Strangeness -2 $(I,J^P)=(\frac{1}{2},\frac{1}{2}^+)$ tribaryon resonance

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I have used the low-energy data of the new Nijmegen ESC08 baryon-baryon interactions for the systems with strangeness 0, -1, and -2 to construct a separable potential model of the $\Lambda\Lambda N$-ΞNN system to study the position and width of the three-body $(I,J^P)=(\frac{1}{2},\frac{1}{2}^+)$ resonance. I found that the $(\frac{1}{2},\frac{1}{2}^+)$ tribaryon has a mass of 3194 MeV, just below the Ξd threshold, and a width of only 0.09 MeV.

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I. INTRODUCTION

The only known three-body bound state with strangeness, the strangeness -1 ΛNN hypertriton, arises as a result of the attractive nature of the S-wave $N\Lambda$ and NN interactions at low energy, in particular to the presence of the NN $^3S_1$ deuteron bound state. In the case of the strangeness -2 $\Lambda\Lambda N$ system the $\Lambda\Lambda$ and ΛN interactions are attractive but not enough to bind the system. However, because the $\Lambda\Lambda N$ system and the ΞNN system are coupled together, the interactions acting in the last component, i.e., ΞN and NN, could provide sufficient attraction to give rise to a strangeness -2 three-body bound state or resonance because in the NN subsystem one has the $^3S_1$ deuteron bound state and the $^1S_0$ virtual state. In the ΞN subsystem the $(i,j)=(1,1)$ channel is bound in the Nijmegen ESC08c model [1] and almost bound in the Salamanca chiral quark model [2]. In addition, in the Salamanca model the $\Xi N(1,0)$ channel and the $\Lambda\Lambda$-ΞN (0,0) channel (the H dibaryon) are bound.

In a series of previous works [3–5] the bound-state problem of the $\Lambda\Lambda N$-ΞNN system was studied using as input the two-body interactions obtained from the Salamanca chiral quark model [6]. When the interactions for the $\Lambda\Lambda$, $N\Lambda$, NN, and $N\Xi$ subsystems given by this model were used in a full three-body calculation of the $\Lambda\Lambda N$-ΞNN system, the channel $(I,J^P)=(\frac{1}{2},\frac{1}{2}^+)$, the so-called strangeness -2 hypertriton, was found to be bound by $\approx$0.5 MeV [3–5].

The very recent baryon-baryon extended-soft-core ESC08c model of the Nijmegen-Wako group [1,7,8], on the other hand, has much less attraction in the strangeness -2 sector such that the $\Lambda\Lambda$-ΞN subsystem is unbound and there is only one $N\Xi$ bound state in the (1,1) channel. This drastic reduction of attraction in the strangeness -2 sector arises because they have incorporated into their analysis the Nagara event [9], which is the most important piece of information for the $\hat{S}=-2$ sector because it identified uniquely the double hypernucleus $^{6}_{\Lambda\Lambda}H$ and determined the binding energy of two Λ hyperons and the $\Lambda\Lambda$ interaction energy which was found to be much smaller than that obtained by previous experiments. Thus, one expects that the $\Lambda\Lambda N$-ΞNN three-body system will also be unbound and it will appear as a resonance.

The purpose of this paper is to extend the study of the three-body bound state into the continuum region, which requires the extension of the integral equations into the complex plane, but this can only be done if the interactions are known in analytical form. This is not easy to do with the interactions obtained from the Nijmegen ESC08 models, which have a large number of terms and different kinds of corrections given in numerical form. I have, therefore, constructed separable potential models of the $\Lambda\Lambda$, $N\Lambda$, NN, and $N\Xi$ subsystems adjusted to the low-energy parameters of each channel. This, first of all, leads to integral equations in one continuous variable for the $\Lambda\Lambda N$-ΞNN system that are easier to handle and, secondly, because they are based on simple analytical functions they allow me to extend the three-body equations into the complex plane.

II. TWO-BODY INTERACTIONS

Because the $\Sigma\Sigma$ and $\Lambda\Sigma$ two-body channels have a very small effect in the strangeness -2 three-body system [5] I do not include them in this work. Thus, the two-body channels that contribute to the $(I,J^P)=(\frac{1}{2},\frac{1}{2}^+)$ three-body state are given in Table I. As one sees from this table the strangenes -2 $\Lambda\Lambda$ and $N\Xi$ two-body systems are coupled together in the $(i,j)=(0,0)$ channel and therefore it is through this interaction that also the $\Lambda\Lambda N$ and ΞNN components get coupled together.

I assume separable potentials for all the uncoupled interactions of the form

$$V_i^\rho = g_i^\rho\rangle\lambda\langle g_i^\rho, \tag{1}$$

such that the two-body t matrices are of the form

$$t_i^\rho = g_i^\rho\rangle\tau_i^\rho\langle g_i^\rho, \tag{2}$$

with

$$\tau_i^\rho = \frac{\lambda}{1-\lambda\langle g_i^\rho|G_0(i)|g_i^\rho\rangle}. \tag{3}$$

In the case of the $\Lambda\Lambda$-$N\Xi$ coupled-channel state $(i,j)=(0,0)$, I follow the approach used by Carr, Afnan, and Gibson [10] for an older version of the Nijmegen potential, i.e.,

$$V_{ij}^{\rho\sigma} = g_i^\rho\rangle\lambda_{ij}\langle g_j^\sigma, \tag{4}$$

such that

$$t_{ij}^{\rho\sigma} = g_i^\rho\rangle\tau_{ij}^{\rho-\sigma}\langle g_j^\sigma, \tag{5}$$

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TABLE I. S-wave two-body channels (i,j) of the various subsystems that contribute to the $(I,J^P)=(\frac{1}{2},\frac{1}{2}^+)$ three-body state.

Subsystem	Strangeness	(i,j)
$\Lambda\Lambda$	−2	(0,0)
$N\Xi$	−2	(0,0),(0,1),(1,0),(1,1)
$N\Lambda$	−1	$(\frac{1}{2},0),(\frac{1}{2},1)$
NN	0	(0,1),(1,0)

with

$$\tau_{11}^{\Lambda\Lambda\text{-}\Lambda\Lambda}=\frac{-\lambda_{13}^2G^{N\Xi}-\lambda_{11}(1-\lambda_{33}G^{N\Xi})}{\lambda_{13}^2G^{\Lambda\Lambda}G^{N\Xi}-(1-\lambda_{11}G^{\Lambda\Lambda})(1-\lambda_{33}G^{N\Xi})},\tag{6}$$

$$\tau_{33}^{N\Xi\text{-}N\Xi}=\frac{-\lambda_{13}^2G^{\Lambda\Lambda}-\lambda_{33}(1-\lambda_{11}G^{\Lambda\Lambda})}{\lambda_{13}^2G^{\Lambda\Lambda}G^{N\Xi}-(1-\lambda_{11}G^{\Lambda\Lambda})(1-\lambda_{33}G^{N\Xi})},\tag{7}$$

$$\tau_{13}^{\Lambda\Lambda\text{-}N\Xi}=\tau_{31}^{N\Xi\text{-}\Lambda\Lambda}=\frac{-\lambda_{13}}{\lambda_{13}^2G^{\Lambda\Lambda}G^{N\Xi}-(1-\lambda_{11}G^{\Lambda\Lambda})(1-\lambda_{33}G^{N\Xi})},\tag{8}$$

and

$$G^{\Lambda\Lambda}=\langle g_1^{\Lambda\Lambda}|G_0(1)|g_1^{\Lambda\Lambda}\rangle,\tag{9}$$

$$G^{N\Xi}=\langle g_3^{N\Xi}|G_0(3)|g_3^{N\Xi}\rangle.\tag{10}$$

I used Yamaguchi form factors [11] for the separable potentials of Eqs. (1) and (4), i.e.,

$$g_i^\sigma(p)=\frac{1}{\alpha^2+p^2}.\tag{11}$$

Thus, for each uncoupled two-body channel I have to fit the two parameters α and λ to the low-energy parameters a and r_0. I give in Table II the low-energy parameters of the different uncoupled channels obtained from the new ESC08 models [1,7,8] and the corresponding separable-potential parameters α and λ. In the case of the low-energy parameters of the ΛN subsystems, I took the average values of Λn and Λp and for the NN subsystem the values of the np. Because these separable potentials are adjusted to the scattering length and effective range of each channel, the deuteron bound state lies at 2.184 MeV and the $N\Xi$ (1,1) D^* bound state of the ESC08c model lies at 1.655 MeV.

TABLE II. Low-energy parameters a and r_0 (in fm) of the ESC08 models [1,7,8] and the corresponding separable potential parameters α (in fm^{-1}) and λ (in fm^{-2}) for uncoupled partial waves.

Subsystem	(i,j)	a	r_0	α	λ
$N\Xi$	(0,1)	−5.357	1.434	2.3168	−2.4537
$N\Xi$	(1,0)	0.579	−2.521	1.1641	0.1837
$N\Xi$	(1,1)	4.911	0.527	5.4067	−39.161
$N\Lambda$	$(\frac{1}{2},0)$	−2.54	3.155	1.3270	−0.3614
$N\Lambda$	$(\frac{1}{2},1)$	−1.725	3.525	1.3414	−0.3190
NN	(0,1)	5.4384	1.7481	1.4198	−1.0336
NN	(1,0)	−23.7316	2.6983	1.1654	−0.3950

TABLE III. Parameters of the separable-potential model of the $(i,j)=(0,0)$ coupled $\Lambda\Lambda$-$N\Xi$ subsystem fitted to the effective-range parameters of the $\Lambda\Lambda$ system, $a=-0.853$ fm and $r_0=5.126$ fm [1], and the $N\Xi$ (complex) effective-range parameters, $a=0.0455-i0.348$ fm and $r_0=-25.38-i1.618$ fm (see the text).

α	β	λ_{11}	λ_{33}	λ_{13}
1.25	4.287	−0.0959	1.302	1.243

In the case of the coupled-channel subsystem (0,0) given by Eqs. (4)–(10) I take

$$g_1^{\Lambda\Lambda}(p)=\frac{1}{\alpha^2+p^2},\tag{12}$$

$$g_3^{N\Xi}(p)=\frac{1}{\beta^2+p^2},\tag{13}$$

so that I have the five parameters α, β, λ_{11}, λ_{33}, and λ_{13}. These five parameters were fitted to the $\Lambda\Lambda$ effective-range parameters $a=-0.853$ fm and $r_0=5.126$ fm and to the $N\Xi$ (complex) effective-range parameters $a=0.0455-i0.348$ fm and $r_0=-25.38-i1.618$ fm, where the last set of parameters were extracted from the $N\Xi$ phase shift and inelasticity of the ESCO8c model given in Fig. 14 of Ref. [1]. I give the parameters of this model in Table III and in Fig. 1 I show its prediction for the $\Lambda\Lambda$ phase shift up to the $N\Xi$ threshold and compare it with the ESC08c phase shift. As one can see from this figure a resonance in this energy region does not exist. This separable potential model of the $\Lambda\Lambda$-$N\Xi$ subsystem takes into account the effect of the $\Sigma\Sigma$ channel

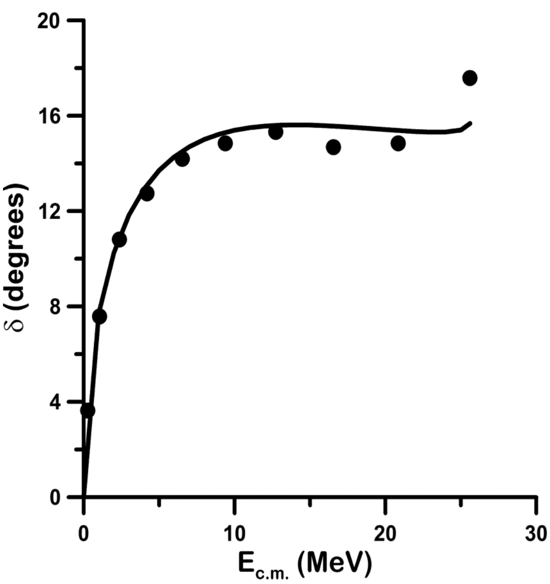

FIG. 1. The $\Lambda\Lambda$ phase shift in the $(i,j)=(0,0)$ channel for energies up to the $N\Xi$ threshold. The dots are the results of the ESC08c model [1].

indirectly because it was fitted to the ESC08c results where this channel has been included.

III. THREE-BODY EQUATIONS

The coupled $\Lambda\Lambda N$-ΞNN system has the peculiarity that each three-body component consists of two identical fermions and a third one that is different. The integral equations of this system were first derived by Miyagawa, Kamada, and Glöckle using the extended Pauli principle [12,13]. An alternative derivation using a graphical method is presented in Ref. [4]. The $\hat{S}=-2$ three-baryon sector has also been approached through the Alt-Grassberger-Sandhas equations to study the breakup process $\Xi d\to\Lambda\Lambda N$ in Ref. [14].

In Ref. [4] we have used the convention that particles 2 and 3 are two identical particles and particle 1 is the different one in each three-body component. After the reduction for identical particles the three-body equations take the following forms:

$$\begin{aligned}
\langle 1|T_1\rangle &= 2\langle 1|t_1^{\Lambda\Lambda}|1\rangle\langle 1|3\rangle G_0(3)\langle 3|T_3\rangle \\
&\quad +\langle 1|t_{13}^{\Lambda\Lambda-N\Xi}|3\rangle\langle 3|1\rangle G_0(1)\langle 1|U_1\rangle \\
&\quad -\langle 1|t_{13}^{\Lambda\Lambda-N\Xi}|3\rangle\langle 2|3\rangle G_0(3)\langle 3|U_3\rangle, \\
\langle 3|T_3\rangle &= -\langle 3|t_3^{N\Lambda}|3\rangle\langle 2|3\rangle G_0(3)\langle 3|T_3\rangle \\
&\quad +\langle 3|t_3^{N\Lambda}|3\rangle\langle 3|1\rangle G_0(1)\langle 1|T_1\rangle, \\
\langle 1|U_1\rangle &= 2\langle 1|t_1^{NN}|1\rangle\langle 1|3\rangle G_0(3)\langle 3|U_3\rangle, \\
\langle 3|U_3\rangle &= -\langle 3|t_3^{N\Xi}|3\rangle\langle 2|3\rangle G_0(3)\langle 3|U_3\rangle \\
&\quad +\langle 3|t_3^{N\Xi}|3\rangle\langle 3|1\rangle G_0(1)\langle 1|U_1\rangle \\
&\quad +2\langle 3|t_{31}^{N\Xi-\Lambda\Lambda}|1\rangle\langle 1|3\rangle G_0(3)\langle 3|T_3\rangle. \qquad (14)
\end{aligned}$$

Using Eqs. (2) and (5) in the integral equations (14) and introducing the transformations $\langle i|T_i\rangle=\langle i|g_i^{\alpha_i}\rangle\langle i|X_i\rangle$ and $\langle i|U_i\rangle=\langle i|g_i^{\alpha_i}\rangle\langle i|Y_i\rangle$, one obtains the one-dimensional integral equations

$$\begin{aligned}
\langle 1|X_1\rangle &= 2\tau_1^{\Lambda\Lambda}\big\langle g_1^{\Lambda\Lambda}\big|1\big\rangle\langle 1|3\rangle G_0(3)\big\langle 3\big|g_3^{N\Lambda}\big\rangle\langle 3|X_3\rangle \\
&\quad +\tau_{13}^{\Lambda\Lambda-N\Xi}\big\langle g_3^{N\Xi}\big|3\big\rangle\langle 3|1\rangle G_0(1)\big\langle 1\big|g_1^{NN}\big\rangle\langle 1|Y_1\rangle \\
&\quad -\tau_{13}^{\Lambda\Lambda-N\Xi}\big\langle g_3^{N\Xi}\big|3\big\rangle\langle 2|3\rangle G_0(3)\big\langle 3\big|g_3^{N\Xi}\big\rangle\langle 3|Y_3\rangle, \\
\langle 3|X_3\rangle &= -\tau_3^{N\Lambda}\big\langle g_3^{N\Lambda}\big|3\big\rangle\langle 2|3\rangle G_0(3)\big\langle 3\big|g_3^{N\Lambda}\big\rangle\langle 3|X_3\rangle \\
&\quad +\tau_3^{N\Lambda}\big\langle g_3^{N\Lambda}\big|3\big\rangle\langle 3|1\rangle G_0(1)\big\langle 1\big|g_1^{\Lambda\Lambda}\big\rangle\langle 1|X_1\rangle, \\
\langle 1|Y_1\rangle &= 2\tau_1^{NN}\big\langle g_1^{NN}\big|1\big\rangle\langle 1|3\rangle G_0(3)\big\langle 3\big|g_3^{N\Xi}\big\rangle\langle 3|Y_3\rangle, \\
\langle 3|Y_3\rangle &= -\tau_3^{N\Xi}\big\langle g_3^{N\Xi}\big|3\big\rangle\langle 2|3\rangle G_0(3)\langle 3|g_3^{N\Xi}\rangle\langle 3|Y_3\rangle \\
&\quad +\tau_3^{N\Xi}\big\langle g_3^{N\Xi}\big|3\big\rangle\langle 3|1\rangle G_0(1)\big\langle 1\big|g_1^{NN}\big\rangle\langle|Y_1\rangle \\
&\quad +2\tau_{31}^{N\Xi-\Lambda\Lambda}\big\langle g_1^{\Lambda\Lambda}\big|1\big\rangle\langle 1|3\rangle G_0(3)\big\langle 3\big|g_3^{N\Lambda}\big\rangle\langle 3|X_3\rangle,
\end{aligned} \qquad (15)$$

where one should keep in mind that

$$\tau_1^{\Lambda\Lambda}=\tau_{11}^{\Lambda\Lambda\text{-}\Lambda\Lambda} \qquad (16)$$

and

$$\tau_3^{N\Xi}=\tau_{33}^{N\Xi\text{-}N\Xi}, \qquad (17)$$

for the $(i,j)=(0,0)$ channel.

IV. RESULTS

To obtain the results I took the nucleon mass as the average of the proton and the neutron masses and the Ξ mass as the average of the Ξ^0 and Ξ^- masses. Thus, the ΞNN and Ξd thresholds are 25.604 and 23.420 MeV above the $\Lambda\Lambda N$ threshold, respectively. I extended the three-body integral equations into the complex plane by following the well-known procedure where the integration path is rotated into the fourth quadrant as $q_i\to q_i e^{-i\phi}$. If any of the two-body amplitudes contains a resonance there is a branch cut starting at the resonant pole and one must be careful that the integration contour dos not cross this cut [15]. However, none of the two-body amplitudes of my model contains a resonance as shown in Fig. 1 for the case of the coupled $\Lambda\Lambda$-$N\Xi$ amplitude. Of course, I checked that the eigenvalue obtained from the integral equations does not change when I use different values for the rotation angle ϕ of the contour, which guarantees that this is a true eigenvalue of the equations.

I found that the three-body resonance lies at

$$E_0=23.408-i0.045\ \ \text{MeV}, \qquad (18)$$

measured with respect to the $\Lambda\Lambda N$ threshold, i.e., just 0.012 MeV below the Ξd threshold. Thus, the binding energy of the state, 12 keV, is smaller than that of the strangeness -1 hypertriton (130 keV). This means that the strangeness -1 hypertriton is a loosely bound state of a Λ and a deuteron while the strangeness -2 hypertriton is a loosely bound state of a Ξ and a deuteron with a small decay width into $\Lambda\Lambda N$.

The result (18) is somewhat intriguing, in particular, the very small width, because the $\Lambda\Lambda N$ threshold is open. Therefore, to understand that result I have studied the effect of the two-body (0,0) channel on the three-body eigenvalue because this two-body channel is responsible for the coupling between the ΞNN and $\Lambda\Lambda N$ three-body channels. First of all, I should point out that near the ΞNN threshold the dominant two-body channels are the $N\Xi$ (1,1) channel with the bound D^* state [1], the NN (0,1) channel with the bound deuteron state, and the NN (1,0) singlet channel with a virtual state just below threshold. If I disconect the $\Lambda\Lambda$ channel, i.e., if I make $\lambda_{13}=0$ in Eq. (8) the three-body eigenvalue becomes $E_0=23.413$ MeV. If I now, in addition, disconect completely the two-body (0,0) channel by making also $\lambda_{33}=0$ I get an eigenvalue of $E_0=23.386$ MeV. Thus, while the coupling to the $\Lambda\Lambda$ channel is important for the $N\Xi$ (0,0) channel, the effect of the full $\Lambda\Lambda$-$N\Xi$ (0,0) channel is negligible in the three-body system near the ΞNN threshold. The change in the mass of the three-body resonance due to the (0,0) channel is just a few keV and therefore it is not surprising that the change in the width should be of the same order of magnitude.

Thus, if I now add the rest masses to the result (18) I get that the three-body $(\frac{1}{2},\frac{1}{2})$ resonance lies at $W_0=3194$ MeV and has a very small width of $\Gamma=0.09$ MeV, so that it is practically a stable bound state like the $(\frac{3}{2},\frac{1}{2})$ state previously discussed [16], both states lying near the ΞNN threshold.

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