Systematic study of suppression of complete fusion in reactions involving weakly bound nuclei at energies above the Coulomb barrier

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Complete fusion excitation functions of reactions involving breakup are studied by using the empirical coupled-channel (ECC) model with breakup effects considered. An exponential function with two parameters is adopted to describe the prompt-breakup probability in the ECC model. These two parameters are fixed by fitting the measured prompt-breakup probability or the complete fusion cross sections. The suppression of complete fusion at energies above the Coulomb barrier is studied by comparing the data with the predictions from the ECC model without the breakup channel considered. The results show that the suppression of complete fusion is roughly independent of the target for the reactions involving the same projectile.

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I. INTRODUCTION

In recent years, the investigation of the breakup effect of weakly bound nuclei on the fusion process has been an interesting topic [1–3]. Different processes can take place in collisions involving weakly bound nuclei. One is direct complete fusion (DCF). In this case, the whole projectile fuses with the target without breakup. Several processes can occur after the breakup of the weakly bound projectile nucleus. When all the fragments fuse with the target, the process is called sequential complete fusion (SCF). If only part of the fragments fuses with the target nucleus, it is called incomplete fusion (ICF). There is also some possibility that none of the fragments is captured by the target. This process is called noncapture breakup (NCBU).

Experimentally, the SCF is difficult to distinguish from the DCF, as the produced compound nuclei from these two processes are the same. Therefore, only the complete fusion (CF) cross section, which includes both DCF and SCF cross sections, i.e., $\sigma_{\rm CF} = \sigma_{\rm SCF} + \sigma_{\rm DCF}$, can be measured. In addition, it is difficult to measure separately ICF and CF cross sections owing to the characteristics of the evaporation of the excited compound nuclei. For light reaction systems, the produced compound nuclei have a large probability for emitting charged particles during the cooling process and consequently residues from ICF coincide with those from CF. Hence, only the sum of the CF and ICF cross sections, which is called the total fusion (TF) cross section, i.e., $\sigma_{\rm TF} \equiv \sigma_{\rm CF} + \sigma_{\rm ICF}$, can be measured. For heavy reaction systems, the evaporation of the excited compound nuclei occurs mainly by the emission of neutrons and α particles. In this case, the separate measurements of CF and ICF cross sections can be achieved [4]. In recent years, many measurements of the CF cross sections have been performed [5–16].

Theoretically, the influence of breakup on the fusion cross section has been an extensively studied topic. The continuum-discretized coupled-channels (CDCC) framework has been very successful [17–22], since it provides a good description of observed NCBU, elastic, and TF cross sections. However, most CDCC calculations have a shortcoming [23,24], as they cannot give the ICF and CF cross sections unambiguously [25]. This shortcoming can be avoided within a new dynamical quantum approach that includes SCF as well as ICF from the bound state(s) of the projectile [26]. In addition, the comparison of experimental fusion cross sections with either the predictions of coupled-channel (CC) calculations without the breakup and transfer channels [12–16,27–31] or the predictions of a single barrier penetration model (SBPM) [9–11] shows that CF cross sections are suppressed owing to the breakup at energies above the Coulomb barrier.

Many efforts have been made to investigate the systematics of the CF suppression [10,32–36]. In Ref. [34], a three-dimensional classical dynamical reaction model [37–39] together with the measured prompt-breakup probabilities was adopted to study the systematics of the suppression for the reactions induced by ^{9}Be. It was found that the CF suppression is nearly independent of the target. In Ref. [33], a large number of CF excitation functions of reactions including the breakup channel were studied by applying the universal fusion function (UFF) prescription [40,41] with the double folding and parameter-free São Paulo potential [42–44]. The authors concluded that the CF cross sections are suppressed owing to the prompt breakup of projectile and that the suppression effect for reactions induced by the same projectile is roughly independent of the target.

Recently, a systematic study of capture (fusion) excitation functions for 217 reaction systems was performed by using

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an empirical coupled-channel (ECC) model [45]. In the ECC model, a barrier distribution is used to take effectively into account the effects of couplings to inelastic excitations and neutron transfer channels [45,46]. However, the coupling to the breakup channel has not been taken into account. The sub-barrier prompt-breakup probabilities for the reactions induced by ^{9}Be have been measured and the radial dependence of the breakup probabilities have been established [34,37,47]. In the present work, the characteristics of the measured prompt-breakup probability and its effect on CF are considered in the ECC model. In Ref. [33], it was found that the suppression of CF is sensitive to the lowest breakup threshold energy of the projectile and there holds an exponential relation between the suppression and the breakup threshold energy. In the present work, a systematic study of the CF excitation functions for reactions involving weakly bound nuclei, as well as of the suppression on CF excitation functions, is investigated.

The paper is organized as follows. In Sec. II, we introduce the ECC model considering the breakup effect (in short, the ECCBU model). In Sec. III, the ECCBU model is applied to analyze the data of different projectile-induced reactions. The suppression of CF excitation functions at energies above the Coulomb barrier is also investigated. Finally, a summary is given in Sec. IV.

II. METHOD

The fusion cross section at a given center-of-mass energy $E_{\mathrm{c.m.}}$ can be written as the sum of the cross section for each partial wave J,

$$\sigma_{\mathrm{Fus}}(E_{\mathrm{c.m.}}) = \pi\lambda^2 \sum_{J}^{J_{\max}} (2J+1)T(E_{\mathrm{c.m.}},J), \tag{1}$$

where $\lambda^2 = \hbar^2/(2\mu E_{\mathrm{c.m.}})$ is the reduced de Broglie wavelength, μ denotes the reduced mass of the reaction system, T denotes the penetration probability of the potential barrier between the colliding nuclei at a given J, and $J_{\max}$ is the critical angular momentum.

When one of the colliding nuclei is weakly bound, the additional breakup degree of freedom makes the colliding process more complicated. A prompt-breakup probability P_{BU} is introduced. Considering the survival of the projectile against breakup before fusion, the CF cross section can be calculated by Eq. (1) with the penetration probability T multiplied by the survival probability $(1 - P_{\mathrm{BU}})$, which is written as [48–51]

$$\sigma_{\mathrm{CF}}(E_{\mathrm{c.m.}}) = \pi\lambda^2 \sum_{J}^{J_{\max}} (2J+1)T(E_{\mathrm{c.m.}},J)[1 - P_{\mathrm{BU}}(E_{\mathrm{c.m.}},J)]. \tag{2}$$

Equation (2) does not include the SCF component, which seems not to be significant at Coulomb barrier energies [25]. Based on both measurements [34,47] and theoretical calculations [37,52], the breakup probability along a given classical orbit can be written as an exponential function of the distance of closest approach, $R_{\min}(E_{\mathrm{c.m.}},J)$,

$$P_{\mathrm{BU}}(E_{\mathrm{c.m.}},J) = \exp[\nu + \mu R_{\min}(E_{\mathrm{c.m.}},J)], \tag{3}$$

where ν and μ are the logarithmic intercept and slope parameters of the function, respectively. These two parameters can be determined by reproducing the measured prompt-breakup probability. $R_{\min}(E_{\mathrm{c.m.}},J) = R_{\mathrm{B}}(J)$ and $R_{\mathrm{B}}(J)$ is the position of the barrier [37].

The penetration probability T in Eqs. (1) and (2) is calculated with the ECC model in which the coupled-channel effects (excluding the breakup channel) are taken into account by introducing a barrier distribution $f(B)$ [45]:

$$T(E_{\mathrm{c.m.}},J) = \int f(B)T_{\mathrm{HW}}(E_{\mathrm{c.m.}},J,B)dB, \tag{4}$$

where B is the barrier height, and T_{HW} denotes the penetration probability calculated by the well-known Hill-Wheeler formula [53]. Note that for very deep sub-barrier penetration, the Hill-Wheeler formula is not valid because of the long tail of the Coulomb potential. In Ref. [54], a new barrier penetration formula was proposed for potential barriers containing a long-range Coulomb interaction and this formula is especially appropriate for the barrier penetration with incident energy much lower than the Coulomb barrier. The implementation of this barrier penetration formula in the ECC model is in progress.

The barrier distribution $f(B)$ is taken to be an asymmetric Gaussian function:

$$f(B) = \begin{cases} \frac{1}{N}\exp\left[-\left(\frac{B-B_{\mathrm{m}}}{\Delta_1}\right)^2\right], & B < B_{\mathrm{m}}, \\ \frac{1}{N}\exp\left[-\left(\frac{B-B_{\mathrm{m}}}{\Delta_2}\right)^2\right], & B > B_{\mathrm{m}}, \end{cases} \tag{5}$$

where $f(B)$ satisfies the normalization condition $\int f(B)dB = 1$; $N = \sqrt{\pi}(\Delta_1 + \Delta_2)/2$ is a normalization coefficient; and Δ_1, Δ_2, and B_{m} denote the left width, the right width, and the central value of the barrier distribution, respectively.

Within the ECC model [45], the barrier distribution is related to the couplings to low-lying collective vibrational states, rotational states, and positive Q-value neutron transfer (PQNT) channels. The vibrational modes are connected to the change of nuclear shape while the nuclear rotational states are related to the static deformations of the interacting nuclei. Furthermore, when the two nuclei come close enough to each other, both nuclei are distorted owing to the attractive nuclear force and the repulsive Coulomb force; thus, dynamical deformation develops [55,56]. Considering the dynamical deformation, a two-dimensional potential energy surface (PES) with respect to relative distance R and quadrupole deformation of the system can be obtained. Based on this PES, empirical formulas for calculating the parameters of the barrier distribution were proposed to take into account the effect of the couplings to inelastic excitations in Ref. [45].

The effect of the coupling to the PQNT channels is simulated by broadening the barrier distribution. Only one neutron pair transfer channel is considered in the present model. When the Q value for one neutron pair transfer is positive, the widths of the barrier distribution are calculated as $\Delta_i \to gQ(2n) + \Delta_i$ $(i = 1,2)$, where $Q(2n)$ is the Q value for one neutron pair transfer, and g is taken as 0.32 for all reactions with positive Q value for one neutron pair transfer channel [45,46].

III. RESULTS AND DISCUSSIONS

To calculate the complete fusion cross sections for a given reaction, two additional parameters ν and μ are needed in the ECCBU model. These two parameters can be determined by reproducing the measured prompt-breakup probability, if such measurements are available. In Ref. [34], the sub-barrier prompt-breakup probabilities for the reactions induced by ^{9}Be were measured and the radial dependence of the sub-barrier breakup probabilities was established. Therefore, for reactions induced by ^{9}Be, both ν and μ can be extracted from the measured breakup probabilities. Next, we first extract ν and μ from the measured prompt-breakup probabilities. Then these ν and μ values are adopted as inputs for the ECCBU calculations.

A. Complete fusion for reactions involving the weakly bound projectile ^{9}Be

In Ref. [34], it was found that when the measured breakup probabilities are presented as a function of the surface separation of the two interacting nuclei, i.e., $R_{\min} - R_0$, the effect of nuclear size is removed and the breakup probabilities for all targets with $62 \leqslant Z \leqslant 83$ overlap. Here $R_0 = R_{\rm P} + R_{\rm T}$ is the summed radius of the interacting nuclei. $R_{\rm P}$ and $R_{\rm T}$ are the radii of the equivalent spherical nuclei and calculated using $R_{\rm P(T)} = r_0 A_{\rm P(T)}^{1/3}$ with $r_0 = 1.2$ fm [34]. This implies that the prompt-breakup probability can be written as

$$P_{\rm BU} = \exp[\nu + \mu R_0 + \mu(R_{\min} - R_0)], \tag{6}$$

where both $\nu + \mu R_0$ and slope μ are independent of the target. Therefore, ν should satisfy $\nu = a - \mu R_0$. Then Eq. (6) becomes

$$P_{\rm BU} = \exp[a + \mu(R_{\min} - R_0)], \tag{7}$$

where μ and a are independent of the target and can be extracted by making a fit to the measured prompt-breakup probabilities. Actually, in Ref. [34], the target-independent slope parameter μ was given as $\bar{\mu} = -0.884 \pm 0.011$ fm^{-1}. Meanwhile, the values of ν for the reactions were obtained by fitting the measured prompt-breakup probabilities using the mean slope $\mu = -0.884$ fm^{-1}, which are shown as a function of R_0 in Fig. 1. As discussed above, ν should satisfy the function of $\nu = a - \mu R_0$, so a can be determined by making a fit to the values of ν, and $a = 0.557$ is obtained. The results of $\nu = 0.557 - \mu R_0$ are displayed as the blue dotted line in Fig. 1.

In the present work, the sub-barrier prompt-breakup probability denoted by Eq. (3) with $\nu = 0.557 - \mu R_0$ and $\mu = -0.884$ fm^{-1} is adopted to perform the ECCBU calculations for the reactions induced by ^{9}Be. The reactions with ^{89}Y [57], ^{124}Sn [58], ^{144}Sm [6,59], ^{169}Tm [13], ^{181}Ta [12], ^{187}Re [13], ^{208}Pb [4], and ^{209}Bi [60,61] as targets have been investigated. The comparison of the calculated CF cross sections with the experimental values is shown in Fig. 2. The pink dash-dotted line (CF) denotes the calculated CF cross sections. Note that after the breakup of ^{9}Be into $\alpha + \alpha + n$, the capture of either of 2α particles by the target or the SCF can also occur and, therefore, contribute to experimental CF cross sections. In the present work, we do not take into account these events

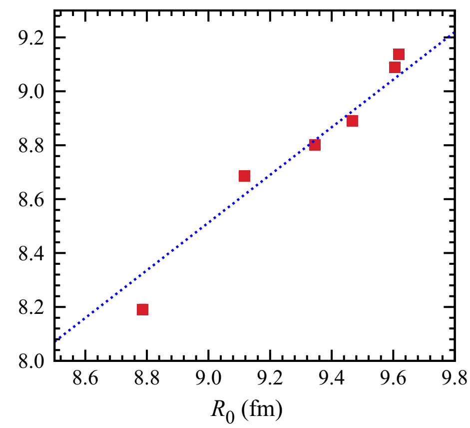

FIG. 1. The fitted results of ν shown as a function of R_0, i.e., $R_0 = R_{\rm P} + R_{\rm T}$. The fitted results of ν taken from Ref. [34] are represented by the solid squares. The blue dotted line denotes the results obtained from the function of $\nu = a - \mu R_0$ with $a = 0.557$ and $\mu = -0.884$ fm^{-1}.

because this is a very complex problem that needs to be further investigated. Hence, the present calculations provide a lower limit for CF cross sections. Comparing this lower limit with the experimental CF cross sections, one can find that the calculated CF cross sections are in good agreement with the data, except the reaction ^{9}Be + ^{144}Sm. Therefore, one can conclude that the capture of all individual components of ^{9}Be by the target, after the ^{9}Be breakup, is not very significant. For convenience, we label the calculated cross sections from the ECC model without the breakup channel considered by the subscript "Fus", i.e., $\sigma_{\rm Fus}$.

As mentioned above, a large number of CF excitation functions of reactions including the breakup channel have been studied by applying the UFF prescription [40,41] in Ref. [33]. It was found that the suppression effect for reactions induced by the same projectile is independent of the target. For the reactions involving ^{9}Be, the suppression factor $F_{\rm BU}$, which is defined as the ratio of the data to the UFF, i.e., the predictions from Wong's formula [62], is 0.68, and the reaction ^{9}Be + ^{144}Sm also did not follow the systematics found in Ref. [33].

Because the inelastic excitation couplings are not important at energies well above the Coulomb barrier, the suppression obtained from ECC calculations without the breakup channel considered should be similar to those obtained from the UFF [33]. To check this, the predictions from the ECC model without the breakup channel considered, which are shown in Fig. 2 by the black line, are used as a reference to be compared with the data. One can find that the CF cross sections are suppressed as compared with $\sigma_{\rm Fus}$ at above-barrier energies. We scale $\sigma_{\rm Fus}$ by the suppression factor $F_{\rm BU} = 0.68$ and show it in Fig. 2 by the blue dotted line. It can be seen that the blue dotted line roughly coincides with the data and the pink dash-dotted line, while there are small deviations from the pink dash-dotted line at high energies. Therefore, comparing $\sigma_{\rm Fus}$ with the CF cross sections, the suppression of CF cross sections for reactions induced by ^{9}Be is independent of the target and

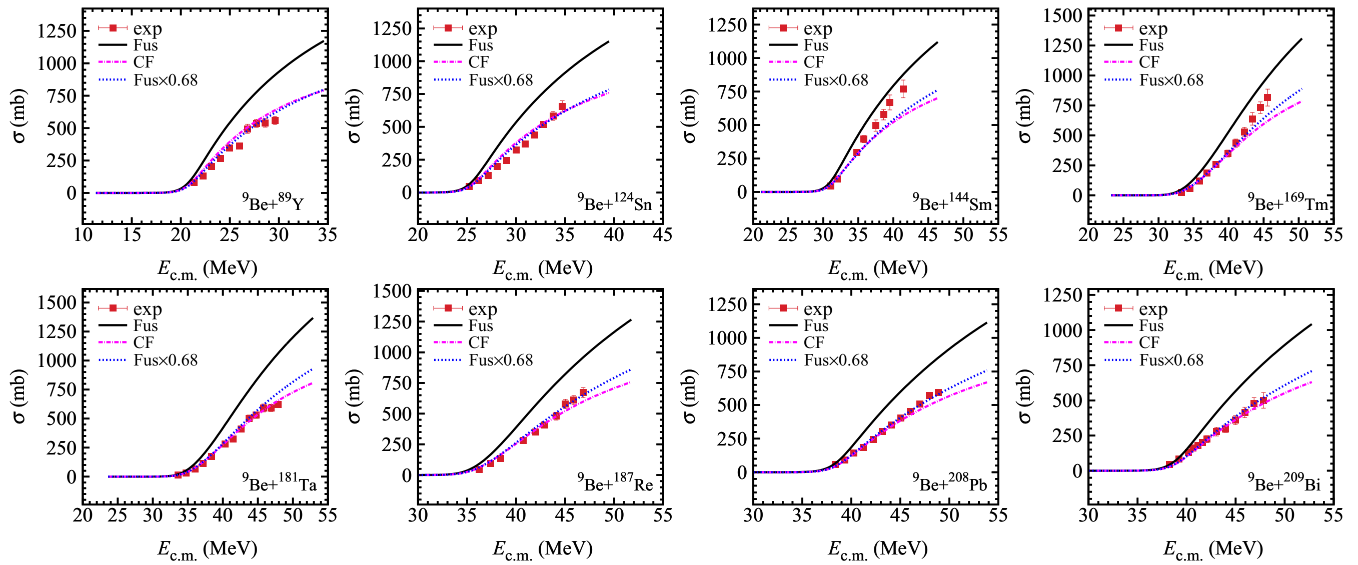

FIG. 2. The experimental complete fusion excitation functions and calculated cross sections for reactions induced by ^{9}Be on ^{89}Y [57], ^{124}Sn [58], ^{144}Sm [6,59], ^{169}Tm [13], ^{181}Ta [12], ^{187}Re [13], ^{208}Pb [4], and ^{209}Bi [60,61]. The black line (Fus) denotes the fusion cross sections obtained from the ECC model without the breakup channel considered, i.e., $\sigma_{\rm Fus}$. The pink dash-dotted line (CF) denotes the calculated complete fusion cross sections obtained from the ECCBU model with $\nu = 0.557 - \mu R_0$ and $\mu = -0.884$ fm^{-1}. The blue dotted line denotes $F_{\rm BU}\sigma_{\rm Fus}$ with the suppression factor $F_{\rm BU} = 0.68$ taken from Ref. [33].

the suppression factor is about 0.68, which is consistent with the result obtained in Ref. [33].

These results are very interesting and somehow unexpected, since it is widely accepted that the Coulomb breakup should increase with the charge of the target. Actually, recent CDCC calculations performed by Otomar *et al.* [20] showed that the total breakup, including the interference between its Coulomb and nuclear components, increases with the target charge and mass, which seems to be contradictory with the conclusion that CF suppression is independent of the target. A possible explanation for this apparent contradiction was recently given in Refs. [12–14,33]. The breakup may be of two kinds, prompt and delayed; the former takes place when the projectile is approaching the target and the latter takes place following direct transfer of nucleons or the excitation of the projectile to a long-lived resonance above the breakup threshold. The experimental results show that the time scale of the delayed breakup is several orders of magnitude longer than the collision time and consequently only the prompt breakup may affect the fusion processes [47,63,64]. In the CDCC calculations made by Otomar *et al.* [20], both the prompt and delayed breakups were included. In the present work, the measured prompt-breakup probabilities are used in the ECCBU calculations and the calculated CF cross sections are in good agreement with data. Such good agreement supports to some extent this explanation. It is highly desirable that one can deal with the prompt and delayed breakups separately in CDCC calculations. Such calculations will provide further insight in the understanding of the target-independent suppression of CF and also provide the prompt-breakup probabilities as inputs for the ECCBU calculations.

B. Complete fusion for reactions involving ^{6,7}Li and ^{10,11}B

Based on the measured prompt-breakup probabilities for the reactions involving ^{9}Be, the function for ν is determined as $\nu = a - \mu R_0$. Meanwhile, the suppression of the CF cross section at energies above the Coulomb barrier and μ are independent of the target. Next, we use the ECCBU model with $\nu = a - \mu R_0$ to study the reactions induced by ^{6}Li, ^{7}Li, ^{10}B, and ^{11}B. We assume that $a = 0.557$ and SCF is not very significant as compared to DCF. For each reaction, the slope parameter μ is obtained by making a fit to the data. Then the systematic behavior of the prompt-breakup probabilities and the suppression of CF cross sections are explored.

First the complete fusion excitation functions for reactions induced by ^{6}Li on ^{90}Zr [27], ^{96}Zr [14], ^{154}Sm [16], ^{159}Tb [29], ^{197}Au [30], ^{198}Pt [7], ^{208}Pb [5], and ^{209}Bi [4] are investigated. The fitted values of μ are listed in Table I. Similar to the results for ^{9}Be, one can find that the slope parameters μ are also roughly independent of the target, with a mean value of $\bar{\mu} = -0.798$ fm^{-1}. Then the mean value

TABLE I. Slope parameter μ obtained by making a least-squares fit to the corresponding CF data using $\nu = 0.557 - \mu R_0$ for reactions with ^{6}Li as projectile.

	^{90}Zr	^{96}Zr	^{154}Sm	^{159}Tb	^{197}Au	^{198}Pt	^{208}Pb	^{209}Bi
μ (fm^{-1})	−0.732	−0.803	−0.915	−0.810	−0.711	−0.810	−0.794	−0.806

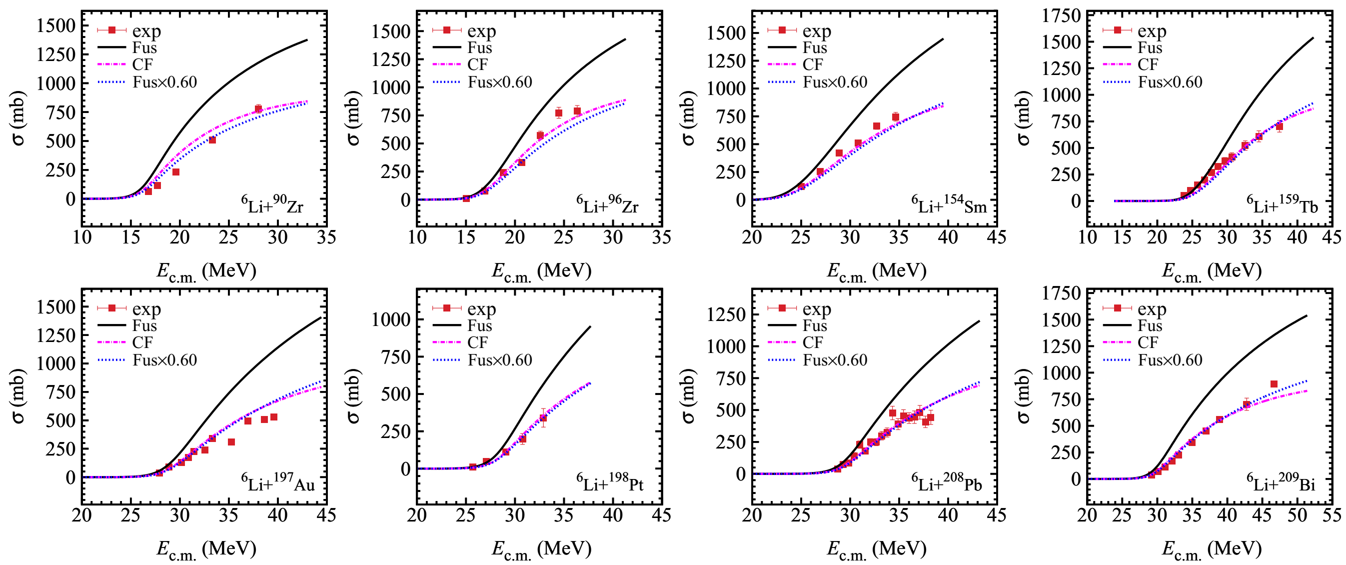

FIG. 3. The experimental complete fusion excitation functions and calculated cross sections for reactions induced by ^{6}Li on ^{90}Zr [27], ^{96}Zr [14], ^{154}Sm [16], ^{159}Tb [29], ^{197}Au [30], ^{198}Pt [7], ^{208}Pb [5], and ^{209}Bi [4]. The black line (Fus) denotes the fusion cross sections obtained from the ECC model without the breakup channel considered, i.e., $\sigma_{\rm Fus}$. The pink dash-dotted line (CF) denotes the calculated complete fusion cross sections obtained from the ECCBU model with $\nu = 0.557 - \mu R_0$ and $\mu = -0.798$ fm^{-1}. The blue dotted line denotes $F_{\rm BU}\sigma_{\rm Fus}$ with the suppression factor $F_{\rm BU} = 0.60$ taken from Ref. [33].

of $\mu = -0.798$ fm^{-1} and $\nu = 0.557 - \mu R_0$ are adopted to perform the ECCBU calculations. The comparison of the calculated CF cross sections to the experimental values is shown in Fig. 3. The calculated CF cross sections are shown by the pink dash-dotted line. It can be seen that the calculated CF cross sections are in good agreement with the data. To explore the suppression of CF cross sections, the theoretical predictions without the breakup channel considered, i.e., $\sigma_{\rm Fus}$, are shown by the black line. Values of $\sigma_{\rm Fus}$ multiplied by the suppression factor $F_{\rm BU} = 0.60$ taken from Ref. [33] are represented by the blue dotted line. One can find that the results denoted by the blue dotted line are in good agreement with the data. It implies that the suppression effect owing to the breakup of ^{6}Li is independent of the target and the suppression factor should be also about 0.60.

For ^{7}Li, the experimental complete fusion excitation functions for the reactions with ^{159}Tb [65], ^{165}Ho [9,66], ^{197}Au [30], ^{198}Pt [67], and ^{209}Bi [4] as targets were measured. The fitted values of μ are listed in Table II. Similar to the results for ^{9}Be and ^{6}Li, one can find that the slope parameter μ is also roughly independent of the target, with a mean value of $\bar{\mu} = -0.964$ fm^{-1}. Again the mean value of $\mu = -0.964$ fm^{-1} and $\nu = 0.557 - \mu R_0$ are adopted to perform the ECCBU calculations. The calculated CF cross sections are shown by the pink dash-dotted line in Fig. 4 and are also in good agreement with the data. $\sigma_{\rm Fus}$ are shown by the black line. We scale $\sigma_{\rm Fus}$ by the suppression factor $F_{\rm BU} = 0.67$ taken from Ref. [33] and show it in Fig. 4 by the blue dotted line. One can find that the results denoted by the blue dotted line coincide with the data. It implies that the suppression of CF cross sections owing to the breakup of ^{7}Li is also independent of the target and the suppression factor is about 0.67. Furthermore, the suppression for ^{7}Li is weaker than that for ^{6}Li.

TABLE II. Slope parameter μ obtained by making a least-squares fit to the corresponding CF data using $\nu = 0.557 - \mu R_0$ for reactions with ^{7}Li as projectile.

	^{159}Tb	^{165}Ho	^{197}Au	^{198}Pt	^{209}Bi
μ (fm^{-1})	−0.956	−0.941	−0.971	−0.993	−0.957

For the reactions involving ^{10}B and ^{11}B, the fitted values of μ for ^{10,11}B + ^{159}Tb [68] and ^{10,11}B + ^{209}Bi [10] are listed in Table III. For the reactions involving ^{10}B, a mean value of $\bar{\mu} = -1.415$ fm^{-1} is obtained, while for ^{11}B, $\bar{\mu} = -1.900$ fm^{-1}. The mean values of $\mu = -1.415$ fm^{-1} and $\mu = -1.900$ fm^{-1} are used to calculate the CF cross sections for the reactions with ^{10}B and ^{11}B as projectiles, respectively. The comparison of the calculated CF cross sections to the experimental values is shown in Fig. 5. For these four reactions, the calculated CF cross sections shown by the pink dash-dotted line are in good agreement with the data. Moreover, $\sigma_{\rm Fus}$ scaled by the suppression factor $F_{\rm BU} = 0.80$ for ^{10}B and $F_{\rm BU} = 0.91$ for ^{11}B taken from Ref. [33] are represented by the blue dotted line, which coincides with the data. One can find that the suppression of CF cross sections for the reactions induced by ^{10}B are independent of the target, as well as the reactions

TABLE III. Slope parameter μ obtained by making a least-squares fit to the corresponding CF data using $\nu = 0.557 - \mu R_0$ for reactions with ^{10}B and ^{11}B as projectiles.

	^{10}B		^{11}B	
	^{159}Tb	^{209}Bi	^{159}Tb	^{209}Bi
μ (fm^{-1})	−1.417	−1.413	−1.87	−1.929

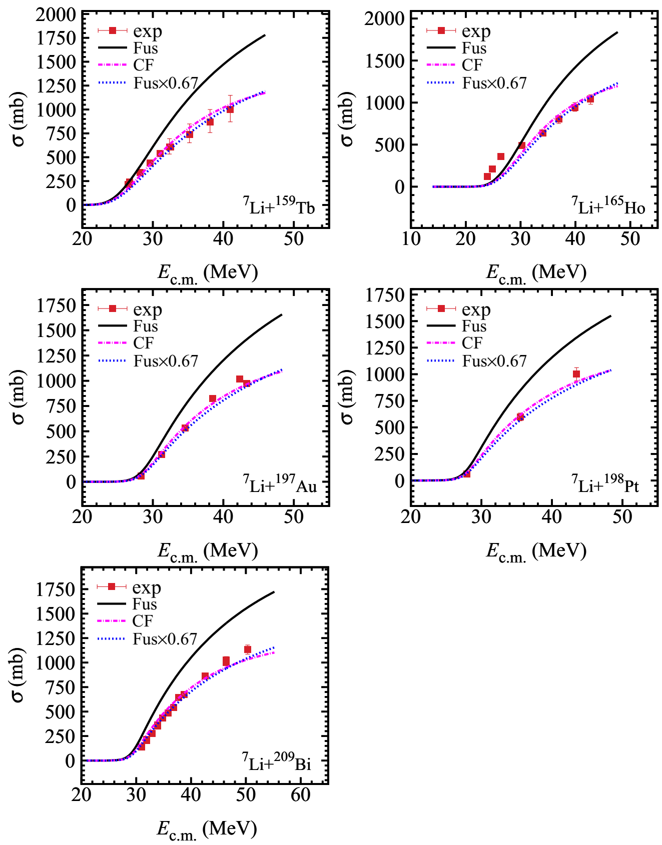

FIG. 4. The experimental complete fusion excitation functions and calculated cross sections for reactions induced by ^{7}Li on ^{159}Tb [65], ^{165}Ho [9,66], ^{197}Au [30], ^{198}Pt [67], and ^{209}Bi [4]. The black line (Fus) denotes the fusion cross sections obtained from the ECC model without the breakup channel considered, i.e., σ_{Fus}. The pink dash-dotted line (CF) denotes the calculated complete fusion cross sections obtained from the ECCBU model with $\nu = 0.557 - \mu R_0$ and $\mu = -0.964$ fm^{-1}. The blue dotted line denotes $F_{\text{BU}}\sigma_{\text{Fus}}$ with the suppression factor $F_{\text{BU}} = 0.67$ taken from Ref. [33].

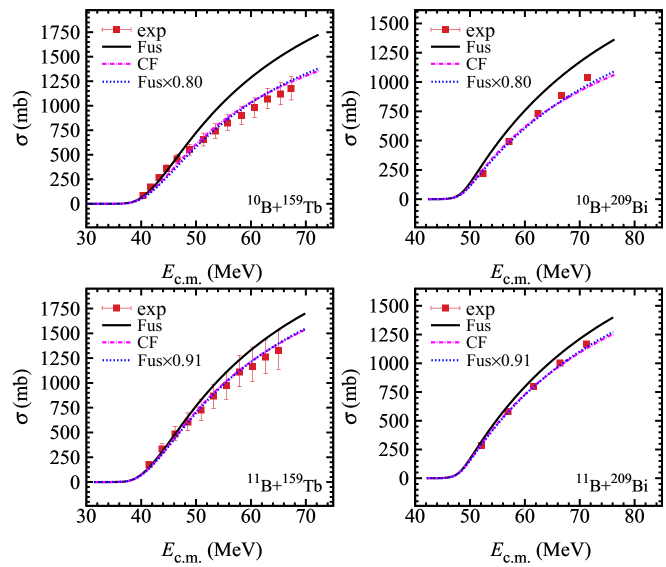

FIG. 5. The experimental complete fusion excitation functions and calculated cross sections for reactions induced by ^{10,11}B on ^{159}Tb [68] and ^{209}Bi [10]. The black line (Fus) denotes the fusion cross section obtained from the ECC model without the breakup channel considered, i.e., σ_{Fus}. The pink dash-dotted line (CF) denotes the calculated complete fusion cross section using the ECCBU model with $\nu = 0.557 - \mu R_0$ and $\mu = -1.415$ fm^{-1} for ^{10}B, while for ^{11}B, $\mu = -1.900$ fm^{-1}. The blue dotted line denotes $F_{\text{BU}}\sigma_{\text{Fus}}$ with the suppression factor $F_{\text{BU}} = 0.80$ for ^{10}B and $F_{\text{BU}} = 0.91$ for ^{11}B taken from Ref. [33].

induced by ^{11}B. Comparing the suppression of ^{10}B with that of its neighboring nucleus ^{11}B, we find that the suppression factor for ^{11}B is larger, as well as its breakup threshold.

C. Systematics of the prompt-breakup probability

Based on the above analysis and discussions, one can find that both the logarithmic slope parameter μ of the prompt-breakup probability and the CF suppression for the reactions induced by the same nucleus are roughly independent of the target. In Ref. [33], it was found that the suppression factor is mainly determined by the lowest breakup threshold energy of the projectile and an exponential relation between the suppression factor and the breakup threshold energy holds. Therefore, it is natural to explore the relation between the logarithmic slope μ and the breakup threshold.

The values of μ for ^{6,7}Li, ^{9}Be, and ^{10,11}B as a function of their lowest breakup threshold energies E_{BU} are shown in Fig. 6. One can find that a linear relation between μ and E_{BU} is fulfilled, at least for $1.474 \leqslant E_{\text{BU}} \leqslant 8.665$ MeV. Furthermore, if the breakup threshold energy is large enough, the breakup effects would not affect the fusion and the absolute value of μ should be large enough to satisfy $P_{\text{BU}} \approx 0$. An analytical formula that satisfies this physical limit is

$$\mu = -x - yE_{\text{BU}}, \tag{8}$$

where x and y are parameters to be determined. By fitting the logarithmic slope parameter μ shown in Fig. 6, we get the values for these two parameters, $x = 0.626$ fm^{-1} and

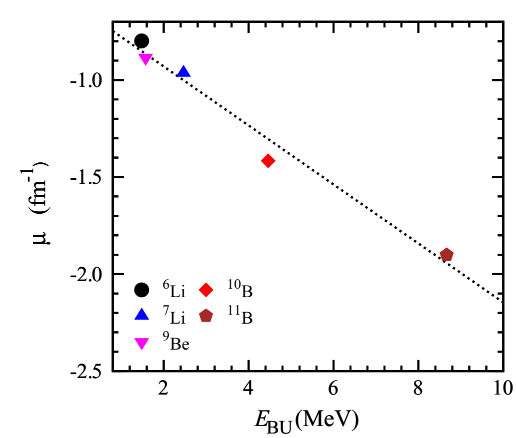

FIG. 6. The values of μ for ^{6,7}Li, ^{9}Be, and ^{10,11}B as a function of the lowest breakup threshold energy E_{BU}. The dotted line denotes the empirical formula (9).

$y = 0.152$ (MeV fm)$^{-1}$. That is, this analytical formula reads

$$\mu = -0.626 - 0.152E_{\mathrm{BU}}, \tag{9}$$

where E_{BU} is in units of MeV. The logarithmic slope parameters obtained by this empirical formula are shown in Fig. 6 by the dotted line. This analytical relation suggests that the effect of breakup on complete fusion may be indeed a threshold effect. With Eq. (9), the ECCBU model can be used to make predictions of complete fusion cross sections for heavy-ion reactions with weakly bound nuclei as projectiles.

IV. SUMMARY

The empirical coupled-channel model is extended by including the breakup effect which is described by a prompt-breakup probability function with two parameters, ν and μ [see Eq. (3)]. The suppression of complete fusion at above-barrier energies in reactions induced by the ^{9}Be, ^{6,7}Li, and ^{10,11}B projectiles on various targets are systematically investigated. For the reactions induced by ^{9}Be, the parameters ν and μ have been extracted from the measured prompt-breakup probabilities, whereas for the other projectiles the parameter μ has been determined by making a fit to the complete fusion data. We found that both μ and the suppression of complete fusion are roughly independent of the target for the reactions induced by the same projectile, μ being mainly determined by the lowest breakup threshold of the weakly bound projectile. An analytical formula which describes well the relation between μ and the breakup threshold energy is proposed. It indicates that the effect of breakup on complete fusion is a threshold effect. Neglecting the SCF component, the present model suggests that the suppression of complete fusion at above-barrier energies is roughly independent of the target in reactions involving the same weakly bound projectile, as this suppression is determined by the prompt-breakup process.

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