Even-odd staggering of the spectroscopic factor as new evidence for α clustering

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We evidence a staggering effect of the experimental spectroscopic factors corresponding to even-even and odd-mass (odd-mass and odd-odd) α emitters. The comparison to the theoretical estimate within the standard Bardeen-Cooper-Schrieffer (BCS) approach reveals a similar staggering, but with a different behavior. It turns out that the ratio between corresponding experimental and theoretical spectroscopic factors is proportional to the experimental reduced decay width. A similar dependence was found in a previous work between the strength of the quadrupole-quadrupole α-core interaction, describing the α-decay fine structure and the reduced width. Thus, the even-odd staggering effect in the spectroscopic factor is a new evidence of the α-clustering phenomenon in medium and heavy nuclei.

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I. INTRODUCTION

The α-decay process is explained by means of the quantum mechanical penetration of the Coulomb barrier by a preformed α particle [1,2]. The first shell-model microscopic estimation of the α-particle formation probability within the R-matrix theory [3], where only one shell model single particle (sp) configuration was included, predicted a decay width by many orders of magnitude smaller in comparison to the corresponding experimental data [4,5]. By increasing the number of sp configurations, one increases substantially the decay width [6,7], but still the absolute value that is found is too small [8–10]. Later on, this result was confirmed by several papers [11–13].

An explanation for the absolute α-decay rate was found in terms of shell-model model plus α-cluster configurations [14]. This can be simulated by a standard Woods-Saxon sp mean field plus an attractive pocket-like potential located on the nuclear surface. This simple model was able to predict a universal linear dependence between the logarithm of the reduced width and the fragmentation potential, confirmed for all strong emission processes, including proton emission and heavy cluster decay [15]. This is strong evidence that the usual sp representation should contain an additional "pocket-like" component to the standard Woods-Saxon mean field [16,17].

The purpose of this paper is to demonstrate that this α-clustering property is evidenced by a new feature, namely the even-odd staggering behavior found in the α-particle spectroscopic factor of even-even, odd-mass, and odd-odd nuclei [18].

II. THEORETICAL BACKGROUND

We will estimate the theoretical amplitude for the splitting process of the parent nucleus into one α cluster and a daughter nucleus, given by the following overlap integral [19]:

$$\mathcal{F}(\mathbf{R}) = \int d\mathbf{x}_\alpha \, d\mathbf{x}_D \big[\psi_\alpha^{(\beta_\alpha)}(\mathbf{x}_\alpha)\Psi_D(\mathbf{x}_D)\big]^* \Psi_P(\mathbf{x}_P), \quad (1)$$

where $\mathbf{R}$ is the center of mass (cm) distance and $\mathbf{x}_i$ denotes the internal coordinates of the fragment i. The internal α wave function is given by the product of proton and neutron singlet gaussian states depending on a harmonic oscillator (ho) parameter $\beta_\alpha \approx 0.5$ fm^{-1} [4]. The antisymmetrization is of course implicit in the wave function of the parent nucleus. Furthermore, antisymmetrization effects in the wave function of the $D+\alpha$ system can be neglected, because we will consider cm distances that are greater than the geometrical touching point and thus Pauli correlations are small. In the analysis, we will use the spherical approach for both parent and daughter nuclei. The overlap integral (1) can be estimated by using the standard coordinate sp representation of the product between two proton and two neutron states in the harmonic oscillator (ho) basis. We use first the recoupling from jj to the LS scheme, then we change from absolute to relative and cm proton and neutron pair coordinates. Finally, by recoupling to relative and cm α coordinates one obtains the following expansion in terms of ho wave functions depending on the α-core cm coordinates [19]:

$$\mathcal{F}(R) = \sum_{N_\alpha} W(N_\alpha)\phi^{(4\beta)}_{N_\alpha 0}(R), \quad (2)$$

where the expansion coefficients,

$$W(N_\alpha) = \sum_{N_\nu, N_\pi} G_\pi(N_\pi)\, G_\nu(N_\nu) \times \langle n_\alpha 0 N_\alpha 0; 0 | N_\pi 0 N_\nu 0; 0\rangle \, \mathcal{I}^{(\beta,\beta_\alpha)}_{n_\alpha 0}, \quad (3)$$

are written in terms of Talmi-Moshinsky brackets and overlap integrals between ho wave functions depending on sp and α parameters. Here, we denoted by N_k the principal quantum numbers and by n_k the radial quantum numbers for $k = \pi, \nu, \alpha$

(proton, neutron, alpha) systems. The G coefficients contain all recoupling transformations

$$G_\tau(N_\tau) = \sum_{n_1n_2j} B_\tau(n_1n_2j) \times \left\langle (ll)0\left(\frac{1}{2}\frac{1}{2}\right)0;0\middle|\left(l\frac{1}{2}\right)j\left(l\frac{1}{2}\right)j;0\right\rangle, \quad (4)$$
$$\tau = \pi,\nu,$$

where we denoted by l the angular momentum and by j the total spin of the sp state. They are expressed in terms of the following coefficients:

$$B_\tau(n_1n_2;j) = X_{\tau j}\, c_{n_1}(\tau j)\, c_{n_2}(\tau j) \times \sum_{n_\tau}(-)^l\langle n_\tau 0 N_\tau 0;0|n_1 l n_2 l;0\rangle\, \mathcal{I}^{(\beta,\beta_\alpha)}_{n_\tau 0}, \quad (5)$$

depending on the expansion coefficients $c_n(\tau j)$ of sp wave functions with respect to the ho basis, labeled by the radial quantum number n.

We will consider in our analysis a schematic pairing Hamiltonian with constant strengths,

$$H = \sum_{\tau=\pi\nu}\sum_j \epsilon_{\tau j}N_{\tau j} - \sum_{\tau=\pi\nu}\frac{G_\tau}{4}\sum_{jj'}P^\dagger_{\tau j}P_{\tau j'}, \quad (6)$$

given in terms of the number of particles and the pair operators

$$N_{\tau j} = \sum_m a^\dagger_{\tau jm}a_{\tau jm},$$
$$P^\dagger_{\tau j} = \sum_m a^\dagger_{\tau jm}a^\dagger_{\tau j-m}(-)^{j-m}. \quad (7)$$

Thus, the BCS pair amplitudes $X_{\tau j}$ are given by

$$X_{\tau j} = \frac{1}{2}\langle D|[a_{\tau j}a_{\tau j}]_0|P\rangle = \frac{\sqrt{2j+1}}{2}u_{\tau j}v_{\tau j}, \quad \tau = \pi\nu, \quad (8)$$

in terms of standard BCS amplitudes.

Let us define the experimental spectroscopic factor for even-even, odd-mass, and odd-odd nuclei as

$$S_{\rm exp} = \frac{\Gamma_{\rm exp}}{\Gamma_{\rm theor}}. \quad (9)$$

This is the standard definition used in this field, see, for example, Refs. [20,21] and references therein. The spectroscopic factor contains the information missing from the theory that is needed to reproduce experimental data. We estimated the decay width $\Gamma_{\rm theor}$ by using a double folding α-core potential plus an attractive pocket-shaped interaction on the nuclear surface as in Ref. [22]. The procedure is identical to the one used in Ref. [23], revolving around Eqs. (6)–(8) therein. In Fig. 1 of Ref. [23] it was shown that there is a very good proportionality between $S_{\rm exp}$ and the experimental reduced width, which excludes the influence of the Coulomb barrier

$$\gamma_0^2(R) = \frac{\Gamma}{2P_0(R)}. \quad (10)$$

Here, $P_0(R)$ is the standard Coulomb monopole penetrability [19] calculated at the geometric touching radius $R = 1.2(A_D^{1/3} + 4^{1/3})$. Notice that this quantity characterizes the amount of α clustering [22].

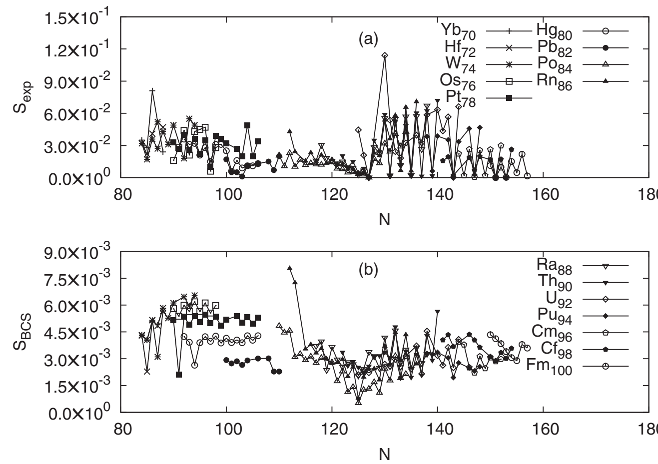

FIG. 1. (a) The experimental spectroscopic factors for even-even and odd-mass nuclei pertaining to various even-Z isotope chains versus the neutron number. (b) Same as in (a), but for the corresponding BCS quantities.

III. APPLICATION

We analyzed all available α-decay data connecting ground states on isotopic chains of reasonable length. In Fig. 1(a) we plotted the experimental spectroscopic factor versus the neutron number N for even-Z isotope chains. Notice the even-odd staggering of this quantity. It becomes much larger when crossing the magic number $N = 126$.

The theoretical reduced width is proportional to the formation amplitude (1) [19]. In Fig. 1(b) we estimated the theoretical counterpart of the spectroscopic factor given by

$$S_{\rm BCS} = \int dR\,R^2|\mathcal{F}(R)|^2. \quad (11)$$

In order to compute the BCS amplitudes defining the pair amplitude $X_{\tau j}$ (8), we used the sp spectrum generated by the

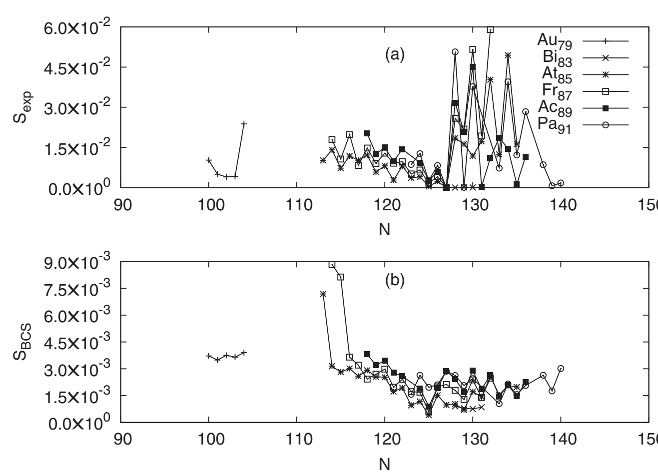

FIG. 2. Same as in Fig. 1, but for odd-Z isotope chains.

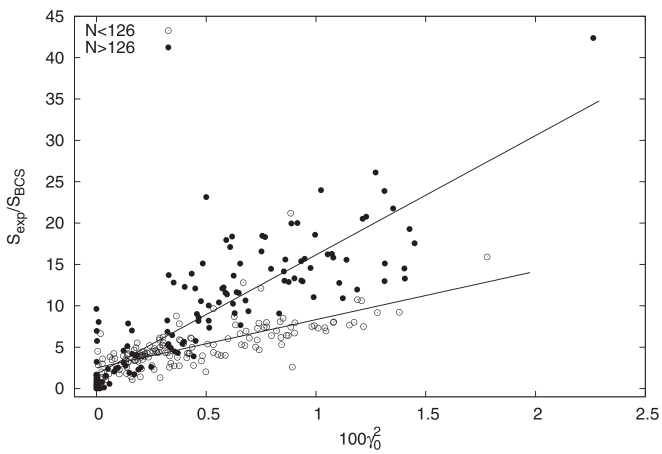

FIG. 3. The ratio between experimental and BCS spectroscopic factors versus the reduced width multiplied by 100.

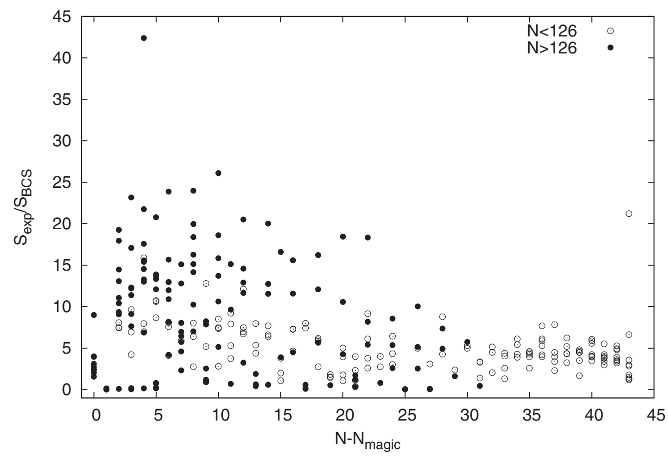

FIG. 4. The ratio between experimental and BCS spectroscopic factors versus the difference between neutron number and the closest magic number.

Woods-Saxon potential with universal parametrization [24] and experimental pairing gaps estimated by using nuclear masses [25]. For systems with an odd number of particles, the unpaired orbital was blocked. We obtained a similar staggered behavior, but with almost constant amplitude when crossing the magic number $N = 126$ along a given isotope chain. This feature is a clear evidence that the mean field plus residual two-body interaction approach is not able to describe α-clustering effects, which are very strong above the double magic nucleus ^{208}Pb [16] as seen in Fig. 1(a).

In Fig. 2(a) and 2(b) we plotted similar dependencies, but for odd-Z isotope chains. One notices a similar behavior, but with significantly smaller experimental amplitudes.

In order to understand this behavior, we plotted the ratio $S_{\text{exp}}/S_{\text{BCS}}$ in Fig. 3, versus the experimental reduced width (10), with filled circles for $N > 126$ and with open circles for $N < 126$. One notices a clear correlation between these quantities. The parameters of the corresponding fitting lines are given in Table I.

Thus, the ratio $S_{\text{exp}}/S_{\text{BCS}}$ is proportional to the α clustering defined by the reduced width (10). It is known that α clustering is stronger for nuclei above the neutron shell closure and decreases by increasing the difference $N - N_{\text{magic}}$ [22]. This property is also supported by the above mentioned ratio, as it is shown in Fig. 4.

In Ref. [22], we analyzed the α-decay fine structure for even-even emitters in terms of the α-core quadrupole-quadrupole (QQ) interaction. There, we evidenced a strong correlation between the reduced width and the strength of the QQ interaction. In Ref. [26], we confirmed this observation for the case of odd-mass nuclei. In Fig. 5 we summarize these results in order to show that the correlation in Fig. 3 is similar to the dependence of the QQ strength C versus the reduced width, given in Fig. 5(a) for even-even emitters and Fig. 5(c) for odd-mass emitters. The dependence of the QQ strength versus $N - N_{\text{magic}}$, given in Figs. 5(b) and 5(d) is similar to the correlation in Fig. 4.

The fitting parameters for the linear dependence of C on the reduced width and its quadratic dependence on the neutron number, in the cases of even-even and odd-mass nuclei having $N > 126$, are given in Table II. It is interesting to observe that the linear correlation with the reduced width is much more evident for favored α transitions to the bandhead of angular momentum projection Ω in the case of odd-mass nuclei than the transition to the ground state in the even-even case. However, the parabolic correlation between C and the neutron number is better established in the even-even case.

TABLE I. Fitting parameters for the lines $ax + b$, ($x = 100\gamma_0^2$) in Fig. 3.

Region	a	b	σ
$N < 126$	5.833	2.498	1.907
$N > 126$	14.425	1.731	3.485

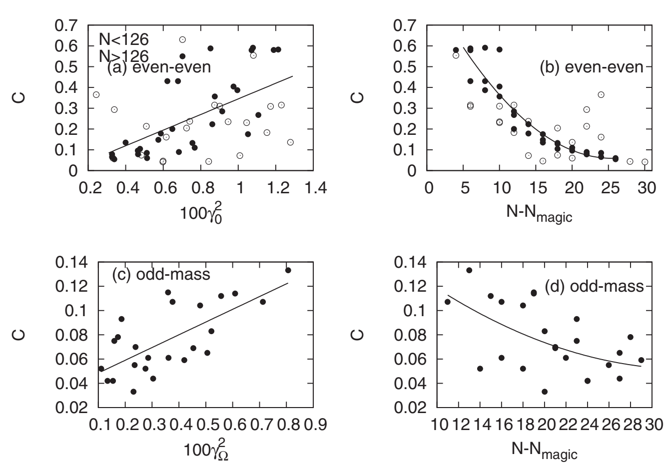

FIG. 5. (a) QQ coupling strength versus the reduced width multiplied by 100 for even-even emitters. (b) QQ coupling strength versus $N - N_{\text{magic}}$ for even-even emitters. (c) Same as in (a), but for odd-mass emitters. (d) Same as in (b), but for odd-mass emitters.

TABLE II. Fitting parameters for the curves $ax^2 + bx + c$ in Fig. 5, in the region $N > 126$.

x	Case	a	b	c	σ
$100\gamma_0^2$	even-even	–	0.376	− 0.031	0.143
$100\gamma_\Omega^2$	odd-mass	–	0.101	0.037	0.021
$N - 126$	even-even	0.00127	− 0.06481	0.88594	0.060
$N - 126$	odd-mass	0.00012	− 0.00806	0.18682	0.023

IV. CONCLUSIONS

We evidenced a staggering effect in the α-particle spectroscopic factors for even-even and odd-mass/odd-mass and odd-odd α emitters. We have shown that the ratio between experimental and BCS spectroscopic factors is proportional to the experimental reduced width. Our calculations evidenced that the standard BCS approach to estimate the α-particle formation probability, shown in Figs. 1(a) and 2(a) is not able to explain the experimental trend shown in Figs. 1(b) and 2(b). We connected this drawback to the missing clustering feature not included in the formalism. An effective way to cure for this deficiency is proposed in Ref. [27], where an additional pocket-like interaction simulating clustering effects is considered in the sp mean field. In Fig. 7 of this reference, it is shown that indeed the strength of this interaction is proportional to the experimental reduced width. This effect is similar to the dependence between the strength of the QQ α-core interaction and the experimental reduced width. Therefore this feature is a new evidence of the α-clustering phenomenon in medium and heavy nuclei.

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