Gaps in nuclear spectra as traces of seniority changes in systems of both neutrons and protons

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There has been a great deal of attention given to the low-lying energy spectrum in a nucleus because of the abundance of experimental data. Likewise, perhaps to a lesser extent but still significant, the high end for a given configuration has been examined. Here, using single j shell calculations as a guide, we examine the middle part of the spectrum resulting from single j shell calculations. Seniority arguments are used to partially explain the midshell behaviors even though in general seniority is not a good quantum number for mixed systems of neutrons and protons.

I. INTRODUCTION

One of the first things the present author learned was that for a system of neutrons and protons the seniority is not a good quantum number. This is when the author joined a collaboration with John McCullen and Ben Bayman on wave functions in the $f_{7/2}$ shell [1]. Only for a system of particles of one kind, e.g., neutrons in the Ca isotopes, is seniority conserved, but once one has mixed systems the neutron-proton interaction strongly mixes states of different seniority. Nevertheless in this work it will be contended that some remnants of the seniority conserving $J = 0$ and $T = 1$ pairing interaction survive.

We have recently performed single j shell studies of a system of three protons and one neutron (or holes), e.g., ^{96}Ag as three $g_{9/2}$ proton holes and one $g_{9/2}$ neutron hole [2,3]. We focused on the yrast $T = 1$ states and came up with a $(2j - 1)$ rule, namely, that states with total angular momentum $I = 2j - 1$ lay very low in energy, sometimes being the ground state. This value of I corresponds to the middle of the calculated spectrum. The spectrum of ^{96}Ag is poorly known but the rule has been verified experimentally for lighter nuclei such as ^{44}Sc and ^{52}Mn.

In the present work we extend the study to even-even nuclei such as ^{44}Ti and ^{96}Cd. Our contention will be that there is a fairly wide gap that separates the lower part of the spectrum from the upper part.

II. CALCULATIONS

In the following four tables (Tables II–V) we present the energies of calculated spectra (MeV) as well as the difference in energies of adjacent spectra (Diff).

In a single j shell calculation for two protons and two neutrons in a single j shell the maximum angular momentum of the two protons is $2j - 1$ and likewise for the two neutrons. Hence for the four particle system the maximum angular momentum I_{max} is equal to $4j - 2$ and the middle angular momentum is $(2j - 1)$.

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$I = 2j - 1$ lay very low in energy, sometimes being the ground state. This value of I corresponds to the middle of the calculated spectrum. The spectrum of ^{96}Ag is poorly known but the rule has been verified experimentally for lighter nuclei such as ^{44}Sc and ^{52}Mn. Results from Ref. [3] are shown in Table I.

In the present work we extend the study to even-even nuclei such as ^{44}Ti, ^{52}Fe , and ^{96}Cd. Our contention will be that there is a fairly wide gap that separates the lower part of the spectrum from the upper part. The spectra are shown in Table II for the INTa (^{44}Ti) and INTb interactions (^{52}Fe) [2,4], Table III for INTd [1,4], and Table IV for the Qi interaction [5] (both for ^{96}Cd). All the states considered have isospin $T = 0$. We focus only on even I in these tables. The lower half of the spectrum consists of states up to $I = 6$ for the $f_{7/2}$ shell and up to $I = 8$ for the $g_{9/2}$ shell. The midshell angular momenta are $I = 6$ and 8, respectively. These gaps, between $I = 2j + 1$ and $I = 2j - 1$ (8–6 and 10–8, respectively), are larger than the neighboring ones. These effects persist for several different interactions. Besides the INTd interaction of Ref. [1] and Qi [3] there is the one of Coraggio *et al.* [6]. They all give qualitatively similar results.

We show more briefly the calculated odd I spectrum for ^{96}Cd with the INTd interaction. We see that the first few levels are spaced very close to each other but these is a sudden gap between $I = 9^+$ and 11^+ of 1.4384 MeV.

Let us make a brief digression to the highest energy levels. We note that in ^{44}Ti the 12^+ state is correctly predicted to be higher in energy than the 10^+ state but in ^{52}Fe the 12^+ state is lower. This is due to the fact that the $J = 7$ $T = 0$ two-body input matrix element in ^{54}Co is smaller than in ^{42}Sc. These results are in agreement with experiment. The consequences are that the 12^+ state in ^{52}Fe has a much longer half-life (45.9 s) than the one in ^{44}Ti (2.1 ns).

In ^{96}Cd the calculated $I = 16^+$ state is lower than $I = 14^+$ (and also 15^+). This implies that the 16^+ state is isomeric. This is in agreement with the experiment of Nara Singh [7].

III. EXPERIMENTAL RESULTS COMPARED WITH THEORY

There is not enough experimental data in the $g_{9/2}$ shell, i.e., ^{96}Cd, to make a comparison of theory and experiment, but such

TABLE I. Experimental and calculated spectra of odd-odd nuclei.

	I	EXP	TH
^{44}Sc	5	1.513	1.276
	6	0.271	0.381
	7	0.968	1.272
^{52}Mn	5	1.254	1.404
	6	0	0
	7	0.870	1.819
^{96}Ag	7	?	0.861
	8	0	0
	9	0.470	0.492
$h_{11/2}$	9	?	1.30
Q.Q	10	?	0.21
	11	?	0.85

TABLE II. Calculated even I spectra of ^{44}Ti (a) and ^{52}Fe (b).

I	INTa	Diffa.	INTb	Diffb
0	0		0	
2	1.1613	1.1613	1.0392	1.0392
4	2.7900	1.6287	2.7737	1.7345
6	4.0618	1.2718	4.2631	1.4634
8	6.0842	2.0244	6.0191	1.7830
10	7.3839	1.3007	7.0903	1.0712
12	7.7022	0.3183	6.9671	−0.1232

TABLE III. Calculated even I spectrum of ^{96}Cd-INTd.

I	E(MeV)	Diff.
0	0.0000	
2	0.2791	0.2971
4	0.9434	0.6463
6	1.8344	0.8905
8	1.9276	0.0923
10	3.1649	1.2373
12	3.9119	0.7470
14	4.1382	0.2263
16	3.4830	−0.6552

TABLE IV. Calculated spectrum of ^{96}Cd-Qi.

I	E(MeV)	Diff.
0	0.000	
2	0.8972	0.8972
4	2.0105	1.1133
6	3.0576	1.0411
8	3.4324	0.3748
10	5.1134	1.6810
12	5.6409	0.5275
14	5.7687	0.1278
16	5.5531	−0.2156

TABLE V. Calculated odd I spectrum of ^{96}Cd-INTd.

I	E(MeV)	Diff.
1	4.1160	
3	4.2220	0.1060
5	4.3708	0.1486
7	4.4944	0.1236
9	4.1256	−0.3688
11	5.5640	1.4384
13	5.8961	0.3311
15	6.2787	0.3836

a comparison can be made in the $f_{7/2}$ region. This is shown in Table VI. The interaction used for ^{44}Ti is INTa from the two-particle spectrum of ^{42}Sc; for ^{52}Fe we use INTb from the two-hole spectrum of ^{54}Co.

In ^{44}Ti we have E(6)-E(4) = 1.561 MeV, E(8)-E(6) = 2.493 MeV, and E(10)-E(8) = 1.163 MeV. In ^{52}Fe we have E(6)-E(4)= 1.941 MeV, E(8) -E(6) =2.035 MeV, and E(10)-E(8) = 1.021 MeV.

These empirical results are in qualitative agreement with the predictions of the single j shell model—that there is indeed a midshell gap in energies of levels below midshell and those above midshell. The effect is not as pronounced in ^{52}Fe as it is in ^{44}Ti but it is there nevertheless. It would be of great interest to find more details of the spectra of ^{96}Cd and ^{96}Ag to see if indeed there is such a gap in these heavier nuclei.

IV. EXPLANATION VIA PAIRING INTERACTION

We feel we can explain the above gap in the spectrum via the pairing interaction of Flowers [8] and Edmonds and Flowers [9]. The two body matrix elements in, say, the $g_{9/2}$ shell from $J = 0$ to 9 are $-A,0,0,0,0,0,0,0,0,0$, with A positive. The expression for the energies is $E = C \{(n-v)/4(4j+8-n-v) - T(T+1) + t(t+1)\}$, with C negative. Here n is the number of valence nucleons, v is the seniority, T is the total isospin, and t is the reduced isospin. The relation between A and C has been discussed by Harper and Zamick [10]. They note that for any j shell $C = -A/(2j+1)$. This can be obtained by noting that the quantum numbers (T,t,v) for the lowest $I = 0, T = 0$ state are $(0,0,0)$ while for the unique $I = 0, T = 2$ state they are $(2,0,0)$.

In the $N = Z$ nuclei we are dealing only with $T = 0$ states while for the odd-odd nuclei in Ref. [1] all states had $T = 1$.

TABLE VI. Comparison of experiment and theory (INT) for the gaps.

^{44}Ti	MeV	EXPT.	INTa
INTa	E(6)-E(4)	1.5611	1.272
	E(8)-E(6)	2.493	2.024
	E(10)-E(8)	1.163	1.307
^{52}Fe		EXPT.	INTb
INTb	E(6)-E(4)	1.941	1.403
	E(8)-E(6)	2.035	1.783
	E(10)-E(8)	1.021	1.072

TABLE VII. Calculated spectra with schematic interactions ^{96}Cd.

I	E(0)	E(9)	E(0,9)
Even			
0	0.0000	0.5294	0.0000
2	1	0.5294	0.6370
4	1	0.5294	0.9292
6	1	0.5294	1.1965
8	1	0.5286	1.2562
10	2.2	0.5253	1.6071
12	2.2	0.4835	1.5674
14	2.2	0.3285	1.4124
16	2.2	0.0000	1.0839
Odd			
1	1.4	1.5298	2.4323
3	1.4	1.5278	2.1862
5	1.4	1.5114	2.1059
7	1.4	1.4247	2.1113
9	1.4	1.0293	1.4943
11	2.2	1.0247	2.1086
13	2.2	0.9700	2.0539
15	2.2	0.7941	1.8787

The resulting yrast spectra for even I are as follows when we choose $A = 1$: E(0) = 0, E(I) = 1 for $I = 2, 4, 6,$ and 8 and E(I) = 2.2 for $I = 10, 12, 14,$ and 16. So we see there is a break after $I = 8$. We can understand this because the energy for this interaction does not depend explicitly on I. But it does depend on seniority. For $I = 2, 4, 6,$ and 8 we can have seniority $v = 2$ states by coupling one pair of nucleons to $J = 0$. But we cannot reach $I = 10, 12, 14,$ or 16 by coupling one pair to $J = 0$. Hence the latter states must have seniority $v = 4$.

We can also look at the odd spectra. For $I = 1, 3, 5, 7,$ and 9 E(I) is equal to 1.4 while for $I = 11, 13,$ and 15 E(I) = 2.2 Hence there is a predicted gap between $I = 9$ and 11. The

TABLE VIII. Calculated spectra with schematic interactions ^{96}Ag.

I	E(0)	E(9)	E(0,9)
0	0.0000	1.735	0.5190
1	0.8	0.7357	0.9380
2	0.6	0.7349	0.5310
3	0.8	0.7337	0.6919
4	0.6	0.7213	0.3025
5	0.8	0.7173	0.6115
6	0.6	0.6325	0.2361
7	0.8	0.6306	0.6170
8	0.6	0.2353	0.0544
9	0.8	0.2352	0.0000
10	1.6	0.2360	0.6196
11	1.6	0.2306	0.6142
12	1.6	0.2504	0.6340
13	1.6	0.1759	0.5596
14	1.6	0.4412	0.8248
15	1.6	0.0000	0.3837

TABLE IX. $f_{7/2}$ matrix elements.

J	^{42}Sc	^{54}Co	Q.Q
0	0	0	0
1	0.6111	0.5723	0.4096
2	1.5863	1.4465	1.1471
3	1.4904	1.8244	2.0483
4	2.8153	2.6450	2.8677
5	1.5101	2.1490	3.2774
6	3.2420	2.9600	2.8677
7	0.6163	0.1990	1.1471

same seniority argument applies for odd I. Such a gap also appears with more realistic interactions as seen in Table V.

It should be emphasized that for most interactions, e.g., INT seniority is not a good quantum number for a system of both neutrons and protons. With the interactions of Edmonds and Flowers [8,9] seniority is a good quantum number. Some remnants of the seniority behavior in their simple interaction seem to have not been completely lost in the more complex INT and other interactions.

For completeness we show results with schematic interactions for yrast states in ^{96}Cd in Table VII and for yrast states in ^{96}Ag in Table VIII.

The interactions are for E(0) $J = 0$ pairing -1.0, 0,0,0,0,0,0,0,0; for E(9) J_{max} pairing 0,0,0,0,0,0,0,0, -1.0; and for E(0,9) sum of above, $-1.0,$ 0,0,0,0,0,0,0, -1.0

Note that with E(0) there is a high degeneracy—the low-lying spectra do not spread out. Also note that with E(9) the ground state does not have $I = 0$. Rather it has $I = 16$, the maximum I, as the ground state. Although both E(0) and E(9) yield terrible spectra when we mix them to form E(0,9) we have the beginning of a reasonable spectrum, with $I = 0$ as the ground state and some spreading out of the energy levels.

APPENDIX: INTERACTIONS USED

The interactions used in the $f_{7/2}$ shell are shown in Table IX. The interactions used in the $g_{9/2}$ shell are shown in Table X. We also show the Q.Q interaction. In some but not all cases a constant has been added so that the $J = 0$ matrix element is zero. This does not affect the spectra.

TABLE X. $g_{9/2}$ matrix elements.

J	INTd	Qi	Corragio	Q.Q
0	0.0000	0.0000	-2.3170	-1.0000
1	1.1387	1.2200	-1.4880	-0.8788
2	1.3947	1.4580	-0.6670	-0.6515
3	1.8230	1.5920	-0.4400	-0.3485
4	2.0283	2.2830	-0.1000	-0.0152
5	1.9215	1.8820	-0.2710	0.2789
6	2.2802	2.5490	0.0660	0.4848
7	1.8797	1.9300	-0.4040	0.4848
8	2.4275	2.6880	0.2100	0.1818
9	0.7500	0.6260	-1.4020	-0.5454

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