New approach for obtaining information on the many-nucleon structure in α decay from accompanying bremsstrahlung emission

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We analyze the nucleon structure of the α-decaying nucleus to see if it can be visible in the experimental bremsstrahlung spectra of the emitted photons which accompany such a decay. We develop a new formalism of the bremsstrahlung model taking into account the distribution of nucleons in the α-decaying nuclear system. We conclude the following. (1) After inclusion of the nucleon structure in the model the calculated bremsstrahlung spectrum is changed very slowly for a majority of the α-decaying nuclei. However, we have observed that visible changes really exist for the ^{106}Te nucleus ($Q_\alpha = 4.29$ MeV, $T_{1/2} = 70\ \mu$s) even for the energy of the emitted photons up to 1 MeV. This nucleus is a good candidate for future experimental study of this task. (2) Inclusion of the nucleon structure in the model increases the bremsstrahlung probability of the emitted photons. (3) We find the following tendencies for obtaining the nuclei, which have bremsstrahlung spectra more sensitive to the nucleon structure: (a) direction to nuclei with smaller Z and (b) direction to nuclei with larger Q_α values.

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I. INTRODUCTION

The bremsstrahlung emission of photons accompanying nuclear reactions has been attracting much interest for a long time (see reviews [1,2] and books [3–5]). This is because such photons provide rich information about the studied nuclear process. Dynamics of the nuclear process, interactions between nucleons, types of nuclear forces, structure of nuclei, quantum effects, and anisotropy (deformations) can be included in the model describing the bremsstrahlung emission. At the same time, measurements of such photons and their analysis provide information about all these aspects and the evaluation of the suitability of the models.

However, progress using dynamics and interactions between nucleons and nuclear forces has been limited. Researchers pointed to such a difficulty and included the realistic potentials of nuclear interactions in calculations of the bremsstrahlung spectra (for example, see Ref. [6]). This is also reflected by the small number of papers in this research area. We explain such a situation by the essential distance between (1) achievement of good agreement of the existing experimental data with the calculated spectra and (2) development of a mathematical formalism sufficiently sensitive to many-nucleon interactions and dynamics, which should give convergent calculations of the spectra and explain the experimental data.

Essential efforts were made to understand emission of the bremsstrahlung photons in nucleon-nucleon and nucleon-nuclear collisions. However, a prevailing idea of the existed models consists in reduction of the complicated interactions between nuclei to two-nucleon interactions, which are assumed to be leading. However, the main emphasis in such papers was on construction of the correct relativistic description of interaction between two nucleons in this task, where the formalism was developed mainly in momentum representation. Here we would like to note two directions of intensive investigations: Refs. [7,8] and Refs. [9–19].

The bremsstrahlung of the emitted photons in nuclear reactions where the nuclei were described on the microscopic level is a traditional research line that has existed for a long time. We note here progress by Baye, Descouvemont, Keller, Sauwens, Liu, Tang, Kanada, Dohet-Eraly, Sparenberg, and their colleagues [20–32]. The nucleon-nucleon scattering data (for example, the Nijmegen data set [33]) and properties of the deuteron are used for controlling parameters of these microscopical cluster models. These microscopic cluster models are based on developments of the resonating group method [34–36] and the generator coordinate method [37].

Microscopic developments of the bremsstrahlung models are attractive because the photons emitted during nuclear reactions provide information about the dynamics of nucleons of a composed nuclear system, which is formed on the basis of interactions (i.e., this is a study of the structure of nuclei). For example, in Ref. [38], from the bremsstrahlung spectra in the scattering of $p + {}^{208}$Pb at the proton incident energies of 140 and 145 MeV and the scattering of $p + {}^{12}$C, $p + {}^{58}$Ni, $p + {}^{107}$Ag, and $p + {}^{197}$Au at the proton incident energy of 190 MeV, we made conclusions about relations (roles) between the momenta of nucleons of the nucleus target and the spin properties of the scattered proton (such relations are not small and they do not form a coherent bremsstrahlung

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contribution to the full emission; this bremsstrahlung emission was measured in Refs. [39–41]). However, the momenta of nucleons are characteristics of internal dynamics of nucleons inside a nucleus. It seems that this information cannot be obtained by standard approaches without measurements of the bremsstrahlung photons. Our current paper is a continuation in this research line.

Note that researchers developed microscopic clusters models, and analyzed bremsstrahlung experimental data mainly for light nuclei (we find data for $p+\alpha$, $\alpha+\alpha$, and $^{3}\mathrm{H}+\alpha$ scattering). Such data were obtained in the coplanar geometry, where maxima in the bremsstrahlung cross sections are observed at some energies (i.e., resonances). Usually, such maxima are explained by resonant states of the compound nuclear system. In frameworks of such a formalism, kinematic relations between the energy of the emitted photon and the energy of relative motion of two nuclei or nucleus and nucleon are used (i.e., such relations put a direct dependence between energy of the scattered proton and energy of the emitted photon).

At the same time, there is essentially a larger volume of the bremsstrahlung experimental information in the nuclear processes that has not been analyzed by those researchers. These data are for the scattering protons of the ^{9}Be, ^{9,12}C, ^{64}Cu, ^{107}Ag, and ^{208}Pb nuclei (for the incident proton energies up to 140 MeV), α decay of ^{210,214}Po, ^{226}Ra, and ^{244}Cm, fission, and fusion. As we see, the accuracy of experimental measurements of photons is essentially higher. For each type of reaction, these bremsstrahlung cross sections (probabilities) are measured inside a wide energy region of the emitted photons for the same fixed energy of moving fragment before photon emission (which is not described by the kinematic relations pointed out above). Such cross sections have smooth continuous shapes where any sharp resonant peaks are not observed.

From such a point of view, it could be interesting for us to analyze these experimental data on the basis of our formalism which is not imposed by the kinematic relations above. To perform many-nucleon analysis, we include some elements of the microscopic theory in our approach. It could be interesting to develop such a formalism for reactions with the participation of heavy nuclei, where there is more evidence (for example, see research papers [39,42–46], reviews [1,2], and Ph.D. thesis [40] for the bremsstrahlung during scattering of protons off nuclei for low and intermediate energies of the emitted photons; theoretical description in Refs [7,38,47–49] and [50–56] for the bremsstrahlung during the α decay of nuclei and fully quantum calculations in Refs. [55–59]; semiclassical calculations in Refs. [60,61] for extraction of information about nuclear deformations of the α-decaying nuclei via analysis of the experimental bremsstrahlung spectra; Refs. [62–68] for the bremsstrahlung in fission of heavy nuclei; theoretical description in Ref. [69]; and Ref. [70] for predictions of the bremsstrahlung during emission of proton from nuclei).

More empirical evidence provides us more possibilities for testing the developed models. However, the first question that should be clarified is how much the nucleon structure of nuclei and incident fragments are visible in the bremsstrahlung spectra. It may appear that inclusion of the many-nucleon structure into the model will barely change visibly the calculated spectra for energies and parameters used in experiments. Moreover, the development of the microscopic formalism for such a problem is difficult. So, before developing an accurate model, its practicality needs to be clarified.

This paper is devoted to analysis and solution of such a question, where the α decay is chosen as the studied reaction. We explain such a choice as follows. (1) The microscopic approaches can be applied for a description of the α-decaying nuclei by natural means (for example, see research papers [71–75], reviews [75–78], and reference therein). (2) The α-decaying nuclei are medium, heavy, and superheavy. (3) Progress has been achieved in the study of the α-nucleus interactions tested experimentally, which makes α decay one of the most deeply studied reactions in nuclear physics. (4) Rich material has been developed in theoretical description of the bremsstrahlung emission of photons in the given reaction [51,52,55–61,79–93]. (5) In the study of the bremsstrahlung photons during the α decay the close agreement between theory and existing experimental data has been achieved [50–56]. (6) Such an agreement (for example, see Ref. [92] for details and demonstrations) is obtained without normalization of the calculated spectra on the experimental data (which has been very rarely achieved for other reactions with good accuracy), which is a significant advance for new predictions and inclusion of all sets of the α-decaying nuclei into analysis (α decay is observed for more than 420 nuclei with $A > 105$ and $Z > 52$). So, in the description of the bremsstrahlung experimental spectra for the α decay on many-nucleon basis, at present, our bremsstrahlung approach has no other alternative models by other people. This is our new contribution to the bremsstrahlung theory in this paper.

II. MODEL

A. Operator of emission of the α-nucleus system

We start from the leading form (7) of the photon emission operator $\hat{H}_\gamma$ in Ref. [38], generalizing it for the system of an α particle composed from four nucleons and a daughter nucleus composed of A nucleons in the laboratory system. Presenting the vector potential of the electromagnetic field in form of Eq. (5) in Ref. [47], we obtain

$$\hat{H}_\gamma - e\sqrt{\frac{2\pi}{w_{\mathrm{ph}}}}\sum_{\alpha=1,2}\mathbf{e}^{(\alpha),*} \times\left\{\sum_{i=1}^{4}\frac{z_i}{m_i}\,e^{-i\mathbf{k}\mathbf{r}_i}\,\mathbf{p}_i+\sum_{j=1}^{A}\frac{z_j}{m_j}\,e^{-i\mathbf{k}\mathbf{r}_j}\,\mathbf{p}_j\right\}. \quad (1)$$

Here the star denotes complex conjugation, z_s and m_s are the electromagnetic charge and mass of the nucleon with number s, respectively, and $\mathbf{p}_s = -i\hbar\,\mathbf{d}/\mathbf{dr}_s$ is the momentum operator for the nucleon with number s ($s = i$ is for nucleon of the α particle and $s = j$ for nucleon of the daughter nucleus; we number nucleons of α particle by index i and nucleons of the nucleus by index j). $\mathbf{e}^{(\alpha)}$ are unit vectors of the polarization of the photon emitted [$\mathbf{e}^{(\alpha),*} = \mathbf{e}^{(\alpha)}$], $\mathbf{k}$ is the wave vector of the photon, and $w_{\rm ph} = kc = |\mathbf{k}|\, c$. Vectors $\mathbf{e}^{(\alpha)}$ are perpendicular to $\mathbf{k}$ in the Coulomb gauge. We have two independent polarizations $\mathbf{e}^{(1)}$ and $\mathbf{e}^{(2)}$ for the photon with momentum $\mathbf{k}$ ($\alpha = 1,2$). In this paper we use the system of units where $\hbar = 1$ and $c = 1$.

Now we turn to the center-of-mass frame. We define coordinate of centers of masses for the α particle as $\mathbf{r}_\alpha$, for the daughter nucleus as $\mathbf{R}_A$, and for the complete system as $\mathbf{R}$ having form $\mathbf{r}_\alpha = \sum_{i=1}^{4} m_i\, \mathbf{r}_{\alpha i}/m_\alpha$, $\mathbf{R}_A = \sum_{j=1}^{A} m_j\, \mathbf{r}_{Aj}/m_A$, and $\mathbf{R} = (m_A \mathbf{R}_A + m_\alpha \mathbf{r}_\alpha)/(m_A + m_\alpha)$, where m_α and m_A are masses of the α particle and daughter nucleus. Introducing new relative coordinates $\boldsymbol{\rho}_{\alpha i}$, $\boldsymbol{\rho}_{Aj}$, and $\mathbf{r}$ as $\mathbf{r}_i = \mathbf{r}_\alpha + \boldsymbol{\rho}_{\alpha i}$, $\mathbf{r}_j = \mathbf{R}_A + \boldsymbol{\rho}_{Aj}$, and $\mathbf{r} = \mathbf{r}_\alpha - \mathbf{R}_A$, we find the corresponding momenta, $\mathbf{p}_i = \mathbf{p}_\alpha + \tilde{\mathbf{p}}_{\alpha i}$, $\mathbf{p}_j = \mathbf{P}_A + \tilde{\mathbf{p}}_{Aj}$, and $\mathbf{p} = \mathbf{p}_\alpha - \mathbf{P}_A$, where $\mathbf{p}_\alpha = -i\hbar\, \mathbf{d}/\mathbf{d}\mathbf{r}_\alpha$, $\tilde{\mathbf{p}}_{\alpha i} = -i\hbar\, \mathbf{d}/\mathbf{d}\rho_{\alpha i}$, $\mathbf{P}_A = -i\hbar\, \mathbf{d}/\mathbf{d}\mathbf{R}_A$, and $\tilde{\mathbf{p}}_{Aj} = -i\hbar\, \mathbf{d}/\mathbf{d}\rho_{Aj}$. Using these formulas, we obtain

$$
\begin{aligned}
&\mathbf{R}_A = \mathbf{R} - c_\alpha\, \mathbf{r}, \quad \mathbf{r}_\alpha = \mathbf{R} + c_A\, \mathbf{r}, \\
&\mathbf{r}_i = \mathbf{R} + c_A\, \mathbf{r} + \boldsymbol{\rho}_{\alpha i}, \quad \mathbf{r}_j = \mathbf{R} - c_\alpha\, \mathbf{r} + \boldsymbol{\rho}_{Aj},
\end{aligned} \tag{2}
$$

where we introduced $c_A = \frac{m_A}{m_A + m_\alpha}$ and $c_\alpha = \frac{m_\alpha}{m_A + m_\alpha}$. Substituting these expressions into Eq. (1), we find ($m_{\rm p}$ is mass of proton)[1]

$$
\begin{aligned}
\hat{H}_\gamma = &-e\sqrt{\frac{2\pi}{w_{\rm ph}}} \sum_{\alpha=1,2} \mathbf{e}^{(\alpha),*}\, e^{-i\mathbf{k}[\mathbf{R} - c_\alpha \mathbf{r}]} \\
&\times \Biggl\{ \Biggl[e^{-i\mathbf{k}\mathbf{r}} \sum_{i=1}^{4} \frac{z_i}{m_i}\, e^{-i\mathbf{k}\boldsymbol{\rho}_{\alpha i}} + \sum_{j=1}^{A} \frac{z_j}{m_j}\, e^{-i\mathbf{k}\boldsymbol{\rho}_{Aj}} \Biggr] \mathbf{P} \\
&+ \Biggl[c_A\, e^{-i\mathbf{k}\mathbf{r}} \sum_{i=1}^{4} \frac{z_i}{m_i}\, e^{-i\mathbf{k}\boldsymbol{\rho}_{\alpha i}} - c_\alpha \sum_{j=1}^{A} \frac{z_j}{m_j}\, e^{-i\mathbf{k}\boldsymbol{\rho}_{Aj}} \Biggr] \mathbf{p} \\
&+ e^{-i\mathbf{k}\mathbf{r}} \sum_{i=1}^{4} \frac{z_i}{m_i}\, e^{-i\mathbf{k}\boldsymbol{\rho}_{\alpha i}}\, \tilde{\mathbf{p}}_{\alpha i} + \sum_{j=1}^{A} \frac{z_j}{m_j}\, e^{-i\mathbf{k}\boldsymbol{\rho}_{Aj}}\, \tilde{\mathbf{p}}_{Aj} \Biggr\}.
\end{aligned} \tag{3}
$$

B. Wave function of the α-nucleus system

Emission of the bremsstrahlung photons is caused by the relative motion of nucleons of the full nuclear system. However, as the most intensive emission of photons is formed by relative motion of the α particle related to the nucleus, it is sensible to represent the total wave function via coordinates of relative motion of these complicated objects. In this paper we follow the formalism given in Ref. [38] for the proton-nucleus scattering, and we add a description of the many-nucleon structure of the α particle. Such a presentation of the wave function allows us to take into account most accurately the leading contribution of the wave function of relative motion into the bremsstrahlung spectrum, while the many-nucleon structure of the α particle and nucleus should provide only minor corrections (such a contribution of the many-nucleon structure follows from good agreement between theory and experiment for α decay obtained without the many-nucleon structure; see [55–57,61]). Before developing a detailed many-nucleon formalism for such a problem, we clarify first if the many-nucleon structure of the α-nucleus system is visible in the experimental bremsstrahlung spectra. In this regard, estimation of the many-nucleon contribution in the full bremsstrahlung spectrum is a well-described task. Thus, we define the wave function of the full nuclear system as

$$
\Psi = \Phi(\mathbf{R}) \cdot \Phi_{\alpha\text{-nucl}}(\mathbf{r}) \cdot \psi_{\rm nucl}(\beta_A) \cdot \psi_\alpha(\beta_\alpha), \tag{4}
$$

where

$$
\begin{aligned}
\psi_{\rm nucl}(\beta_A) &= \psi_{\rm nucl}(1 \cdots A) \\
&= \frac{1}{\sqrt{A!}} \sum_{p_A} (-1)^{\varepsilon_{p_A}}\, \psi_{\lambda_1}(1)\psi_{\lambda_2}(2) \cdots \psi_{\lambda_A}(A), \\
\psi_\alpha(\beta_\alpha) &= \psi_\alpha(1 \cdots 4) \\
&= \frac{1}{\sqrt{4!}} \sum_{p_\alpha} (-1)^{\varepsilon_{p_\alpha}}\, \psi_{\lambda_1}(1)\psi_{\lambda_2}(2)\psi_{\lambda_3}(3)\psi_{\lambda_4}(4).
\end{aligned} \tag{5}
$$

Here β_α is the set of numbers $1 \cdots 4$ of nucleons of the α particle, β_A is the set of numbers $1 \cdots A$ of nucleons of the nucleus, $\Phi(\mathbf{R})$ is the function describing motion of center of mass of the full nuclear system, $\Phi_{\alpha\text{-nucl}}(\mathbf{r})$ is the function describing relative motion of the α particle concerning to nucleus (without description of internal relative motions of nucleons in the α particle and nucleus), $\psi_\alpha(\beta_\alpha)$ is the many-nucleon function dependent on nucleons of the α particle (it determines space state on the basis of relative distances $\boldsymbol{\rho}_1 \cdots \boldsymbol{\rho}_4$ of nucleons of the α particle concerning to its center of mass), $\psi_{\rm nucl}(\beta_A)$ is the many-nucleon function dependent on nucleons of the nucleus. Summation in Eqs. (5) is performed over all $A!$ permutations of coordinates or states of nucleons. One-nucleon functions $\psi_{\lambda_s}(s)$ represent the multiplication of space and spin-isospin functions as $\psi_{\lambda_s}(s) = \varphi_{n_s}(\mathbf{r}_s)\, |\, \sigma^{(s)}\tau^{(s)}\rangle$,where φ_{n_s} is the space function of the nucleon with number s, n_s is the number of state of the space function of the nucleon with number s, and $|\, \sigma^{(s)}\tau^{(s)}\rangle$ is the spin-isospin function of the nucleon with number s.

We include the many-nucleon structure into wave functions $\psi_{\rm nucl}$ and ψ_α of the nucleus and the α particle, while we assume that wave function of relative motion $\Phi_{\alpha\text{-nucl}}(\mathbf{r})$ is calculated without them but with maximal orientation of the α-nucleus potential extracted from experimental data of α decay, α-nucleus scattering, and α capture. So, ψ_α and $\psi_{\rm nucl}$ describe only the internal states of the α particle and nucleus. The motion of nucleons of the nucleus relative to each other does not influence the internal states of the α particle and, therefore, such a representation of the wave function can be considered as an approximation. However, both relative internal motions of nucleons of the α particle and the nucleus provide their contributions to the full bremsstrahlung spectrum and can be estimated. In such a sense we take into account the

[1]Because we have only three independent variables $\boldsymbol{\rho}_{\alpha i}$ and $A - 1$ independent variables $\boldsymbol{\rho}_{Aj}$, Eq. (3) can be rewritten without variables $\boldsymbol{\rho}_{\alpha 4}$, $\boldsymbol{\rho}_{AA}$ and $\tilde{\mathbf{p}}_{\alpha 4}$, $\tilde{\mathbf{p}}_{AA}$.

internal nucleon structure of the α particle and nucleus. We calculate the matrix element of the photon emission as

$$\begin{aligned}
&\langle \psi_f(1\cdots A)|\hat{H}_\gamma|\psi_i(1\cdots A)\rangle \\
&= \frac{1}{A(A-1)} \sum_{k=1}^{A} \sum_{m=1,m\neq k}^{A} \{\langle \psi_k(i)\psi_m(j)|\hat{H}_\gamma|\psi_k(i)\psi_m(j)\rangle \\
&\quad - \langle \psi_k(i)\psi_m(j)|\hat{H}_\gamma|\psi_m(i)\psi_k(j)\rangle\}. \qquad (6)
\end{aligned}$$

C. Matrix element of emission and effective charge

We assume $\Phi_{\bar{s}}(\mathbf{R}) = e^{-i\,\mathbf{K}_{\bar{s}}\cdot\mathbf{R}}$, where $\bar{s} = i$ or f (indexes i and f denote the initial state, i.e., the state before emission of photon, and the final state, i.e., the state after emission of photon) and $\mathbf{K}_s$ is the momentum of the total system [6]. Suggesting $\mathbf{K}_i = 0$, we calculate the matrix element,

$$\begin{aligned}
\langle \Psi_f|\,\hat{H}_\gamma|\Psi_i\rangle = -\frac{e}{m_\mathrm{p}}\sqrt{\frac{2\pi}{w_\mathrm{ph}}} \sum_{\alpha=1,2} \mathbf{e}^{(\alpha),*} \\
\times \{M_1 + M_2 + M_3 + M_4\}, \qquad (7)
\end{aligned}$$

where

$$\begin{aligned}
M_1 &= \langle \Psi_f|e^{i(\mathbf{K}_f-\mathbf{k})\cdot\mathbf{R}} e^{ic_\alpha \mathbf{kr}} \Bigg[e^{-i\mathbf{kr}} \sum_{i=1}^{4} z_i \frac{m_\mathrm{p}}{m_i} e^{-i\mathbf{k}\boldsymbol{\rho}_{\alpha i}} \\
&\quad + \sum_{j=1}^{A} z_j \frac{m_\mathrm{p}}{m_j} e^{-i\mathbf{k}\boldsymbol{\rho}_{Aj}} \Bigg] \mathbf{P}|\Psi_i\rangle, \\
M_2 &= \langle \Psi_f|e^{i(\mathbf{K}_f-\mathbf{k})\cdot\mathbf{R}} e^{ic_\alpha \mathbf{kr}} \Bigg[e^{-i\mathbf{kr}} c_A \sum_{i=1}^{4} z_i \frac{m_\mathrm{p}}{m_i} e^{-i\mathbf{k}\boldsymbol{\rho}_{\alpha i}} \\
&\quad - c_\alpha \sum_{j=1}^{A} z_j \frac{m_\mathrm{p}}{m_j} e^{-i\mathbf{k}\boldsymbol{\rho}_{Aj}} \Bigg] \mathbf{p}|\Psi_i\rangle, \\
M_3 &= \langle \Psi_f|e^{i(\mathbf{K}_f-\mathbf{k})\cdot\mathbf{R}} e^{ic_\alpha \mathbf{kr}} e^{-i\mathbf{kr}} \sum_{i=1}^{4} z_i \frac{m_\mathrm{p}}{m_i} e^{-i\mathbf{k}\boldsymbol{\rho}_{\alpha i}} \tilde{\mathbf{p}}_{\alpha i}|\Psi_i\rangle, \\
M_4 &= \langle \Psi_f|\,e^{i(\mathbf{K}_f-\mathbf{k})\cdot\mathbf{R}} e^{ic_\alpha \mathbf{kr}} \sum_{j=1}^{A} z_j \frac{m_\mathrm{p}}{m_j} e^{-i\mathbf{k}\boldsymbol{\rho}_{Aj}} \tilde{\mathbf{p}}_{Aj}|\Psi_i\rangle. \qquad (8)
\end{aligned}$$

We do not use the first term M_1 (because we study decay in the center-of-mass system and neglect possible response), the third term M_3 (because we neglect the contribution of photon emission caused by the deformation of the α particle as it leaves), and the fourth term M_4 (because we do not study the contribution of photon emission caused by the deformation of the daughter nucleus during decay). On this basis we obtain

$$\begin{aligned}
\langle \Psi_f|\hat{H}_\gamma|\Psi_i\rangle = -\frac{e}{m_\mathrm{p}}\sqrt{\frac{2\pi}{w_\mathrm{ph}}} \sum_{\alpha=1,2} \mathbf{e}^{(\alpha),*}\delta(\mathbf{K}_f - \mathbf{k}) \\
\times \langle \Phi_f(\mathbf{r})|Z_\mathrm{eff}(\mathbf{r},\mathbf{k})\,e^{-i\mathbf{kr}}\,\mathbf{p}|\Phi_i(\mathbf{r})\rangle, \qquad (9)
\end{aligned}$$

where we introduced the effective charge of the system composed from the α particle and the daughter nucleus, the charged form factor of the α particle, and the charged form factor of the daughter nucleus as

$$Z_\mathrm{eff}(\mathbf{r},\mathbf{k}) = e^{i\mathbf{kr}}\{e^{-ic_A\mathbf{kr}} c_A Z_\alpha(\mathbf{k}) - e^{ic_\alpha\mathbf{kr}} c_\alpha Z_\mathrm{A}(\mathbf{k})\}, \qquad (10)$$

$$\begin{aligned}
Z_\alpha(\mathbf{k}) &= \langle \psi_{\alpha,f}|\sum_{i=1}^{4} z_i \frac{m_\mathrm{p}}{m_i} e^{-i\mathbf{k}\boldsymbol{\rho}_{\alpha i}}|\psi_{\alpha,i}\rangle, \\
Z_\mathrm{A}(\mathbf{k}) &= \langle \psi_\mathrm{nucl,f}|\sum_{j=1}^{A} z_j \frac{m_\mathrm{p}}{m_j} e^{-i\mathbf{k}\boldsymbol{\rho}_{Aj}}|\psi_\mathrm{nucl,i}\rangle. \qquad (11)
\end{aligned}$$

In the first approximation (called *dipole*) $\exp(i\mathbf{kr}) \to 1$ we have

$$Z_\mathrm{eff}^\mathrm{(dip)}(\mathbf{k}) = c_A Z_\alpha(\mathbf{k}) - c_\alpha Z_\mathrm{A}(\mathbf{k}). \qquad (12)$$

One can see that in such an approximation the effective charge becomes independent of the relative distance between centers of masses of the α particle and the daughter nucleus. In further calculations we restrict ourselves by application of the dipole approximation for determination of the effective charge in form of Eq. (12), while we calculate the matrix element without such an approximation. Such a way allows us to take the multipole corrections into account (in contrast to the dipole approach, for example, see Ref. [58]).

D. Electromagnetic form factors of the α particle and daughter nucleus

For further calculation of the electromagnetic form factors (11), we need to know the full wave functions before and after the emission of the photon (which corresponds to the unperturbed Hamiltonian). For such functions, we use the general formula (5), where one-nucleon wave functions are represented in a form of multiplication of the space and spin-isospin functions. In this paper we assume that the space wave function of one nucleon should determine probability of displacement of this nucleon relative to its most probable spatial position, which is not concentrated in the center of mass of the fragment, but on a particular distance. We develop such a consideration on the basis of the following simple idea: The most probable positions of nucleons of the α particle (described by the space one-nucleon wave functions) in the ground state should not coincide with the center of mass of the α particle. In such a case, they represent vertexes of the tetrahedron, while the oscillating space wave functions of the first four states give maximal probabilities in the joint center. Such a consideration is more naturally extended on the many-nucleon systems, where the most probable positions of nucleons of nucleus in the ground state should be correlated with a uniformly distributed density of the nuclear matter (and with its saturation). We take information about the most probable positions of nucleons (i.e., data about radius vectors $\boldsymbol{\rho}_{0,s}$) from other methods. By such motivations, we reformulate many-nucleon formalism in our previous model applied for the proton-nucleus scattering in Ref. [38]. We show below that such a presentation of the space one-nucleon wave function allows more accurate analysis of a dependence of the bremsstrahlung spectra on the size of the emitted α particle.

Thus, we rewrite the vector of position of nucleon with number s relatively to the center-of-mass of the fragment as

$$\boldsymbol{\rho}_s = \boldsymbol{\rho}_{0,s} + \tilde{\boldsymbol{\rho}}_s, \tag{13}$$

where $\boldsymbol{\rho}_{0,s}$ is the radius vector from the center of mass of the fragment to the point of the most probable location of nucleon with number s, $\tilde{\boldsymbol{\rho}}_s$ is the displacement of nucleon relative to this point of its most probable location. Thus, we construct the full one-nucleon wave function in the form

$$\psi_{\lambda_s}(s) = \varphi_{\lambda_s}(\boldsymbol{\rho}_s - \boldsymbol{\rho}_{s,0})|\sigma^{(s)}\tau^{(s)}\rangle, \tag{14}$$

where λ_s denotes number of state of nucleon with number s. Also, we assume that space function of nucleon is normalized by the condition

$$\int |\varphi_\lambda(\tilde{\boldsymbol{\rho}}_s)|^2 \mathbf{d}\tilde{\boldsymbol{\rho}}_s = 1. \tag{15}$$

Using a one-nucleon representation for the wave function, we find for the α particle the form of the form factor (see Appendix A)

$$Z_\alpha(\mathbf{k}) = \frac{Z_\alpha}{4}\sum_{i=1}^{4} e^{-i\mathbf{k}\boldsymbol{\rho}_{i,0}} \tag{16}$$

and the form factor for the daughter nucleus obtains the form

$$Z_{\rm d}(\mathbf{k}) = 2e^{-(a^2k_x^2+b^2k_y^2+c^2k_z^2)/4}\, f_1\big(\mathbf{k},n_1\cdots n_{A_{\rm d}}\big) \times f_2\big(\mathbf{k},\rho_1\cdots\rho_{A_{\rm d}}\big), \tag{17}$$

where

$$f_1\big(\mathbf{k},n_1\cdots n_{A_{\rm d}}\big) = \sum_{n_x,n_y,n_z=0}^{n_x+n_y+n_z\leqslant N} L_{n_x}\big[a^2k_x^2/2\big] \times L_{n_y}\big[b^2k_y^2/2\big]L_{n_z}\big[c^2k_z^2/2\big], \tag{18}$$

$$f_2\big(\mathbf{k},\rho_1\cdots\rho_{A_{\rm d}}\big) = \frac{1}{A_{\rm d}}\sum_{j=1}^{A_{\rm d}} e^{-i\mathbf{k}\boldsymbol{\rho}_{j,0}}.$$

Here function f_1 is the summation over all states of a one-nucleon space wave function; function f_2 describes space distribution of nucleons inside the nucleus.

E. The effective charge and bremsstrahlung probability

Let us calculate the effective charge in the dipole approximation. Substituting formulas (16) and (17) for the form factors of the α particle and the daughter nucleus into Eq. (12), we obtain

$$Z_{\rm eff}^{(\rm dip)}(\mathbf{k}) = 2e^{-(a^2k_x^2+b^2k_y^2+c^2k_z^2)/4}\{c_A f_{2\alpha}(\mathbf{k},\rho_1\cdots\rho_4) - c_\alpha f_{1,{\rm d}}(\mathbf{k},n_1\cdots n_{A_{\rm d}}) f_{2,{\rm d}}(\mathbf{k},\rho_1\cdots\rho_{A_{\rm d}})\}. \tag{19}$$

Now we rewrite the matric element of the photon emission as

$$\langle\Psi_f|\hat{H}_\gamma|\Psi_i\rangle = -\frac{e}{m_{\rm p}}\sqrt{\frac{2\pi}{w_{\rm ph}}}\cdot p_{fi}\,\delta(\mathbf{K}_f - \mathbf{k}), \tag{20}$$

where

$$p_{fi} = 2e^{-(a^2k_x^2+b^2k_y^2+c^2k_z^2)/4}\sum_{\alpha=1,2}\mathbf{e}^{(\alpha),*} \cdot \langle\varphi_f(\mathbf{r})|\tilde{Z}_{\rm eff}^{\rm dip}(\mathbf{k})e^{-i\mathbf{kr}}\mathbf{p}|\varphi_i(\mathbf{r})\rangle,$$
$$\tilde{Z}_{\rm eff}^{(\rm dip)}(\mathbf{k}) = c_A f_{2\alpha}(\mathbf{k},\rho_1\cdots\rho_4) - c_\alpha f_{1,{\rm d}}\big(\mathbf{k},n_1\cdots n_{A_{\rm d}}\big) \times f_{2,{\rm d}}\big(\mathbf{k},\rho_1\cdots\rho_{A_{\rm d}}\big). \tag{21}$$

We define the probability of the emitted photons on the basis of matrix element (20) in frameworks of formalism given in Ref. [47] and we do not repeat it in this paper. In result, we obtain the bremsstrahlung probability as[2]

$$\frac{d^2P(\theta_f)}{dw_{\rm ph}d\cos\theta_f} = \frac{e^2}{2\pi c^5}\frac{w_{\rm ph}E_i}{m_{\rm p}^2k_i}\left\{p_{fi}\frac{dp_{fi}^*(\theta_f)}{d\cos\theta_f} + \text{c.c.}\right\}, \tag{22}$$

where c.c. is complex conjugation, p_{fi} is proportional to the electrical component $p_{\rm el}$ in Eqs. (10) in Ref. [47] [with the additional factor of $2\,e^{-(a^2k_x^2+b^2k_y^2+c^2k_z^2)/4}$ and the included effective charge $\tilde{Z}_{\rm eff}^{(\rm dip)}$] and $dp_{fi}(\theta_f)/d\cos\theta_f$ is defined by the same way as $dp(k_i,k_f,\theta_f)/d\cos\theta_f$ in Ref. [47].

III. CALCULATIONS AND ANALYSIS

We apply the method to calculate the spectrum of photons emitted during the α decay. We started our calculations from the ^{210}Po, ^{214}Po, and ^{226}Ra nuclei, for which there are experimental data of the bremsstrahlung spectra [50–56], and our previous developments of the model and results [55–57,61,89,92,93] were tested. Of course, we were initially interested in analysis of the ^{210}Po nucleus, where the experimental data [53,54] were obtained with the best accuracy. However, the difference between calculations with the included many-nucleon structure and without it is practically not visible [see Fig. 1(a)]. Both calculations describe these experimental data enough well. It turns out that for all these nuclei above, where we have any experimental information, the inclusion of the nucleon structure of the α particle and the daughter nucleus is practically not visible in the bremsstrahlung spectra (the second digit of the calculated spectrum is varied) for the energy region of the emitted photons below 1 MeV (such a limit is the highest in the experimental data). However, we find that such an inclusion increases the full bremsstrahlung probability of the emitted photons for each studied nucleus. This is the first conclusion that we have obtained.

[2]We obtain the formula (22) in dependence on the mass of proton $m_{\rm p}$, while in Ref. [47] we had the bremsstrahlung probability (49) in dependence on the reduced mass μ. Such a difference is explained by the fact that in the current paper we develop a formalism on the basis of the emission operator of the many-nucleon system (1), while in Ref. [47] we started the formalism on the basis of the operator of emission (4) of the proton-nucleus system defined via the reduced mass of proton and nucleus.

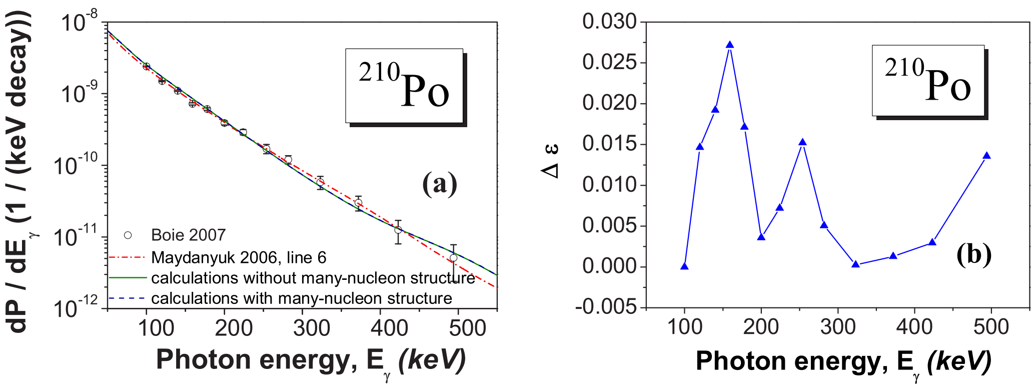

FIG. 1. (a) The bremsstrahlung probabilities of the emitted photons in the α decay of the ^{210}Po nucleus and experimental data [53] [in calculations $\boldsymbol{\rho}_{i,0} = 1.7$ fm for the α particle and $\theta_f = 90^\circ$ are used; θ_f is the angle between direction of the α particle motion (or its tunneling) after emission of photon and the direction of the photon emission]. Here the experimental data given by open circles are extracted from Ref. [53], the solid green line represents the calculations without the included nucleon structure, the dashed blue line represents the calculations with the included nucleon structure, the dash-dotted red line represents calculations for the pointlike α particle taken from Ref. [57] (see line 6 in Fig. 1 in that paper). The spectrum for calculations with inclusion of the many-nucleon structure is located above the spectrum without this structure, but such a difference is practically not visible. (b) The difference of the functions of errors, $\Delta\varepsilon(E_k)$, defined in Eqs. (24) and that obtained after comparative analysis between the new calculations with and without the nucleon structure and experimental data given in panel (a).

In such a situation, one can add the following analysis. We define the functions of errors

$$\varepsilon^{(s)}(E_k) = \frac{|\ln[\sigma^{(\mathrm{theor},s)}(E_k)] - \ln[\sigma^{(\exp)}(E_k)]|}{|\ln[\sigma^{(\exp)}(E_1)]|}, \tag{23}$$

define the difference between these functions, and calculate the summation

$$\begin{gathered}\Delta\varepsilon(E_k) = \varepsilon^{(\mathrm{no-micro})}(E_k) - \varepsilon^{(\mathrm{micro})}(E_k),\\ \Delta\bar{\varepsilon} = \frac{1}{N}\sum_{k=1}^{N}\Delta\varepsilon(E_k).\end{gathered} \tag{24}$$

Here $\sigma^{(\mathrm{theor},s)}(E_k)$ and $\sigma^{(\exp)}(E_k)$ are the theoretical and experimental bremsstrahlung probabilities in the α decay at energy E_k of the emitted photon, s indicates the inclusion of the many-nucleon structure into calculations (we denote such calculations by the index micro) or calculations without such a many-nucleon structure (we shall use the index no-micro for such a case), and the summation is performed over experimental data. Such definitions are based on a minimization method (see Ref. [94]).

To estimate if inclusion of the many-nucleon structure into calculations provides a better description of the experimental data, we have to find a difference $\Delta\varepsilon(E_k)$ between functions $\varepsilon^{(\text{no-micro})}(E_k)$ and $\varepsilon^{(\mathrm{micro})}(E_k)$. Such calculations for the α decay of the ^{210}Po nucleus are given in Fig. 1(b). One can see that the function is positive inside the whole photon energy region, which indicates that inclusion of the many-nucleon structure into calculations is more successful in description of the experimental data [53,54]. A general estimation can be obtained via the summarizing characteristic in Eqs. (24), and we obtain $\Delta\bar{\varepsilon} = 0.000\,010\,15$ (that is positive also). In Fig. 1(b) one can observe a slight oscillatory dependence of the curve on thet energy of the emitted photon with a period of about 200 KeV in the low-energy part of the spectrum. However, there is less information about these oscillations and their period than there is about positive values for the function $\Delta\varepsilon$; i.e., it requires obtaining more accurate (stable) bremsstrahlung spectra [according to Fig. 1(b), we should be sure in the first three to five digits of the calculated bremsstrahlung data for ^{210}Po[3]].

As the next step, we began to search other α-decaying nuclei, for which this effect (of influence of the nucleon structure on the bremsstrahlung spectra) could be visible practically. In selection of appropriate nuclei we have chosen the following basis.

(1) Possible new measurements of the bremsstrahlung photons will have smaller experimental errors for the α-decaying nuclei with higher emitted probabilities

[3]One can suppose that the form factors describing nucleon distribution in the nonpointlike α particle and nucleus have very slight oscillatory dependence on the energy of the emitted photon with such a period [in particular, the form factor for the α particle in Eq. (A8) has harmonic behavior in dependence on the energy of photons; at the pointlike limit this formula transforms to Eq. (A9) and such an oscillator dependence is lost; the form factor for the nucleus in form (A20) has similar behavior] . In such a case, their inclusion into the calculations introduces the small oscillator changes of the bremsstrahlung spectrum. Then from Fig. 1(b) one can suppose that at some energies separated on such a period, the calculations with the included nucleon structure describe the experimental spectrum most accurately for such reason (in comparison on calculations without such a structure). Another possibility explained that such an effect could be that experimental data have some slight oscillations that are not taken into account by the present model, and at some energies the calculations with the included nucleon structure are in the best agreement with this experiment. In any case, such a way could make it possible to investigate more deeply nucleon distribution in the α particle and nucleus during the α decay via analysis of the bremsstrahlung spectra, which looks to be an interesting task for future research.

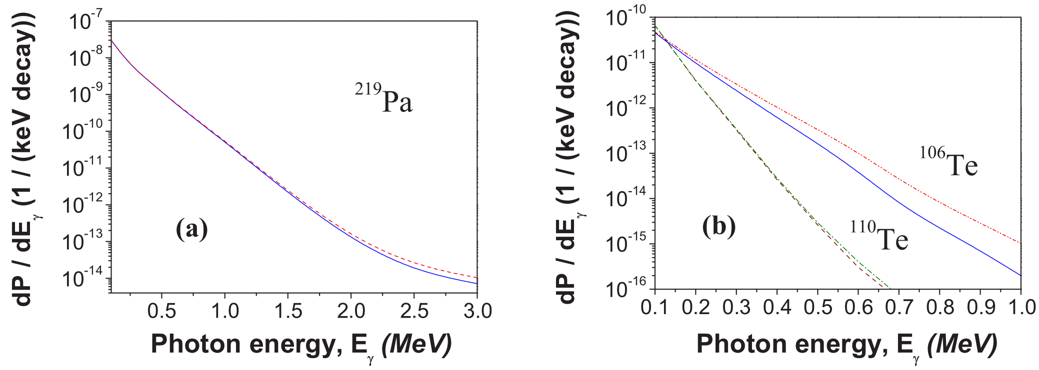

FIG. 2. The bremsstrahlung probabilities of the emitted photons at the α decay of the nuclei ^{219}Pa (a) and ^{106}Te and ^{110}Te (b) [in calculations $\boldsymbol{\rho}_{i,0} = 1.7$ fm for the α particle and $\theta_f = 90^\circ$ are used; θ_f is the angle between the direction of the α particle motion (or its tunneling) after emission of photon and the direction of the photon emission]. (a) The dashed red line is for calculations for ^{219}Pa with included nucleon structure, the solid blue line is for calculations for ^{219}Pa without included nucleon structure. (b) The dash-double dotted red line is for calculations for ^{106}Te with included nucleon structure, the solid blue line is for calculations for ^{106}Te without nucleon structure, the dash-dotted green line is for calculations for ^{110}Te with included nucleon structure, and the dashed brown line is for calculations for ^{110}Te without included nucleon structure. The curve for each nucleus after inclusion of the nucleon structure is located above in comparison with the curve without this structure. The nucleon structure is visible more strongly in the spectra for nuclei with higher Q_α values and smaller Z.

(at the same energies of the emitted photons), which corresponds to higher Q_α values of these nuclei.

(2) Calculations of the bremsstrahlung spectra are more stable and give more convergent results at higher Q_α values of the α-decaying nuclei and at lower energies of the emitted photons.

The form factors are included into the effective charge Z_{eff} and, therefore, this effective charge should determine the degree of variations of the bremsstrahlung spectrum after inclusion of the nucleon structure into the model and calculations. However, analyzing the different nuclei, we have found that such spectra variations are sufficiently slow. Moreover, Q_α value influences the probability of the bremsstrahlung photons. This parameter is larger and the emission of photons is more intensive. Hence, one can conclude that the nucleon structure should be visible for nuclei with higher Q_α values. This idea allows us to extend the region of our search of proper nuclei, which was our next step. In general, Q_α value is gradually increased with increasing mass of the nucleus. Therefore, we have looked for nuclei in the direction of the heaviest where nucleon structure could be visible in the bremsstrahlung spectra. However, the next calculations have shown that for heavy and superheavy nuclei such an influence of the nucleon structure is not visible again [see Fig. 2(a)].

However, there are other parameters which play important roles in such a search. In particular, this is the Coulomb barrier determined by the electromagnetic charge of the daughter nucleus. For light nuclei this parameter is essentially smaller and, therefore, the probability of the emitted photons for such nuclei should be larger. Analyzing distributions of Q values for the lightest α emitters, we go up to an island of α emission, covering neutron-deficient isotopes from tellurium ($Z = 52$) to cesium ($Z = 55$), which have very fast decay. From the literature we find that these α emitters and their α decays were subjects of recent intensive experimental [95–97] and theoretical [98,99] investigations. In particular, the α-decay half-lives of the ^{110}Xe and ^{106}Te nuclei were measured in Ref. [96] at the GSI on-line mass separator (here, half-times were determined to be $T_{1/2} = 105^{+35}_{-25}$ ms for ^{110}Xe and $T_{1/2} = 70^{+20}_{-10}$ μs for ^{106}Te). Authors of that paper used that information for estimation of reduced α widths and analysis of the α formation amplitudes in the emitters above ^{100}Sn. An α-decay Q value of $Q_\alpha = 4900(50)$ keV and half-life of $T_{1/2} = 0.70^{+0.25}_{-0.17}$ μs of the neutron-deficient nuclide ^{105}Te were measured using the implantation-decay correlation technique in Argonne National Laboratory [97]. Wang, Gu, and Hou in Ref. [98] provided systematics for theoretical study and measured half-lives and Q_α values for the different α-decay channels for isotopes near $N = Z = 50$ (from $Z = 52$ to $Z = 55$) and estimated a preformation factor for α particles for these α emitters. Misicu and Rizea in Ref. [99] studied the α decay of the ^{106}Te nucleus theoretically under the influence of an ultraintense laser field. Providing a detailed analysis on the example of this nucleus, they determined the tunneling probability and rates for the α decay for various laser intensities and frequencies and showed an enhancement of tunneling probability.

To support these investigations, we estimated the bremsstrahlung emission for the α decay of ^{106}Te and ^{110}Te, and results of such calculations are presented in Fig. 2(b). We observe that such a visible change of the bremsstrahlung spectra after inclusion of the nucleon structure into the model and calculations is really present for the ^{106}Te nucleus ($Q_\alpha = 4.29$ MeV, $T_{1/2} = 70$ μs) even for the photons energies below 1 MeV. However, for the ^{110}Te nucleus with smaller Q_α value ($Q_\alpha = 2.73$ MeV) this visible role of nucleon structure in spectra is already lost. We hope our analysis of these nuclei in our paper will support those researchers by supplementing some additional new information. As a possible new idea, we suppose that our current research of the many-nucleon structure of the α-nucleus system for this nucleus could be further reinforced by adding the external strong electromagnetic fields.

Now we see that in Figs. 1 and 2 the bremsstrahlung spectra with the included many-nucleon structure are above the corresponding spectra without such a structure. Such a difference can be explained in formalism by the fact that the spectrum can be changed as a result of change of the effective charge $Z_{\rm eff}(\mathbf{r},\mathbf{k})$ [see Eqs. (10) and (11)], which is used in calculation of the matrix element of emission (9). Here the effective charge in Eq. (10) is not constant and dependent on momentum $\mathbf{k}$ of the emitted photon and two form factors $Z_\alpha(\mathbf{k})$ and $Z_A(\mathbf{k})$.

If we use a pointlike consideration of the α particle and nucleus and do not use a many-nucleon description of the α particle and nucleus, then we have constant form factors for the α particle and nucleus in Eq. (11), which corresponds simply to electromagnetic charges of the α particle and nucleus. If we use the dipole approximation, then we completely lose a dependence of the effective charge on the momentum of photon [see Eq. (12)]. Such an approximation corresponds to more simple bremsstrahlung models. We demonstrate this case in Fig. 2 by the lower curves [see the solid blue line for ^{219}Pa in panel (a) and the solid blue line for ^{106}Te and the dashed brown line for ^{110}Te in panel (b)]. In nonpointlike consideration of the α particle and nucleus, each form factor gives its own dependence on the momentum of the emitted photons to the effective charge. We demonstrate this case in Fig. 2 by the upper curves [see the dashed red line for ^{219}Pa in panel (a) and the dash-double dotted red line for ^{106}Te and the dash-dotted green line for ^{110}Te in panel (b)].

However, we define interactions between the α particle and nucleus on the basis of the chosen α-nucleus potential (we use a spherically symmetric approximation), which has Coulomb barrier and was parametrized in Refs. [100–103] for a large number of nuclei on the basis of rich analysis of experimental information. That is, we suppose that this potential provides us realistic interactions between the α particle and the nucleus given by experiments. On the basis of this potential, we calculate wave functions $\Phi_{\alpha\text{-nucl}}(\mathbf{r})$ numerically [see Eq. (4)], which are defined relatively a distance $\mathbf{r}$ between centers of masses of the α particle and nucleus and characterize a relative motion of these two objects. On the basis of such wave functions, then we calculate the matrix element (9) of emission. So the nucleus is not pointlike in calculations of such a wave function, but we can include or exclude the many-nucleon description of the nucleus and α particle in the effective charge (which appears from the many-nucleon operator of emission). This changes the bremsstrahlung spectra.

In current formalism, the bremsstrahlung spectrum with included many-nucleon structure is above the bremsstrahlung spectrum without the many-nucleon structure for all α-emitting nuclei. This can be explained in formulas by the fact that the effective charge $\tilde{Z}_{\rm eff}^{\rm (dip)}$ with the many-nucleon structure is larger than this effective charge without such structure [see Eqs. (21)]. For a more clear understanding of this aspect, we calculated the ratio $\tilde{Z}_{\rm eff}^{\rm (dip,\,micro)}/\tilde{Z}_{\rm eff}^{\rm (dip,\,no\text{-}micro)}$ between the effective charges calculated with inclusion of many-nucleon structure and without it in dependence on the energy of the emitted photon. In Fig. 3 one can see that a role of the many-nucleon description in the calculation of the effective charge is increased with increasing energy of the emitted photon.[4]

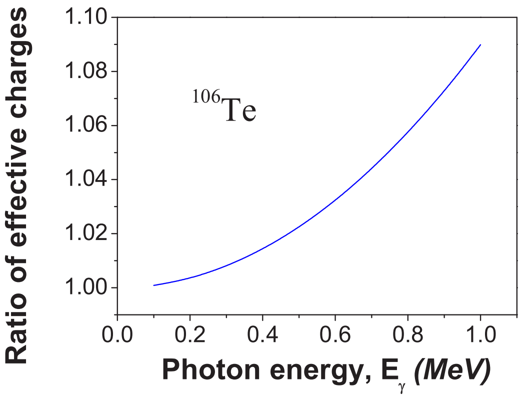

FIG. 3. Ratio $\tilde{Z}_{\rm eff}^{\rm (dip,\,micro)}/\tilde{Z}_{\rm eff}^{\rm (dip,\,no\text{-}micro)}$ between the effective charges calculated with inclusion of many-nucleon structure and without it in dependence on the energy of the emitted photon [here $\tilde{Z}_{\rm eff}^{\rm (dip,\,micro)}$ is the effective charge with the included many-nucleon structure defined in Eq. (21) and $\tilde{Z}_{\rm eff}^{\rm (dip,\,no\text{-}micro)}$ is the effective charge calculated without such a structure, i.e., for the pointlike α particle and nucleus]. One can see that the role of the many-nucleon description in the calculation of the effective charge $\tilde{Z}_{\rm eff}^{\rm (dip,\,micro)}$ is increased with increasing energy of the emitted photon. This explains why the bremsstrahlung spectra with the included many-nucleon structure are larger than the bremsstrahlung spectra without such a structure in Figs. 2(a) and 2(b), and such a difference only increased with increasing energy of the emitted photons.

IV. CONCLUSIONS AND PERSPECTIVES

In this paper we have studied if the nucleon structure of the α-decaying nucleus can be visible in the experimental bremsstrahlung spectra of the emitted photons that accompany such a decay. In this regard, we have developed a new formalism which takes into account the distribution of nucleons in the α-decaying nuclear system in the model of bremsstrahlung. Conclusions from analysis on the basis of this model are as follows.

(1) After inclusion of the nucleon structure into the model the calculated bremsstrahlung spectrum is changed very slowly for the majority of the α-decaying nuclei [see Fig. 2(a) for the α decay of ^{219}Pa]. However, we have observed that visible changes really exist for the ^{106}Te nucleus ($Q_\alpha = 4.29$ MeV, $T_{1/2} = 70$ μs) even

[4]There is also exponent $\exp[-(a^2k_x^2 + b^2k_y^2 + c^2k_z^2)/4]$ in matrix element p_{fi} in Eqs. (21), which suppresses the bremsstrahlung spectrum (i.e., it has the opposite influence on the spectrum than the effective charge). However its role is practically negligible for energies of the emitted photons in the α decay. For example, for the α particle we have $a,b,c \leqslant 1.5$ fm $= 1.5 \times 0.005$ MeV^{-1} $=$ 0.0075 MeV^{-1}, $k_x,k_y,k_z \leqslant k = E_{\rm ph} \leqslant 8$ MeV and we obtain $ak_x \leqslant$ 0.06 (in numerical calculations we write all variables in units MeV and MeV^{-1}).

for the energy of the emitted photons up to 1 MeV [see Fig. 2(b)].

(2) Inclusion of the nucleon structure into the model increases the bremsstrahlung probability of the emitted photons.

(3) We find the following tendencies for obtaining the nuclei, which have the bremsstrahlung spectra more sensitive to the nucleon structure: (a) direction to nuclei with smaller Z and (b) direction to nuclei with larger Q_α values.

Summarizing, we note that the visibility of the many-nucleon effects in the bremsstrahlung spectra for the α decays is enough small; however, we demonstrated a way in which it can be studied by means of the bremsstrahlung photons in nuclear reactions in a general way. By such a reason, one can naturally suppose that the many-nucleon structure should be more clearly visible in the bremsstrahlung spectra in cluster decays, fission, scattering of protons, and light charged particle off nuclei (energies should be essentially higher than Q values of the α-decaying nuclei). Another example where many-nucleon structure can be studied by means of the approach given in this paper involves nuclear reactions in astrophysics. At present, there are strong reasons why proton-rich isotopes are believed to be created in the rapid proton-capture process in stars. Such a process is accompanied by the bremsstrahlung emission, which can be used for the study of the many-nucleon structure. Taking into account high interest in such a task, we add some of our preliminary analysis of it in Appendix B, because qualitative results should be obtained from new independent deep research. All these ideas are perspectives for future research.

ACKNOWLEDGMENTS

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APPENDIX A: THE FORM FACTORS OF THE α-NUCLEUS SYSTEM

1. Form factor of the α particle

We calculate the matrix element (11) in the form

$$Z(\mathbf{k}) = \frac{1}{A}\sum_{i=1}^{A}\sum_{k=1}^{A}\langle\psi_k(i)|\frac{Z_k m_\mathrm{p}}{m_k}e^{-i\mathbf{k}\boldsymbol{\rho}_i}|\psi_k(i)\rangle, \quad \text{(A1)}$$

where we take into account the orthogonality between wave functions $\langle\psi_k(j)|\psi_m(j)\rangle = \delta_{mk}$. Taking into account zero charge of neutron, we sum Eq. (A1) over spin-isospin states. For even-even fragments we obtain

$$Z_\mathrm{A}(\mathbf{k}) = \frac{2}{A}\sum_{i=1}^{A}\sum_{k=1}^{B}\langle\varphi_k(\tilde{\boldsymbol{\rho}}_i)|e^{-i\mathbf{k}\boldsymbol{\rho}_i}|\varphi_k(\tilde{\boldsymbol{\rho}}_i)\rangle, \quad \text{(A2)}$$

where B is the number of states of the space wave function of nucleon. Taking into account spin-isospin states, we obtain $B = A/4$. In particular, for the α particle we have $B = 1$.

We choose the space wave function of one nucleon in the Gaussian form, according to formalism in Appendix A in Ref. [38]. Substituting it into Eq. (A2), we find the form factor for the α particle,

$$Z_\alpha(\mathbf{k}) = \frac{1}{2}\sum_{i=1}^{4} I_x(n_x)I_y(n_y)I_z(n_z), \quad \text{(A3)}$$

where

$$I_x(n_x,x_{i,0},a) = N_{\alpha,x}^2\int e^{-\frac{(x_i-x_{i,0})^2}{a^2}}e^{-ik_x x_i}H_{n_x}^2\left(\frac{x_i-x_{i,0}}{a}\right)dx_i, \quad \text{(A4)}$$

H_{n_x} are Hermitian polynomials (see Ref. [104], p. 749), and solutions for $I_y(n_y)$ and $I_z(n_z)$ are obtained after change of indexes $x \to y$ and $x \to z$. After simplification of this integral we obtain

$$\begin{aligned} I_x &= N_{\alpha,x}^2\exp\left[-a^2k_x^2/4 - ik_x x_{i,0}\right] \\ &\quad\times\int\exp\left[-\frac{(x_i - x_{i,0} + ia^2k_x/2)^2}{a^2}\right]H_{n_x}^2 \\ &\quad\times\left(\frac{x_i - x_{i,0}}{a}\right)dx_i. \end{aligned} \quad \text{(A5)}$$

Now let us consider a case when the α particle is in the ground state ($n_x = n_y = n_z = 0$). We have $H_{n_x=0} = 1$, $H_{n_y=0} = 1$, $H_{n_z=0} = 1$. In approximation, integral (A5) over complex variable $\tilde{x} = x_i - \rho_{i,x} + ia^2k_x/2$ has solution

$$\begin{aligned} &\int\exp\left[-\frac{(x_i - \rho_{i,x} + ia^2k_x/2)^2}{a^2}\right]dx_i \\ &\quad= \int\exp\left[-\frac{x_i^2}{a^2}\right]dx_i = N_{\alpha,x}^{-2} \end{aligned} \quad \text{(A6)}$$

and we obtain

$$I_{\alpha,x}(n_x = 0) = \exp\left[-a^2k_x^2/4 - ik_x x_{i,0}\right]. \quad \text{(A7)}$$

Now we calculate form factor (A3):

$$Z_\alpha(\mathbf{k}) = \frac{1}{2}e^{-(a^2k_x^2+b^2k_y^2+c^2k_z^2)/4}\sum_{i=1}^{4}e^{-i\mathbf{k}\boldsymbol{\rho}_{i,0}}. \quad \text{(A8)}$$

In the limit of the pointlike α particle (at $\boldsymbol{\rho}_{0,i} = 0$) we obtain

$$Z_\alpha(\mathbf{k};\boldsymbol{\rho}_{i,0} \to 0) = 2e^{-(a^2k_x^2+b^2k_y^2+c^2k_z^2)/4}. \quad \text{(A9)}$$

One can see that the charged form factor depends on the energy of the emitted photon, the direction of its emission, and also the parameters of the wave function of the nucleon. To make the form factor unambiguous, we impose the following condition: that the form factor of the α particle at pointlike limit should correspond to its electromagnetic charge Z_α as

$$2e^{-(a^2k_x^2+b^2k_y^2+c^2k_z^2)/4} \equiv Z_\alpha. \tag{A10}$$

Applying such a condition, we obtain

$$Z_\alpha(\mathbf{k}) = \frac{Z_\alpha}{4} \cdot \sum_{i=1}^{4} e^{-i\mathbf{k}\boldsymbol{\rho}_{i,0}}. \tag{A11}$$

On such a basis we construct the following logic. If the photon is not emitted by the nucleon of the α particle, then $|k| = 0$ and we directly obtain fulfillment of property (A10). However, if the photon is emitted by this nucleon, then the exponent suppresses the form factor of the α particle. This effect appears after taking into account the internal structure of the α particle, composed of four nucleons.

Now, if on remembers that a, b, and c determine the space size of the localization of the wave function that describes the most probable location of each nucleon inside the α particle, then one concludes that the parameters are larger and the emitted photon suppresses the electromagnetic charge of the α particle more strongly. Also, the parameters a, b, and c smaller, so the factors in the wave function like $\exp(-x/a)$ are closer to the δ function, and then the emission of the photon does not influence the charge of the α particle practically. According to our preliminary estimations, for the α particle for energies of the emitted photon up to 10 MeV, the charge is not changed essentially. However, this is not so for high energies (close to 100 MeV and higher) or for heavy ions and nuclei.

2. Form factor of the daughter nucleus

In determining the form factor of the nucleus we have to take into account nonzero states of the one-nucleon space wave function. At first, we find the integral $I_x(n_x \neq 0)$. Here one can apply the formulas of summation of Hermitian polynomials

$$\begin{aligned} &\frac{(a_1^2+a_2^2)^{\mu/2}}{\mu!} H_\mu\left(\frac{a_1x_1+a_2x_2}{\sqrt{a_1^2+a_2^2}}\right) \\ &\quad = \sum_{m_1+m_2=\mu} \frac{a_1^{m_1}}{m_1!}\frac{a_2^{m_2}}{m_2!} H_{m_1}(x_1)\, H_{m_2}(x_2), \\ &\int_{-\infty}^{+\infty} e^{-(x-y)^2} H_m(x) H_n(x) dx \\ &\quad = 2^n\sqrt{\pi}\, m!\, y^{n-m} L_n^{n-m}(-2y^2) \end{aligned} \tag{A12}$$

at $m \leqslant n$ and where L_n^{n-m} is generalized Laguerre polynomial. At $n = m$ we find

$$\int_{-\infty}^{+\infty} e^{-(x-y)^2} H_n^2(x) dx = 2^n\sqrt{\pi}\, n!\, L_n(-2y^2), \tag{A13}$$

where $L_n = L_n^0$ is the Rodrigues polynomial, defined by the Rodrigues formula

$$L_n(x) = \sum_{k=0}^{n} \frac{(-1)^k}{k!}\binom{b}{k} x^k. \tag{A14}$$

However, for computer calculations a recurrent formula could be more useful,

$$L_{k+1}(x) = \frac{1}{k+1}\big[(2k+1-x)L_k(x) - kL_{k-1}(x)\big] \quad \text{at } k \geqslant 1, \tag{A15}$$

where the first two polynomials equal

$$L_0(x) = 1, \quad L_1(x) = 1 - x. \tag{A16}$$

Using formula (A13), we calculate the integral (A5) for an arbitrary state,

$$I_x = L_{n_x}\big[a^2k_x^2/2\big] \cdot \exp\big[-a^2k_x^2/4 - ik_x\rho_{i,x}\big], \tag{A17}$$

and calculate the form factor of the daughter nucleus,

$$\begin{aligned} Z_\mathrm{d}(\mathbf{k}) = &\frac{2e^{-(a^2k_x^2+b^2k_y^2+c^2k_z^2)/4}}{A_\mathrm{d}} \sum_{n_x,n_y,n_z=0}^{n_x+n_y+n_z\leqslant N} L_{n_x}\big[a^2k_x^2/2\big] \\ &\times L_{n_y}\big[b^2k_y^2/2\big]\, L_{n_z}\big[c^2k_z^2/2\big] \cdot \sum_{i=1}^{A_\mathrm{d}} e^{-i\mathbf{k}\rho_i}. \end{aligned} \tag{A18}$$

This solution can be rewritten as

$$\begin{aligned} Z_\mathrm{d}(\mathbf{k}) = &2e^{-(a^2k_x^2+b^2k_y^2+c^2k_z^2)/4} f_1\big(\mathbf{k},n_1\cdots n_{A_\mathrm{d}}\big) \\ &\times f_2\big(\mathbf{k},\rho_1\cdots\rho_{A_\mathrm{d}}\big), \end{aligned} \tag{A19}$$

where

$$\begin{aligned} f_1\big(\mathbf{k},n_1\cdots n_{A_\mathrm{d}}\big) &= \sum_{n_x,n_y,n_z=0}^{n_x+n_y+n_z\leqslant N} L_{n_x}\big[a^2k_x^2/2\big]L_{n_y} \\ &\quad\times\big[b^2k_y^2/2\big]L_{n_z}\big[c^2k_z^2/2\big], \\ f_2\big(\mathbf{k},\rho_1\cdots\rho_{A_\mathrm{d}}\big) &= \frac{1}{A_\mathrm{d}}\sum_{i=1}^{A_\mathrm{d}} e^{-i\mathbf{k}\rho_i}. \end{aligned} \tag{A20}$$

Here the function f_1 is the summation over all states of the one-nucleon space wave function, function f_2 describes space distribution of nucleons inside the nucleus (i.e., it characterizes the density of nucleons in the nucleus).

APPENDIX B: THE BREMSSTRAHLUNG PHOTONS EMITTED DURING THE PROTON-CAPTURE REACTIONS IN STARS

Proton-capture reactions at low energy play a very important role in nucleosynthesis in the p process, which includes proton capture, charge exchange, and photodisintegration and is important for the production of certain so-called p nuclei, which can have short lifetimes, and many of them cannot be obtained on Earth. So, experimental study of these nuclei is difficult, which has stimulated intensive theoretical investigations of this topic [105–109]. In general, we know about 2000 p nuclei and 20 000 reactions and decays connected with such nuclei (for example, see textbook [110] and review [111–113]).

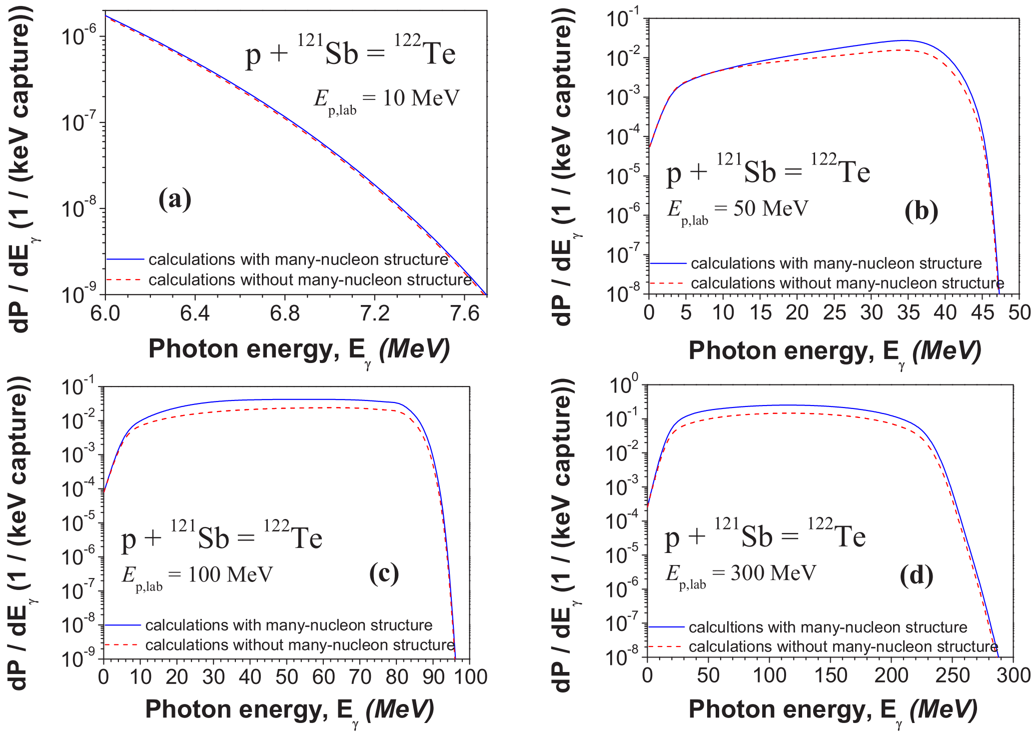

FIG. 4. The bremsstrahlung probabilities of the emitted photons during capture of proton by the ^{121}Sb nucleus at the incident proton energy of 10 MeV (a), 50 MeV (b), 100 MeV (c), and 300 MeV (d) in the laboratory frame. Here solid blue lines are for calculations with many-nucleon structure and dashed red lines are for calculations without included nucleon structure. One can see that the curve for the nucleus after inclusion of the many-nucleon structure at each proton incident energy is located above in comparison with curve without such a structure. From the figures one can estimate how the bremsstrahlung probability (for the same photon energy) is increased with increasing energy of the incident proton. In all figures one can see that the bremsstrahlung probabilities for emission of the hard photons at kinematic limit (defined by the energy of the incident proton) tends to zero.

In this connection, we are interested in applying of our formalism for better understanding of these reactions. In particular, we pay attention to the proton-rich isotopes near the $N = Z$ line. In this section we estimate the probability of the emitted photons which should accompany the capture of protons by nuclei with forming of isotopes of tellurium. In Fig. 4 we present our new calculations for reaction of $^{121}\text{Sb} + p = {}^{122}\text{Te}$ [109] for different energies of the incident proton.[5] We see that for energies of the incident protons up to 4 MeV (such proton energies were used in experimental and theoretical study of this capture reaction [109]) the effect of inclusion of the many-nucleon structure is practically not visible. However, with increasing of the energy of the protons starting from 50 MeV (or some less), this many-nucleon effect is observable, and all bremsstrahlung probabilities are essentially larger.

[5]We use the potential of interaction between the proton and the nucleus defined in Eqs. (26) and (27) with parameters calculated by Eqs. (28) and (29) in Ref. [70]; parametrization is based on analysis from Ref. [114]. Using such a potential, we calculate the wave functions $\varphi_i(\mathbf{r})$ and $\varphi_f(\mathbf{r})$ numerically. We find that it is very difficult to achieve convergence in calculations of the bremsstrahlung spectra in the multipole approach for the studied reaction. By such a reason, and to obtain the first estimations of the spectra for the different energies of the incident proton, we apply dipole approximation (see Ref. [58] for details and main transformations) for calculation of the matrix element in Eqs. (21), adapted for the proton-capture process. However, as we found that the dipole approximation shifts essentially the spectra along the bremsstrahlung probability axis, so should the spectra be renormalized. So, we find a factor of normalization from ratio between the spectra calculated in the multipole and dipole approaches for the incident proton energy of 3 MeV (we find this factor to be equal to $7.5 \times 10^{+24}$). Then we use the same factor for normalization of the spectra calculated in the dipole approach for other energies of the incident proton. Our calculations of the bremsstrahlung spectra in the multipole approach without normalization for the α decay were in enough good agreement with experimental data (see Ref. [92] for details) that we use the multipole approach to test for the normalization of the spectra. For analysis, we consider the incident proton in state $s_{1/2}$ before the emission of a photon and in state $p_{1/2}$ after the emission of a photon (as the proton-nucleus potential includes spin-orbital term defined on the basis of quantum numbers j and l).

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