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Nonperturbative collisional energy loss of heavy quarks in quark-gluon plasma

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We suggest a new mechanism for the energy loss of fast heavy quarks in quark-gluon plasma. This mechanism is based on pion production caused by the anomalous chromomagnetic quark-gluon-pion interaction induced by strong topological fluctuations of the gluon fields represented by instantons. We found that this mechanism makes a considerable contribution to the collisional energy loss of a heavy quark in quark-gluon plasma, which shows a nontrivial role of nonperturbative phenomena in strongly interacting quark-gluon plasma.

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The large energy loss of heavy quarks observed in high energy heavy ion collisions at the BNL Relativistic Heavy Ion Collider (RHIC) and CERN Large Hadron Collider (LHC) is one of the exciting puzzles in the physics of quark-gluon plasma (QGP). In spite of the significant progress in understanding the possible mechanisms for the energy loss of fast heavy quarks, the complete mechanism of this effect is yet to be explored. So far, there are many suggestions on the origins of heavy quark energy loss. The first is the collisional energy loss that was considered in the pioneering work of Bjorken [1]. (See also Refs. [2–4].) Another mechanism is the radiative energy loss due to the gluon radiation by a heavy quark induced by the interactions with QGP partons [5–11]. Other suggestions include the so-called dead cone effect that comes from the suppression of gluon emission by the quark mass at small angles [12]. The detailed discussions on the different mechanisms of the heavy quark energy suppression in heavy ion collisions can be found, for example, in recent publications [13–16] and references therein.

Although the afore-mentioned effects must be crucial to understanding the energy loss phenomena, these ideas are based on perturbative QCD (pQCD). This means that the system is assumed to be in the domain of pQCD for heavy quark interactions within QGP. However, it is now widely accepted that the QGP produced in relativistic heavy ion collisions is not weakly interacting pQCD-like QGP but is strongly interacting QGP (sQGP) with nonperturbative interactions between quarks and gluons [17]. Therefore, it is natural to speculate that nonperturbative QCD effects might have a crucial role in unravelling the heavy quark energy loss problem. For example, it was suggested in Ref. [18] that thermal monopoles in QGP may lead to a large enhancement of the parton radiative energy loss in QGP.

In this Rapid Communication we investigate a nonperturbative mechanism for heavy quark energy loss that is related to the anomalous chromomagnetic quark-gluon interaction induced by instantons [19]. Instantons, being strong topological fluctuations of gluon fields in the QCD vacuum, play an important role in hadron physics and give strong influence to the properties of QGP. (For a review, see, for example, Refs. [20,21].) Furthermore, instantons lead to various nontrivial effective interactions such as a very specific quark-quark interaction [22], the chromomagnetic quark-gluon interaction [19], and the quark-gluon-pion interaction [21,23]. Recently, it was demonstrated that the interaction of the last type has an important role in understanding the cross sections of the inclusive pion production in high energy proton-proton collisions [24]. Furthermore, it may have a nontrivial role in unpolarized and polarized gluon distributions of nucleons at small Bjorken x [25]. Therefore, it is natural to expect that such nonperturbative strong interactions would have a nontrivial role in high energy heavy ion collisions as well. The purpose of the present work is thus to investigate the role of the chromomagnetic quark-gluon and quark-gluon-pion interactions in the heavy quark energy loss in QGP.

In Ref. [19], it was shown that instantons generate a new type of chromomagnetic quark-gluon interaction as

$$\mathcal{L}_I = -i\frac{g_s\mu_a}{4M_q}\,\bar{q}\,\sigma^{\mu\nu}t^a q\,G^a_{\mu\nu}, \tag{1}$$

where t^a is the SU(3) Gell-Mann matrices, μ_a is the anomalous quark chromomagnetic moment, M_q is the effective quark mass in the instanton vacuum, g_s is the strong coupling constant, and $G^a_{\mu\nu}$ is the gluon field strength tensor. Within the instanton model the value of μ_a is estimated as

$$\mu_a = -\frac{3\pi(M_q\rho_c)^2}{4\alpha_s(\rho_c^{-2})}, \tag{2}$$

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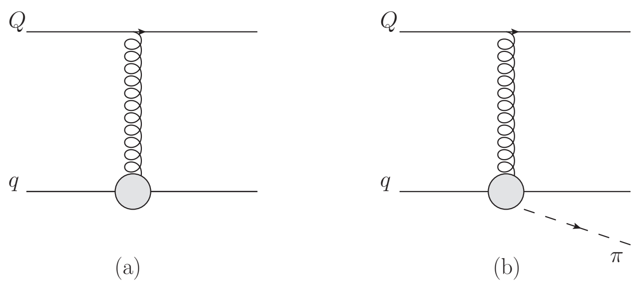

FIG. 1. Diagrams for heavy quark energy loss in QGP due to (a) the quark-gluon and (b) the quark-gluon-pion chromomagnetic interaction. Here, Q and q stand for a heavy quark and a light quark, respectively.

where ρ_c is the average instanton size in QCD vacuum, and $\alpha_s(\rho_c^{-2}) = g_s^2(\rho_c)/(4\pi)$ is the strong coupling constant at the scale of ρ_c. For details, we refer the reader to Refs. [21,26].

However, the Lagrangian of Eq. (1) does not respect chiral symmetry and needs to be modified by introducing the pion field for preserving chiral symmetry [21,23]. Then, in the single instanton approximation, the effective Lagrangian becomes

$$\mathcal{L}_I = -i\frac{g_s\mu_a}{4M_q}\bar{q}\sigma^{\mu\nu}t^a q G^a_{\mu\nu} + \frac{g_s\mu_a}{4M_q F_\pi}\bar{q}\sigma^{\mu\nu}t^a\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\phi}_\pi q\, G^a_{\mu\nu} \tag{3}$$

with the pion decay constant $F_\pi = 93$ MeV. Using Eq. (2), it is rewritten as

$$\mathcal{L}_I = i\frac{3\pi^2\rho_c^2 M_q}{4g_s(\rho_c)}\bar{q}\sigma^{\mu\nu}t^a q G^a_{\mu\nu} - \frac{3\pi^2\rho_c^2}{4g_s(\rho_c)}g_{\pi qq}\bar{q}\sigma^{\mu\nu}t^a\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\phi}_\pi q G^a_{\mu\nu}, \tag{4}$$

where $g_{\pi qq} = M_q/F_\pi$ is the pion-quark coupling constant at zero temperature, $T = 0$. Equipped with the effective Lagrangian of the quark-gluon and quark-pion-gluon interactions, we now consider their role in the heavy quark energy loss in QGP. The relevant diagrams contributing to the nonperturbative heavy quark energy loss are presented in Fig. 1.

Our starting point is the Bjorken's formula for the collisional energy loss [1] with a t-channel exchange, which reads

$$\frac{dE}{dx} = \int d^3k n_i(k,T)\mathcal{F}\int d|t|\frac{d\sigma}{d|t|}\nu, \tag{5}$$

where $\mathcal{F}$ is the flux factor and $\nu = E - E'$ with E' being the energy of the emergent parton. The parton density in QGP at temperature T is given by

$$n_i(k,T) = \frac{N_i}{(2\pi)^3}\frac{1}{\exp\left(\sqrt{k^2+m_i^2}/T\right)\pm 1}, \tag{6}$$

where the positive sign corresponds to the quark density ($i = q$) with $N_q = 12n_f$ and n_f being the number of active quark flavors in QGP, while the negative sign corresponds to the gluon density ($i = g$) with $N_g = 16$. In our estimation, we use $n_f = 2$ by considering the light u and d quarks. The flux factor is $\mathcal{F} = 1 - \cos\theta$, where θ is the angle between the momenta of two incident partons.

The cross section for the diagram of Fig. 1(a) is then calculated as

$$\frac{d\sigma}{dt} = \frac{\pi^3(M_q\rho_c)^2\rho_c^2F_g^2(\sqrt{|t|}\rho_c)}{8|t|}, \tag{7}$$

where $F_g(y) = 4/y^2 - 2K_2(y)$ is the instanton form factor [26] with $K_2(y)$ being the modified Bessel function of the second kind of order 2, and we assume that, in our nonperturbative calculation, the scale in the running strong coupling constant is determined by the instanton size. Therefore, the nonperturbative contribution of this diagram to the energy loss is

$$\frac{dE^{\text{np(a)}}}{dx} = \frac{\pi^3(M_q\rho_c)^2\rho_c^2}{8}\int d^3k\frac{n_i(k,T)}{2k} \times\int_{|t|_{\min}}^{|t|_{\max}} d|t|F_g^2(\sqrt{|t|}\rho_c), \tag{8}$$

which leads to

$$\frac{dE^{\text{np(a)}}}{dx} = \frac{\pi^3(M_q\rho_c)^2\rho_c^2T^2}{16}\int_{|t|_{\min}}^{|t|_{\max}} d|t|\, F_g^2(\sqrt{|t|}\rho_c). \tag{9}$$

For the diagram of Fig. 1(b), consideration of the kinematics of the process gives the relation

$$\nu = \frac{M_X^2 - t}{2k(1-\cos\theta)}, \tag{10}$$

where M_X is the invariant mass of the final system of the light quark and pion. Therefore, the collisional energy loss can be rewritten as

$$\frac{dE}{dx} = \int\frac{d^3k}{2k}n_i(k,T)\int d|t|\frac{d\sigma}{d|t|}\left(|t| + M_X^2\right). \tag{11}$$

By decomposing the momentum in the longitudinal and transversal components [24], the differential cross section becomes

$$d\sigma = \frac{3g_{\pi qq}^2\rho_c^4}{2^7\pi}\frac{F_g^2(|\boldsymbol{q}|\rho_c)}{\boldsymbol{q}^2}\frac{dz}{z}d^2\boldsymbol{q}\,d^2\boldsymbol{k}_\pi, \tag{12}$$

where z is the fraction of the initial light quark momentum carried by the pion in the center-of-momentum frame, $\boldsymbol{k}_\pi$ is the transverse momentum of the pion, and $\boldsymbol{q}$ is the transverse momentum of the exchanged gluon, which leads to $t \approx -\boldsymbol{q}^2$. Here the isospin factor 3 is included for pion production. By substituting $\tilde{\boldsymbol{q}} = z\boldsymbol{q} - \boldsymbol{k}$ and using the relations

$$\tilde{\boldsymbol{q}}^2 = z(1-z)M_X^2, \quad d^2\boldsymbol{k}_\pi = d^2\tilde{q} = \pi z(1-z)\,dM_X^2, \tag{13}$$

the integrals over z and $\boldsymbol{k}_\pi$ can be performed.

Since instantons describe the subbarrier transitions between classical QCD vacua with different topological charges, the integration over invariant mass M_X of the light-quark–pion system is restricted by the so-called sphaleron energy defined as

$$E_{\text{sph}} = \frac{3\pi}{4\rho_c\alpha_s\left(\rho_c^{-2}\right)}, \tag{14}$$

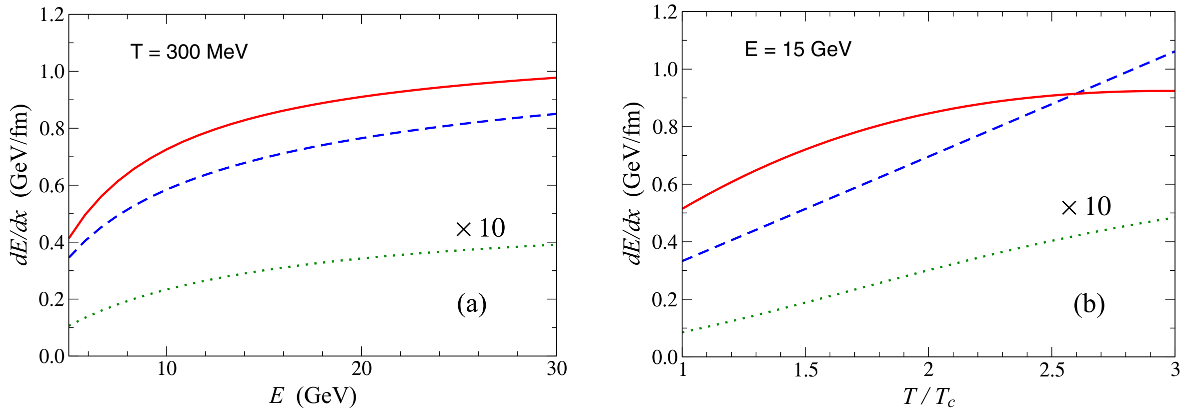

FIG. 2. (a) Energy dependence of the collisional energy loss of a charm quark in QGP at $T = 2\,T_c$. (b) Temperature dependence of the energy loss at $E = 15$ GeV. The solid lines are the nonperturbative contribution of Eq. (9) with the pion field, while the dotted lines, multiplied by 10, are nonperturbative contributions of Eq. (15) without the pion field. The dashed lines are the perturbative results given in Eq. (16).

which is the height of the potential barrier between the vacua [21]. The expression for the heavy quark energy loss induced by the nonperturbative quark-gluon-pion interaction then becomes

$$\frac{dE^{\mathrm{np(b)}}}{dx} = \frac{3^3\pi^3 g_{\pi qq}^2(T)}{2^{13}}\frac{\rho_c^2 T^2}{\alpha_s^2(\rho_c^{-2})} \times \int_{|t|_{\min}}^{|t|_{\max}} d|t| F_g^2(\sqrt{|t|}\rho_c)\left(1 + \frac{E_{\mathrm{sph}}^2}{2|t|}\right). \quad (15)$$

This result should be compared with the pQCD collisional energy loss, which is given by [4]

$$\frac{dE^{\mathrm{pert}}}{dx} = \frac{4\pi T^2}{3}\alpha_s(M_D^2)\alpha_s(ET)\left[\left(1+\frac{n_f}{6}\right)\ln\frac{ET}{M_D^2} + \frac{2}{9}\frac{\alpha_s(M_Q^2)}{\alpha_s(M_D^2)}\ln\frac{ET}{M_Q^2} + c(n_f)\right], \quad (16)$$

where $c(n_f) \approx 0.146\,n_f + 0.05$ and M_D is the Debye mass.

For our numerical estimate, we neglect the possible weak temperature dependence of the quark-pion coupling constant $g_{\pi qq}$ in the range of $T = (1 \sim 3)\,T_c$ following Ref. [27]. This temperature range is expected to cover the temperatures achieved at the relativistic heavy ion collisions at RHIC and LHC. For the deconfinement temperature, we use $T_c = 150$ MeV, and the charm quark mass is $M_c = 1.3$ GeV. The Debye mass is a function of temperature and, following the lattice QCD calculations of Refs. [28–30], we write it as

$$M_D(T) \approx 3\,T. \quad (17)$$

The integrals for $dE^{\mathrm{np(a)}}/dx$ and $dE^{\mathrm{np(b)}}/dx$ are performed with $|t|_{\min} = M_D^2$ and $|t|_{\max} = ET$.

The parameters of our model are determined as follows. For the running strong coupling constant, we use the widely used form given as [31]

$$\alpha_s(Q^2) = \frac{1}{\beta_0}\left[\frac{1}{\ln(Q^2/\Lambda^2)} + \frac{\Lambda^2}{\Lambda^2 - Q^2}\right], \quad (18)$$

where $\beta_0 = (33 - 2N_F)/12\pi$ and $\Lambda = 200$ MeV for $N_F = 4$. The average size of the instanton is taken to be $\rho_c = \frac{1}{3}$ fm, which is supported by both phenomenology and lattice calculations [20]. The strong coupling constant is then $\alpha_s(\rho_c^{-2}) \simeq 0.5$, which also agrees with the value of the instanton model of Ref. [21].

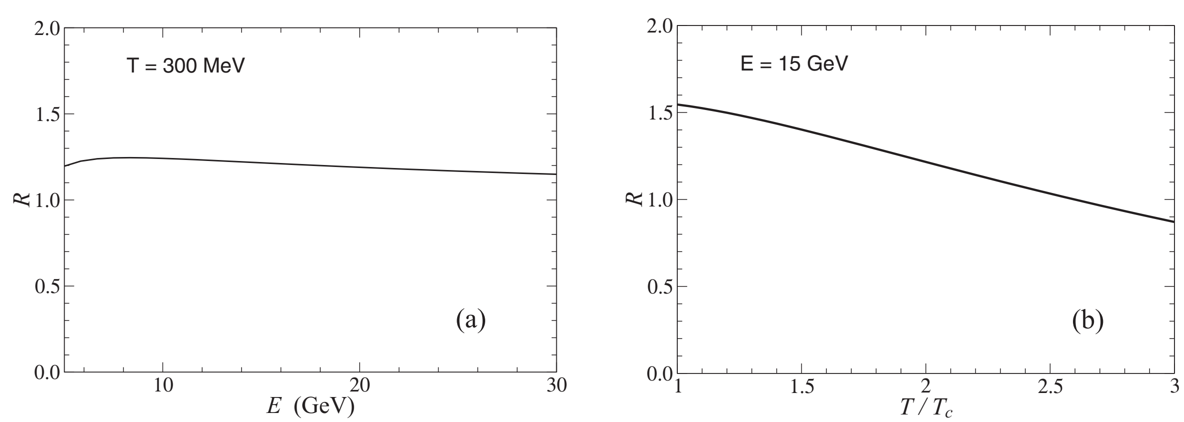

FIG. 3. (a) Energy dependence of the ratio $\mathcal{R}$ of the nonperturbative to the perturbative contributions defined in Eq. (19) at $T = 2\,T_c$. (b) Temperature dependence of the ratio $\mathcal{R}$ at $E = 15$ GeV.

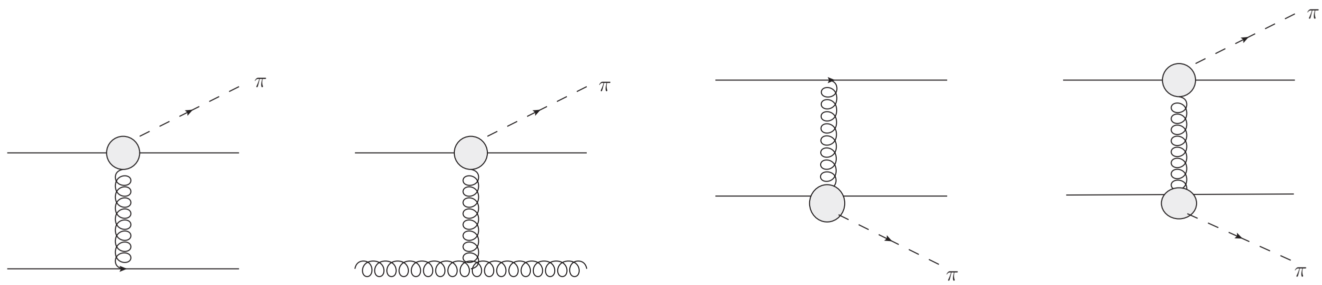

FIG. 4. Diagrams that can contribute to the nonperturbative light quark energy loss in QGP.

An important parameter of our model is the effective quark mass in instanton vacuum. The Lagrangians of Eqs. (1) and (4) are obtained in the effective single instanton approximation and, in this approximation, multi-instanton effects are included in the effective quark mass in the zero-mode-like propagator in the instanton field. Then a careful analysis on various correlation functions within the instanton model leads to $M_q = 86$ MeV [32]. We will use this value in the present work. But, since our results are rather dependent on the quark mass, we also discuss the results with a higher mass, $M_q = 170$ MeV, which was estimated in the earlier work of Ref. [33].

Shown in Figs. 2 and 3 are the numerical results of the present calculation. In Fig. 2, the charm quark energy losses from the perturbative and nonperturbative parts are shown as functions of energy and temperature. We find that the contribution from the quark-gluon interaction without pion emission [Fig. 1(a)] is very small as shown by the dotted lines in Fig. 2 and, therefore, can be safely neglected. However, the nonperturbative contribution with pion emission [Fig. 1(b)] to the collisional heavy quark energy loss is found to be similar or even larger than that of the pQCD contribution. To quantify the difference, in Fig. 3 we plot the ratio $\mathcal{R}$ defined as

$$\mathcal{R} = \frac{dE^{\text{np(b)}}}{dx} \bigg/ \frac{dE^{\text{pert}}}{dx}. \tag{19}$$

Figure 3 shows that $R = 1.2 \sim 1.3$ in the considered region of energy and temperature. We also found that the energy dependence of the ratio is very weak in the energy range between 10 and 30 GeV, while it has sensitive dependence on temperature. This is because the energy dependence is determined by the cross sections that are almost saturated in the considered energy region, while the temperature dependence is governed by the parton density distribution.

As stated before, the radiative energy loss is expected to be one of the major sources of heavy quark energy loss. However, there are large uncertainties in the estimation of the radiative energy loss [5–11]. In Refs. [34,35], it was claimed that a phenomenological factor $K = 3.5$ for the ratio of the total energy loss to the pQCD collisional energy loss is needed to explain the measured data. This means that the contributions from mechanisms other than the pQCD contribution should be larger than the pQCD contribution by a factor of 2.5. Therefore, we conclude that a smaller radiative energy loss, namely, about 1.3 times the pQCD collisional contribution, would be enough to resolve the heavy quark energy loss puzzle due to the nonperturbative contribution considered in the present work.[1]

We finally make a qualitative comment on the nonperturbative energy loss of light quarks. In this case, the virtuality of the fast quark is not large and, therefore, additional diagrams with pion radiation from the fast light quark can give nontrivial contributions to the total energy loss. Examples of such diagrams are shown in Fig. 4. In this case, it is evident that the direct pion production from fast quarks will give enhancement of total pion production in heavy ion collisions which, therefore, will increase the nuclear modification factor R_{AA}. However, on the other hand, the direct pion emission should lead to the energy loss of the fast quark similar to the heavy quark case, and this will decrease R_{AA}. Therefore, we have two contributions of opposite roles and the empirical R_{AA} will be determined by the competition between them. This requires more careful and complex analyses and will be discussed elsewhere.

To summarize, we suggest a new nonperturbative mechanism for heavy quark energy loss in QGP. It was shown that the nonperturbative chromomagnetic quark-gluon-pion interaction in QGP may give a nontrivial contribution to the heavy quark collisional energy loss. Therefore, this mechanism will be important to understanding the mechanisms of the heavy quark energy loss combined with other mechanisms. Our finding again shows the important role of nonperturbative phenomena in the understanding of the dynamics of QGP observed in high energy heavy ion collisions.

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[1]We found that the ratio $\mathcal{R}$ is rather sensitive to the effective quark mass M_q. If we use $M_q = 170$ MeV, it becomes as large as $\sim$4. Therefore, the ambiguity in M_q brings in another uncertainty originating from the nonperturbative calculation.

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