{in\phi_i} e^{-in\phi_k} \frac{1}{N_C} \sum_{j\in C} (p_j - \langle [p_\perp] \rangle) \right\rangle}{\sqrt{\mathrm{Var}\left(\frac{1}{N_A N_B} \sum_{i\in A,k\in B} e^{in\phi_i} e^{-in\phi_k}\right) \mathrm{Var}([p_\perp]_C)}}. \quad (4)$$

The Pearson coefficient can be calculated from the experimental data to estimate correlations between an observable and the magnitude of the flow. However, the result depends on multiplicities in the intervals where the quantities are calculated, so changing the rapidity intervals or transverse-momentum cuts introduces a spurious effect due to self-correlations, not related to correlations of the collective quantities.

III. SELF-CORRELATIONS

The Pearson correlation coefficient [Eq. (4)], normalized by the variances of $v_n\{2\}^2$ and $[p_\perp]$, depends strongly on the choice of the kinematic range, as the multiplicities can change. In the presence of collective flow, one is rather interested in extracting the correlation coefficient of the event-by-event

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characteristics of the spectra, the flow coefficient squared, and the average transverse momentum.

The correlation coefficient can be normalized by the standard deviation of the flow coefficient and of the average transverse momentum. For the average transverse momentum it amounts to using the dynamical transverse-momentum fluctuations [13]

$$C_{p_\perp} = \left\langle \frac{1}{N(N-1)} \sum_{i \neq j} (p_i - \langle [p_\perp] \rangle)(p_j - \langle [p_\perp] \rangle) \right\rangle. \quad (5)$$

The variance of the flow coefficient squared can be estimated from

$$\mathrm{Var}(v_n^2)_{\mathrm{dyn}} = \left\langle \frac{1}{N_A(N_A-1)N_B(N_B-1)} \times \sum_{i\neq j \in A} \sum_{k \neq l \in B} e^{in\phi_i + in\phi_j} e^{-in\phi_k - in\phi_l} \right\rangle - \left\langle \frac{1}{N_A N_B} \sum_{i \in A, k \in B} e^{in\phi_i} e^{-in\phi_k} \right\rangle^2 \quad (6)$$

or equivalently

$$\mathrm{Var}\left(v_n^2\right)_{\mathrm{dyn}} = v_n\{2\}^4 - v_n\{4\}^4. \quad (7)$$

The correlation coefficient of the collective parameters in the events is

$$\rho(v_n\{2\}^2,[p_\perp]) = \frac{\mathrm{cov}(v_n\{2\}^2,[p_\perp])}{\sqrt{\mathrm{Var}\left(v_n^2\right)_{\mathrm{dyn}} C_{p_\perp}}}. \quad (8)$$

The correlation coefficient defined above has two desired features. First, the correlation coefficient (8) is a very good estimate of the true correlation of the collective parameters. This can be checked by comparing results using realistic finite multiplicity events against those obtained by integration of the final spectra. Second, the correlation coefficient does depend very weakly on the choice of the kinematic range in pseudorapidity. Note that such a small dependence is possible due to nonflow effects or to the pseudorapidity dependence of the flow [14]. In the following I call $\rho(v_n\{2\}^2,[p_\perp])$ the transverse-momentum–flow correlation coefficient.

Unlike the Pearson coefficient (4) the correlation coefficient (8) is not necessarily limited to the range $[-1,1]$. However, if genuine nonstatistical fluctuations of v_n and $[p_\perp]$ stem from fluctuations of collective parameters of the spectra, the correlation coefficient $\rho(v_n\{2\}^2,[p_\perp])$ measures the correlation between these parameters and should be in the range $[-1,1]$.

IV. RESULTS FROM THE HYDRODYNAMIC MODEL

Viscous hydrodynamic model simulations in (3+1) dimensions were performed for Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV and p-Pb collisions at 5.02 TeV [15]. The initial conditions were generated event by event from the Glauber Monte Carlo model. At the positions of the participant nucleons in the transverse plane x_i, y_i entropy is deposited with a Gaussian profile of width $\sigma = 0.4$ fm. The transverse profile is given by a sum of contributions from all participant nucleons:

$$S(x,y) \propto \sum_i \left[(1-\alpha) + N_i^{\mathrm{coll}} \alpha\right] e^{-\frac{(x-x_i)^2+(y-y_i)^2}{2\sigma^2}}, \quad (9)$$

where the deposited strength has a contribution $1-\alpha$ ($\alpha = 0.15$) times the number of collisions for nucleon i; more details are given in [12].

In each event, after hydrodynamic evolution, statistical emission of hadrons is performed giving events with realistic multiplicities. The Pearson correlation coefficient (4) and the correlation of the transverse momentum and flow (8) are calculated in several centrality classes from central to midperipheral. The centrality classes in the calculation are defined by the number of participant nucleons. The Pearson coefficient is always smaller in magnitude than the transverse-momentum–flow correlation coefficient (Figs. 1 and 2). It is due to contributions from self-correlation in the denominator of Eq. (4). These contributions are important for small multiplicities, and get larger for peripheral events or for a narrow

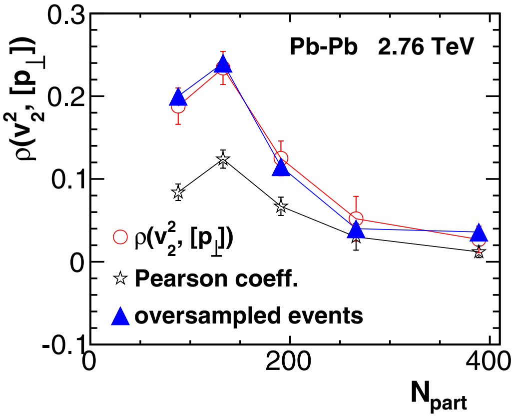

FIG. 1. Correlation coefficient between the elliptic flow coefficient squared $v_2\{2\}^2$ and the average transverse momentum of charged particles in an event for different centralities. The stars denote the Pearson coefficient [Eq. (4)], the circles denote the correlation coefficient without self-correlations [Eq. (8)] and the triangles denote the correlation coefficient calculated from oversampled events.

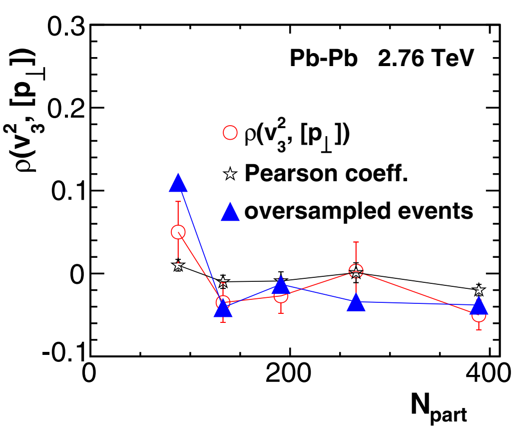

FIG. 2. Same as Fig. 1, but for the triangular flow $v_3\{2\}^2$.

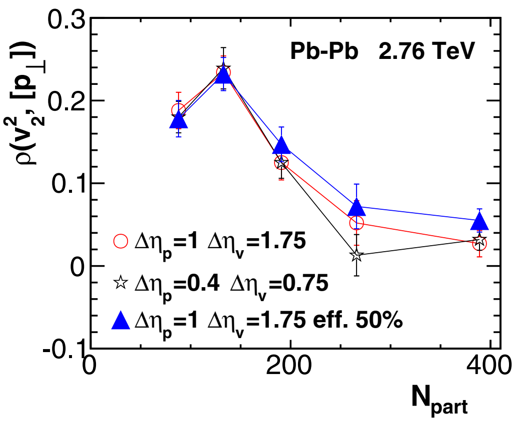

FIG. 3. Correlation coefficient between the elliptic flow coefficient squared $v_2\{2\}^2$ and the average transverse momentum of charged particles in an event for different centralities. The stars denote correlations calculated in the range $|\eta| \in [1.75,-2.5]$ for v_2^2 and $\eta \in [-0.2,0.2]$ for $[p_\perp]$, the circles and triangles denote the correlation coefficient calculated with $|\eta| \in [1.75,-2.5]$ for v_2^2, and $\eta \in [-0.2,0.2]$ with 100% and 50% efficiency, respectively.

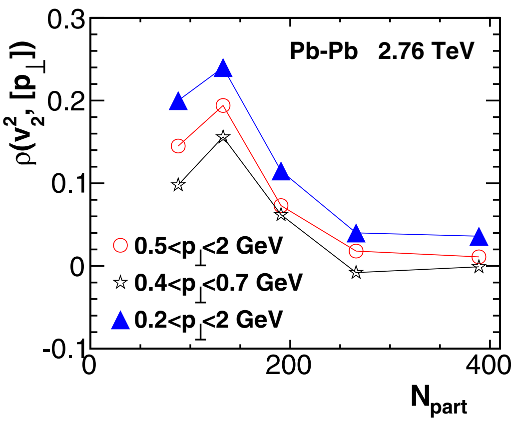

FIG. 4. Correlation coefficient between the elliptic flow coefficient squared $v_2\{2\}^2$ and the average transverse momentum of charged particles in an event for different centralities. The triangles, circles, and stars denote the correlation coefficient for the flow calculated in the ranges $0.2 < p_\perp < 2$ GeV, $0.5 < p_\perp < 2$ GeV, and $0.4 < p_\perp < 0.7$ GeV, respectively. The average transverse momentum of charged particles in the event $[p_\perp]$ is calculated in the range $0.2 < p_\perp < 2$ GeV in all the cases.

pseudorapidity range. The correlation calculated from Eq. (8) is a quantity that is defined to be independent of the multiplicity, except for small nonflow effects. In Figs. 1 and 2 (triangles) are shown the results for the transverse-momentum–flow correlations obtained by integrating the spectra in each event. Technically, these numbers are calculated using oversampled events, where the multiplicity is increased by a factor 100–300 depending on centrality. As can be observed from the results in Figs. 1 and 2, the transverse-momentum–flow correlation coefficient (8) is very close to the result for oversampled events. It means that Eq. (8) can be used in practice to estimate the genuine transverse-momentum–flow correlations, without self-correlations and with only small nonflow contributions. The approximate independence of the pseudorapidity range or efficiency is explicitly shown in Fig. 3. The results for the correlation coefficient $\rho(v_2\{2\}^2,[p_\perp])$ do change when the multiplicity changes, due to finite efficiency or different range in pseudorapidity.

The correlation between the elliptic flow and the transverse momentum is positive (Fig. 1), it is small for central but increases for midcentral events. The correlation coefficient reaches 0.25 for centrality 30–40% which indicates a significant positive correlation. The increase of the mean transverse momentum indicates a stronger transverse flow and a stronger collective response to the initial geometry of the source. The results are qualitatively consistent with the results of the ALICE Collaboration obtained using the event shape engineering technique [9]. A stronger transverse push yields a stronger hydrodynamic response of the spectra to the initial azimuthal deformation. Such an effect is also largely responsible for the observed energy dependence of the integrated elliptic flow [16]. A less important, reverse effect is present in the initial state from the Glauber Monte Carlo model. The initial ellipticity is negatively correlated to the inverse rms radius. Smaller, more compact sources give larger transverse momentum, but a smaller deformation. The strength of that negative correlation depends on the width of Gaussian smearing of the deposited density from each participant nucleon [Eq. (9)]. The increase of the transverse-momentum–flow correlation for midcentral events indicates that it comes from a stronger hydrodynamic response to the large deformation in such events; this is consistent with arguments based on the principal component analysis of the elliptic and transverse flow [6].

The triangular flow shows almost no correlation with the transverse flow (Fig. 2). The negative correlation of the initial triangularity with the inverse of the rms radius is stronger than for the elliptic flow. Also, the triangular deformation is more sensitive to the initial Gaussian smoothing [Eq. (9)] than the elliptic deformation. Unlike for the elliptic flow, the magnitude of hydrodynamic transverse push is not identifiable as a predictor for the triangular flow [3].

The transverse-momentum–flow correlation coefficient [Eq. (8)] is defined to be independent of the range in pseudorapidity. On the other hand, the elliptic and triangular flows depend on transverse momentum. The shift of the integrated elliptic or triangular flow with the change of the average transverse momentum depends on the $p_\perp$ range chosen to calculate v_n^2. In Fig. 4 the correlation coefficients are compared for three different $p_\perp$ ranges used to calculate the integrated flow coefficient, [0.2,2] GeV (typical range where predictions of the hydrodynamic model are justified), [0.5,2] GeV [preferred range in view of the efficiency of the ATLAS and CMS detectors at the CERN Large Hadron Collider (LHC)], [0.4,0.7] GeV (range where the average transverse momentum lies). The transverse-momentum average is calculated for charged hadrons with $0.2 < p_\perp < 2.0$ GeV. The calculated transverse-momentum–flow correlation coefficient depends on the $p_\perp$ integration range.

The collective flow observed in p-Pb collisions can be described fairly well using relativistic hydrodynamics [17].

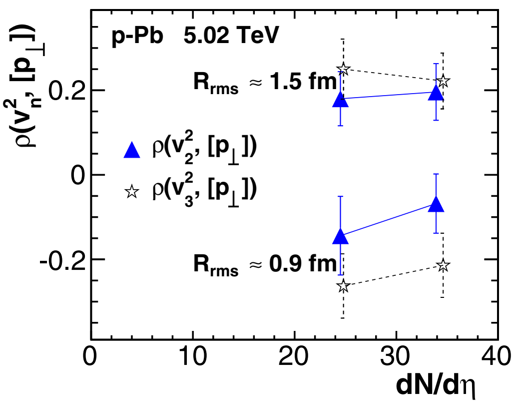

FIG. 5. Correlation coefficient between the elliptic flow coefficient squared (triangles) or the triangular flow coefficient squared (stars) and the average transverse momentum for p-Pb collisions at 5.02 TeV for two centralities, 0–3% and 10–20%. The hydrodynamic evolution is performed for two scenarios of energy deposition in the initial fireball: compact source (lower symbols) and standard Glauber model (upper symbols).

The predicted flow depends strongly on assumptions concerning the initial density fluctuations [18]. Two simple scenarios of the entropy deposition in the transverse plane are studied: the standard Glauber model, with entropy deposited at the positions of the participant nucleons, and the compact source scenario, with entropy deposited in between the two colliding nucleons [19]. For centralities in the range 0–20% the rms radius of the fireball in the first scenario is around 1.5 fm, while in the second case it is much smaller, 0.9 fm.

The correlation between the final average transverse momentum $[p_\perp]$ and the initial eccentricities has a different sign in the two scenarios. For centrality 0–3%, one finds $\rho([p_\perp],\epsilon_2) = -0.04 \pm 0.03$ and $\rho([p_\perp],\epsilon_3) = -0.13 \pm 0.04$ for the compact source model, while $\rho([p_\perp],\epsilon_2) = 0.14 \pm 0.03$ and $\rho([p_\perp],\epsilon_3) = 0.05 \pm 0.03$ for the standard Glauber model. The final transverse-momentum–flow correlation is very different in the two scenarios. For the larger source it is positive and for the compact source it is negative (Fig. 5). The measured value of the transverse-momentum–flow correlation in small systems is very sensitive to the mechanism of the entropy deposition in the initial state of hydrodynamics. It would be also interesting to check if this observable could be used to distinguish between the hydrodynamic expansion and the partonic cascade mechanism [20] of generating flowlike correlations in small systems.

V. CONCLUSIONS

An observable testing the hydrodynamic response of the particle spectra to the initial eccentricity is proposed. It provides a simple quantitative measure of the correlation between the transverse flow and the coefficients of the azimuthal asymmetry of the spectra. A correlation coefficient can be defined between the square of the flow coefficient v_n^2 and the average transverse momentum in the event $[p_\perp]$. Excluding self-correlations in the calculation of the covariance and the variances, one obtains a good estimator of the correlation coefficient of the average transverse momentum and the flow coefficients of the spectra, with no significant nonflow effects. Explicit calculations in the relativistic hydrodynamic model show that the transverse-momentum–flow correlation can be measured for heavy-ion collisions at the LHC. The same is true for collisions at energies currently available at the BNL Relativistic Heavy Ion Collider, but the smaller pseudorapidity acceptance and smaller multiplicity would make the interpretation more difficult due to nonflow effects.

The azimuthal asymmetry in the spectra of particles emitted in relativistic heavy-ion collisions is formed during the collective transverse expansion of the fireball. The strength of the response depends on the gradients of the source density. For smaller sources a stronger transverse flow is generated. On the other hand, fireballs with a smaller initial size tend to have smaller eccentricities, especially for the triangular flow. Hydrodynamic model calculations give a significant positive correlation between the average transverse flow and the elliptic flow, increasing from central to midcentral collisions. This is qualitatively consistent with the experimental results using the event shape engineering [9] and the analysis of the nonlinear response in Ref. [6]. The hydrodynamic model with the Glauber model initial condition using a smoothing scale of 0.4 fm predicts almost no correlations between the triangular flow and the average transverse momentum. It would be interesting to check if the transverse-momentum–flow correlations in peripheral A-A or in p-A collisions could be used as an additional constraint in studies trying to estimate the smoothing scale in the initial entropy deposition in the fireball [21]. The sensitivity of the proposed correlation measure to viscosity coefficients of matter in the fireball is left for further studies. In this context it should be noted that transverse-momentum fluctuations are sensitive to the effective equation of state [22] and could be sensitive to the increase of bulk viscosity near the critical temperature.

In small system collisions the magnitude of the transverse push in the expansion is very sensitive to the duration of the collective dynamics and the size of the initial fireball. Moreover, if the system size fluctuates to be small, the smoothing in the initial entropy deposition yields a stronger reduction of the initial eccentricities. The hydrodynamic model gives very different predictions for the transverse-momentum–flow correlation in two scenarios: the standard Glauber model and the compact source scenario in p-Pb collisions at the LHC. This observable could be used to probe the mechanism of energy deposition at small scales in the the first stage of the collision. Finally, it would be interesting to compare the predictions of the hydrodynamic and the cascade AMPT models [20,23] for the transverse-momentum–flow correlation.

In summary, the paper proposes to study correlations between the flow, or specifically the square of the flow coefficient, and other observables, as an alternative to the event shape engineering technique. Hydrodynamic model calculations for the correlation of the flow and the transverse momentum demonstrate the practical feasibility of the procedure. The transverse-momentum–flow correlation could be used to study fluctuations in the initial stage of the collision.

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