

**$\bar{K}$ -induced formation of the  $f_0(980)$  and  $a_0(980)$  resonances on proton targets**Ju-Jun Xie,<sup>1,2,\*</sup> Wei-Hong Liang,<sup>3,†</sup> and E. Oset<sup>1,4,‡</sup><sup>1</sup>*Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, China*<sup>2</sup>*State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China*<sup>3</sup>*Department of Physics, Guangxi Normal University, Guilin 541004, China*<sup>4</sup>*Departamento de Física Teórica and IFIC, Centro Mixto Universidad de Valencia-CSIC Institutos de Investigación de Paterna, Aptdo. 22085, 46071 Valencia, Spain*

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We perform a calculation of the cross section for nine reactions induced by  $\bar{K}$  scattering on protons. The reactions studied are  $K^-p \rightarrow \Lambda\pi^+\pi^-$ ,  $K^-p \rightarrow \Sigma^0\pi^+\pi^-$ ,  $K^-p \rightarrow \Lambda\pi^0\eta$ ,  $K^-p \rightarrow \Sigma^0\pi^0\eta$ ,  $K^-p \rightarrow \Sigma^+\pi^-\eta$ ,  $\bar{K}^0p \rightarrow \Lambda\pi^+\eta$ ,  $\bar{K}^0p \rightarrow \Sigma^0\pi^+\eta$ ,  $\bar{K}^0p \rightarrow \Sigma^+\pi^+\pi^-$ , and  $\bar{K}^0p \rightarrow \Sigma^+\pi^0\eta$ . We find that in the reactions producing  $\pi^+\pi^-$ , a clear peak for the  $f_0(980)$  resonance is found, while no trace of  $f_0(500)$  appears. Similarly, in the cases of  $\pi\eta$  production, a strong peak is found for the  $a_0(980)$  resonance, with the characteristic strong cusp shape. Cross sections and invariant mass distributions are evaluated which should serve, by comparing them with future data, to test the dynamics of the chiral unitary approach used for the evaluations and the nature of these resonances.

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**I. INTRODUCTION**

Kaon beams are becoming a good source for new investigations in hadron physics. At intermediate energies the Japan Proton Accelerator Research Complex (J-PARC) offers good intensity secondary kaon beams up to about 2 GeV/c [1,2]. The Phi-Factory at Frascati provides low energy kaon beams [3,4]. Very recently plans have been made for a secondary meson beam facility at Jefferson Lab, which includes kaons, both charged and neutral [5]. One of the aims is to produce hyperons ( $\equiv Y$ ), which are not as well studied as nucleons or  $\Delta$ 's [6], and also cascade states, which are even less known [7,8]. In the present paper we address a different problem using kaon beams, which is the kaon-induced production of the  $f_0(980)$  and  $a_0(980)$  resonances. The reactions proposed are  $\bar{K}p \rightarrow \pi\pi Y$  and  $\bar{K}p \rightarrow \pi\eta Y$ , which produce the  $f_0(980)$  and  $a_0(980)$  resonances, respectively. These two resonances are the most emblematic scalar resonances of low energy which have generated an intense debate as to their nature, as  $q\bar{q}$ , tetraquarks, meson molecules, glueballs, dynamically generated states, etc. [9]. By now it is commonly accepted that these mesons are not standard  $q\bar{q}$  states but “extraordinary” states [10]. The coupling of some original  $q\bar{q}$  state to meson-meson components demanding unitarity has as a consequence that the meson cloud eats up the original seed becoming the largest component [11–14]. The advent of chiral dynamics in its unitarized form in coupled channels, the chiral unitary approach, has brought new light into the subject and the resonances appear from the interaction of pseudoscalar mesons, usually taken into account by coupled Bethe Salpeter equations with a kernel, or potential [15–18] extracted from

the chiral Lagrangians [19], or equivalent methods like the inverse amplitude method [20,21]. A recent review on this issue makes a detailed comparative study of work done on these issues, strongly supporting this latter view [22].

The study of  $B$  and  $D$  decays [23,24] has also offered a new valuable source of information on these states and has stimulated much theoretical work [25–32] (see also a recent review [33]). Yet, little is done in reactions involving baryons, with the exception of  $f_0(980)$  photoproduction, done in Refs. [34,35], for which predictions had been done in Ref. [36], which also have been addressed theoretically lately [37,38]. With this scarce information, the use of proton targets to produce these states, now induced by kaons, is bound to be a new good source of information which should narrow our scope on the nature of these resonances.

One of the outcomes of the chiral unitary theories is that the  $f_0(980)$  couples strongly to  $K\bar{K}$  although it decays into  $\pi\pi$  which is an open channel. On the other hand, the  $a_0(980)$  couples both to  $K\bar{K}$  and to  $\pi\eta$ , which becomes the decay channel. The use of kaon beams to produce these resonances offers one new way in which to test these ideas, since the original kaon, together with a virtual kaon that will act as a mediator of the process, will produce the resonances using the entrance channel to which they couple most strongly. We will study different processes, having  $f_0(980)$  or  $a_0(980)$  in the final state, together with a  $\Lambda$  or a  $\Sigma$  and we will use both  $K^-$  or  $\bar{K}^0$  to initiate the reaction. In total we study nine reactions for which we evaluate  $d^2\sigma/dM_{\text{inv}}d\cos(\theta)$  and make predictions for the dependence on the energy of the beam, the invariant mass of the final two mesons, and the scattering angle,  $\theta$ .

The contents of the article are organized as follows. In Sec. II, we revisit the chiral unitary approach for the  $f_0(980)$  and  $a_0(980)$  resonances. In Sec. III, we present the formalism and main ingredients of the model. In Sec. IV, we present our main results and, finally, in the last section we summarize our approach and main findings.

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## II. THE CHIRAL UNITARY APPROACH FOR THE $f_0(980)$ AND $a_0(980)$ RESONANCES

Following Refs. [15,39], we start from the coupled channels,  $\pi^+\pi^-$ ,  $\pi^0\pi^0$ ,  $\pi^0\eta$ ,  $\eta\eta$ ,  $K^+K^-$ , and  $K^0\bar{K}^0$ , and evaluate the transition potentials from the lowest order chiral Lagrangians of Ref. [19]. Explicit expressions for an  $S$  wave, which we consider here, can be seen in Refs. [25,26]. Then, by using the on-shell factorization of the Bethe-Salpeter equation in coupled channels [40,41], one has in matrix form

$$T = V + VGT; \quad T = [1 - VG]^{-1}V, \quad (1)$$

where  $V$  is the transition potential and  $G$  the loop function for two intermediate meson propagators which must be regularized. Following Ref. [25] we take a cutoff in three-momenta of 600 MeV, demanded when the  $\eta\eta$  channel is considered explicitly. Equation (1) provides the transition  $T$  matrix  $t_{ij}$  from any one to the other channels, and we shall only need the  $t_{K^+K^- \rightarrow \pi^+\pi^-}$ ,  $t_{K^0\bar{K}^0 \rightarrow \pi^+\pi^-}$ ,  $t_{K^+K^- \rightarrow \pi^0\eta}$ , and  $t_{K^0\bar{K}^0 \rightarrow \pi^0\eta}$  matrix elements. The first two matrix elements contain a pole associated with the  $f_0(980)$ , while the latter two contain the pole of the  $a_0(980)$ , although this resonance is quite singular and appears as a big cusp around the  $K\bar{K}$  threshold, both in the theory as in experiments [42,43]. The  $f_0(980)$  couples strongly to the  $K\bar{K}$  channel with  $\pi\pi$  the decay channel, and the  $a_0(980)$  couples strongly to  $K\bar{K}$  and  $\pi\eta$  channels.

## III. FORMALISM

From the perspective that the  $f_0(980)$  and  $a_0(980)$  resonances are generated from the meson-meson interaction, the picture for  $f_0(980)$  and  $a_0(980)$  antikaon-induced production proceeds via the creation of one  $K$  by the  $\bar{K}p$  initial state in a primary step and the interaction of the  $K$  and  $\bar{K}$  generating the resonances. This is provided by the mechanism depicted in Fig. 1 by means of a Feynman diagram.

Let us study the  $K^-p \rightarrow \Lambda(\Sigma^0)\pi^+\pi^-(\pi^0\eta)$  as a reference. From this reaction we shall be able to construct the other five reactions with minimal changes. In this case, we want to couple the  $K^-$  with another  $K^+$  to form the resonances. The first thing one observes is that one of the kaons (the  $K^+$ ) is necessarily off shell, since neither the  $\Lambda$  nor the  $\Sigma^0$  can decay into  $\bar{K}p$ . Then, in principle one needs the  $K^+K^- \rightarrow \pi^+\pi^-(\pi^0\eta)$  amplitude with the  $K^+$  leg off shell, which can be evaluated from the chiral Lagrangians. Yet, the structure of these Lagrangians is

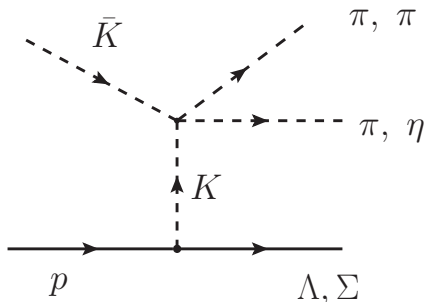


FIG. 1. Feynman diagram for the  $\bar{K}p \rightarrow \pi\pi(\pi^0\eta)Y$  reaction.

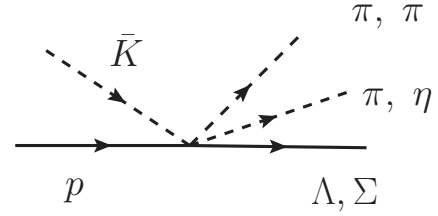


FIG. 2. Contact term stemming from the Feynman diagram of Fig. 1 from the off-shell part of the  $K^+K^- \rightarrow \pi^+\pi^-(\pi^0\eta)$  transition potential.

such that the potential can be written as [15]

$$V_{K^+K^- \rightarrow \pi^+\pi^-}(p_{K^-}, q) = V_{K^+K^- \rightarrow \pi^+\pi^-}^{\text{on}}(M_{\text{inv}}) + b(q^2 - m_{K^+}^2), \quad (2)$$

where  $p_{K^-}$  and  $q$  are the four-momenta of  $K^-$  and  $K^+$  mesons, respectively, while  $M_{\text{inv}} = \sqrt{(p_{K^-} + q)^2}$  is the invariant mass of the  $K^+K^-$  system. The term with  $b$  depends on the representation of the fields taken in the chiral Lagrangian, while the part of  $V^{\text{on}}$  does not depend upon this representation. In this sense the  $b$  term is not physical, and observables cannot depend upon it. The same chiral Lagrangians have a means to cure this, since the term  $b(q^2 - m_{K^+}^2)$  multiplied by the  $K^+$  propagator of Fig. 1 leads to a contact term as depicted in Fig. 2.

However, the chiral Lagrangian for the meson baryon [44,45], upon expanding on the number of pion fields, contains also contact terms with the same topology as the one generated from the off-shell part of the amplitude [46] which cancel this latter term. The result is that one can take just the on-shell  $K\bar{K} \rightarrow \pi\pi(\pi\eta)$  amplitude in the diagram of Fig. 1 and ignore the contact terms stemming from the meson baryon Lagrangian. These cancellations were observed before in Ref. [47] in the study of the  $\pi N \rightarrow \pi\pi N$  reaction and in Ref. [48] for the study of the pion cloud contribution to the kaon nucleus optical potential.

The other ingredient that we need for the evaluation of the diagram of Fig. 1 is the structure of the Yukawa meson-baryon-baryon vertex. Using chiral Lagrangians [44] and keeping linear terms in the meson field, the Lagrangian can be written as

$$\begin{aligned} \mathcal{L} &= \frac{D}{2} \langle \bar{B} \gamma^\mu \gamma_5 \{u_\mu, B\} \rangle + \frac{F}{2} \langle \bar{B} \gamma^\mu \gamma_5 [u_\mu, B] \rangle \\ &= \frac{D+F}{2} \langle \bar{B} \gamma^\mu \gamma_5 u_\mu B \rangle + \frac{D-F}{2} \langle \bar{B} \gamma^\mu \gamma_5 B u_\mu \rangle, \end{aligned} \quad (3)$$

where the symbol  $\langle \rangle$  stands for the trace of SU(3). The term linear in the meson field gives

$$u_\mu \simeq -\sqrt{2} \frac{\partial_\mu \Phi}{f}, \quad (4)$$

with  $f$  the pion decay constant,  $f = f_\pi = 93$  MeV, and  $\Phi$  and  $B$  the meson and baryon SU(3) field matrices

TABLE I. Coefficients for the  $\bar{K}NY$  couplings of Eq. (7).

	$K^-p \rightarrow \Lambda$	$K^-p \rightarrow \Sigma^0$	$K^-n \rightarrow \Sigma^-$
$\alpha$	$-\frac{2}{\sqrt{3}}$	0	0
$\beta$	$\frac{1}{\sqrt{3}}$	1	$\sqrt{2}$
	$\bar{K}^0n \rightarrow \Lambda$	$\bar{K}^0n \rightarrow \Sigma^0$	$\bar{K}^0p \rightarrow \Sigma^+$
$\alpha$	$-\frac{2}{\sqrt{3}}$	0	0
$\beta$	$\frac{1}{\sqrt{3}}$	-1	$\sqrt{2}$

given by

$$\Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}, \quad (5)$$

$$B = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}. \quad (6)$$

We take  $D = 0.795$  and  $F = 0.465$  in this work at the tree level, consistent with the findings of Ref. [49]. The explicit evaluation of the SU(3) matrix elements of Eq. (3) leads to the following expression

$$\mathcal{L} \rightarrow i \left( \alpha \frac{D+F}{2f} + \beta \frac{D-F}{2f} \right) \bar{u}(p', s'_B) \not{q} \gamma_5 u(p, s_B), \quad (7)$$

where  $u(p, s_B)$  and  $\bar{u}(p', s'_B)$  are the ordinary Dirac spinors of the initial and final baryons, respectively, and  $p, s_B$  and  $p', s'_B$  are the four-momenta and spins of the baryons, while  $q = p - p'$  is the four-momentum of the meson. The values of  $\alpha$  and  $\beta$  are tabulated in Table I.

Altogether we can write the amplitude for the diagram of Fig. 1 as

$$T = -it_{K\bar{K} \rightarrow MM} \frac{1}{q^2 - m_{\bar{K}}^2} \left( \alpha \frac{D+F}{2f} + \beta \frac{D-F}{2f} \right) \times \bar{u}(p', s'_{\Lambda/\Sigma}) \not{q} \gamma_5 u(p, s_p) F(q^2), \quad (8)$$

where we have added the customary Yukawa form factor that we take of the form

$$F(q^2) = \frac{\Lambda^2}{\Lambda^2 - q^2}, \quad (9)$$

with typical values of  $\Lambda$  of the order of 1 GeV.

The sum and average of  $|T|^2$  over final and initial polarizations of the baryons is easily written as

$$\begin{aligned} \overline{\sum_{s_p} \sum_{s'_{\Lambda/\Sigma}} |T_i|^2} &= |t_{K\bar{K} \rightarrow MM}^{(i)}|^2 \left( \frac{1}{q^2 - m_{\bar{K}}^2} \right)^2 \\ &\times \frac{(M_p + M')^2}{4M_p M'} [(M_p - M')^2 - q^2] \\ &\times \left( \alpha_i \frac{D+F}{2f} + \beta_i \frac{D-F}{2f} \right)^2 F^2(q^2), \quad (10) \end{aligned}$$

where  $M_p, M'$  are the masses of the proton and the final baryon ( $\Lambda$  or  $\Sigma$ ). The subindex  $i$  stands for different reactions.

We can write  $q^2$  in terms of the variables of the external particles and have

$$q^2 = M_p^2 + M'^2 - 2EE' + 2|\vec{p}||\vec{p}'| \cos \theta, \quad (11)$$

where  $\vec{p}, \vec{p}'$  and  $E, E'$  are the momenta and energies of the proton and the final baryon, and  $\theta$  is the angle between the direction of the initial and final baryon, all of them in the global center of mass frame (CM). The  $\vec{p}, \vec{p}'$  and  $E, E'$  have the form as

$$|\vec{p}| = \frac{\lambda^{1/2}(s, m_{\bar{K}}^2, M_p^2)}{2\sqrt{s}}, \quad (12)$$

$$|\vec{p}'| = \frac{\lambda^{1/2}(s, M_{\text{inv}}^2, M'^2)}{2\sqrt{s}}, \quad (13)$$

$$E = \sqrt{M_p^2 + |\vec{p}|^2}, \quad (14)$$

$$E' = \sqrt{M'^2 + |\vec{p}'|^2}, \quad (15)$$

where  $s$  is the invariant mass square of the  $\bar{K}p$  system and  $\lambda$  is the Källén function with  $\lambda(x, y, z) = (x - y - z)^2 - 4yz$ .

We can write the differential cross section as

$$\frac{d^2\sigma}{dM_{\text{inv}} d\cos\theta} = \frac{M_p M'}{32\pi^3} \frac{|\vec{p}'| |\vec{p}|}{|\vec{p}| s} \overline{\sum_{s_p} \sum_{s'_{\Lambda/\Sigma}} |T|^2}, \quad (16)$$

with  $|\vec{p}|$  the momentum of one of the mesons in the frame where the two final mesons are at rest,

$$|\vec{p}| = \frac{\lambda^{1/2}(M_{\text{inv}}^2, m_1^2, m_2^2)}{2M_{\text{inv}}}, \quad (17)$$

where  $M_{\text{inv}}$  is the invariant mass of the two mesons system, and  $m_1$  and  $m_2$  are the masses of the two mesons, respectively. Note that the  $K\bar{K} \rightarrow MM$  scattering amplitudes  $t_{K\bar{K} \rightarrow MM}$  depend on  $M_{\text{inv}}$  only.

We want to study nine reactions

$$\begin{aligned} K^-p &\rightarrow \Lambda\pi^+\pi^-, & K^-p &\rightarrow \Sigma^0\pi^+\pi^-, & K^-p &\rightarrow \Lambda\pi^0\eta, \\ K^-p &\rightarrow \Sigma^0\pi^0\eta, & K^-p &\rightarrow \Sigma^+\pi^-\eta, & \bar{K}^0p &\rightarrow \Lambda\pi^+\eta, \\ \bar{K}^0p &\rightarrow \Sigma^0\pi^+\eta, & \bar{K}^0p &\rightarrow \Sigma^+\pi^+\pi^-, & \bar{K}^0p &\rightarrow \Sigma^+\pi^0\eta. \end{aligned} \quad (18)$$

The Yukawa vertices for  $KBB$  are summarized in Table I. The  $K\bar{K} \rightarrow MM$  amplitudes are discussed above. However, only the  $I_3 = 0$  components are studied there, corresponding to zero charge. We have three cases with  $\pi\eta$  where the charge is nonzero,  $K^-p \rightarrow \Sigma^+\pi^-\eta$ ,  $\bar{K}^0p \rightarrow \Lambda\pi^+\eta$ , and  $\bar{K}^0p \rightarrow \Sigma^0\pi^+\eta$ . We can easily relate the  $K\bar{K} \rightarrow \pi\eta$  amplitudes to the  $K^+K^- \rightarrow \pi^0\eta$ , which is evaluated in the case of zero charge, using isospin symmetry. Indeed, recalling the phases  $|K^- \rangle = -|1/2, -1/2\rangle$ ,  $|\pi^+ \rangle = -|1, 1\rangle$ , we can write in terms of the total isospin

$$\begin{aligned} |K^+K^- \rangle &= -\frac{1}{\sqrt{2}}|1, 0\rangle - \frac{1}{\sqrt{2}}|0, 0\rangle, \\ |K^0K^- \rangle &= -|1, -1\rangle, & |K^+\bar{K}^0 \rangle &= |1, 1\rangle, \\ |\pi^+\eta \rangle &= -|1, 1\rangle, & |\pi^-\eta \rangle &= |1, -1\rangle, \end{aligned} \quad (19)$$

TABLE II. Matrices  $t_{K\bar{K}\rightarrow MM}$ ;  $\alpha$ ,  $\beta$  used in each reaction; and resonance obtained.

Reaction	$t_{K\bar{K}\rightarrow MM}$	$\alpha$	$\beta$	Resonance
$K^- p \rightarrow \Lambda \pi^+ \pi^-$	$t_{K^+ K^- \rightarrow \pi^+ \pi^-}$	$-\frac{2}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	$f_0(980)$
$K^- p \rightarrow \Sigma^0 \pi^+ \pi^-$	$t_{K^+ K^- \rightarrow \pi^+ \pi^-}$	0	1	$f_0(980)$
$K^- p \rightarrow \Lambda \pi^0 \eta$	$t_{K^+ K^- \rightarrow \pi^0 \eta}$	$-\frac{2}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	$a_0(980)$
$K^- p \rightarrow \Sigma^0 \pi^0 \eta$	$t_{K^+ K^- \rightarrow \pi^0 \eta}$	0	1	$a_0(980)$
$K^- p \rightarrow \Sigma^+ \pi^- \eta$	$\sqrt{2} t_{K^+ K^- \rightarrow \pi^0 \eta}$	0	$\sqrt{2}$	$a_0(980)$
$\bar{K}^0 p \rightarrow \Lambda \pi^+ \eta$	$\sqrt{2} t_{K^+ K^- \rightarrow \pi^0 \eta}$	$-\frac{2}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	$a_0(980)$
$\bar{K}^0 p \rightarrow \Sigma^0 \pi^+ \eta$	$\sqrt{2} t_{K^+ K^- \rightarrow \pi^0 \eta}$	0	1	$a_0(980)$
$\bar{K}^0 p \rightarrow \Sigma^+ \pi^+ \pi^-$	$t_{K^0 \bar{K}^0 \rightarrow \pi^+ \pi^-}$	0	$\sqrt{2}$	$f_0(980)$
$\bar{K}^0 p \rightarrow \Sigma^+ \pi^0 \eta$	$t_{K^0 \bar{K}^0 \rightarrow \pi^0 \eta}$	0	$\sqrt{2}$	$a_0(980)$

and then we find

$$\begin{aligned}
 t_{K^+ K^- \rightarrow \pi^0 \eta} &= -\frac{1}{\sqrt{2}} t_{K\bar{K} \rightarrow \pi \eta}^{I=1}, \\
 t_{K^0 \bar{K}^0 \rightarrow \pi^- \eta} &= \sqrt{2} t_{K^+ K^- \rightarrow \pi^0 \eta}, \\
 t_{K^+ \bar{K}^0 \rightarrow \pi^+ \eta} &= \sqrt{2} t_{K^+ K^- \rightarrow \pi^0 \eta}.
 \end{aligned} \tag{20}$$

With these ingredients we will use Eq. (16) to evaluate the cross section in each case, and all we must do is change the  $t_{K\bar{K}, MM}$  in each case and the values of  $\alpha$  and  $\beta$ . These magnitudes are summarized in Table II.

#### IV. RESULTS

We have a dependence of the cross section in the energy,  $M_{\text{inv}}$ , and scattering angle  $\theta$  given by Eq. (11). We first evaluate the cross section for  $\theta = 0$ , in the forward direction. In Fig. 3, we show the numerical results of  $d\sigma/dM_{\text{inv}}d\cos\theta$  for  $\cos(\theta) = 1$  as a function of  $M_{\text{inv}}$  of the  $\pi^+ \pi^-$  for  $K^- p \rightarrow \Lambda(\Sigma^0)\pi^+ \pi^-$  reactions. We have chosen  $\sqrt{s} = 2.4$  GeV,

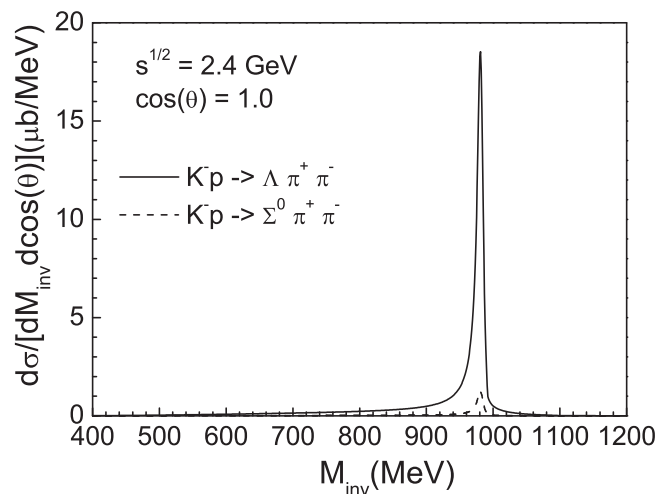


FIG. 3. Theoretical predictions for  $S$  wave  $\pi^+ \pi^-$  mass distributions for  $K^- p \rightarrow \Lambda(\Sigma^0)\pi^+ \pi^-$  reactions at  $\sqrt{s} = 2.4$  GeV and  $\cos(\theta) = 1$ .

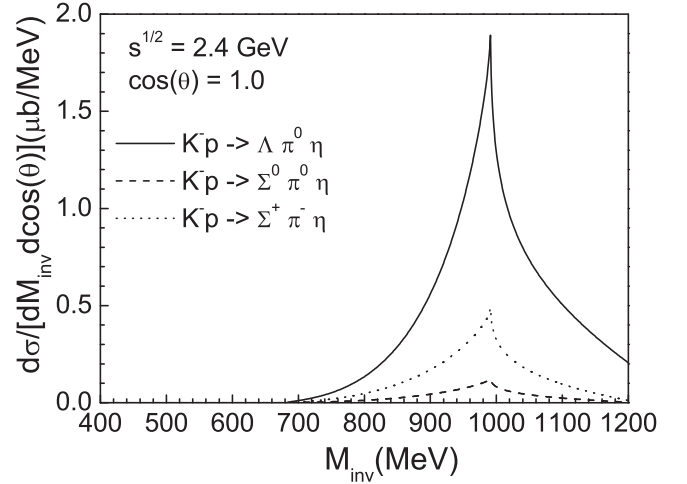


FIG. 4. Theoretical predictions for  $S$  wave  $\pi \eta$  mass distributions for  $K^- p \rightarrow \Lambda(\Sigma^0)\pi^0 \eta$  and  $K^- p \rightarrow \Sigma^+ \pi^- \eta$  reactions at  $\sqrt{s} = 2.4$  GeV and  $\cos(\theta) = 1$ .

corresponding to the  $K^-$  momentum  $p_{K^-} = 2.42$  GeV in the laboratory frame.<sup>1</sup> One can see that there is a clear peak around  $M_{\text{inv}} = 980$  MeV which is the signal for the  $f_0(980)$  resonance that was produced by the initial  $K^+ K^-$  coupled channel interactions and decaying into  $\pi^+ \pi^-$  channel. On the other hand, the magnitude of the cross section for  $\Lambda$  production is of the order of 10 times larger than for  $\Sigma^0$  production, because the coupling of  $KN\Lambda$  is stronger than the  $KN\Sigma$  coupling.

In Fig. 4, we show the numerical results of  $d\sigma/dM_{\text{inv}}d\cos\theta$  for  $\cos(\theta) = 1$  as a function of  $M_{\text{inv}}$  of the  $\pi \eta$  for  $K^- p \rightarrow \Lambda(\Sigma^0)\pi^0 \eta$  and  $K^- p \rightarrow \Sigma^+ \pi^- \eta$  reactions. In this case we see also a clear peak or cusp around  $M_{\text{inv}} = 980$  MeV which corresponds to the  $a_0(980)$  state.

Similarly, we show our results for  $\bar{K}^0 p$  reactions in Fig. 5. One can see again the clear peaks for  $a_0(980)$  and  $f_0(980)$  resonances around  $M_{\text{inv}} = 980$  MeV.

In all the reactions mentioned above, we observe clear peaks for the  $f_0(980)$  in the case of the  $\pi^+ \pi^-$  production or for the  $a_0(980)$  in the case of  $\pi \eta$  production. It is remarkable that in the case of the  $f_0(980)$  production there is no trace of the  $f_0(500)$  ( $\sigma$ ) production. This is reminiscent of what happens in  $B_s^0 \rightarrow J/\psi \pi^+ \pi^-$ , where a clear peak is seen for the  $f_0(980)$  but no trace is observed of the  $f_0(500)$  [23]. The chiral unitary approach of Ref. [25] offered an explanation for this fact. Indeed, in this reaction at the quark level, one produces  $c\bar{c}$ , that makes the  $J/\psi$ , and an  $s\bar{s}$  pair. This pair hadronizes into two mesons which are not  $\pi\pi$ , but mostly  $K\bar{K}$  or  $\eta\eta$ . Then these particles undergo a final state interaction producing the resonances. However, the  $K\bar{K}$  couples strongly to the  $f_0(980)$  resonance and very weakly to the  $f_0(500)$ , and this explains the observed features. In this case we have the  $K\bar{K}$  producing the resonances, and, similarly, we find a production of the  $f_0(980)$  and not of the  $f_0(500)$ .

<sup>1</sup>In the laboratory frame,  $s = m_K^2 + m_p^2 + 2m_p \sqrt{m_K^2 + p_K^2}$ .

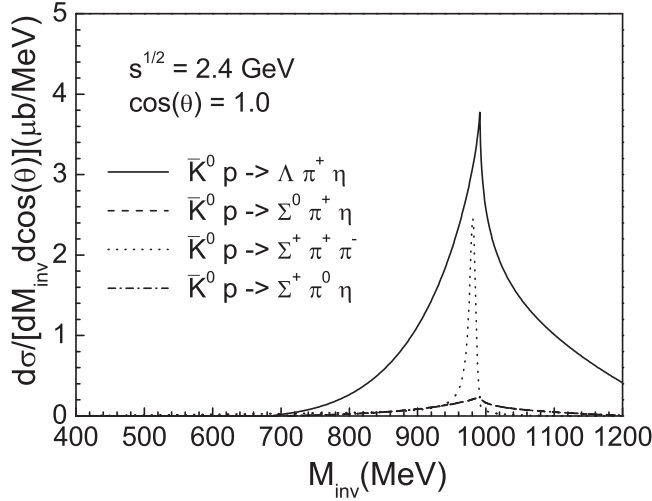


FIG. 5. Theoretical predictions for  $S$  wave  $\pi\eta$  and  $\pi^+\pi^-$  mass distributions for  $\bar{K}^0 p \rightarrow \Lambda(\Sigma^0)\pi^+\eta$  and  $\bar{K}^0 p \rightarrow \Sigma^+\pi^+\pi^-$  reactions at  $\sqrt{s} = 2.4$  GeV and  $\cos(\theta) = 1$ .

The reactions with  $\pi\eta$  in the final state produce the  $a_0(980)$  resonance. It is interesting to observe the shape. It is nearly a cusp around the  $K\bar{K}$  threshold, but with a large strength. As we remarked earlier, this feature is common to all reactions where the  $a_0(980)$  is produced with good statistics [42,43].

Furthermore, in Figs. 6 to 8 we show the results for  $d\sigma/dM_{\text{inv}}d\cos\theta$  for the  $\bar{K}p$  reactions at the peak of the invariant mass, for  $f_0(980)$ ,  $a_0(980)$  production as a function of  $\cos\theta$ . Because we considered only the contributions from the  $t$  channel  $K$  exchange, the reactions peak forward and one can see a fall down of about a factor of 10 in the cross section from forward to backward angles, where contributions from  $s$  and  $u$  channels could be dominant. On the other hand, the results obtained with our model are same for  $\bar{K}^0 p \rightarrow \Sigma^0\pi^+\eta$  and  $\bar{K}^0 p \rightarrow \Sigma^+\pi^0\eta$  reactions as shown in Fig. 6 by dashed and dash-dotted curves, respectively.

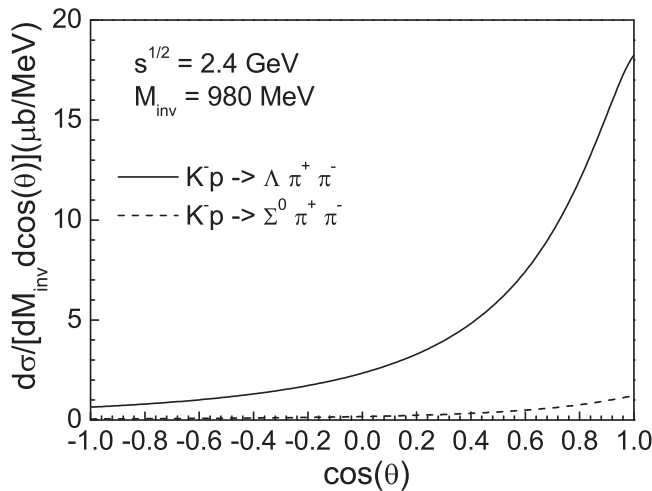


FIG. 6. Theoretical predictions for  $d\sigma/dM_{\text{inv}}d\cos\theta$  as a function of  $\cos(\theta)$  for  $\bar{K}^0 p \rightarrow \Lambda(\Sigma^0)\pi^+\pi^-$  reactions at  $\sqrt{s} = 2.4$  GeV and  $M_{\text{inv}} = 980$  MeV.

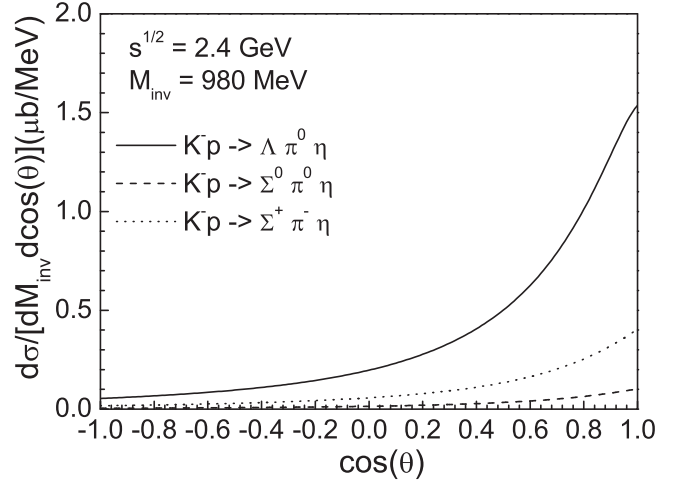


FIG. 7. Theoretical predictions for  $d\sigma/dM_{\text{inv}}d\cos\theta$  as a function of  $\cos(\theta)$  for  $\bar{K}^0 p \rightarrow \Lambda(\Sigma^0)\pi^0\eta$  and  $\bar{K}^0 p \rightarrow \Sigma^+\pi^-\eta$  reactions at  $\sqrt{s} = 2.4$  GeV and  $M_{\text{inv}} = 980$  MeV.

Finally, we now fix  $M_{\text{inv}} = 980$  MeV at the peak of the resonance and  $\cos\theta = 1$  and look at the dependence of the cross section with the energy of the  $\bar{K}$  beam. Because the  $\Lambda$  production is larger than the  $\Sigma$  production, we show only the results for the  $\Lambda$  production in Fig. 9. We observe that the cross section grows fast from the reaction threshold and reaches a peak around  $p_{\bar{K}} = 2.5$  GeV.

In the formalism developed here we have used explicitly the  $K\bar{K} \rightarrow \pi\pi$  and  $K\bar{K} \rightarrow \pi\eta$  amplitudes evaluated in the chiral unitary approach. These amplitudes encode the couplings of the  $f_0(980)$  to  $\pi\pi$  and  $K\bar{K}$  and  $a_0(980)$  to  $\pi\eta$  and  $K\bar{K}$ . The values of these couplings, together with the masses of the resonances have been used in other schemes [50] to see if the pattern obtained agrees better with a scalar nonet with ideal mixing of Okubo [51] or responds better to a dual ideal mixing [50]. The first case, followed by the vector mesons,

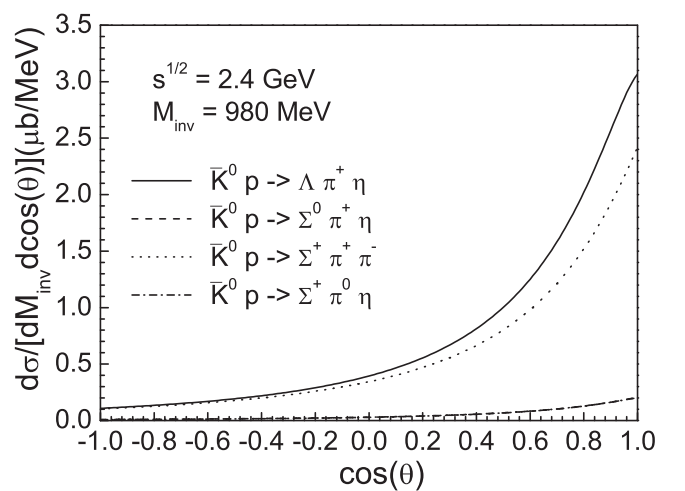


FIG. 8. Theoretical predictions for  $d\sigma/dM_{\text{inv}}d\cos\theta$  as a function of  $\cos(\theta)$  for  $\bar{K}^0 p \rightarrow \Lambda(\Sigma^0)\pi^+\eta$  and  $\bar{K}^0 p \rightarrow \Sigma^+\pi^0\eta(\pi^+\pi^-)$  reactions at  $\sqrt{s} = 2.4$  GeV and  $M_{\text{inv}} = 980$  MeV.

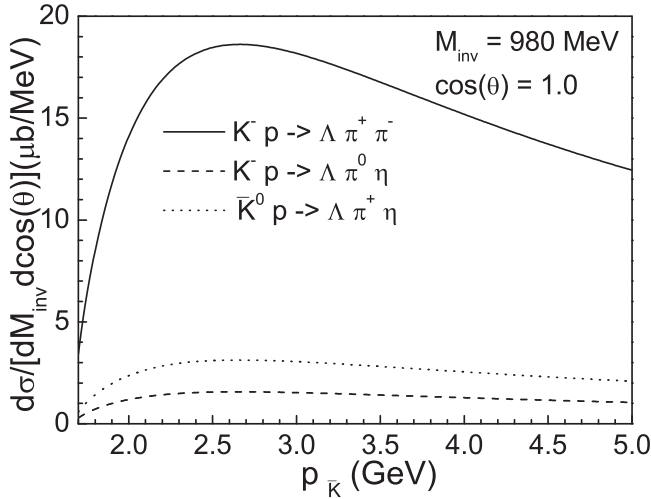


FIG. 9. Theoretical predictions for  $d\sigma/dM_{\text{inv}}d\cos\theta$  as a function of  $p_K$  for  $K^-p \rightarrow \Lambda\pi^+\pi^-$  ( $\pi^0\eta$ ) and  $\bar{K}^0p \rightarrow \Lambda\pi^+\eta$  reactions at  $\cos\theta = 1$  and  $M_{\text{inv}} = 980$  MeV.

would allow one to classify the states as  $q\bar{q}$ , while in the second case one would rather have a tetraquark nature,  $qq\bar{q}\bar{q}$ . The couplings with our model can be seen in Ref. [40] (see Table I of that paper). We reproduce these results which give

$$|g_{f_0(980)K\bar{K}}| = 3.63 \text{ GeV}, \quad (21)$$

$$|g_{f_0(500)K\bar{K}}| = 1.08 \text{ GeV}, \quad (22)$$

and

$$\frac{|g_{f_0(980)\pi\pi}|}{|g_{f_0(980)K\bar{K}}|} = 0.51, \quad (23)$$

$$\frac{|g_{f_0(500)\pi\pi}|}{|g_{f_0(500)K\bar{K}}|} = 3.94. \quad (24)$$

These results are at odds with predictions with the ordinary ideal mixing where the  $f_0(500)$  couples more strongly to  $K\bar{K}$  than to  $\pi\pi$  and the  $f_0(980)$  couplings to  $\pi\pi$  and  $K\bar{K}$  are similar [50]. They are closer to the dual mixing results [50] where the  $f_0(980)$  couples more strongly to  $K\bar{K}$  than  $\pi\pi$  and the  $f_0(500)$  coupling to  $\pi\pi$  becomes bigger. Our couplings give preference to a four quark interpretation of these states, which is hardly surprising when they have been dynamically generated from the meson-meson interaction. We should, however, note that the coupling of  $f_0(500)$  to  $\pi\pi$  is about four times bigger than to  $K\bar{K}$  in our model, while in the dual scenario of Ref. [50] they are still similar. The Large Hadron Collider beauty (LHCb) experiment of Ref. [23], where practically no trace of the  $f_0(500)$  is seen in the  $B_s \rightarrow J\psi\pi^+\pi^-$ , together with the interpretation of Ref. [25], showing that  $\pi^+\pi^-$  should come from the hadronization of an  $s\bar{s}$  pair, which gives  $K\bar{K}$  but no  $\pi\pi$ , clearly are telling us that the  $f_0(500)$  couples very weakly to  $K\bar{K}$ . Indeed,  $\pi^+\pi^-$  would come from rescattering of  $K\bar{K}$ , implying the  $g_{f_0(500)K\bar{K}} \cdot g_{f_0(500)\pi\pi}$  product, and the experiment is telling us this should be very small.

There is another issue worth mentioning. In the present work we have studied the production of  $f_0(500)$ ,  $f_0(980)$ , and  $a_0(980)$ . One may ask whether the method could be extended to deal with the production of other resonances. There are indeed other resonances that couple to  $K\bar{K}$ , only more indirectly. In Ref. [52] it was found that resonances like the  $f_2(1270)$  and  $f_0(1320)$  are mostly made from the interaction of  $\rho$  mesons. In Ref. [53] the idea was extended to SU(3), and, in addition to the  $f_2(1270)$  and  $f_0(1320)$ , other states were found, like the  $f_2'(1525)$  and  $f_0(1710)$ , which couple mostly to  $K^*\bar{K}^*$ . This  $K^*\bar{K}^*$  can go to  $K\bar{K}$  exchanging a virtual  $\pi$ , and this means that these states could also be reached with the kaon beams. Other states like the  $f_0(1500)$  could not be reproduced in Ref. [53], indicating a different nature, so we cannot say much about the  $\bar{K}$ -induced production of this state.

Concerning the  $a_0(980)$  there are other issues like the mixing with the  $a_0(1450)$ . This issue has had some attention in the context of the dynamical generation of resonances. Indeed in Refs. [12,54,55], effective Lagrangians were used by putting explicitly some  $q\bar{q}$  seed states in the Lagrangians and allowing them to couple to meson-meson components. The mechanism leads, depending on the strength of the parameters of the theory, to an extra dynamically generated state of meson-meson nature, together with another state, remnant of the original seed. The idea has been extended recently in Ref. [56], including derivative couplings typical of the chiral Lagrangians [19], and it has been shown that, with a choice of suitable parameters, a picture emerges where the  $a_0(980)$  would be the partner state dynamically generated from a seed of a  $q\bar{q}$  state that turns out to be the  $a_0(1450)$  at the end. As quoted in Ref. [56] this picture is similar to the one used in Ref. [40], where an octet of bare resonances around 1.4 GeV were introduced, which upon coupling to meson-meson components using chiral Lagrangians, gave rise to the  $a_0(980)$  as a dynamically generated state plus the  $a_0(1450)$ . Of relevance to the present work is the fact that the effect of the  $a_0(1450)$  on the  $a_0(980)$  was small [40], meaning that its omission did not change the properties of the  $a_0(980)$  obtained solely from the interaction of pseudoscalar mesons in coupled channels, as it has been done here. On the other hand, the existence of these pictures also means that the work done here could be extended to study the kaon production of the  $a_0(1450)$ .

## V. CONCLUSIONS

In this work, we study the production of  $f_0(980)$  and  $a_0(980)$  resonances in the  $\bar{K}p$  reaction with the picture that these two resonances are dynamically generated within the coupled pseudoscalar-pseudoscalar channels interaction in  $I = 0$  and 1, respectively. This is the first evaluation of the cross section for these reactions. In the cases of  $\pi^+\pi^-$  production we find a neat peak for the  $f_0(980)$  production and no production of the  $f_0(500)$ . This feature is associated with the fact that the resonance is created from  $K\bar{K}$  and the  $f_0(980)$  has a strong coupling  $K\bar{K}$  while the  $f_0(500)$  has a very small coupling to this component. Thus, in spite of the fact that the  $f_0(980)$  is observed in  $\pi^+\pi^-$ , to which the  $f_0(500)$  couples strongly, one finishes with a negligible signal for  $f_0(500)$  in this

reaction. This feature is also observed in the  $B_s \rightarrow J/\psi \pi^+ \pi^-$  reaction, and we find a natural explanation of both reactions within the chiral unitary approach to the nature of these resonances. It would be good to have the reactions proposed implemented in actual experiments to narrow the scope of possible interpretations of the nature of these resonances. Some alternative explanations for the features observed in the  $B_s \rightarrow J/\psi \pi^+ \pi^-$  reaction are given, for instance, in Ref. [57], and it would be good to see what these pictures would predict for the reactions studied here.

The reactions with the  $\pi \eta$  production give rise also to a clear peak corresponding to the  $a_0(980)$ . This resonance appears as a border line in the chiral unitary approach, corresponding to a state slightly unbound, or barely bound. The fact is that it shows up clearly in the form of a strong cusp around the  $K \bar{K}$  threshold, and this feature is observed in recent experiments with large statistics. It would be good to see what happens when the experiment is done. We should also note that our theoretical approach provides the absolute strength for both the  $f_0(980)$  and  $a_0(980)$  production, and this is also a consequence of the theoretical framework that generates dynamically these two resonances.

We have assumed a  $t$ -channel dominance, based on the strong coupling of the resonances to  $K \bar{K}$ . This has as a consequence that the nine reactions that we have studied have a definite weight, the largest differences coming from

the Yukawa meson-baryon-baryon couplings which are well known. Comparison of the strength of these reactions could serve to assert the dominance of the production model that we have assumed.

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