Photoproduction of $a_0(980)$ and $f_0(980)$

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There is no generally accepted view of the structure of the light-quark nonstrange scalar mesons. A variety of models has been proposed that encompass $qq\bar{q}q$, molecular, $q\bar{q}$, and glueball states in various combinations. Previously we considered scalar-meson photoproduction in a simple Regge-pole model and showed that it was experimentally viable. Recent data on the photoproduction of $a_0(980)$ and $f_0(980)$ confirm this. We extend our model to incorporate Regge cuts, based on our knowledge of π^0 photoproduction. The theoretical predictions are compared to the $a_0(980)$ and $f_0(980)$ photoproduction data.

DOI: [10.1103/PhysRevC.93.025203](http://dx.doi.org/10.1103/PhysRevC.93.025203)

I. INTRODUCTION

The scalar sector of light-quark spectroscopy remains poorly understood, and various phenomenological models have been suggested to describe the light scalars. Simple ground or excited ${}^{3}P_0 q\bar{q}$ states and tetraquark models have been discussed, as well as a glueball admixture for the isoscalar-scalar states. Large branching ratios of scalars to pseudoscalar meson pairs suggest also the possibility of scalar resonances generated dynamically. The latter has been widely discussed in connection with the $a_0(980)$ and $f_0(980)$ states which are very close to the $K\bar{K}$ threshold and, as such, could contain a large admixture of a $K\overline{K}$ molecule.

The photoproduction of $\pi \pi / \pi \eta$ and $K \bar{K}$ pairs near the $K\bar{K}$ threshold is recognized as a powerful tool to study the properties of the $f_0(980)$ and $a_0(980)$ mesons and to help discriminate among models for scalars. If there is a large admixture of pseudoscalar meson pairs, the resonance will be seen in the final state interaction of the produced $\pi \pi / \pi \eta$ and $K\bar{K}$ pairs, a mechanism that was considered in Refs. [\[1,2\]](#page-7-0). If, however, the scalar contains significant admixture of a compact $q\bar{q}$ state, the resonance can be produced directly via the $q\bar{q}$ component. It is this possibility that is discussed in the present paper. In reality the scalar meson wave function contains both $q\bar{q}$ and $K\bar{K}$ admixtures. We do not argue that the $q\bar{q}$ component is the sole or even dominant part of the $a_0(980)$ and $f₀(980)$ wave function but do attempt to put reasonable limits on its contribution to the photoproduction cross section. The calculation does not exclude the possibility of a $K\bar{K}$ component in either meson or indeed a glueball component in $f_0(980)$.

The photoproduction of the light-quark scalar mesons $a_0(980)$, $f_0(980)$, $f_0(1370)$, $a_0(1450)$, $f_0(1500)$, and $f_0(1710)$ was calculated in Ref. [\[3\]](#page-7-0). The principal objective was to demonstrate that the cross sections are sufficiently large to be measurable and that they provide a viable mechanism to probe the structure of the scalar mesons. A simple model was used, assuming the dominant mechanism to be Reggeized ρ and ω exchange, both of which are well understood in pion photoproduction. The electromagnetic couplings γSV of a vector meson to a scalar meson were calculated assuming that the scalar mesons are bound $q\bar{q}^{3}P_0$ states, so the radiative decays $V \rightarrow S\gamma$ and $S \rightarrow V\gamma$ proceed via a quark loop and the corresponding matrix element can be estimated in the quark model $[4,5]$. These estimates were used in Ref. $[3]$ to calculate the $a_0(980)$ and $f_0(980)$ photoproduction amplitude in the quark loop mechanism and it was shown that even with a modest admixture of $q\bar{q}$ component in the scalar wave function the direct mechanism will dominate the cross section.

The latter result can be explained with the findings of Ref. [\[6\]](#page-7-0): the radiative transitions between vector mesons and $a_0(980)/f_0(980)$ exhibit a distinct hierarchy pattern so that the closer is the mass of the vector meson to the $K\overline{K}$ threshold, the larger is the transition via an intermediate kaon loop. For example, the $\phi \rightarrow \gamma a_0(980)/f_0(980)$ amplitude is dominated by the kaon loop mechanism, while $a_0(980)/f_0(980) \rightarrow \gamma \rho/\omega$ transition widths are much smaller in the $K\bar{K}$ molecular model for scalars than in the $q\bar{q}$ model. Clearly, photoproduction kinematics with t-channel vector meson exchange discriminates even more strongly in favor of the quark loop mechanism.

The conclusions of Ref. [\[3\]](#page-7-0) were that light-quark scalar meson photoproduction on protons is a practical proposition given the luminosities available to modern photoproduction experiments. However, contributions from lower-lying trajectories, particularly that associated with the $b₁(1235)$, and Regge cuts were not considered. The resulting differential cross sections have a deep minimum in the vicinity of $t = -0.5$ GeV² due to the wrong-signature zeros in the ρ and ω trajectories. Subsequently data on the photoproduction of $a_0(980)$ [\[7\]](#page-8-0) in the range $2.0 < E_{\gamma} < 2.85$ GeV and of $f_0(980)$ [\[8\]](#page-8-0) in the range 3.0 < E_ν < 3.8 GeV have become available. Neither cross section shows the minimum expected from the wrong-signature zeros in the ρ and ω trajectories and both are larger than the predictions of Ref. [\[3\]](#page-7-0) at small t. This is analogous to the situation in π^0 photoproduction where strong cuts are required in both natural and unnatural parity exchange [\[9\]](#page-8-0). The analysis of Ref. [\[9\]](#page-8-0) made use of finite energy sum rules (FESRs) and it was possible to make a clear separation between the Regge-pole and Regge-cut contributions. The cuts do not conform to any particular Regge-cut model and have to be treated phenomenologically. A similar result holds in π^+ photoproduction [\[10\]](#page-8-0). It is logical to assume that the discrepancy between the results of Ref. [\[3\]](#page-7-0) and the data of Refs. [\[7,8\]](#page-8-0) for scalar photoproduction are due primarily to the same kind of cut effects that occur in π^0 photoproduction.

It is not practical to analyze the data on $a_0(980)$ and $f_0(980)$ photoproduction directly as both occur at only one energy and with rather low statistics. Thus we employ the following strategy: as Regge-cut effects cannot be calculated *a priori* we propose a simple phenomenological model for cut effects that gives a reasonable description of π^0 photoproduction. We do not claim that this model of π^0 photoproduction is on a par with more sophisticated approaches as, for example, that of Ref. [\[9\]](#page-8-0). We use the existing data on π^0 photoproduction to find the parameters of our simple model which is readily transportable to scalar-meson photoproduction.

A brief overview of Regge-cut phenomenology is given in Sec. II and is applied to π^0 photoproduction in Sec. III. The model is extended to $a_0(980)$ and $f_0(980)$ photoproduction in Sec. [IV.](#page-4-0) The implications of these results for exploring the nature of the scalar mesons in photoproduction are discussed in Sec. [V.](#page-7-0)

II. REGGE-CUT PHENOMENOLOGY

The aim is to construct a simple model of π^0 photoproduction that can be transported to scalar-meson photoproduction. This approach can be justified as the Regge terms are the same in both cases: dominant natural parity ω and ρ exchange plus a small contribution from unnatural parity $b_1(1235)$ exchange. We know [\[9\]](#page-8-0) that cut effects are important in π^0 photoproduction so we give a brief discussion of the relevant phenomenology. A fuller discussion of Regge cuts may be found in Ref. $[11]$.

Regge cuts arise from rescattering, the simplest being the exchange of two Reggeons R_1 and R_2 although there is no reason to exclude multi-Reggeon cuts. The exchange of two Reggeons with linear trajectories $\alpha_i(t) = \alpha_i(0) + \alpha'_i t, i = 1, 2,$ are known [\[12\]](#page-8-0) to yield a cut with a linear trajectory $\alpha_c(t)$:

$$
\alpha_c(t) = \alpha_c(0) + \alpha'_c t,\tag{1}
$$

where

$$
\alpha_c(0) = \alpha_1(0) + \alpha_2(0) - 1,
$$

\n
$$
\alpha'_c = \frac{\alpha'_1 \alpha'_2}{\alpha'_1 + \alpha'_2}.
$$
\n(2)

Cuts cannot be calculated with any precision and none of the numerous models proposed agree with all aspects of the data. Hence, cuts are best treated phenomenologically, although even then there is no consistency among different reactions. Good examples of this are provided by $\pi^-p \to \pi^0 n$, dominated by ρ exchange, and $\gamma p \to \pi^0 p$, dominated by $ω$ exchange. Effective $ρ$ and $ω$ trajectories can be obtained directly from the data: for example, see Figs. 5.1 and 5.7 in Ref. [\[13\]](#page-8-0). In the case of the ρ , the effective trajectory agrees rather well with the extrapolation from the physical region, so cut effects are small in $\pi^- p$ charge exchange. In the case of the ω there is essentially no agreement with the extrapolation from the physical region, so cut effects are significant. The FESR analysis of π^0 photoproduction by Ref. [\[9\]](#page-8-0) demonstrates this latter point explicitly.

Because there is angular momentum associated with the two-Reggeon system, a Regge cut will occur in natural and unnatural parity-exchange amplitudes irrespective of the intrinsic parities of the two Reggeons. Naturality is defined as $+1$ if the spin and parity of the mesons on it have natural parity and as -1 if they have unnatural parity. It was shown [\[14\]](#page-8-0) that if the exchanged Reggeons have naturalities n_1 and n_2 then amplitudes of naturality $-n_1n_2$ are suppressed relative to amplitudes of naturality $+n_1n_2$ and this suppression grows with increasing energy.

In the case of π^0 photoproduction one would expect the leading process to be π^0 production followed by π^0 elastic scattering. The latter is dominated by f_2 and Pomeron exchange, both of which have natural parity, so the natural parity cut should dominate over the unnatural parity. This is again in accord with the analysis of Ref. [\[9\]](#page-8-0). Two cut terms are included in that analysis with trajectories $\alpha_3(t)$ = $0.447 + 0.333t$ and $\alpha_4(t) = 0.177 + 0.5t$.

As in Ref. [\[3\]](#page-7-0) we assume linear nondegenerate ω and ρ trajectories,

$$
\alpha_{\omega} = 0.44 + 0.9t,\n\alpha_{\rho} = 0.55 + 0.8t.
$$
\n(3)

We take the Pomeron trajectory to be [\[11\]](#page-8-0)

$$
\alpha_{I\!\!P} \sim 1.08 + 0.25t,\tag{4}
$$

so from (2) the trajectory of the ω -*IP* cut is $\alpha_p^c = 0.52 +$ 0.196t and that of the ρ -*IP* cut is $\alpha_p^c = 0.64 + 0.160t$. The second state on the f_2 trajectory is the $f_4(2050)$ with a mass of 2018 MeV [\[15\]](#page-8-0). Taking a mass of 1275 MeV for the $f_2(1270)$ [\[15\]](#page-8-0) gives the f_2 trajectory as $\alpha_{f_2} = 0.672 + 0.817t$, and the trajectories of the associated ω - f_2 and ρ - f_2 cuts are $\alpha_{\text{IR}}^c = 0.112 + 0.428t$ and $\alpha_{\text{IR}}^c = 0.222 + 0.404t$. Thus it is reasonable to conclude that the $\alpha_3(t)$ cut of Ref. [\[9\]](#page-8-0) corresponds roughly to the combined ω -*IP* and ρ -*IP* cuts and the $\alpha_4(t)$ cut corresponds roughly to the combined ω - f_2 and ρ - f_2 cuts.

III. *π***⁰ PHOTOPRODUCTION**

A. Vector exchange

Let q, p_1 , k, and p_2 be respectively the 4-momenta of the photon, initial proton, pion, and recoil proton. The hadronic current for vector exchange is

$$
J_{\mu}^{V} = -e^{\frac{g_{V\pi\gamma}}{m_{\pi}} \epsilon_{\mu\nu\rho\sigma} q_{\nu} p_{\rho} g_{\sigma\tau}} \times \bar{u}(p_2) \{ig_V \gamma_{\tau} - g_T \sigma_{\tau\lambda} p_{\lambda} \} u(p_1) D_V(s, t), \quad (5)
$$

where m_π is the π^0 mass, $p = p_2 - p_1$, and $D_V(s,t)$ is the full Regge propagator for vector exchange:

$$
D_V(s,t) = \left(\frac{s}{s_0}\right)^{\alpha_V(t)-1} \frac{\pi \alpha'_V}{\sin(\pi \alpha_V(t))} \frac{-1 + e^{-i\pi \alpha_V(t)}}{2} \frac{1}{\Gamma(\alpha_V(t))},\tag{6}
$$

with $\alpha_V(t) = \alpha_{V0} + \alpha_V' t$ the Regge trajectory. As in Ref. [\[3\]](#page-7-0) we use $g_V^{\omega} = 15$, $g_T^{\omega} = 0$, $g_V^{\rho} = 3.4$, and $g_T^{\rho} = 11 \text{ GeV}^{-1}$. The electromagnetic coupling constants $g_{V\pi\gamma}$ are obtained from the electromagnetic decay width:

$$
\Gamma_{(V \to \pi \gamma)} = \frac{\alpha}{24} \left\{ \frac{g_{V\pi\gamma}}{m_{\pi}} \right\}^2 m_V^3 \left\{ 1 - \left(\frac{m_{\pi}}{m_V} \right)^2 \right\}^3. \tag{7}
$$

This gives $g_{\omega \pi \nu} = 0.322$ for a width of 75.6 keV [\[15\]](#page-8-0) and $g_{\rho^0 \pi \gamma} = 0.119$ for a width of 89.6 keV [\[15\]](#page-8-0).

Note that the vector-exchange contributions vanish at the wrong-signature points given by $\alpha_V(t) = 0$, i.e., at $t =$ −0.49 GeV² for the ω and $t = -0.69$ GeV² for the ρ . These zeros result in a pronounced dip in the differential cross section.

The cross section for the exchange of a single vector meson is

$$
\frac{d\sigma}{dt} = -\frac{T_V}{16\pi \left(s - m_p^2\right)^2},\tag{8}
$$

where

$$
T_V = \frac{4\pi\alpha g_{V\pi\gamma}^2}{m_{\pi}^2} \left\{ \frac{1}{2} \left[s(t - t_1)(t - t_2) + \frac{1}{2}t(t - m_{\pi}^2)^2 \right] a a^* + \frac{1}{2} m_p s(t - t_1)(t - t_2)(a b^* + a^* b) + \frac{1}{8} s (4m_p^2 - t)(t - t_1)(t - t_2) b b^* \right\} |D_V(s, t)|^2. \tag{9}
$$

Here t_1 and t_2 are the kinematical boundaries,

$$
t_{1,2} = \frac{1}{2s} \Big\{ - \left(m_p^2 - s\right)^2 + m_\pi^2 \left(m_p^2 + s\right)
$$

$$
\pm \left(m_p^2 - s\right) \sqrt{\left(m_p^2 - s\right)^2 - 2m_\pi^2 \left(m_p^2 + s\right) + m_\pi^4} \Big\}, (10)
$$

and

$$
a = (g_V + 2m_p g_T), \quad b = -2g_T. \tag{11}
$$

B. Axial-vector exchange

It is possible to separate natural-parity and unnaturalparity exchange in pion photoproduction by using a planepolarized photon beam. The polarized-beam asymmetry (σ_{\perp} – σ_{\parallel})/($\sigma_{\perp} + \sigma_{\parallel}$) is close to unity for natural-parity exchange, and any deviation from this indicates the presence of unnaturalparity exchange. This is clearly the case in π^0 photoproduction as can be seen in Figs. [1](#page-3-0) and [2](#page-4-0) below.

The Regge-pole exchange is that associated with the $b_1(1235)$. C-parity requires that the coupling of the $b_1(1235)$ to the nucleon is the axial-tensor coupling $\sigma_{\mu\nu}\gamma_5$ [\[16\]](#page-8-0). The hadronic current for b_1 exchange may be written as

$$
J_{\mu}^{b} = -e g_{b} g_{b_{1}NN} \{ (p.q)g_{\mu\beta} - p_{\mu}q_{\beta} \}
$$

× $(p_{2\beta} + p_{1\beta})\bar{u}(p_{2})\gamma_{5}u(p_{1})D_{b}(s,t),$ (12)

where

$$
g_b = \frac{g_{b_1 \pi \gamma}}{m_\pi}.\tag{13}
$$

The axial-vector Regge propagator $D_b(s,t)$ has the same form as [\(6\)](#page-1-0) with $\alpha_V(t)$ replaced by the b_1 trajectory $\alpha_b(t) \approx -0.013 + 0.664t$. Note that there is no interference

between vector and axial-vector exchange in the cross section or polarized-beam asymmetry.

The cross section is

$$
\frac{d\sigma}{dt} = -\frac{T_A}{16\pi \left(s - m_p^2\right)^2},\tag{14}
$$

where

$$
T_A = -4\pi\alpha \frac{g_{b_1\pi\gamma}^2}{m_{\pi}^2} g_{b_1NN}^2 \frac{st}{2}(t-t_1)(t-t_2)|D_b(s,t)|^2.
$$
 (15)

The value of $g_{b_1\pi\gamma}$ can be found from the radiative decay width $\Gamma_{b_1^+\pi^+\gamma}$ which is given by

$$
\Gamma_{b_1^+ \pi^+ \gamma} = \frac{\alpha}{24} \left\{ \frac{g_{b_1 \pi \gamma}}{m_{\pi}} \right\}^2 m_{b_1}^3 \left\{ 1 - \left(\frac{m_{\pi}}{m_{b_1}} \right)^2 \right\}^3. \tag{16}
$$

The radiative decay width of $b_1^+ \rightarrow \pi^+ \gamma$ is $\Gamma_{b_1^+ \pi^+ \gamma} =$ 228 ± 57 keV [\[15\]](#page-8-0), so $g_{b_1 \pi \gamma}/m_\pi = 0.648 \pm 0.081$ GeV⁻¹. Although the value of g_b is rather well established, little is known about g_{b_1NN} .

We adopt the suggestion of Ref. [\[16\]](#page-8-0) that $g_{b_1NN} =$ $G_{b_1NN}/2m_p$ with $G_{b_1NN} \approx G_{a_1NN} \approx 7$.

The contribution of $b_1(1235)$ exchange is negligibly small for this choice of coupling and indeed remains small for any reasonable value of g_{b_1NN} . Thus the unnatural-parity-exchange contribution must come primarily from the unnatural-parity contribution arising from the ω and ρ cuts.

C. The cut contributions

Because a physical mass cannot be be associated with a cut, the simplest form of amplitude for a cut term is

$$
A_c(s,t) = C_c D_c(s,t),\tag{17}
$$

where C_c is a constant and

$$
D_c(s,t) = e^{d_c t} e^{-i\frac{1}{2}\pi\alpha_c(t)} s^{\alpha_c(t)-1},
$$
\n(18)

where we retained only the Regge phase and absorbed the rest of the t dependence in the exponential, $\alpha_c(t)$ is the cut trajectory, and the constants C_c and d_c for each cut term are obtained by fitting data.

We need a mechanism to allow us to transfer the π^0 cut model to scalar photoproduction. The simplest way is to take the cut terms proportional to the dominant ω and ρ exchanges, retaining the kinematical structure and replacing $g_{V\pi\gamma}g_{VNN}D_V(s,t), V=\omega,\rho,$ by

$$
g_{V\pi\gamma}g_{VNN}\big(D_V(s,t)+C_{n_1}^V D_{c_1}^V(s,t)+C_{n_2}^V D_{c_2}^V(s,t)\big),\quad(19)
$$

where $g_{V\pi\gamma}$ and g_{VNN} are respectively the $V\pi\gamma$ and relevant *VNN* coupling constants and $C_{n_1}^V$ and $C_{n_2}^V$ are respectively the natural-parity constants for the V - f_2 and V - $I\!P$ cuts. These cuts also feed into the unnatural-parity-exchange term and are much larger than any cuts generated by $b_1(1235)$ exchange due to its small contribution. So $g_b g_{B_1NN} D_b(s,t)$ is replaced by

$$
g_b g_{b_1 NN} D_b(s,t) + \sum_V g_{V\pi\gamma} g_{VNN}
$$

$$
\times (C_{u_1}^V D_{c_1}(s,t) + C_{u_2}^V D_{c_2}(s,t)),
$$
 (20)

FIG. 1. Fits to the differential π^0 photoproduction cross sections in μ b GeV⁻² at $E_\gamma = 4$, 6, and 9 GeV and the polarized-beam asymmetry Σ at $E_y = 4, 6$, and 10 GeV. Predictions without cuts are shown as a dotted line. The differential cross section data at 4 GeV are from Ref. [\[17\]](#page-8-0) (squares) and Ref. [\[18\]](#page-8-0) (dots). The differential cross section data at 6 and 9 GeV and the polarized-beam asymmetry data are from Ref. [\[19\]](#page-8-0).

where the $C_{u_i}^V$ are the unnatural-parity constants. It turns out that the cuts dominate unnatural-parity exchange so in practice the b_1 pole term could be omitted.

The parameters for ρ and ω were taken to be the same, i.e., $C_{n_i}^{\rho} = \dot{C}_{n_i}^{\omega}$ and $C_{u_i}^{\rho} = \dot{C}_{u_i}^{\omega}, i = 1, 2$. Also the argument d_c of the exponential in [\(18\)](#page-2-0) was taken to be the same for all terms. So in practice we have only five free parameters to describe π^0 photoproduction.

This approach is plausible and has the merit of simplicity, although obviously it is not unique. However, as the aim is to provide a reasonable qualitative description of π^0 photoproduction rather than a precise fit it is perfectly adequate and, as is shown in the next section, is surprisingly good.

D. Fits to π^0 photoproduction

For the fit we use the differential cross section data of the Liverpool group at $E_{\gamma} = 4$ GeV [\[17\]](#page-8-0), of Braunschweig *et al.* [\[18\]](#page-8-0) at $E_\gamma = 4, 5$, and 5.8 GeV, and of Anderson *et al.* [\[19\]](#page-8-0) at $E_{\gamma} = 6, 9$, and 12 GeV. The polarized-beam asymmetry data are from Anderson *et al.* [\[19\]](#page-8-0) at $E_y = 4, 6$, and 10 GeV. A typical fit to some of the differential cross-section data and the polarized-beam asymmetry is shown in Fig. 1, which also gives the result for the cross section without the cut terms.

Figure [2](#page-4-0) compares the predictions of the model with differential cross-section data at $E_{\gamma} = 2.0$ GeV [\[18,20\]](#page-8-0) and 3.0 GeV [\[18\]](#page-8-0) and polarized-beam data at $E_{\gamma} = 2.0$ GeV [\[21\]](#page-8-0) and 3.0 GeV [\[22\]](#page-8-0). Although these energies are rather low

FIG. 2. Comparison of the model with the differential π^0 photoproduction cross sections in μ b GeV⁻² and with the polarized-beam asymmetry Σ at $E_y = 2$ and 3 GeV. Predictions without cuts are shown as the dotted line. The differential cross-section data at 2 GeV are from Ref. [\[18\]](#page-8-0) (dots) and Ref. [\[20\]](#page-8-0) (squares) and at 3 GeV from Ref. [\[18\]](#page-8-0). The polarized-beam data at 2 GeV are from Ref. [\[21\]](#page-8-0) and at 3 GeV from Ref. [\[22\]](#page-8-0).

for the model, particularly $E_{\gamma} = 2.0$ GeV, the $a_0(980)$ and $f₀(980)$ photoproduction data are close to these energies. Nonetheless, the extrapolation of the model to these low energies is sufficiently satisfactory for our present purpose. The forward peak is reasonably well reproduced, although rather low at 2 GeV, but the model retains some dip structure that is not apparent in the data.

The contribution from unnatural-parity exchange is small, except in the dip region. Elsewhere, as there is no interference between natural-parity and unnatural-parity exchange, naturalparity exchange dominates. This is immediately apparent from the polarized-beam asymmetry.

IV. $a_0(980)$ AND $f_0(980)$ PHOTOPRODUCTION

A. Vector exchange

The hadronic current for vector exchange is [\[3\]](#page-7-0)

$$
J_{\mu}^{V} = \{g_{\mu\nu}(p.q) - p_{\mu}q_{\nu}\}\bar{u}(p_2)\{a\gamma_{\nu} + bp_{1\nu}\}u(p_1)D_{V}(s,t),
$$
\n(21)

where $D_V(s,t)$ is the Regge propagator [\(6\)](#page-1-0), $a = g_S(g_V +$ $2m_p g_T$), $b = -2g_S g_T$, with g_V and g_T as before, and g_S is the coupling at the $SV\gamma$ vertex, defined in terms of the radiative decay width by

$$
\Gamma(S \to V\gamma) = g_S^2 \frac{m_S^3}{32\pi} \left(1 - \frac{m_V^2}{m_S^2} \right)^3.
$$
 (22)

For the radiative decay width we use the results of Ref. [\[3\]](#page-7-0), which were based on the model of Refs. [\[4,5\]](#page-7-0). It is assumed that the scalar mesons are $q\bar{q}^{3}P_0$ bound states with the radiative decay proceeding via a quark loop. Specifically

$$
\Gamma(a_0(980) \to \gamma \rho) = 14 \text{ keV},
$$

$$
\Gamma(f_0(980) \to \gamma \rho) = 125 \text{ keV}.
$$
 (23)

The radiative widths to $\gamma \omega$ are a factor of 9 larger for $a_0(980)$ and a factor of 9 smaller for $f_0(980)$.

It was shown in Refs. $[4,5]$ that the model gives good agreement with existing data on radiative decays of $q\bar{q}$ mesons. The cross section is [\[3\]](#page-7-0)

$$
\frac{d\sigma}{dt} = -\frac{\tilde{T}_V}{16\pi \left(s - m_p^2\right)^2},\tag{24}
$$

where

$$
\tilde{T}_V = \frac{1}{8} g_S^2 \{ 4aa^* \left[s(t - t_1)(t - t_2) + 2t(t - m_S^2)^2 \right] + 4(ab^* + a^*b)m_p s(t - t_1)(t - t_2) + bb^* s(4m_p^2 - t)(t - t_1)(t - t_2) \} |D_V(s, t)|^2, \quad (25)
$$

a and b are given by (11) , and t_1 and t_2 are the kinematical boundaries.¹

B. Axial-vector exchange

The structure of the current for b_1 exchange is

$$
J^A_\mu = -\tilde{g}_b \epsilon_{\mu\nu\lambda\rho} p_\lambda q_\rho (p_{1\nu} + p_{2\nu}) \bar{u}(p_2) \gamma_5 u(p_1) D_b(s,t), \tag{26}
$$

where $D_b(s,t)$ is the Regge propagator, and \tilde{g}_b contains $b_1S\gamma$ and b_1NN couplings. The cross section is

$$
\frac{d\sigma}{dt} = -\frac{\tilde{T}_A}{16\pi \left(s - m_p^2\right)^2},\tag{27}
$$

where

$$
\tilde{T}_A = -\frac{1}{2}\tilde{g}_b^2 st(t - t_1)(t - t_2)|D_b(s, t)|^2.
$$
 (28)

Because we know even less about the $b_1(1235)$ couplings in scalar-meson photoproduction than in π^0 photoproduction, the pole term was again omitted, Eqs. (27) and (28) providing the kinematical structure for the cut terms. As in the case of π^0 photoproduction there is no interference between naturalparity and unnatural-parity exchange in the cross section.

C. The cut contributions

We adopt exactly the same approach for the cut contributions as for π^0 photoproduction, with the constants $C_{n_i}^V$ and $C_{u_i}^V$ having the same values as in π^0 photoproduction. That is, the relative strengths of the cut and pole terms are the same as in π^0 photoproduction and the structure of the cross section is the same as for scalar photoproduction without the cut terms.

D. Mass distributions

To obtain mass distributions for $a_0(980)$ and $f_0(980)$ we represent them as Breit-Wigner resonances with energydependent partial widths. The signal cross section for the final state i is given by [\[3\]](#page-7-0)

$$
\frac{d\sigma}{dt dM} = \frac{d\sigma_0(t, M)}{dt} \frac{2m_S^2}{\pi} \frac{\Gamma_i(M)}{(m_S^2 - M^2)^2 + M^2 \Gamma_{\text{Tot}}^2},\tag{29}
$$

where $d\sigma_0(t,M)/dt$ is the narrow-width differential cross section at a scalar mass M which is straightforward to calculate in our model. In practice the narrow-width cross section varies very little over the width of the scalar meson so it can be evaluated at the scalar mass and weighted with the integral over the Breit-Wigner line shape. However, a problem arises in the choice of the Breit-Wigner amplitude.

The Breit-Wigner amplitudes that have been used to describe the decays $a_0(980) \rightarrow \pi^0 \eta, K\overline{K}$ and $f_0(980) \rightarrow$ $\pi \pi$, K \overline{K} are those employed by the KLOE Collaboration to analyze their data on $\phi \to \pi \eta \gamma$ [\[24\]](#page-8-0) and $\phi \to \pi^0 \pi^0 \gamma$ [\[25\]](#page-8-0) and are based either on a "kaon-loop" (KL) model [\[26\]](#page-8-0) in

which the radiative transition proceeds via kaon loop, or on a 'no-structure" (NS) model [\[27\]](#page-8-0) in which the coupling is pointlike. In the case of $a_0(980)$ the $\pi \eta$ line shape in the KL model is

$$
B_a = t_{\pi\eta}^a t_{\pi\eta}^{a*} \frac{k_{\pi\eta}}{4\pi^2},\tag{30}
$$

where

$$
k_{\pi\eta} = \sqrt{\frac{(M^2 - (m_{\pi} + m_{\eta})^2)(M^2 - (m_{\pi} - m_{\eta})^2)}{4M^2}}
$$
(31)

and

$$
t_{\pi\eta}^{a} = \frac{g_{\pi\eta}}{D_{a}},
$$
\n
$$
D_{a} = m_{S}^{2} - M^{2} + \text{Re}\Pi_{\pi\eta}(m_{S}) - \Pi_{\pi\eta}(M)
$$
\n
$$
+ \text{Re}\Pi_{K^{+}}(m_{S}) - \Pi_{K^{+}}(M)
$$
\n
$$
+ \text{Re}\Pi_{K^{0}}(m_{S}) - \Pi_{K^{0}}(M),
$$
\n(33)

$$
\Pi_{\pi\eta}(M) = \frac{g_{\pi\eta}^2}{16\pi} \left(\frac{m_+ m_-}{\pi M^2} \log \frac{m_\pi}{m_\eta} + \rho_{\pi\eta}(M) \times \left(i + \frac{1}{\pi} \log \frac{1 - \rho_{\pi\eta}(M)}{1 + \rho_{\pi\eta}(M)} \right) \right), \tag{34}
$$

$$
m_{\pm} = m_{\eta} \pm m_{\pi},\tag{35}
$$

$$
\rho_{\pi\eta}(M) = \sqrt{(1 - m_+^2/M^2)(1 - m_-^2/M^2)},\tag{36}
$$

$$
\Pi_{K}(M) = \frac{1}{2}\theta(M - 2m_{K})\frac{g_{K}^{2}}{16\pi}\rho_{K}(M)
$$

$$
\times \left(i + \frac{1}{\pi}\log\frac{1 - \rho_{K}(M)}{1 + \rho_{K}(M)}\right)
$$

$$
-\frac{1}{2}\theta(2m_{K} - M)\frac{g_{K}^{2}}{16\pi^{2}}|\rho_{K}(M)|
$$

$$
\times (\pi - 2\arctan|\rho_{K}(M)|),
$$
(37)

$$
\rho_K = \sqrt{1 - \frac{4m_K^2}{M^2}}.
$$
\n(38)

Here $g_K = \sqrt{2}g_{K^+K^-}$.

The NS line shape is given by the same expressions (30) and (31) with the replacement $D_a \rightarrow D_m$, where

$$
D_m = m_S^2 - M^2 - \Sigma,
$$
\n
$$
\Sigma = i \frac{g_{\pi \eta}^2}{16\pi} \rho_{\pi \eta} (M) + \frac{i}{2} \theta (M - 2m_{K^+}) \frac{g_K^2}{16\pi} \rho_{K^+}(M)
$$
\n
$$
- \frac{1}{2} \theta (2m_K^+ - M) \frac{g_K^2}{16\pi} |\rho_{K^+}(M)| + \frac{i}{2} \theta (M - 2m_{K^0})
$$
\n
$$
\times \frac{g_K^2}{16\pi} \rho_{K^0}(M) - \frac{1}{2} \theta (2m_K^0 - M) \frac{g_K^2}{16\pi} |\rho_{K^0}(M)|.
$$
\n(40)

The corresponding expressions for $f_0 \to \pi \pi$ are the same with the obvious replacement of m_{η} with m_{π} and $g_{\pi\eta}$ with $g_{\pi} = \sqrt{3/2}g_{\pi^+\pi^-}$.

Both the NS and KL forms were used by KLOE to analyze their data on $\phi \to \pi \eta \gamma$ [\[24\]](#page-8-0) and $\phi \to \pi^0 \pi^0 \gamma$ [\[25\]](#page-8-0), updated

¹ It was pointed out in Ref. $[23]$ that there is a typographical error in Eqs. [\(17\)](#page-2-0) and (A.8) of Ref. [\[3\]](#page-7-0). The results presented in Ref. [\[3\]](#page-7-0) used the correct formula, as given here.

in Ref. [\[28\]](#page-8-0), giving two parameter sets for each of $a_0(980)$ and $f_0(980)$. The KL formalism has also been applied to the reactions $\gamma \gamma \rightarrow \eta \pi$ [\[29\]](#page-8-0) to give two somewhat different results for $a_0(980)$ and to $\gamma \gamma \rightarrow \pi \pi$, together with the $I = 0$ S-wave phase shift $[30,31]$, giving eight further parameters sets for $f_0(980)$.

Formally, there is no contradiction between the KL and NS forms of Breit-Wigner amplitudes and it can be easily verified that the KL form reduces to the NS one near the KK threshold. However, there is a significant difference when they are applied to the production mechanism. In the key reactions $\phi \to \gamma S$ the transition mechanism is via a quark loop in the case of the NS model *versus* a kaon loop in the case of the KL model. In other words, in the absence of defined couplings, the model assumed for the transition mechanism automatically leads to a difference in the g_K coupling, it being significantly larger in the KL case.

Additional information is available on the ratio g_K^2/g_π^2 for the decay of $f_0(980)$, principally from the analysis of $\pi \pi \to \pi \pi$ and $\pi \pi \to K \bar{K}$ [\[32–35\]](#page-8-0), and from $J/\psi \to$ $\phi(1020)\pi^+\pi^-$ and $J/\psi \to \phi(1020)K^+K^-$ [\[36\]](#page-8-0). The results from these different analyses are consistent and average to $g_K^2/g_\pi^2 = 4.0 \pm 0.3$ [\[37\]](#page-8-0). None of these experiments quotes a width, so the actual values of g_{π} and g_K are unknown. Consequently we do not use these results for predictions in our model.

None of the ten parameter sets for $f_0(980)$ from Refs. [\[28,30,31\]](#page-8-0) satisfies this constraint explicitly, but nine have $g_K^2/g_\pi^2 > 4.0$ and one has $g_K^2 \ll g_\pi^2$. For definiteness in the KL model we take [\[31\]](#page-8-0)

$$
m_{f_0} = 0.9783 \text{ GeV}, \quad g_K = 5.006, \quad g_\pi = 1.705, \quad (41)
$$

which is the one with the lowest g_K^2/g_π^2 value. The NS model yields $g_K^2/g_\pi^2 < 1$, and, to illustrate the sensitivity of our results to the ratio of couplings, we include it with the parameters [\[28\]](#page-8-0)

$$
m_{f_0} = 0.9847 \text{ GeV}, \quad g_K = 0.556, \quad g_\pi = 1.600.
$$
 (42)

There is no additional information on the $a₀(980)$ couplings, so for consistency we again take the KL model set with the lowest $g_K^2/g_{\pi\eta}^2$ value. This is [\[24\]](#page-8-0)

$$
m_{a_0} = 0.9825 \text{ GeV}, \quad g_K = 3.05, \quad g_{\pi \eta} = 2.82.
$$
 (43)

The corresponding result for the NS model is [\[24\]](#page-8-0)

$$
m_{a_0} = 0.9825 \text{ GeV}, \quad g_K = 2.22, \quad g_{\pi \eta} = 2.16.
$$
 (44)

E. Prediction of $a_0(980)$ and $f_0(980)$ photoproduction

In principle there are no free parameters in this calculation, because the couplings are known and the constants defining the cut terms are determined by the fit to π^0 photoproduction. However, the range of values for the constants for $a_0(980)$ and $f₀(980)$, particularly the latter, is such that a unique prediction is not possible.

In Ref. [\[3\]](#page-7-0) three scenarios for the scalars were considered in which the lowest $n\bar{n}$ nonet contains $a_0(980)$ and $f_0(980)$ as members. In two of these, $f_0(980)$ is mixed with either $f₀(1370)$ or $f₀(1500)$ such that they are octets and $f₀(980)$ $f_0(1370)$ or $f_0(1500)$ such that they are octets and $f_0(980)$
is the singlet $(u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}$. In the third, $f_0(980)$ is the is the singlet $(uu + aa + ss)/\sqrt{2}$. In the third, $J_0(980)$ is the standard $(u\bar{u} + d\bar{d})\sqrt{2}$ member of the octet. For definiteness we use the latter.

The $a_0(980)$ and $f_0(980)$ Breit-Wigner line shapes were integrated over the relevant experimental ranges and the resulting cross sections for the KL and NS model parameters of (43) and (44) and (41) and (42) , respectively, are given in Fig. 3, together with the ELSA [\[7\]](#page-8-0) and CLAS [\[8\]](#page-8-0) small-t data. In both cases the NS prediction is the upper curve.

There are several reasons why one expects the predicted cross sections to be smaller than the data. There is a coherent continuum background in the $\pi^0 \eta^0$ and $\pi^+ \pi^-$ channels that will interfere with direct production of $a_0(980)$ and $f_0(980)$, respectively. The effect of this is to increase the cross section and distort the resonance line shape. This is discussed in detail in Ref. [\[3\]](#page-7-0). Furthermore, there is the possibility of background from the decay of high-mass baryon resonances. This is explicit in the case of $a_0(980)$ from $P_{33}(1232)\eta$ and $S_{11}(1535)\pi$; see Fig. [4\(i\)](#page-7-0) of Ref. [\[8\]](#page-8-0). Sideband subtraction certainly removes incoherent background but not coherent background. Finally we note that our results are sensitive to the

FIG. 3. Predictions of the differential cross sections for $a_0(980)$ photoproduction in the $\pi^0\eta^0$ channel (left) and $f_0(980)$ photoproduction in the $\pi^+\pi^-$ channel (right). The data are from the ELSA [\[7\]](#page-8-0) and CLAS [\[8\]](#page-8-0) experiments, respectively, and the units are μ b GeV⁻². In each plot the upper line is the NS model prediction and the lower line is the KL model prediction.

FIG. 4. NS model predictions of the differential cross sections for $a_0(980)$ photoproduction in the $\pi^0\eta^0$ channel (left) and $f_0(980)$ photoproduction in the $\pi^0\pi^0$ channel (right). The photon energies are 3.5 (top), 5.0 (center), and 9.0 (bottom) GeV and the units are μ b GeV⁻². The KL model predictions are about a factor of 3/4 lower for the $a_0(980)$ and a factor of 2 lower for the $f_0(980)$.

choice of the rescattering terms, particularly in natural-parity exchange through interference with the leading terms.

The results show that the principal objective of Ref. [3], namely to demonstrate that scalar photoproduction cross sections are sufficiently large to be measured, has been attained. They also show that the production mechanism is more complicated than that considered in Ref. [3]; in particular, cut effects are not negligible. This is not particularly surprising, but it does make more difficult the extraction of radiative widths from photoproduction data. Within the context of the present model, the pole and cut terms can be separated via the energy dependence of the cross section.

Predictions of the cross sections for $a_0(980)$ (in the $\pi^0 \eta^0$ channel) and octet $f_0(980)$ photoproduction in the $\pi^0 \pi^0$ channel at $E_y = 3.5, 5.0,$ and 9.0 GeV are given in Fig. 4. The advantage of the $\pi^0 \pi^0$ channel for $f_0(980)$ is that it automatically excludes any vector-meson contribution to the final state.

The multiplicity of parameters for $a_0(980)$ and $f_0(980)$ lead to very different KK and total widths. In principle $K^+K^$ photoproduction would provide a check on our model through interference with $\phi(1020)$, just as the $f_0(980)$ is seen in $\pi^+\pi^$ photoproduction through interference with $\rho(770)$. There are two relevant experiments [\[38,39\]](#page-8-0) that provide indications of a scalar amplitude, albeit with large errors. A theoretically based analysis [\[40\]](#page-8-0) of these data also indicates the presence of a scalar amplitude, although again with large errors. The prediction from our model for the K^+K^- scalar cross section lies within the range of these analyses but their errors are too large to provide any constraint.

V. CONCLUSIONS

We presented a simple parameter-free model for $a_0(980)$ and $f₀(980)$ photoproduction. Within the context of the radiative decay model of Refs. [4,5] the results imply that $a_0(980)$ and $f_0(980)$ have a significant compact $n\bar{n}$ scalar ground state component in their wave function. This assignment looks quite natural in naive quark model calculations. For example in Ref. [\[41\]](#page-8-0) and more recently in Ref. [\[42\]](#page-8-0), 1^3P_0 states made of light quarks are predicted to exist at 1 GeV, and $a_0(980)$ and $f₀(980)$ are natural candidates for such states.

The $K\bar{K}$ threshold proximity should lead to a significant admixture of a $K\bar{K}$ molecular component in the wave function of scalar resonances, so that a bare $n\bar{n}$ scalar seed is to be coupled to pseudoscalar meson pairs. The relative weight of the molecular component depends on the strength of this coupling, a scenario advocated in Ref. [\[43\]](#page-8-0). Our findings are in favor of such a scenario. Although we cannot quantify the relative weights of quark and molecular components, we indicate strongly the presence of the former.

The incompatibility of the $f_0(980)$ couplings to $\pi\pi$ and $K\bar{K}$ between Refs. [\[28,30,31\]](#page-8-0) on the one hand and Refs. [\[32–](#page-8-0) [35,37\]](#page-8-0) on the other is an issue that clearly needs to be resolved. Photoproduction of $\pi^0 \pi^0$ and K^+K^- could achieve this. The minimum experimental requirements are differential cross sections, plane-polarized beam asymmetries (to obtain information on the unnatural-parity exchange), the resonance line shape (to obtain information on the background), and sufficiently high energy to eliminate contamination from N^* production and decay.

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