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Particle-rotor versus particle-vibration features in *g* factors of ^{111}Cd and ^{113}Cd

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The emergence and evolution of collective excitations in complex nuclei remains a central problem in the quest to understand the nuclear many-body problem. Nuclear quadrupole collectivity is usually investigated via electric quadrupole observables. Here, however, we measure the g factors of low-excitation states in ^{111}Cd and ^{113}Cd and show that they are sensitive to the nature of the collectivity in these nuclei in ways that the electric quadrupole observables are not. The particle-vibration model, which assumes spherical core excitations, cannot explain the g factors, whereas a particle-rotor model with a small, nonzero core deformation does. The contrast of the two models is made stark by the fact that they begin from the same limiting g-factor values: It is shown that when an odd nucleon occupies a spherical orbit with angular momentum $j = 1/2$, or a deformed orbit with $j = 1/2$ parentage, the particle-vibration model and the particle-rotor model both reduce to the same g-factor value in their respective limits of zero particle-vibration coupling or zero deformation.

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Nuclei are many-body quantum systems in which the correlated motions of many individual nucleons can give rise to collective excitations. Conceptually and historically, collective nuclei have been classified as either spherical or deformed. Nuclei having a proton or neutron number close to a magic number tend to be spherical with low-energy vibrational excitations. As the numbers of protons and neutrons both depart from magic numbers, nuclei develop permanently deformed shapes and the lowest excitations become rotations. The development of nuclear collectivity and the emergence of simple excitation patterns such as quadrupole vibrations and rotations in complex nuclei remain active areas of investigation. In such studies the focus is usually on electric quadrupole ($E2$) transition rates and moments as indicators of the onset of collectivity and deformation. In contrast, the focus here is on magnetic dipole moments (or equivalently g factors), which are generally sensitive to single-particle features of the nuclear wave function. It is demonstrated that the g factors can give unique insight into the development of nuclear collectivity by distinguishing between spherical vibrations and rotations at weak deformation, in ways that $E2$ observables cannot.

With $Z = 48$ protons, two less than magic $Z = 50$, the isotopes ^{111}Cd and ^{113}Cd are odd-A neighbors of the even isotopes ^{110,112,114}Cd, which are often cited as textbook examples of spherical vibrational nuclei [1,2]. Recent work has begun to challenge this interpretation, particulary in relation to states in the two- and three-phonon multiplets and concerning the role of shape-coexisting so-called intruder configurations [3,4]. These even-A Cd isotopes have also been considered recently in terms of an effective field theory for nuclear vibrations, suggesting that the $E2$ properties of the low-lying states are consistent with anharmonic quadrupole vibrations [5].

The structures of the odd-A Cd isotopes result from an interplay between the odd-nucleon and the collective core. The core-excitation model of de Shalit [6], wherein the motion of a single-odd nucleon in a unique orbit is weakly coupled to a vibrational core, provides a useful benchmark. The weak-coupling states also serve as the basis states for the particle-vibration model, which represents a more general approach [7–9].

In this Rapid Communication we report g-factor measurements on excited states of the isotopes ^{111,113}Cd. It is found that the g factors of the $I^\pi = 1/2^+$ ground states and the strongly Coulomb-excited $5/2^+_2$ states *cannot* be explained by the particle-vibration model, which assumes spherical core excitations. It has long been known, however, that the static quadrupole moments of the 2^+_1 states in the vibrational Cd isotopes near $A = 110$ are nonzero [10,11], and with $Q_0{\sim}140\,e$ fm^2 [12,13], they correspond to a small deformation of $\epsilon_2{\sim}0.1$. The effect of the core deformation on the electromagnetic properties of the natural parity states of these isotopes is therefore explored through comparisons between the particle-vibration and Nilsson-based particle-rotor models. In the particle-rotor approach the odd nucleon moves in a deformed field which rotates. The wave function of the deformed orbit becomes a deformation-dependent mixture of spherical-orbit wave functions, with further mixing due to Coriolis interactions.

The two Cd isotopes of interest straddle the $N = 64$ subshell gap and both have $I^\pi = 1/2^+$ ground states, based on the $s_{1/2}$ configuration. The states for which the particle-vibration model fails to describe the g factors are formed by coupling the odd-neutron predominantly in the $s_{1/2}$ orbit to the core excitations. It will be shown that in cases where an odd nucleon occupies a spherical $s_{1/2}$ or $p_{1/2}$ orbit (i.e., $j = \frac{1}{2}$), or the Nilsson orbit with $s_{1/2}$ or $p_{1/2}$ parentage, the g factors of the two models (particle-vibration and particle-rotor) both reduce to the weak-coupling model in their respective limits of

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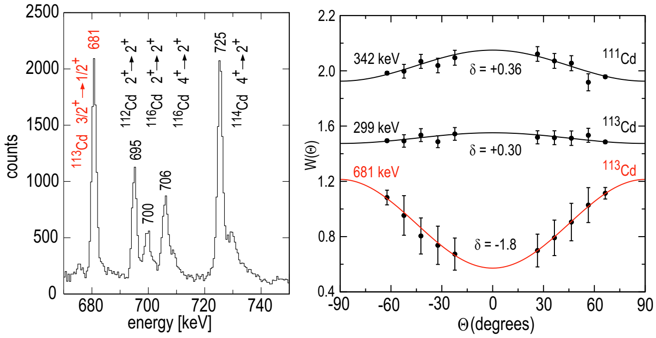

FIG. 1. Left: Spectrum of γ rays near 700 keV observed at $+65^\circ$ to the beam axis in coincidence with backscattered ^{32}S beam ions. Right: Experimental and calculated particle-γ angular correlations for $3/2^+ \rightarrow 1/2^+$ transitions in ^{111}Cd and ^{113}Cd. Θ is the angle between the γ-ray detector and the beam axis. (Data have been offset for presentation.)

zero particle-vibration coupling strength or zero deformation. In ^{111}Cd and ^{113}Cd, core deformation introduces the mixing needed to explain the g factors.

The g factors of the $3/2^+_1$ (342 keV) and $5/2^+_2$ (620 keV) states in ^{111}Cd and the $5/2^+_2$ (584 keV) and $3/2^+_2$ (681 keV) states in ^{113}Cd were measured by the transient-field technique with a natural Cd target (12.8% ^{111}Cd and 12.2% ^{113}Cd). The states of interest were populated by Coulomb excitation with 95 MeV ^{32}S beams from the ANU 14UD Pelletron accelerator. Experimental details have been given elsewhere [14].

A portion of the γ-ray spectrum observed in coincidence with backscattered beam ions is shown in Fig. 1. Along with the 681-keV $3/2^+ \rightarrow 1/2^+$ transition in ^{113}Cd, for which the g factor was measured for the first time, there are lines from $2^+_2 \rightarrow 2^+_1$ and $4^+_1 \rightarrow 2^+_1$ transitions in the even isotopes. As seen in Fig. 1, the 706- and 725-keV transitions, from states with half lives of 1.7 and 1.2 ps, respectively, show prominent Doppler-broadened line shapes, whereas the 681-keV line does not, implying the $3/2^+$ level at 681 keV must have a longer lifetime. The previously reported $B(E2)$ measurements [15,16] correspond to a half-life of 4.9(6) ps, which is consistent with the line shape observed. The half-life of 12(3) fs adopted in Ref. [13] is evidently in error.

Angular correlations between backscattered beam ions and de-excitation γ rays were measured. Those for the decays of the low-excitation $3/2^+$ states to the $1/2^+$ ground state are shown in Fig. 1. These data can be well fitted by calculated angular correlations based on previously published $E2/M1$ mixing ratios [12,13]. In the case of the 299-keV transition in ^{113}Cd, the angular correlation is almost isotropic and a meaningful g factor could not be extracted.

Results of the g-factor measurement are summarized in Table I. The precession angle is $\Delta\Theta = \epsilon/S$, where the effect, ϵ, and slope, S, were determined by the procedures described in Ref. [14]. The g factors were determined from $\Delta\Theta$ in the same way as for the even Cd isotopes [14]. There is agreement between the previous [17] and present experiments. The important result for the following discussion is that $g(5/2^+_2) > 0$ by more than three standard deviations in both isotopes.

The isotopes ^{111}Cd and ^{113}Cd were considered in terms of the particle-vibration (PV) model, in which the odd neutron is coupled to the quadrupole vibrations of a spherical core [7,9], and the strength of the coupling is determined by a dimensionless coupling parameter denoted ξ. The formalism can be found in Refs. [8,18–21].

The energy of the core vibration ($\hbar\omega$) was initially set by reference to the neighboring even isotopes, and then adjusted to best fit the low-excitation levels. For example, in the case of ^{111}Cd, $\hbar\omega$ was initially set at the midpoint between the energies of the 2^+_1 states in ^{110}Cd and ^{112}Cd. It was then varied between these two 2^+_1 energy values to improve the fit to the low-excitation levels of ^{111}Cd. A similar procedure was followed for ^{113}Cd. States in the even core up to the two-phonon triplet were included, as were all four positive-parity single-particle orbits in the $N = 50$ to $N = 82$ shell, namely: $s_{1/2}, d_{3/2}, d_{5/2}$, and $g_{7/2}$. The single-particle energies were initially set by reference to the Sn isotones and then adjusted to improve agreement with the experimental spectra of ^{111}Cd and ^{113}Cd. Following the usual procedure, the strength of the coupling between the odd-nucleon and the quadrupole vibration of the core was also set by comparison with the experimental levels. Good fits were obtained for both isotopes with $\xi \sim 1$. Gyromagnetic ratios were evaluated taking $g_c = Z/A$ for the core and quenching g_s for the odd neutron to 0.70 of that of a free neutron.

The comparison of PV model g factors with experimental results in Table II shows reasonable agreement for the $5/2^+_1$ level in ^{111}Cd and for the $3/2^+$ levels in both nuclides. However the calculated ground-state g factors are twice the experimental values, and the calculated moments of the $5/2^+_2$ levels in both nuclei have the wrong sign. Figure 2 shows that the g factors are relatively insensitive to the coupling strength. It is impossible to reach a positive g factor for the $5/2^+_2$ state in

TABLE I. Measured g factors in ^{111,113}Cd.

Isotope	Level		$\epsilon(65^\circ) \times 10^3$	$S(65^\circ)$ (rad^{-1})	$\Delta\Theta$ (mrad)	$g(I^\pi_i)$		
	E_x (keV)	I^π_i				Present	Ref. [17]	Adopted
^{111}Cd	342	$3/2^+_1$	+6(3)	−0.18(4)	−32(19)	+0.6(4)	+0.03(110)	+0.6(4)
	620	$5/2^+_2$	+18(5)	−1.43(7)	−13(4)	+0.25(7)	+0.15(6)	+0.19(5)
^{113}Cd	299	$3/2^+_1$	−3(3)	−0.04(10)			−0.36(74)	−0.4(7)
	584	$5/2^+_2$	+11(3)	−1.27(6)	−8.5(23)	+0.17(5)	+0.08(7)	+0.14(4)
	681	$3/2^+_2$	−33(9)	+0.46(4)	−72(20)	+1.4(4)		+1.4(4)

TABLE II. g factors in ^{111,113}Cd.

Isotope	I^π	g factor		
		PV[a]	PR[a]	Experiment[b]
^{111}Cd	$1/2_1^+$	−2.686	−1.251	−1.1898
	$5/2_1^+$	−0.488	−0.374	−0.306
	$3/2_1^+$	+0.663	+0.586	+0.6(4)
	$5/2_2^+$	−0.218	+0.315	+0.19(5)
^{113}Cd	$1/2_1^+$	−2.676	−1.130	−1.2446
	$3/2_1^+$	+0.686	+0.570	−0.4(7)
	$5/2_1^+$	−0.432	−0.326	
	$5/2_2^+$	−0.265	+0.259	+0.14(4)
	$3/2_2^+$	+0.896	+0.780	+1.4(4)

[a]PV, particle-vibration model; PR, particle-rotor model.
[b]Experimental data from Nuclear Data Sheets [12,13] and Table I.

either nuclide with any reasonable parameters. The $5/2_2^+$ states are strongly populated by Coulomb excitation, which confirms their association with the collective core 2_1^+ state. In the PV model the wave function of the $5/2_2^+$ state is predominantly due to the configuration $2^+ \otimes s_{1/2}$, with the next most prominent admixture being $0^+ \otimes d_{5/2}$; both of these configurations have negative g factors near −0.2.

Because there are difficulties in accounting for the g factors in the spherical PV model, particle-plus-rotor (PR) model calculations [22] based on the Nilsson potential were performed to assess the effect of deformation. The quadrupole deformation $\epsilon_2 = 0.1$ was set to reproduce $Q(2^+)$ in the neighboring Cd isotopes, with hexadecapole deformation $\epsilon_4 = 0.0$. The moment of inertia was set by reference to the 2_1^+ excitation in the neighboring isotopes and Coriolis interactions were attenuated by a standard factor of 0.7 [22]. Axially symmetric prolate deformation was assumed. The core and odd-neutron g factors were the same as in the PV calculation. No attempt was made to tune either the potential parameters or the triaxiality parameter to better fit the energy levels; the g factors are much less sensitive to triaxiality than the excitation energies. Results are given in Table II. The g factors are much better described by the PR model. In particular, the ground-state moments are within 10% of experiment and the $g(5/2_2^+)$ values are positive in agreement with experiment. For other states, the two models are in about equal agreement with experiment.

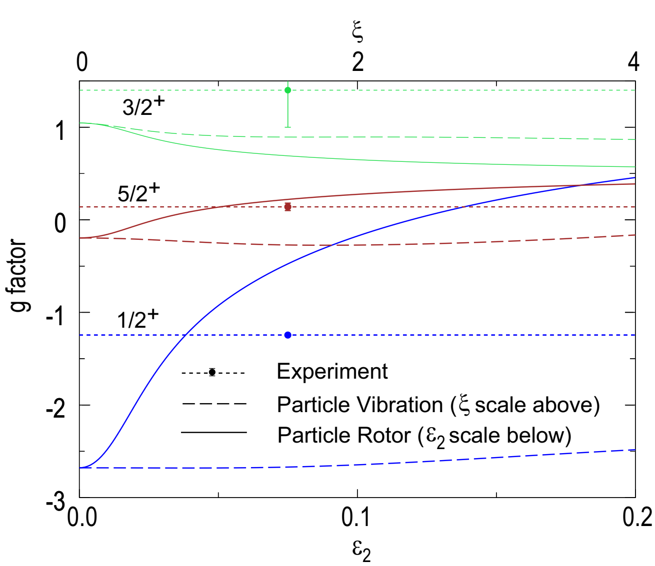

FIG. 2. Comparison of g-factor variations with deformation or particle-vibration coupling strength. These schematic particle-rotor calculations ignore Coriolis coupling.

To explore the differences between the PV and PR models and to investigate the dependence of the g factors on deformation, Nilsson model calculations were performed as a function of ϵ_2 with $\epsilon_4 = 0$. No Coriolis interactions were included; thus these calculations correspond to the strong-coupling limit of the PR model. The dependence of the g factors on deformation is shown in Fig. 2. In the PR model, the $1/2_1^+$,$3/2_1^+$, and $5/2_2^+$ states in both ^{111}Cd and ^{113}Cd are members of the $1/2^+[411]$ band. As seen in Fig. 2, the g factor of the $1/2_1^+$ level is strongly dependent on small core deformations, and the g factor of the $5/2_2^+$ level becomes positive at modest deformations.

Figure 2 indicates that the states associated with the $1/2^+[411]$ band have the same g factors at $\epsilon_2 = 0$ as the corresponding states in the PV model at $\xi = 0$. It can be shown in general that g factors of Nilsson states with $s_{1/2}$ or $p_{1/2}$ parentage reduce to the weak-coupling model values in the spherical limit. For $K = 1/2$ bands the Nilsson model g factors are

$$g(I) = g_R + \frac{(g_K - g_R)}{4I(I+1)}\left[1 + (2I+1)(-1)^{I+\frac{1}{2}}b\right], \quad (1)$$

where g_K and g_R are the single-particle and collective g factors, respectively, and b is the magnetic decoupling parameter [9]. For Nilsson orbits with $j = 1/2$ parentage, $b \to -1$ as the deformation goes to zero. Also in the spherical limit, $g_K \to g_j$, where g_j is the g factor of the spherical orbit. At first sight, there is little resemblance to the weak-coupling model expression

$$g(I) = \frac{1}{2}(g_c + g_j) + \frac{1}{2}(g_c - g_j)\frac{J_c(J_c+1) - j(j+1)}{I(I+1)}, \quad (2)$$

where j and J_c are the angular momentum of the single particle and the core, respectively; g_j and g_c represent the corresponding g factors. However, by substituting $j = 1/2$ and $I = J_c \pm 1/2$, Eq. (2) can be rewritten in the same form as the Nilsson model expression, Eq. (1), but with g_c in place of g_R. Whether the collective core excitation is a vibration or rotation, the g factor has much the same value, $g_c \approx g_R \approx Z/A$. Thus in the spherical limit, the Nilsson model g factors in the $1/2^+[411]$ band approach the weak-coupling model values.

The fact that both the PV and PR models begin with the same unperturbed states emphasizes the differences between the two models: The nature of the configuration mixing is very different when they depart from this limit. Even a small deformation has a huge impact on the ground-state g factor. The origin of this dependence on deformation can be seen

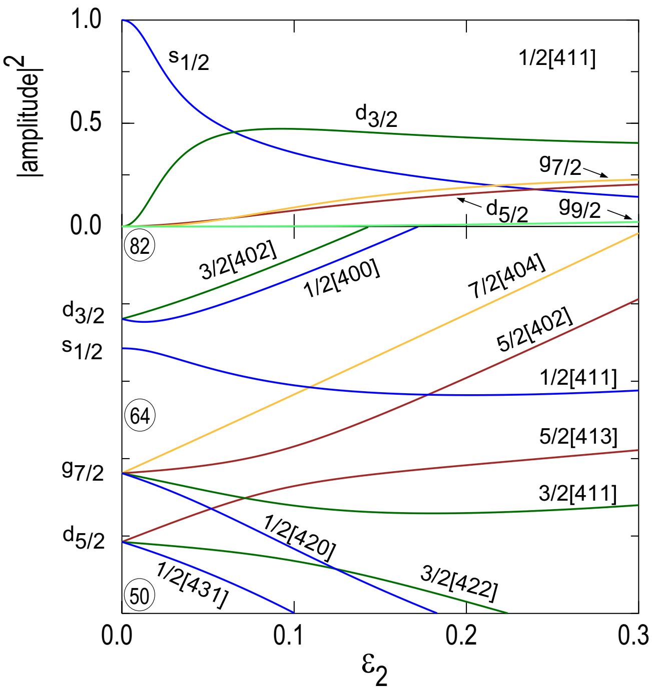

FIG. 3. Lower panel: Nilsson scheme for positive-parity neutron orbits in the $50 < N < 82$ shell. Upper panel: Composition of the wave function of the $1/2^{+}[411]$ Nilsson orbit in terms of spherical basis states.

in Fig. 3. The Nilsson diagram in the lower panel shows a repulsion between the $1/2^{+}[411]$ and $1/2^{+}[400]$ orbits at small deformations, which indicates a mixing of the spherical $s_{1/2}$ and $d_{3/2}$ orbits. This mixing is confirmed by the composition of the $1/2^{+}[411]$ wave function displayed in the upper panel. Thus the difference between the models comes about mainly because the particle-vibration coupling mixes configurations associated with the $d_{5/2}$ neutron (negative g factor), whereas the onset of deformation mixes configurations associated with the $d_{3/2}$ neutron (positive g factor).

It is noteworthy that the g factors of these odd-A nuclei show a sensitivity to deformation that is not evident from the $E2$ transition rates, which stem predominantly from the collective excitations of the core. As pointed out by Grodzins, in even-even nuclei it is not possible to distinguish between a quadrupole vibration and a quadrupole rotation on the basis of the $2_1^+ \to 0_1^+$ $E2$ transition rate alone [23]. This difficulty applies to these odd-A systems as well. For example, the ratio of reduced transition rates for the decays of the core-coupled $3/2^+$ and $5/2^+$ states, $B(E2; 5/2^+ \to 1/2^+)/B(E2; 3/2^+ \to 1/2^+)$, is unity in both the weak-coupling and rotor models.

In summary, it has been shown that core deformation introduces the mixing needed to explain the g factors of ^{111}Cd and ^{113}Cd whereas particle-vibration coupling does not. The approach taken has been to explore the implications of a spherical vibrational core versus a weakly deformed rotational core on the wave function of the odd nucleon. The contrast between the two scenarios is marked because they both start from the same g-factor values in their respective limits. A more microscopic approach would need to explain the configuration mixing required by the g factors while also illuminating the correlations that give rise to collectivity. From either a macroscopic or microscopic perspective, it is important to recognize that g factors must be considered along with $E2$ observables in studying the origins, nature, and evolution of nuclear collectivity.

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