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$B(E2)$ values in neutron-excess nuclei near $A = 16$

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A simple model is used to compute $B(E2)$'s in several nuclei that have one or two sd-shell neutrons and no sd-shell protons. The model works well for all six nuclei if I use later experimental values for ^{16}C for which the measured $B(E2)$ is about four to eight times earlier values.

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I. INTRODUCTION

In ^{16}C, the $B(E2)$ from the first-excited 2^+ state to the ground state (g.s.) has had an interesting history, both experimentally and theoretically. The first experiments reported values that were remarkably small. A value of 0.63(11)(16 syst) $e^2\,\mathrm{fm}^4$ was inferred from the 2^+ lifetime of 77(14)(19 syst) ps measured by a recoil shadow method [1]. In another work from about the same time, inelastic scattering of ^{16}C from ^{208}Pb was used (via Coulomb-nuclear interference) to obtain the ratio of mass to charge deformation lengths, which were, in turn, converted into neutron and proton $E2$ matrix elements from which the $B(E2)$ was computed as $B(E2) = M_p^2$ [2]. This result is stated as $B(E2) = 0.28(6)$ Weiskopf units (W. u.) to be compared with 0.26(5)(7 syst) W. u. from the lifetime mentioned above. In terms of W. u. this $B(E2)$ is the smallest ever measured for a 2_1^+ to g.s. transition in any nucleus.

The ground and first-excited states of ^{16}C are very well described [3] in terms of excitations of two neutrons in the sd shell coupled to two proton holes in the $1p$ shell. Within this space, the $E2$ involves only neutron transitions and hence will depend on the neutron effective charge. It is a certainty, then, that the $B(E2)$—no matter how small—can be fitted by some value of the neutron effective charge as has been demonstrated [4]. Suzuki *et al.* [4], in a $^{14}\mathrm{C} + n + n$ model, found they could fit the experimental value if they used a neutron effective charge of $e_n = 0.10\,e$, compared to their value of $e_n = 0.16\,e$ in ^{15}C. Imai *et al.* [1] had suggested that different $E2$ amplitudes interfered destructively. Heyde *et al.* [5] explained that such destructive interference was extremely unlikely for the $E2$ between the lowest 0^+ and 2^+ states. They proposed the existence of an undiscovered 0^+ state about 800 keV below the 2^+ state that had the majority of the $E2$ strength. This was also an extremely unlikely scenario.

Horiuchi and Suzuki [6], also in a $^{14}\mathrm{C} + n + n$ model, found that their calculated $B(E2)$ was about twice the experimental value if they used the same neutron effective charge in ^{16}C and ^{15}C. They pointed out that the inclusion of $S = 1$ components in the 0^+ and 2^+ wave functions caused a slight reduction in the calculated $B(E2)$. Hagino and Sagawa [7] used the same e_n in ^{15}C and ^{16}C and were able to compute values in the range of 0.94–1.07 $e^2\,\mathrm{fm}^4$. They pointed out that the dominance of the $(d_{5/2})^2$ configuration in the g.s. plays a crucial role in the agreement and that such $(d_{5/2})^2$ dominance was in disagreement with other works [4,6]. It also disagrees with our wave functions [3].

In a no-core shell-model calculation, Fujii *et al.* [8] found that they needed to use "dressed" single-particle energies in order to get the correct level ordering in ^{16}C. Their dressed $B(E2)$ was 0.84 $e^2\,\mathrm{fm}^4$ (undressed was 1.30). Other groups [9–11] performed calculations that came close to reproducing the extremely small $E2$ strength in ^{16}C. Thus, many different sets of workers were able to produce a small value for ^{16}C.

On the other hand, systematics of other nearby nuclei would have led one to expect a much larger $B(E2)$. A calculation by Sagawa *et al.* [12] provided values from 3.80 to 6.85 $e^2\,\mathrm{fm}^4$. Fortunately, another set of experiments soon followed, and they drastically changed the situation. Ong *et al.* [13] measured the 2^+ mean life to be 18.3(1.4)(4.8 syst) ps, about 25% of the previous value, and thus $B(E2) = 2.6$ (0.2)(0.7 syst), about four times larger than earlier. Elekes *et al.* [14] reexamined their analysis and found a similar result of $B(E2) = 3.04\,e^2\,\mathrm{fm}^4$. Then, Wiedeking *et al.* [15] used the recoil distance method following the fusion reaction $^{9}\mathrm{Be}(^{9}\mathrm{Be},2p)$ and measured $\tau = 11.7(20)$ ps, giving $B(E2) = 4.15(73) e^2\,\mathrm{fm}^4$. This value is larger even than those of Ong *et al.* and the reexamination of Elekes *et al.* The simple average of the three is about $3.26(43) e^2\,\mathrm{fm}^4$, approximately 5.2 times the first values. These experimental values and the average of the last four are listed in Table I.

II. CALCULATIONS AND RESULTS

Weak-coupling considerations have worked extremely well in accounting for $B(E2)$'s in a particle-hole nucleus in terms of those in the particle and hole nuclei separately. For example, in the $1/2^-$ band of ^{19}F, the $B(E2)$'s [17] are in excellent agreement [18] with those expected for a $4p$-$1h$ band in which the $4p$'s are the 0^+ band of ^{20}Ne [19]. Many other examples exist.

A proton inelastic-scattering experiment [20] on ^{16}C has confirmed that the g.s.-2_1^+ transition is predominantly a neutron excitation. The deformation parameter $\beta_{pp'} = 0.47(5)$ determined in their experiment is consistent with the trend in other nearby nuclei. Because the ^{16}C transition primarily involves sd-shell neutron transitions, I have examined low-lying $B(E2)$'s in other nearby nuclei for which the "action" is mostly in the sd-shell neutron space. I restrict attention to pure $E2$'s connecting the g.s. to a low-lying state whose J value does not allow the presence of a competing $M1$. These $B(E2)$'s [17,21,22] are listed in Table II along with values of $M(E2)$ defined as $B(E2 : i \to f) = M^2/(2J_i + 1)$.

TABLE I. Experimental $B(E2: 2^+ \to 0^+)$ (e^2 fm^4) in ^{16}C.

Reference	Value
Imai *et al.* [1]	0.63[11(15)]
Elekes *et al.* [2]	0.67(14)
Ong *et al.* [13]	2.6[2(7)]
Elekes *et al.* reexamination [14]	3.04
Wiedeking *et al.* [15]	4.15(73)
Petri *et al.* [16]	$4.21^{+0.34}_{-0.26}$(stat)$^{+0.28}_{-0.24}$ syst
Simple average of the last four	3.50(30)

The cases of ^{18}O and (to a lesser extent) ^{17}O require special mention. The lowest 0^+ and 2^+ states of ^{18}O have long been known [23,24] to contain significant core-excited (mostly $4p$-$2h$) components. Even though these collective components are only about 10% of the wave functions of the first two states, the strong $4p$ $E2$ causes a much larger contribution to the $B(E2)$. In a fit [24] to many of the properties of the low-lying states of ^{18}O, the fitted value of the $(sd)^2$ part of $M(E2)$ was 3.91 e fm^2 Re-doing the least-squares fit, fit with the updated experimental value from Table II, provides $M(E2) \approx 4.03\, e$ fm^2 for the $(sd)^2$ part. As this value arises from only 90% of the wave functions (91% for the g.s. and 88% for 2_1^+), the "complete" $M(E2)$ for two sd-shell neutrons would thus be $4.5e$ fm^2. Even before this renormalization, I would have expected $M(^{16}\mathrm{C})/M(^{15}\mathrm{C}) \geqslant M[^{18}\mathrm{O}\,(sd)^2]/M(^{17}\mathrm{O})$ for several reasons:

(1) The non-$(sd)^2$ part of ^{16}C likely involves $(sd)^4 \times {}^{12}$C, where $(sd)^4$ represents ^{20}O, and ^{20}O($2^+ \to 0^+$) is reasonably large [$M(E2) = 5.43(11)e$ fm^2].

(2) The $E2$ in ^{17}O is likely more collective (from core excitation) than is ^{15}C.

(3) The ^{15}C value may be slightly suppressed because the $5/2^+$ state is not pure single particle. Its spectroscopic factor [25] is only 0.69, and some other amplitudes are present [26]. But they are not expected to have an appreciable impact on the $M(E2)$. The $(sd)^3 \times {}^{12}$C component will involve an $E2$ in ^{19}O that is also reduced by a $5/2^+$ spectroscopic factor [27] considerably less than unity.

TABLE III. Experimental and weak-coupling values of $M(E2)$ (in e fm^2) and their ratios.

Transition	Nucleus	M_{exp}	M_{wc}	$M_{\mathrm{exp}}/M_{\mathrm{wc}}$
$s \leftrightarrow d$	^{17}O($s \leftrightarrow d$)	2.90 [a]		$\equiv 1.00$
	^{16}N	2.06(1)	2.05	1.00
			1.91[c]	1.08[c]
	^{15}C	2.42(4)	2.90	0.834(14)
			2.49[c]	0.972(16)[c]
$(sd)^2 \leftrightarrow (sd)^2$	^{18}O $(sd)^2$[a,b]	4.50		
	^{17}N	3.67(37)	4.90	0.75(8)
			4.60[c]	0.80(8)[c]
	^{16}C	4.18(18)	4.50	0.93(4)
			3.86[c]	1.08(5)[c]

[a]After removing a collective contribution.
[b]Renormalized to 100% of the wave function.
[c]Values on the second line use $e_n = (Z/A)e$, where Z, A refer to the core.

However, from the initial values in Table I (with $M_{18} = 4.5e$ fm^2) values are $M_{^{16}\mathrm{C}}/M_{^{15}\mathrm{C}} = 0.73$ and $M_{^{18}\mathrm{O}}(sd)^2/M_{^{17}\mathrm{O}} = 1.27$, clearly demonstrating that the earlier value of ^{16}C $B(E2)$ was less than one-third of the expected value. With the average of the more recent values, the ^{16}C/^{15}C ratio is 1.67, clearly satisfying the expected inequality.

For a more quantitative computation, I have removed the collective part of the ^{17}O $E2$ to get a "local" value for $M(s \leftrightarrow d)$, viz. $2.9e$ fm^2. With these two values now of $M[(sd)^2 \leftrightarrow (sd)^2]$ and $M(s \leftrightarrow d)$, I can compute $M(E2)$ for the other

TABLE II. Experimental $B(E2)$'s in relevant nuclei.[a]

Nucleus	J_i^π	J_f^π	$E\gamma$ (MeV)	$\Gamma\gamma$ (eV)	$B(E2)$ (e^2 fm^4)	$M(E2)$ (e fm^2)
^{18}O[b]	2^+	0^+	1.982	$2.35(6) \times 10^{-4}$	9.3(3)	6.8(1)
^{17}O[c]	$1/2^+$	$5/2^+$	0.8707	$2.55(3) \times 10^{-6}$	6.3(1)	3.55(2)
^{17}N[c]	$5/2^-$–	$1/2^-$–	1.907	$4.6(9) \times 10^{-5}$	2.25(44)	3.67(37)
^{16}N[c]	0^-–	2^-–	0.1204	$8.7(1) \times 10^{-11}$	4.25(5)	2.06(1)
^{16}C	2^+	0^+	1.766	τ(ps) = 77(14)(19 syst) [d]	3.50(30) [g]	4.18(18)
				18.3(1.4)(4.8 syst) [e]		
				11.7(20) [f]		
				$11.4^{+0.8}_{-0.9}$(7 syst)		
^{15}C[h]	$5/2^+$	$1/2^+$	0.7400	$1.75(5) \times 10^{-7}$	0.98(2)	2.42(4)

[a]Nuclei that have one or two sd-shell neutrons outside a p-shell core.
[b]Reference [17].
[c]Reference [19].
[d]Reference [1].
[e]Reference [13].
[f]Reference [15].
[g]Average from Table I.
[h]Reference [21].

TABLE IV. Wave functions of relevant states in ^{18}O, ^{17}N, and ^{16}C.

State	Wave-function amplitudes[a]					Reference
	d^2	ds	Coll	dd'	$d's$	
^{18}O(2^+_1)	0.774	0.485	−0.347	0.056	−0.204	24
^{17}N($5/2^-_1$)	0.787	0.572		0.054	−0.226	30
^{16}C(2^+_1)	0.606	0.755		0.054	−0.244	3
	d^2	s^2	Coll			
^{18}O(g.s.)	0.848	0.438	−0.297			24
^{17}N(g.s.)	0.845	0.536				30
^{16}C(g.s.)	0.682	0.731				3

[a]The $1d_{5/2}$, $2s_{1/2}$, and $1d_{3/2}$ are denoted by d, s, and d', respectively.

nuclei in Table II within the weak-coupling approximation and compare with the experimental values. The relevant formula is [28] as follows:

$$B[E2:(jJ_c)I_i \to (jJ_c{}')I_f] = (2I_f+1)(2J_c+1)W^2(jJ_cI_f2;I_iJ_c{}')B(E2:J_c \to J_c{}'),$$

where W is a Racah coefficient. Results are listed in Table III. We see that agreement with weak coupling is reasonably good for all cases—even for ^{16}C with the newer experimental values. The agreement for ^{17}N and ^{15}C is made better if I use local neutron effective charges to renormalize those cases relative to the others, viz. if I use $e_n = (Z/A)e$, where Z and A refer to the core. The M's from this renormalization are on the second line for each nucleus in Table III. The values for ^{15,16}C and ^{16,17}N are all within expectations for such a simple model. For ^{17}N, our ratio of experimental to calculated $M(E2)$ is 0.75(8) or 0.80(8). A realistic shell-model calculation [29] gave 0.82(8) for this ratio. So, a remeasurement of the ^{17}N γ width might be warranted. In ^{16}C the ratio of d^2 to s^2 in the g.s. and the ratio of d^2 to ds in the 2^+ [3] (Table IV) are somewhat different from those in ^{18}O [24], but computing $M(E2)$ with the different wave functions makes only a small difference. [The calculated ^{16}C $M(E2)$ becomes larger by 3.7%.] Table IV also lists the relevant wave functions in ^{17}N [30].

TABLE V. Dimensionles $M(E2)$ ratios in ^{15,16}C and ^{16,17}N.

Transition	Quantity	Calculated ratio		Expt, ratio
$s \leftrightarrow d$	$M(^{15}\text{C})/M(^{16}\text{N})$	1.41[a]	1.30[b]	1.17(2)
$(sd)^2 \leftrightarrow (sd)^2$	$M(^{16}\text{C})/M(^{17}\text{N})$	0.913 [a]	0.838 [b]	1.14(10)

[a]Using the constant effective charge and oscillator parameter.
[b]Using the local effective charge $e_n = (Z/A)e$.

It is possible to estimate the value of $M(E2)$ in ^{16}C without reference to ^{18}O and ^{17}O, both of which have collective admixtures of a type that is much less likely in the other nuclei in Table II. And, one can make this estimate by forming dimensionless ratios. The $E2$'s in both ^{16}N and ^{15}C involve the transition $s \leftrightarrow d$. Simply from coupling coefficients, I expect to have $M(^{15}\text{C})/M(^{16}\text{N}) = \sqrt{2}$. If I use the same n effective charge and oscillator parameter in the two nuclei, this ratio does not depend on either quantity. If I use the local effective charge $e_n = (Z/A)e$, this ratio is reduced by a factor of $(6/14)/(7/15) = 0.918$, resulting in an expected $M(E2)$ ratio of 1.30 to be compared with the experimental ratio of $2.42/2.06 = 1.17$.

In a similar fashion, the $E2$'s in both ^{17}N and ^{16}C involve the transitions $(sd)^2 \leftrightarrow (sd)^2$. If I temporarily ignore the small differences in the $(sd)^2$ amplitudes in these two nuclei (Table IV), I expect to find $M(^{16}\text{C})/M(^{17}\text{N}) = \sqrt{(5/6)}$ with a constant effective charge and oscillator parameter. With local effective charges the expectation is 0.838 for the $M(E2)$ ratio. The experimental ratio is 1.14(10). Accounting for the differences in $(sd)^2$ amplitudes in Table IV causes an increase of 4% in ^{16}C relative to ^{17}N. These expectations are listed in Table V.

III. SUMMARY

In conclusion, I have used a simple model to compute the $B(E2)$'s from the ground state of several nuclei in which the transition is dominated by transitions within the neutron sd-shell space. The model is satisfactory for ^{17,18}O, ^{16,17}N, and ^{15,16}C. For ^{16}C, the most recent experimental $B(E2)$ values agree with the expectation from weak coupling, whereas the earlier values were about a factor of 4–6 too small.

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