

# Widths and structure of unbound states in $^{12}\text{Be}$

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I have estimated the energies of several unbound states of  $^{12}\text{Be}$  and their spectroscopic factors for decay to the  $1/2^+$  ground state and  $1/2^-$  first-excited state of  $^{11}\text{Be}$ . These are then used to estimate the expected widths for such decays. Results for likely  $3^-$  and  $4^+$  states are good. I find that only  $0^+$  and/or  $2^+$  states can account for the width recently observed for a state decaying with a centroid neutron energy of 1.24 MeV.

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## I. INTRODUCTION

Above an excitation energy of 3.171(2)(3.49) MeV, states of  $^{12}\text{Be}$  can decay by  $1n$  emission [1] to the  $1/2^+$  ground state (g.s.) ( $1/2^-$ , 0.32-MeV state) of  $^{11}\text{Be}$ . Above 3.672(2) MeV,  $2n$  emission to the g.s. of  $^{10}\text{Be}$  is allowed. Very few unbound states of  $^{12}\text{Be}$  are known. Here, I compare their experimental widths with ones calculated using a variety of assumptions about their structure.

In the reaction  $^{10}\text{Be}(t,p)$  [2,3], four states were strongly excited: the  $0^+$  g.s.,  $2^+$  at 2.11 MeV, probable  $3^-$  at 4.58 MeV, and likely  $4^+$  at 5.72 MeV. A weaker state at 2.70 MeV was later determined [4] to be  $1^-$ , and an excited  $0^+$  state was discovered at 2.25 MeV [5]. In the  $(t,p)$  reaction, a weak broad state was observed near 5.4 MeV and a hint of another just below the 5.72-MeV peak....

Very recently, a state (or collection of states) was populated in proton removal from  $^{13}\text{B}$  and observed to decay by  $1n$  emission to the g.s. and/or first-excited state of  $^{11}\text{Be}$  with a centroid decay energy of  $E_n = 1.243(21)$  MeV and an extracted width of 634(60) keV [6]. The large width required  $\ell \leq 1$  if a single state and the peak shape appeared to eliminate  $\ell = 0$ . Experimental results thus require  $J^\pi = 0^\pm, 1^\pm$ , or  $2^\pm$  for the state exhibiting the bulk of the decays. Reference [6] expressed a preference for  $2^-(1^-)$ . Because the identity of the final state could not be determined, the excitation of the  $^{12}\text{Be}$  state is not known, but it is in the range of 4.4–4.8 MeV.

In an earlier paper [7], I discussed (in general terms) those results as they relate to widths expected for  $2^+$  and  $2^-$  states. Here, I present calculated energies, spectroscopic factors, and widths for all the states expected in the energy region of the state observed in Ref. [6].

## II. CALCULATIONS AND RESULTS

First, I have computed the widths to be expected for the  $3^-$  and  $4^+$  states, and then for other unbound states expected in this energy region, I use a Woods-Saxon nuclear potential, having  $r_0, a = 1.26, 0.60$  fm with the well depth adjusted to reproduce the energy of the state. The single-particle width is then computed from the phase shifts.

The state at 5.72 MeV has the energy and  $(t,p)$  strength appropriate to the lowest  $4^+$  state in this nucleus [3,8]. In a simple  $(sd)^2$  shell-model calculation, its configuration is predominantly  $d^2$  with a small amount of  $dd'$ , where  $d$  and  $d'$  refer to  $d_{5/2}$  and  $d_{3/2}$ , respectively. Thus, its allowed decay

is to the  $5/2^+$  state at 1.78 MeV in  $^{11}\text{Be}$ . Decay to states of lower  $J$  would require the presence of neutrons in the  $1f$  and/or higher orbitals and can thus be ignored. The observed width is 86(15) keV [2,3], and the neutron decay energy is 0.77 MeV for which the computed single-particle width for  $\ell = 2$  decay is 54 keV, resulting in a spectroscopic factor of  $S = \Gamma_{\text{exp}}/\Gamma_{\text{sp}} = 1.59(28)$ . The rigorous upper limit on this spectroscopic factor is 2.0, and the  $S$  for  $\ell = 2$  to a pure  $d_{5/2}$  state is about 1.95 [8]. However, the 1.78-MeV state of  $^{11}\text{Be}$  is not pure  $d_{5/2}$ . Its spectroscopic factor computed from its width is 0.58(8) [9] so that in the simplest model, the  $S$  for  $4^+ \rightarrow 5/2^+$  would be 1.13(16), in reasonable agreement with the value deduced here.

The state at 4.58 MeV is now thought [10–12] to be the first  $3^-$  state in  $^{12}\text{Be}$ . In the reaction  $^{10}\text{Be}(t,p)$  [3], it was suggested to have  $J^\pi = 2^+$  or  $3^-$  on the basis of the shape of its angular distribution. Millener [10] was the first to point out that it was much too strong to be  $2^+$  because nearly all the  $2^+$  strength expected in the  $psd$  space is exhausted by the first  $2^+$  state at 2.1 MeV. He suggested a  $3^-$  or a  $3^-/2^+$  doublet. A later calculation [11] demonstrated the correctness of his argument. Within a few percent, this state has the appropriate cross section for a  $3^-$  state reached by stripping of a  $p_{1/2}d_{5/2}$  pair. Its energy agrees very well [12] with that calculated in a simple model that treats it as being of the structure  $^{11}\text{Be}(1/2^-) \otimes d$ . Thus, it should decay exclusively to the  $1/2^-$  state with  $S = 1$ . Decay to the g.s. would require participation of the  $1f$  orbital, which we can ignore. The single-particle width for this energy of 1.09 MeV is 118 keV, giving  $S = 0.91(14)$ —in good agreement with the model. The  $3^-$  and  $4^+$  results are summarized in Table I.

Concerning the new unbound state [6] that decays with a neutron energy of 1.24 MeV, we need to consider several possibilities for its  $J^\pi$  and structure. Except for  $0^-$ , no

TABLE I. Decays of first  $3^-$  and  $4^+$  states in  $^{12}\text{Be}$  (energies in MeV and widths in keV).

Initial state	$J^\pi$	Final state		Decay					
		$E_x$	$J^\pi$	$\ell$	$E_n$	$\Gamma_{\text{exp}}$	$\Gamma_{\text{sp}}$	$S_{\text{exp}} = \Gamma_{\text{exp}}/\Gamma_{\text{sp}}$	$S_{\text{th}}$
4.58	$3^-$	0.320	$1/2^-$	2	1.09	107(17)	118	0.91(14)	1.0
5.72	$4^+$	1.78	$5/2^+$	2	0.77	86(15)	54	1.59(28)	1.13–1.95 <sup>a</sup>

<sup>a</sup>See the text.

TABLE II. Dominant configurations and predicted excitation energies (MeV) and spectroscopic factors of relevant negative- and positive-parity states of  $^{12}\text{Be}$ .

$J^\pi$	Dominant configuration	$E_x$	$S(1/2^+)$	$S(1/2^-)$
$2^-$	$^{11}\text{Be}(1/2^-) \otimes d$	5.12	0	$\sim 1.0$
$1_2^-$	$^{11}\text{Be}(3/2^-) \otimes s$	5.09	$\sim 0$	$\sim 0$
$2_2^+$	$^{10}\text{Be}(2^+) \otimes (sd)_0^2$	4.03	0.009	0
$0_3^+$	Second $^{10}\text{Be}(\text{g.s.}) \otimes (sd)^2 0^+$	4.35	0.31	$1.8\varepsilon^{2a}$
$2_3^+$	Second $^{10}\text{Be}(\text{g.s.}) \otimes (sd)^2 2^+$	4.68	0.09	0.020
$0_4^+$	$^{10}\text{Be}(2^+) \otimes (sd)^2 2^+$	5.48	0.24	0.02
$2_4^+$	$^{12}\text{Be } p\text{-shell } 2^+$	5.46	0.12	0.05 or 0.14 <sup>b</sup>

<sup>a</sup> $\varepsilon^2$  is the intensity of  $p$ -shell  $0^+$  in this state, assumed to be 0.02 here.

<sup>b</sup>See the text.

unobserved states are predicted below 4 MeV, but in the region of 4 to 5.5 MeV, several states are expected—including the third (and perhaps fourth)  $0^+$ , the second (and perhaps third)  $2^+$ , the first  $2^-$ , and perhaps a second  $1^-$  state. These possibilities are listed in Table II. States above this region, such as the  $2^-$  with configuration  $^{11}\text{Be}(3/2^-) \otimes s$  are not included. I have estimated the expected excitation energies of these states in a model that includes shell-model and weak-coupling considerations. These are also listed in Table II.

The aim now is to evaluate spectroscopic factors connecting these states to the first two states of  $^{11}\text{Be}$ . The  $2^-$  state has  $S = 1$  to  $1/2^-$ , whereas  $1_2^-$  has  $S = 0$ . Of course, both have  $S = 0$  to the g.s. The lowest undiscovered positive-parity state is  $2_2^+$  with an unperturbed energy of 4.03 MeV. Its structure is two  $sd$ -shell neutrons with  $J = 0$ , coupled to the  $2^+$  state of  $^{10}\text{Be}$ . It can decay to the g.s. via the small component of  $2^+ \otimes d$  in the g.s. The spectroscopic factor for  $2^+ \rightarrow \text{g.s.}$  is then 0.009. It has no strength to the  $1/2^-$ . Consideration of  $B(E2)$ 's in  $^{12}\text{Be}$  concluded [13] that the first  $2^+$  contained about 19% of this configuration so that mixing would put the  $2_2^+$  state somewhat higher. The third  $0^+$  state is primarily the second  $(sd)^2 0^+$  state and is predicted at 4.35 MeV. It has  $S = 0.31$  to the g.s. This  $0^+$  state could contain a small component  $\varepsilon$  of the  $p$ -shell  $0^+$ , although the first two  $0^+$  states exhaust most of this strength. But, because the  $p$ -shell  $S$  for

$0^+ \rightarrow 1/2^-$  is so large,  $S$  for  $0_3^+$  to  $1/2^-$  is  $1.8\varepsilon^2$ . For present purposes, I take  $\varepsilon^2$  to be about 0.02. The third  $2^+$  state is predominantly the second  $(sd)^2 2^+$  state and is expected near 4.68 MeV. It has  $S = 0.09$  for the g.s. and  $S = 0.02$  for  $1/2^-$ .

The fourth  $0^+$  and  $2^+$  states are both expected near 5.5 MeV. This  $0^+$  configuration is an  $(sd)^2 2^+$  state coupled to the  $2^+$  of  $^{10}\text{Be}$ . It has  $S = 0.24$  to  $1/2^+$  and  $S = 0.02$  to  $1/2^-$ . The  $2^+$  state has about 80% of the  $p$ -shell  $2^+$  state and about 20% of the lowest  $(sd)^2 2^+$  state. It has  $S = 0.12$  to the g.s. [from the  $(sd)^2$  component]. In a pure  $p$ -shell calculation,  $S$  is about 0.05 for decay to the  $1/2^-$ . However, in the  $^9\text{Be}(t, p)$  reaction [14], the  $1/2^-$  state was populated with a cross section that was about twice that expected for the pure  $p$ -shell state. The conclusion there was that it contained about 9% of the configuration  $^9\text{Be}(\text{g.s.}) \otimes [(sd)^2 2^+]$ . That impurity component would increase  $S$  from 0.05 to 0.14 for  $2_4^+ \rightarrow 1/2^-$ . In what follows, I give results for both possibilities. Both of these latter states may be too high in energy to be candidates for the state(s) observed in proton removal, but I include them for completeness.

The energies of the other states in Table II are all close enough that they could be considered as candidates for the state(s) observed by Ref. [6]. I have computed single-particle widths for all these states to decay by  $\ell = 1$  to one of the first two states of  $^{11}\text{Be}$  and by  $\ell = 0$  and/or 2 to the other one. These are listed in Table III. As pointed out in Ref. [6], if the decaying state has negative parity, the observed  $\ell = 1$  decay must be to the g.s. so that the other allowed decay (with  $\ell = 0$  and/or 2) would be to the first-excited state—thus having a smaller decay energy. However, if the decaying state has positive parity, its observed decay is to the first-excited state, and the other allowed decay has higher energy. These are all listed in Table III.

Single-particle widths for  $s$ -wave neutron resonances are difficult to calculate, but they should vary as  $\sqrt{E_n}$ . For the present purposes, a rough estimate suffices as we will see. The  $sp$  width for  $p$ -wave decay in this case is also very large and thus difficult to compute. Again, a rough estimate turns out to be sufficient. The expected spectroscopic factors vary over quite a wide range from 0 to 1. However, they are small for all but one of the expected  $\ell = 1$  decays, namely, the decay from the fourth  $2^+$  state, whose configuration is dominated

TABLE III. Estimated widths for various assumptions about the identity of the  $^{12}\text{Be}$  state(s) [6] decaying via  $\ell = 1$  and  $E_n = 1.24$  MeV (energies in MeV and widths in keV).

$J^\pi$	Decay to $1/2^+$					Decay to $1/2^-$					$\Gamma_{\text{tot}}$ (calc)
	$E_n$	$\ell$	$S$	$\Gamma_{\text{sp}}$	$\Gamma_{\text{calc}}$	$E_n$	$\ell$	$S$	$\Gamma_{\text{sp}}$	$\Gamma_{\text{calc}}$	
$2^-$	1.24	1	0	$\sim 1600$	0	0.92	2	$\sim 1.0$	93	93	93
$1_2^-$	1.24	1	0	$\sim 1600$	0	0.92	2	$\sim 0$	93	$\sim 0$	Small
						0.92	0	$\sim 0$	950	$\sim 0$	Small
$0_3^+$	1.56	0	0.31	(1230)	(381)	1.24	1	0.04	$\sim 1600$	$\sim 64$	445?
$0_4^+$	1.56	0	0.24	(1230)	(295)	1.24	1	0.02	$\sim 1600$	$\sim 32$	327?
$2_2^+$	1.56	2	0.009	290	3	1.24	1	0	$\sim 1600$	0	3
$2_3^+$	1.56	2	0.09	290	26	1.24	1	0.020	$\sim 1600$	32	58
$2_4^+$	1.56	2	0.12	290	35	1.24	1	0.05;0.14	$\sim 1600$	80;224	115;260

TABLE IV. Energies (MeV) and widths (keV) if primary decay is  $\ell = 0$ , rather than  $\ell = 1$ .

$J^\pi$	Decay to $1/2^+$					Decay to $1/2^-$					$\Gamma_{\text{tot}}$ (calc)
	$E_n$	$\ell$	$S$	$\Gamma_{\text{sp}}$	$\Gamma_{\text{calc}}$	$E_n$	$\ell$	$S$	$\Gamma_{\text{sp}}$	$\Gamma_{\text{calc}}$	
$0_3^+$	1.24	0	0.31	(1100)	(341)	0.92	1	0.04	950	38	379?
$0_4^+$	1.24	0	0.24	(1100)	(264)	0.92	1	0.02	950	19	283?

by the  $p$ -shell component. For a pure  $p$ -shell  $1/2^-$  state,  $S$  is only 0.05, but results of the  $^9\text{Be}(t, p) ^{11}\text{Be}$  reaction [14] suggested an  $(sd)^2$  component in this state. The addition of that component (only about 9% in intensity) increases  $S$  to about 0.14. Results are given for both values.

For each decay, I have computed the expected width as  $\Gamma_{\text{calc}} = S\Gamma_{\text{sp}}$ . The last column in Table III lists the sum of these expected widths for the two decays. Note that these are small for both negative-parity states and for two of the three  $2^+$  states. Only the  $0^+$  states and the fourth  $2^+$  have expected total widths consistent with the reported width of 634(60) keV. If an apparent enhancement factor of about 1.6 [15] is removed, the experimental result is reduced to about 400(40) keV. Even

so, the present calculations require the decaying state(s) to have  $J^\pi = 0^+$  and/or  $2^+$ . Note that for the  $2^+$  state, most of the expected width is to the  $1/2^-$  state, whereas for both  $0^+$  states, most of the predicted width is to the g.s. Thus, if the final state in the decay could be identified in a future experiment, the  $0^+/2^+$  ambiguity could be removed.

For completeness, I have also computed the expected widths if the observed primary decay actually has  $\ell = 0$ , rather than  $\ell = 1$ . The results are listed in Table IV.

### III. SUMMARY

I have computed single-particle widths for probable  $3^-$  and  $4^+$  states in  $^{12}\text{Be}$ . With theoretical spectroscopic factors from a simple shell model, the expected widths  $\Gamma_{\text{calc}} = S\Gamma_{\text{sp}}$  are in good agreement with the experimental results for these two states. For several other expected but unknown states in the region of 4.0 to 5.5 MeV, I have estimated the energies, spectroscopic factors, and  $sp$  widths. I then compared the expected widths with the reported width [6] for states(s) decaying by  $\ell = 1$  to one or both of the first two states of  $^{11}\text{Be}$  with a centroid decay energy of 1.24 MeV. The results eliminate negative parity as the source of the decay and rule out two  $2^+$  states as the decaying state—leaving one or both  $0^+$  states and one  $2^+$  as candidates.

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