# Probing postsaddle nuclear dissipation with excitation energy at scission

W. Ye\* and J. Tian

Department of Physics, Southeast University, Nanjing 210096, Jiangsu Province, People's Republic of China (Received 1 December 2015; revised manuscript received 24 February 2016; published 4 April 2016)

Using the stochastic Langevin model coupled with a statistical decay model, we study postsaddle dissipation properties in fission by analyzing the excitation energy at scission  $(E_{sc}^*)$  measured in fissioning nuclei <sup>179</sup>Re and <sup>254,256</sup>Fm. The postsaddle dissipation strength ( $\beta$ ) required to fit  $E_{sc}^*$  data is found to be larger for <sup>254,256</sup>Fm than light <sup>179</sup>Re which has a smaller postsaddle deformation compared to heavy <sup>254,256</sup>Fm, showing a rise of nuclear dissipation strength at a greater deformation. Furthermore, we explore the influence of initial excitation energy of a fissioning system <sup>246</sup>Cf on the sensitivity of its  $E_{sc}^*$  to  $\beta$ , and find that the sensitivity is significantly enhanced with increasing the initial excitation energy. Our finding suggests that, on the experimental side, to more accurately probe the postsaddle dissipation strength through the measurement of  $E_{sc}^*$ , it is best to yield those fissioning systems with high energy.

DOI: 10.1103/PhysRevC.93.044603

# I. INTRODUCTION

Nuclear dissipation plays an important role in mechanisms responsible for quasifission [1], the synthesis of superheavy nuclei [2], and the decay of hot nuclei [3]. It hinders fission that leads to a significant discrepancy between measured particle multiplicity and fission probability and prediction of standard statistical models, as clearly shown in a great number of experimental works [4-12]. Numerous theoretical investigations have indicated that stochastic approaches [13-21] to fission are a suitable framework to address the discrepancy. A systematic study [22] has demonstrated that by assuming a weak friction inside saddle and a strong postsaddle friction, the Langevin model can provide a consistent, comprehensive description of different types of fission data, but assuming a full one-body dissipation (OBD) strength gives a too-small fission probability compared to experimental values. However, after reducing the strength for wall formula, the modified OBD strength (a decreasing function of deformation) can better reproduce experimental results [23].

While the two types of deformation-dependent friction give a similar presaddle friction strength, they predict a quite different strength for postsaddle friction. The shape dependence of nuclear dissipation [24] is identified as a key ingredient in Langevin models when the model is applied to treat fission of excited nuclei. Currently more efforts have been spent on the accurate determination of the strength of presaddle friction by analyzing measured data that are sensitive to presaddle dissipation effects only, such as fission probability [22,23,25], evaporation residue cross section [26], and its spin distributions [27]. Consequently, presaddle friction is severely constrained. Therefore, getting the precise knowledge of the magnitude of postsaddle friction becomes very crucial for probing the deformation dependence of friction in nuclear fission. However, to date, less attention has been paid to this issue.

Light particles in fission processes can be affected by postsaddle dissipation [22,23,28], because they are evaporated along the entire fission path. However, prescission particles

consist of both presaddle and saddle-to-scission emissions, and an experimental separation of these two contributions is fraught with difficulty, and, as a result, the difficulty of making a direct comparison between experimental and theoretical postsaddle particles. This is unfavorable for stringently constraining the strength of postsaddle dissipation.

Experimentally, by a multicomponent, moving-source model fit to measured particle energy spectra in coincidence with fission fragments, apart from prescission particles, the number of emitted particles associated with two fission fragments can be extracted as well. Moreover, these postscission particle multiplicities can be utilized to determine the excitation energy at scission ( $E_{sc}^*$ ) quite well [4], a quantity that is closely related to nuclear dissipation. The reason is that  $E_{sc}^*$  is not only a function of the number of prescission neutrons, but also it depends on the energy that these emitted particles take away. The two aspects are connected with the properties of nuclear dissipation. As an independent information source, the  $E_{sc}^*$  thus constitutes an alternative tool of exploiting postsaddle dissipation.

Until now, few have used experimental information of  $E_{\rm sc}^*$  to pin down the properties of postsaddle dissipation. In this context, in the present work the data of  $E_{sc}^*$  of heavy  $^{254,256}$ Fm and light  $^{179}$ Re nuclei will be employed to probe deformation dependence of nuclear dissipation. While the scission configuration is independent of the masses of the fissioning nuclei, postsaddle deformation is a function of the size of the fissioning nuclei, which provides an opportunity for exploring the deformation dependence of friction with  $E_{sc}^*$ . Furthermore, to instruct experimental explorations, we are devoted to the study of the favorable experimental conditions through which postsaddle nuclear dissipation can be better revealed with  $E_{sc}^*$ . Toward that goal, we survey the influence of (initial) excitation energy of fissioning nuclei on  $E_{sc}^*$  as a probe of postsaddle friction in the framework of the Langevin model [29]. This approach was shown to reproduce a volume of experimental data of particle emission, fission probability, etc., for a lot of compound systems over a broad domain of excitation energy, angular momentum, and fissility.

<sup>\*</sup>Corresponding author: yewei@seu.edu.cn

## **II. THEORETICAL MODEL**

An account of the combination of the dynamical Langevin equation with a statistical decay model (CDSM) is given. We refer the reader to Refs. [22,29] for further details. The dynamic part of the CDSM is described by the Langevin equation that is expressed by entropy. We employ the following one-dimensional overdamped Langevin equation [22] to perform the trajectory calculations:

$$\frac{dq}{dt} = \frac{T}{M\beta} \frac{dS}{dq} + \sqrt{\frac{T}{M\beta}} \Gamma(t).$$
(1)

Here *q* is the dimensionless fission coordinate and is defined as half the distance between the center of mass of the future fission fragments divided by the radius of the compound nucleus, *M* is the inertia parameter [30], and  $\beta$  is the dissipation strength. The temperature in Eq. (1) is denoted by *T* and  $\Gamma(t)$  is a fluctuating force with  $\langle \Gamma(t) \rangle = 0$  and  $\langle \Gamma(t)\Gamma(t') \rangle = 2\delta(t - t')$ . The driving force of the Langevin equation is calculated from the entropy:

$$S(q, E^*, A, Z, \ell) = 2\sqrt{a(q, A)[E^* - V(q, A, Z, \ell)]}.$$
 (2)

 $E^*$  is the excitation energy of the system. A and Z are the mass number and charge number of the fissioning nucleus. Equation (2) is constructed from the Fermi-gas expression with a finite-range liquid-drop potential [31] V(q) in the  $\{c,h,\alpha\}$  parametrization [32]. Because only symmetrical fission is considered, the parameter describing the asymmetry of the shape is set to  $\alpha = 0$  [29,33]. The deformation coordinate q is obtained by the relation  $q(c,h) = (3c/8)\{1 + \frac{2}{15}[2h + (c - 1)/2]c^3\}$  [22,34], where c and h correspond to the elongation and neck degrees of the freedom of the nucleus, respectively. The q-dependent surface, Coulomb, and rotation energy terms are included in the potential  $V(q, A, Z, \ell)$ .

In constructing the entropy, the deformation-dependent level density parameter is used:

$$a(q, A) = a_1 A + a_2 A^{2/3} B_s(q),$$
(3)

where  $a_1 = 0.073$  and  $a_2 = 0.095$  are taken from Ignatyuk *et al.* [35].  $B_s$  is the dimensionless surface area (for a sphere  $B_s = 1$ ) which can be parametrized by the analytical expression [36],

$$B_s(q) = \begin{cases} 1 + 2.844(q - 0.375)^2, & \text{if } q < 0.452, \\ 0.983 + 0.439(q - 0.375), & \text{if } q \ge 0.452. \end{cases}$$
(4)

In the CDSM evaporation of prescission light particles along Langevin fission trajectories from their ground state to their scission point was taken into account. The emission width of a particle of kind  $v (= n, p, \alpha)$  is given by [37]

$$\Gamma_{\nu} = (2s_{\nu} + 1) \frac{m_{\nu}}{\pi^2 \hbar^2 \rho_c(E^*)} \\ \times \int_0^{E^* - B_{\nu}} d\varepsilon_{\nu} \rho_R(E^* - B_{\nu} - \varepsilon_{\nu}) \varepsilon_{\nu} \sigma_{\text{inv}}(\varepsilon_{\nu}), \quad (5)$$

where  $s_v$  is the spin of the emitted particle v, and  $m_v$  its reduced mass with respect to the residual nucleus. The level densities of the compound and residual nuclei are denoted by  $\rho_c(E^*)$  and

 $\rho_R(E^* - B_\nu - \varepsilon_\nu)$ .  $B_\nu$  are the liquid-drop binding energies.  $\varepsilon$  is the kinetic energy of the emitted particle and  $\sigma_{inv}(\varepsilon_\nu)$  is the inverse cross sections [37].

Light-particle evaporation is coupled to the fission mode by a Monte Carlo way. The present simulation allows for the discrete emission of light particles. The procedure is in the following. We calculate the decay widths for light particles at each Langevin time step  $\tau$ . Then the emission of particle is allowed by asking along the trajectory at each time step  $\tau$  whether a random number  $\zeta$  is less than the ratio of the Langevin time step  $\tau$  to the decay time  $\tau_{dec} = \hbar/\Gamma_{tot}$ :  $\zeta < \tau/\tau_{dec}$  ( $0 \leq \zeta \leq 1$ ), where  $\Gamma_{tot}$  is the sum of light particles decay widths. If this is the case, a particle is emitted and we ask for the kind of particle  $\nu$  ( $\nu = n, p, \alpha$ ) by a Monte Carlo selection with the weights  $\Gamma_{\nu}/\Gamma_{tot}$ . This procedure simulates the law of radioactive decay for the different particles.

After each emission act of a particle of kind  $\nu$  the energy of the emitted particle, which is the sum of its separation energy and the kinetic energy for the neutron case, as well as the inclusion of an additional term accounting for the Coulomb emission barrier (which is calculated using the formula given in Ref. [30]) for the case of light charged particles such as protons and  $\alpha$  particles, is calculated by a hit and miss Monte Carlo procedure [15,38] that uses the integrand of the formula for the corresponding decay width as weight function. Then the intrinsic energy, the entropy, and the temperature in the Langevin equation are recalculated and the dynamics is continued. The procedure above was shown to nicely reproduce the energy spectra of prescission particles measured by Rossner *et al.* [39].

The CDSM describes the fission process as follows: At early times, the decay of the system is modeled by means of the Langevin equation. After the fission probability flow over the fission barrier attains its quasistationary value, the decay of compound systems is described by the statistical part of the CDSM, which allows for multiple emissions of light particles and higher-chance fission. In case fission is decided there, one switches again to the Langevin equation for computing the evolution from saddle to scission. Prescission particle multiplicities are calculated by counting the number of corresponding evaporated particle events registered in the dynamic and statistical branch of the CDSM. To accumulate sufficient statistics, 10<sup>7</sup> Langevin trajectories are simulated.

Regarding the excitation energy at scission  $(E_{sc}^*)$ , it is determined by using energy conservation law,

$$E^* = E_{\rm sc}^* + E_{\rm coll} + V(q) + E_{\rm evap}(t_{\rm sc}),$$
 (6)

where  $E^*$  and V(q) have the same meaning mentioned earlier.  $E_{coll}$  is the kinetic energy of the collective degrees of freedom [22], and  $E_{evap}(t_{sc})$  is the energy carried away by all evaporated particles by the scission time  $t_{sc}$ .

For starting a Langevin trajectory an orbit angular momentum value is sampled from the fusion spin distribution, which reads

$$\frac{d\sigma(\ell)}{d\ell} = \frac{2\pi}{k^2} \frac{2\ell+1}{1 + \exp[(\ell-\ell_c)/\delta\ell]}.$$
(7)

The parameters  $\ell_c$  and  $\delta \ell$  are the critical angular momentum for fusion and diffuseness, respectively. For different systems,

they are found to follow a scaling [30], which is in accordance with the surface friction model [40]. Namely,

$$\ell_c = \sqrt{A_p A_T / A_{\rm CN}} (A_p^{1/3} + A_T^{1/3}) (0.33 + 0.205 \sqrt{E_{\rm c.m.} - V_c}),$$
(8)

when  $0 < E_{c.m.} - V_c < 120$  MeV; and when  $E_{c.m.} - V_c > 120$  MeV the term in the last bracket is put equal to 2.5. In Eq. (8),  $A_T$ ,  $A_P$ , and  $A_{CN}$  represent the mass of target, projectile, and compound nucleus. The barrier  $V_c = \frac{5}{3}c_3 \frac{A_P A_T}{A_P^{1/3} + A_T^{1/3} + 1.6}$  with  $c_3 = 0.7053$  MeV. The diffuseness  $\delta l$  scales as

$$\delta l = \begin{cases} \left[ (A_P A_T)^{3/2} \times 10^{-5} \right] \left[ 1.5 + 0.02 (E_{c.m.} - V_c - 10) \right] & \text{for } E_{c.m.} > V_c + 10, \\ \left[ (A_P A_T)^{3/2} \times 10^{-5} \right] \left[ 1.5 - 0.04 (E_{c.m.} - V_c - 10) \right] & \text{for } E_{c.m.} < V_c + 10. \end{cases}$$
(9)

These scaling values have been widely tested by successfully fitting fusion cross sections [22], particle emission, fission probability, the kinetic-energy distributions of fission fragments [23], and evaporation residue spin distributions [41], etc., and hence used in our calculations.

Previous studies [22,23] based on the Langevin description of fission of hot nuclei have been demonstrated to be very successful, so that except for the friction parameter, other main model parameters are severely constrained. These parameters are as follows: a scaling description of fusion spin distribution [Eqs. (7)–(9)] supplemented by experimental fusion data if available, the choice of the weak deformation dependence of the level-density parameter [Eq. (3)], the adoption of the procedure of a discrete particle emission and particle emission width formula [Eq. (5)], and the free energy or entropy (evaluated via the Fermi-gas model [Eq. (2)]) rather than bare potential as the driving force of Langevin equations. They have been widely tested and applied in one- and multidimensional Langevin calculation [18,22,23,33,41,42] to assure a consistence with previous results obtained by analyzing measured prescission particle multiplicity, fission probability, and evaporation residue cross section, etc., [22,23]. The difference between theory and experiment is mainly ascribed to friction effect, including its strength and possible dependence on deformation [22,23], temperature [43], and angular momentum [44], in addition to the known dimension factor when involving the comparison of the calculated and measured fragment characteristics.

### **III. RESULTS**

Because the particle evaporation channel can compete with fission channel for many times before the destine of a compound nucleus is determined, i.e., it fissions or survives as an evaporation residue, the friction strength thus deduced by evaporation residue and fission cross sections or other newly suggested observables [9,27,45] stands for its average magnitude before saddle-point deformation. In other words, these observables are determined by the mean presaddle friction strength, and not by the friction strength at a presaddle deformation point. This could be the main reason that these



FIG. 1. Fits to measured excitation energy at scission in (a) <sup>16</sup>O  $(E_{lab} = 288 \text{ MeV}) + {}^{236}\text{U} \longrightarrow {}^{254}\text{Fm}$  and (b) <sup>18</sup>O  $(E_{lab} = 159 \text{ MeV}) + {}^{238}\text{U} \longrightarrow {}^{256}\text{Fm}$  systems. Data [47] are denoted by the shaded band. Solid lines denote model calculations. Note that  $\beta$  represents the postsaddle friction strength.

measured observables can be reproduced equally well by assuming a rising [22], a decreasing [23], a constant [9,25,45] friction with increasing deformation or this kind of friction that modifies the standard OBD friction formula by taking into account chaos factor [42]. One can note that while these works assumed different deformation dependence of friction in their calculations, the average friction strength they predict for presaddle deformation is comparable. A constant friction value before saddle-point deformation used here should be considered as representing the average friction strength before the saddle-point deformation region. The same is true of the meaning for postsaddle friction strength extracted below by comparing theory with experiment, which denotes the average magnitude of the friction strength in the saddle-to-scission deformation region, and not the magnitude at the scission deformation.

To better reveal postsaddle dissipation with  $E_{sc}^*$ , in this work the presaddle friction strength is set as  $3 \text{ zs}^{-1}$  (1 zs =  $10^{-21}$ s), in agreement with recent theoretical estimates and experimental analyses [9,19,22,23,29,41,45,46], while the postsaddle friction strength  $\beta$  is determined by reproducing measured  $E_{sc}^*$  in the reactions  ${}^{16}\text{O} + {}^{238}\text{U}$  [47],  ${}^{18}\text{O} + {}^{238}\text{U}$ [47], and  ${}^{20}\text{Ne} + {}^{159}\text{Tb}$  [48].

Figure 1(a) displays a comparison between experimental  $E_{sc}^*$  of the <sup>254</sup>Fm system and theoretical ones which are calculated based on the Langevin model considering different values of  $\beta$ . Two principle observations are made. First,  $E_{sc}^*$  becomes smaller with increasing  $\beta$ . This is because of the consequence of more particle emission stemming from the saddle-to-scission region, as seen in Fig. 2(a) where postsaddle neutron multiplicity as a function of  $\beta$  is shown. A strong friction retards the fission process more severely, providing more time for particle emission. Consequently, more excitation energy will be carried away, leaving a lower excitation energy at scission at larger  $\beta$ . Second, at a low postsaddle friction strength of 5 zs<sup>-1</sup>, the calculated  $E_{sc}^*$  is much higher than the measured one. A friction strength of over 10 zs<sup>-1</sup> is required to explain data, and the best-fit value of  $\beta$  is found around 12.5 zs<sup>-1</sup> (denoted by solid squares in Fig. 1).



FIG. 2. (a) Postsaddle neutron multiplicity as a function of postsaddle dissipation strength ( $\beta$ ) for the reaction <sup>16</sup>O ( $E_{\text{lab}} = 288 \text{ MeV}$ ) + <sup>238</sup>U. (b) Postsaddle neutron multiplicity of <sup>246</sup>Cf at various  $\beta$  calculated at angular momentum  $\ell = 40\hbar$  and at three initial excitation energies  $E^* = 50 \text{ MeV}$ , 80 MeV, and 180 MeV.

For the <sup>18</sup>O + <sup>238</sup>U reaction, which yields <sup>256</sup>Fm at a lower bombarding energy, the best-fit value [denoted by solid squares in Fig. 1(b)] obtained for  $\beta$  is 11.5 zs<sup>-1</sup>, a magnitude similar to that of the <sup>254</sup>Fm nucleus.

Overall, the strength of postsaddele friction ( $\sim 12 \text{ zs}^{-1}$ ) of the two heavy Fm nuclei is evidently greater than that of presaddle friction, indicating a rising friction strength at a larger deformation.

With a statistical model that is improved to take account of dissipation effects, Shaw *et al.* [43] showed that a large postsaddle friction during the saddle to scission path (close to  $20 \text{ zs}^{-1}$ ) together with a small presaddle friction (~4 zs<sup>-1</sup>) described the experimental giant dipole resonance  $\gamma$ -ray spectra of heavy <sup>240</sup>Cf nuclei very well. In addition, Aleshin [49] has indicated within the framework of the nonequilibrium statistical-operator theory that the friction strength rises with deformation, lending a certain support to a strong postsaddle friction.

When a nucleus fissions, it will experience deformation. The lighter the fissioning nucleus, the shorter the distance between its saddle and its scission points; that is, a light fissioning system undergoes a smaller postsaddle deformation than a heavy one. This fact implies that using the  $E_{sc}^*$  data of light fissioning nuclei could provide valuable information about postsaddle nuclear dissipation.

To that end, we choose the <sup>179</sup>Re system produced in <sup>20</sup>Ne + <sup>159</sup>Tb. The comparison between experimental and calculated  $E_{sc}^*$  are presented in Fig. 3. One can notice that calculations performed at  $\beta = 4 \text{ zs}^{-1}$  ( $\beta = 6 \text{ zs}^{-1}$ ) can fit the measured  $E_{sc}^*$  at the lower (higher) energy, but they apparently overestimate (underestimate) those data at higher (lower) energy. Further raising  $\beta$  up to 7 zs<sup>-1</sup> causes a more pronounced deviation from data. We find in Fig. 3 that the case that gives the best agreement with all data points is represented by the black solid line ( $\beta = 5 \text{ zs}^{-1}$ ).

In the stochastic description of the fission process of a hot nucleus, the driving force of the Langevin equations is



FIG. 3. The excitation energy at scission calculated for the system  ${}^{20}\text{Ne} + {}^{159}\text{Tb} \longrightarrow {}^{179}\text{Re}$  versus laboratory energy per nucleon. Curves represent theoretical calculation at postsaddle friction strength  $\beta = 4 \text{ zs}^{-1}$  (dashed black line),  $5 \text{ zs}^{-1}$  (solid black line),  $6 \text{ zs}^{-1}$  (dashed double-dot black line), and  $7 \text{ zs}^{-1}$  (solid blue line). Data (solid red circle) are taken from [48]. Note that  $\beta$  represents the friction strength throughout the postsaddle fission process, and it is the only adjustable parameter in CDSM.

the thermodynamical quantity entropy  $S(q, E^*, \ell)$ , which is a function of excitation energy  $E^*$  and angular momentum  $\ell$ . A change in  $E^*$  and  $\ell$  thus shifts the position of the stationary points of the entropy. It was noted [22,44] that the location of the saddle point becomes far away from the scission-point configuration with increasing  $E^*$  and  $\ell$ , which is much more prominent for light decaying systems than for heavy ones, because the latter have a longer saddle-to-scission distance.

Heavy-ion-induced fusion reactions can deposit a large energy and spin into the populated light compound nucleus, which causes a significant shift of its saddle point from its scission point, as pointed out before. Thus, the postsaddle friction strength  $\beta$  for the light <sup>179</sup>Re system extracted here actually corresponds to a deformation region, where the magnitude of deformation that postsaddle neutrons (which affect the amplitude of  $E_{sc}^*$ ) mainly stem from is much smaller than that of the scission configuration of the light decaying nucleus. In addition, the short saddle-to-scission distance of the light nucleus makes it quickly fission after it gets across the saddle, which constrains postsaddle emission at a large deformation. A strong presaddle evaporation [22,42,44] further removes most of the excitation energy from the light decaying nucleus. So, its postsaddle neutrons are usually from the early stage of the postsaddle decay chain, i.e., they are chiefly evaporated at a relatively small postsaddle deformation position [22]; accordingly, they (and hence  $E_{sc}^*$ ) carry information of postsaddle friction at a small deformation.

In contrast, the heavy nucleus has a longer saddle-toscission descent, resulting in its postsaddle emission as the main source of prescission particles [13,43]. As a consequence, neutrons can be emitted along the entire saddle-to-scission deformation region (including the large deformation position near the scission configuration), and not confined to the small deformation region around the saddle point. This indicates that the postsaddle friction strength extracted with  $E_{sc}^*$  (which is closely related to postsaddle multiplicity) of heavy nuclei thus corresponds to a postsaddle deformation region that is far larger than its saddle-point deformation, which is clearly different from the case of light fissioning nuclei.

Moreover, in a stochastic process like nuclear fission, because of the presence of random forces in the equations of motion, a long saddle-to-scission distance of heavier nucleus considerably increases the times that it walks to and fro along the saddle-to-scission path before it reaches the scission. This extends time that the heavy decaying system stays in a larger deformation region, which further enhances neutron emission and its (and hence  $E_{\rm sc}^*$ ) dependence on the friction strength at a larger deformation.

It is clear from Figs. 1 and 3 that the strength of postsaddle friction obtained for light <sup>179</sup>Re is much weaker than that for heavy <sup>254</sup>Fm and <sup>256</sup>Fm which have a longer saddle-to-scission distance and thereby correspond to a larger postsaddle deformation than that of the former nuclei. Such a comparison is an indication that nuclear dissipation strength could have a deformation.

In addition, several studies [23,42] have shown that, when adopting the modified OBD strength (which assumes that postsaddle friction is weaker than presaddle friction), the Langevin calculation obviously underpredicts data of the prescission particle multiplicity of heavy fissioning nuclei (A > 250), where a longer descent from the saddle to the scission points and hence a larger deformation is involved, hinting at the necessity of introducing a strong saddle-toscission friction in model calculation.

Previous calculations (as shown in Figs. 1 and 3) illustrate that the Langevin model can provide a reasonable, quantitative prediction of the  $E_{sc}^*$ . In addition, the initial excitation energy  $(E^*)$  is a key parameter controlling deexcitation modes of a hot nucleus. So, to better help experimentalists explore postsaddle dissipation, we survey the role of (initial) excitation energy in the sensitivity of  $E_{sc}^*$  vs  $\beta$ . We use the <sup>246</sup>Cf system as an illustration (Fig. 4).

The most prominent feature observed in Fig. 4 is that the steepness of  $E_{sc}^*$  vs  $\beta$ , which reflects the sensitivity of the  $E_{sc}^*$  to the variation of postsaddle friction strength, differs very much with a variation in  $E^*$ . Specifically speaking, at low initial excitation energy of 50 MeV, the  $E_{sc}^*$  only has a little change with increasing  $\beta$ , showing a low sensitivity to  $\beta$ , but this change becomes appreciable at  $E^* = 80$  MeV, meaning a rise of the sensitivity. When  $E^*$  reaches 180 MeV, the  $E_{sc}^*$  has a more marked difference at different  $\beta$ . This demonstrates that the steepness of  $E_{sc}^*$  with respect to  $\beta$  is clearly larger at high  $E^*$ , indicating a more sensitive dependence of  $E_{sc}^*$  on  $\beta$  at high energy.

The physical understanding for this excitation energy dependence is as follows: At a low energy, particles emitted prior to saddle remove energy, leaving a very small portion of excitation energy for saddle-to-scission evaporation. This lowers the effects of postsaddle friction on particle emission and, hence, on  $E_{sc}^*$ . But at high  $E^*$ , even if presaddle particles carry away excitation energy, a considerable part of the excitation energy is still left for postsaddle particle emission. Postsaddle neutrons



FIG. 4. Comparison of the sensitivity of excitation energy at scission  $E_{sc}^*$  versus postsaddle friction strength  $\beta$  for <sup>246</sup>Cf nuclei at angular momentum  $\ell = 40\hbar$  but at three different initial excitation energies  $E^* = 50$  MeV, 80 MeV, and 180 MeV.

are an increasing function of excitation energy  $E^*$  [Fig. 2(b)]. An increasing neutron emission will remove more energy, and at high  $E^*$  the light charged-particle emission further removes the excitation energy from the decaying system. In addition, a strong friction yields a long fission delay, which affects particle emission more strongly at high energy because of a shortened particle evaporation time. These result in a rapid cooling of the fissioning nucleus at high energy and, correspondingly, a greater sensitivity to  $\beta$ . Thus, a measurement of  $E_{sc}^*$  at high energy could place more stringent constraints on the magnitude of postsaddle nuclear friction in fission.

#### **IV. DISCUSSION**

In the literature, several representative proposals [22,23,42] were given about the deformation-dependent friction, but the specific form that friction depends on deformation is still precisely unknown. Also, as mentioned previously, in a decay process of a hot compound nucleus, particle emission can compete with fission for many times, which makes fission observables sensitive to the average friction strength along the fission path, and not sensitive to the specific detail that friction is taken as a possible function form of deformation. Under this circumstance, we do not assume a new and complicated deformation-dependent friction form in calculation.

Because of these reasons above, one constant friction value before saddle-point deformation and another one beyond the saddle are considered in the present work. A use of a steplike function or a continuous function form about the deformation dependence of friction in calculation may have an effect on the deduced specific postsaddle numerical value. However, the result that a larger postsaddle friction required to fit  $E_{sc}^*$  data of heavy systems compared to that for light systems obtained here is not altered, because the magnitude of  $E_{sc}^*$  is determined by the average strength of postsaddle friction throughout the saddle-to-scission deformation region.

Different from the case treating the decay of highly excited compound nuclei where a new element, i.e., nuclear dissipation is introduced into the decay model, the standard statistical model employed to handle the de-excitation of fission fragments was well developed and widely tested in previous studies [4,5,43]. In addition to postscission neutron data, the standard decay model is further constrained by the systematic  $\gamma$ -ray data of fission fragments. This means that while the  $E_{sc}^*$  is deduced by fitting experimental postscission neutron multiplicity with a standard decay model, its dependence on the model is weak. However, it is still interesting to further develop the standard decay model describing fragment decays to obtain high precision  $E_{sc}^*$  data through which the average postsaddle friction strength can be determined more accurately.

Experimentally, the three-source model approach [4,11,28,39,43,47,48] was a standard method to extract preand postscission neutron multiplicities. However, it is not even completely clear how correctly one may divide the total measured multiplicity on pre- and postscission, because some neutrons can be evaporated during the fragment acceleration stage that is beyond the standard deconvolution method. The neglect of the extra neutron evaporation source may affect the accuracy for both prescission and postscission neutrons extracted and, correspondingly, the accuracy of the friction strength deduced based on the experimental prescission or postscission neutrons reported in previous and the present works. Thus, further investigations for the issue are still necessary.

The principle experimental probes currently employed to explore nuclear dissipation are particle observables (i.e., neutrons, light charged particles, and GDR  $\gamma$  rays) and fragment observables (i.e., evaporation residue and fission cross section, fission fragment mass, angle, and energy distributions). It is well known that earlier studies based on fragment characteristics do not yield a definite conclusion on the nature and magnitude of nuclear dissipation and that the relevant study was greatly advanced by available information of measured particle emission in fission, indicating the importance of utilizing particle-type observables in exploiting nuclear dissipation.

On the theoretical side, the one-dimensional Langevin model mainly deals with particle-type observables from different fissioning systems. It is unsuitable for describing fragment characteristics, because it does not explicitly treat the neck degree of freedom as a collective coordinate (for the case of symmetric fission  $\alpha = 0$  discussed here), but the deformation coordinate q is precisely determined by the elongation and neck degrees of the freedom of the nucleus. The multidimensional model is applied to explain both particle-type and fragmenttype observables and expected to provide clear conclusions on the friction strength. Unfortunately, still no definite and precise conclusion is reached. One may note that the friction strength deduced from comparing multidimensional calculation with particle observables and with fragment observables are not always inconsistent. The contrast results on the magnitude of the nuclear friction, i.e., a significantly reduced (with a reduction factor  $k_s = 0.25 - 0.5$  [23]) and a full (with  $k_s = 1.0$ 



FIG. 5. Fits to measured prescission neutron multiplicity in (a)  ${}^{18}\text{O}$  ( $E_{\text{lab}} = 159 \text{ MeV}$ ) +  ${}^{238}\text{U} \longrightarrow {}^{256}\text{Fm}$  and (b)  ${}^{20}\text{Ne}$  ( $E_{\text{lab}} = 320 \text{ MeV}$ ) +  ${}^{159}\text{Tb} \longrightarrow {}^{179}\text{Re}$  systems. Data [47,48] are denoted by the shaded band. Solid lines denote model calculations. Note that  $\beta$  represents the postsaddle friction strength.

[50]) OBD friction strength, are found to be needed to account for fission data from fissioning systems with different size. This situation indicates that more efforts should be made toward further developing the multidimensional model to give a consistent result of the friction strength for different fissioning systems. The one-dimensional model was shown to provide a consistent and systematic description of fission data over a broad range of compound systems [22]. Thus, as far as the presently developed state for practical theoretical simulations is concerned, calculation of particle-type observables based on the one-dimensional model is still indispensable, as was done here.

To make more reliable and robust conclusions on the properties of nuclear dissipation, in addition to further improving theoretical simulations, two other issues are worth carefully investigating.

First, how to improve the quality of experimental data is a crucial issue. In this respect, a sensitive analysis (as was done in the present work) of physical quantities under different experimental conditions becomes very important, because it can help experimentalists obtain those high-quality experimental data. This can significantly enhance the constraint that experiment places on the model parameters used in calculation.

Second, fitting more types of data favors a stringent constraint on the model parameters including the friction parameter discussed here. As an illustration, we show in Fig. 5 the comparison between theoretical and experimental prescission neutrons for <sup>256</sup>Fm and <sup>179</sup>Re. As can be seen, the best-fit postsaddle friction value (denoted by solid squares in the figure) required for heavy <sup>256</sup>Fm ( $\beta \sim 11 \text{ zs}^{-1}$ ) is larger than that for light <sup>179</sup>Re ( $\beta \sim 4.5 \text{ zs}^{-1}$ ). However, we note that these best-fit  $\beta$  values obtained from prescission neutron data are somewhat different from those obtained from  $E_{\text{sc}}^*$  data. One main cause for the difference is because of the fact that different observables could have different sensitivities to  $\beta$ , and their sensitivity could have different changes with experimental conditions, such as excitation energy, angular momentum, and system size. These yield an influence on the accuracy of the extracted friction parameter. A difference in

the deduced  $\beta$  values from different quantities reveals that identifying those most sensitive experimental signatures and investigating the evolution of their sensitivity to  $\beta$  with the controllable experimental conditions are key tasks for making further progress.

# **V. CONCLUSIONS**

In the framework of the Langevin model of fission dynamics, by comparing calculated and measured  $E_{sc}^*$  data for <sup>254</sup>Fm and <sup>256</sup>Fm nuclei, the postsaddle friction strength extracted is found to be apparently greater than that extracted for the light <sup>179</sup>Re system which has a smaller postsaddle deformation than heavy Fm systems, showing a rising nuclear dissipation strength with increasing deformation. Furthermore,

we investigate the influence of initial excitation energy on the observable  $E_{sc}^*$  as a tool of postsaddle dissipation strength  $\beta$ . We find that increasing excitation energy can substantially enhance the sensitivity of  $E_{sc}^*$  to  $\beta$ . This finding suggests that, experimentally, to precisely probe postsaddle dissipation effects by measuring  $E_{sc}^*$ , it is best to populate fissioning systems with high energy.

#### ACKNOWLEDGMENTS

The authors thank the anonymous referee for comments and suggestions, which led to an improved version of this paper. This work is supported by National Natural Science Foundation of China under Grant No. 11575044.

- K. Siwek-Wilczyńska, J. Wilczyński, H. K. W. Leegte, R. H. Siemssen, H. W. Wilschut, K. Grotowski, A. Panasiewicz, Z. Sosin, and A. Wieloch, Phys. Rev. C 48, 228 (1993); 54, 325 (1996); J. Velkovska, C. R. Morton, R. L. McGrath, P. Chung, and I. Diószegi, *ibid.* 59, 1506 (1999).
- [2] Y. Arimoto, Nucl. Phys. A 780, 222 (2006).
- [3] Y. Abe, S. Ayik, P. G. Reinhard, and E. Suraud, Phys. Rep. 275, 49 (1996); D. Boilley *et al.*, Nucl. Phys. A 556, 67 (1993).
- [4] D. Hilscher and R. Rossner, Ann. Phys. (Paris) 17, 471 (1992).
- [5] P. Paul and M. Thoennessen, Annu. Rev. Nucl. Part. Sci. 44, 65 (1994).
- [6] B. Lott, F. Goldenbaum, A. Böhm, W. Bohne, T. von Egidy, P. Figuera, J. Galin, D. Hilscher, U. Jahnke, J. Jastrzebski, M. Morjean, G. Pausch, A. Péghaire, L. Pienkowski, D. Polster, S. Proschitzki, B. Quednau, H. Rossner, S. Schmid, and W. Schmid, Phys. Rev. C 63, 034616 (2001).
- [7] I. Mukul *et al.*, Phys. Rev. C **92**, 054606 (2015); H. Singh,
  B. R. Behera, G. Singh, I. M. Govil, K. S. Golda, A. Jhingan,
  R. P. Singh, P. Sugathan, M. B. Chatterjee, S. K. Datta, S. Pal,
  R. S. Mandal, P. D. Shidling, and G. Viesti, *ibid.* **80**, 064615 (2009); **76**, 044610 (2007).
- [8] D. Jacquet and M. Morjean, Prog. Part. Nucl. Phys. 63, 155 (2009).
- [9] C. Schmitt, K.-H. Schmidt, A. Kelić, A. Heinz, B. Jurado, and P. N. Nadtochy, Phys. Rev. C 81, 064602 (2010); Phys. Rev. Lett. 99, 042701 (2007).
- [10] D. Mancusi, R. J. Charity, and J. Cugnon, Phys. Rev. C 82, 044610 (2010).
- [11] V. Singh, B. R. Behera, M. Kaur, A. Kumar, P. Sugathan, K. S. Golda, A. Jhingan, M. B. Chatterjee, R. K. Bhowmik, D. Siwal, S. Goyal, J. Sadhukhan, S. Pal, A. Saxena, S. Santra, and S. Kailas, Phys. Rev. C 87, 064601 (2013); R. Sandal, B. R. Behera, V. Singh, M. Kaur, A. Kumar, G. Singh, K. P. Singh, P. Sugathan, A. Jhingan, K. S. Golda, M. B. Chatterjee, R. K. Bhowmik, S. Kalkal, D. Siwal, S. Goyal, S. Mandal, E. Prasad, K. Mahata, A. Saxena, J. Sadhukhan, and S. Pal, *ibid.* 87, 014604 (2013).
- [12] Y. Ayyad, J. Benlliure, J. L. Rodríguez-Sánchez *et al.*, Phys. Rev. C **91**, 034601 (2015); **89**, 054610 (2014).
- [13] I. I. Gontchar et al., Nucl. Phys. A 551, 495 (1993).

- [14] T. Wada, Y. Abe, and N. Carjan, Phys. Rev. Lett. 70, 3538 (1993).
- [15] K. Pomorski, J. Bartel, J. Richert, and K. Dietrich, Nucl. Phys. A 605, 87 (1996); 679, 25 (2000).
- [16] V. V. Sargsyan, Yu. V. Palchikov, Z. Kanokov, G. G. Adamian, and N. V. Antonenko, Phys. Rev. C 76, 064604 (2007).
- [17] H. Eslamizadeh and M. Pirpour, Chin. Phys. C 38, 064101 (2014); Eur. Phys. J. A 47, 134 (2011); S. M. Mirfathi and M. R. Pahlavani, Phys. Rev. C 78, 064612 (2008).
- [18] K. Mazurek, C. Schmitt, and P. N. Nadtochy, Phys. Rev. C 91, 041603(R) (2015).
- [19] J. Sadhukhan and S. Pal, Phys. Rev. C 78, 011603(R) (2008).
- [20] V. M. Kolomietz and S. V. Radionov, Phys. Rev. C 80, 024308 (2009).
- [21] W. Ye, Phys. Lett. B 681, 413 (2009); Phys. Rev. C 76, 021604(R) (2007).
- [22] P. Fröbrich and I. I. Gontchar, Phys. Rep. 292, 131 (1998).
- [23] P. N. Nadtochy, G. D. Adeev, and A. V. Karpov, Phys. Rev. C 65, 064615 (2002).
- [24] H. J. Krappe and K. Pomorski, *Theory of Nuclear Fission*, Lecture Notes in Physics Vol. 838 (Springer-Verlag, Berlin, 2012).
- [25] W. Ye and N. Wang, Phys. Rev. C 87, 014610 (2013); P. V. Laveen *et al.*, J. Phys. G 42, 095105 (2015).
- [26] B. B. Back, D. J. Blumenthal, C. N. Davids, D. J. Henderson, R. Hermann, D. J. Hofman, C. L. Jiang, H. T. Penttilä, and A. H. Wuosmaa, Phys. Rev. C 60, 044602 (1999); R. Sandal, B. R. Behera, V. Singh, M. Kaur, A. Kumar, G. Kaur, P. Sharma, N. Madhavan, S. Nath, J. Gehlot, A. Jhingan, K. S. Golda, H. Singh, S. Mandal, S. Verma, E. Prasad, K. M. Varier, A. M. Vinodkumar, A. Saxena, J. Sadhukhan, and S. Pal, *ibid.* 91, 044621 (2015).
- [27] P. D. Shidling, N. M. Badiger, S. Nath, R. Kumar, A. Jhingan, R. P. Singh, P. Sugathan, S. Muralithar, N. Madhavan, A. K. Sinha, S. Pal, S. Kailas, S. Verma, K. Kalita, S. Mandal, R. Singh, B. R. Behera, K. M. Varier, and M. C. Radhakrishna, Phys. Rev. C 74, 064603 (2006); S. K. Hui, C. R. Bhuinya, A. K. Ganguly, N. Madhavan, J. J. Das, P. Sugathan, D. O. Kataria, S. Mutlithar, L. T. Baby, V. Tripathi, A. Jhingan, A. K. Sinha, P. V. Madhusudhana Rao, N. V. S. V. Prasad, A. M. VinodKumar, R. Singh, M. Thoennessen, and G. Gervais, *ibid.* 62, 054604 (2000).

- [28] K. Ramachandran, A. Chatterjee, A. Navin, K. Mahata, A. Shrivastava, V. Tripathi, S. Kailas, V. Nanal, R. G. Pillay, A. Saxena, R. G. Thomas, S. Kumar, and P. K. Sahu, Phys. Rev. C 73, 064609 (2006).
- [29] P. Fröbrich, Nucl. Phys. A 787, 170c (2007).
- [30] I. I. Gontchar, L. A. Litnesvsky, and P. Fröbrich, Comput. Phys. Commun. 107, 223 (1997).
- [31] H. J. Krappe, J. R. Nix, and A. J. Sierk, Phys. Rev. C 20, 992 (1979); A. J. Sierk, *ibid.* 33, 2039 (1986); P. Möller, W. D. Myers, W. J. Światecki, and J. Treiner, At. Data Nucl. Data Tables 39, 225 (1988).
- [32] M. Brack et al., Rev. Mod. Phys. 44, 320 (1972).
- [33] G. Chaudhuri and S. Pal, Eur. Phys. J. A 18, 9 (2003).
- [34] R. W. Hass and W. D. Myers, Geometrical Relationships of Macroscopic Nuclear Physics (Springer, New York, 1988), and references therein.
- [35] A. V. Ignatyuk et al., Sov. J. Nucl. Phys. 21, 612 (1975).
- [36] I. I. Gontchar, P. Fröbrich, and N. I. Pischasov, Phys. Rev. C 47, 2228 (1993).
- [37] M. Blann, Phys. Rev. C 21, 1770 (1980).
- [38] P. Fröbrich et al., Nucl. Phys. A 563, 326 (1993).
- [39] H. Rossner, D. J. Hinde, J. R. Leigh, J. P. Lestone, J. O. Newton, J. X. Wei, and S. Elfström, Phys. Rev. C 45, 719 (1992).
- [40] P. Fröbrich, Phys. Rep. 116, 337 (1984).

- [41] W. Ye, H. W. Yang, and F. Wu, Phys. Rev. C 77, 011302(R) (2008).
- [42] G. Chaudhuri and S. Pal, Phys. Rev. C 65, 054612 (2002).
- [43] N. P. Shaw, I. Diośzegi, I. Mazumdar, A. Buda, C. R. Morton, J. Velkovska, J. R. Beene, D. W. Stracener, R. L. Varner, M. Thoennessen, and P. Paul, Phys. Rev. C 61, 044612 (2000).
- [44] W. Ye, Phys. Rev. C 84, 034617 (2011); K. Pomorski and H. Hofmann, Phys. Lett. B 263, 164 (1991).
- [45] B. Jurado, C. Schmitt, K. H. Schmidt, J. Benlliure, T. Enqvist, A. R. Junghans, A. Kelić, and F. Rejmund, Phys. Rev. Lett. 93, 072501 (2004).
- [46] E. G. Ryabov et al., Phys. Rev. C 78, 044614 (2008).
- [47] D. J. Hinde, D. Hilscher, H. Rossner, B. Gebauer, M. Lehmann, and M. Wilpert, Phys. Rev. C 45, 1229 (1992).
- [48] J. Cabrera, Th. Keutgen, Y. El Masri, Ch. Dufauquez, V. Roberfroid, I. Tilquin, J. Van Mol, R. Régimbart, R. J. Charity, J. B. Natowitz, K. Hagel, R. Wada, and D. J. Hinde, Phys. Rev. C 68, 034613 (2003).
- [49] V. P. Aleshin, Nucl. Phys. A 781, 363 (2007).
- [50] E. Vardaci, P. N. Nadtochy, A. Di Nitto, A. Brondi, G. La Rana, R. Moro, P. K. Rath, M. Ashaduzzaman, E. M. Kozulin, G. N. Knyazheva, I. M. Itkis, M. Cinausero, G. Prete, D. Fabris, G. Montagnoli, and N. Gelli, Phys. Rev. C 92, 034610 (2015).