# Shape coexistence and evolution in $^{98}$Sr

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(Received 11 June 2015; revised manuscript received 14 December 2015; published 25 January 2016)

Shape coexistence between the strongly deformed ground state and the weakly deformed $0_2^+$ state in $^{98}$Sr has been a major topic of interest due to the energy difference of 215 keV, which is the smallest in all even-even nuclei. The electric monopole transition strength $\rho^2(E0)$ is an important quantity that can relate the deformation difference and the shape mixing between the two $0^+$ states, which are admixtures of the vibrational (S) and the rotational (D) states in a simple mixing model. In a $\beta$-decay spectroscopy experiment, the experimental $\rho^2(E0)$ was measured. A value of 0.053(5) is consistent with the previous measurement and was combined with known electric quadrupole transition strengths $B(E2)$ in calculations of a two-state mixing model. Based on a systematic study on neighboring Kr, Zr, and Mo isotopes, the mixing of the $0^+$ and $2^+$ states in $^{98}$Sr was determined to be 8.6% and 1.3%, respectively, corresponding to deformation parameters $\beta_{\rm D} = 0.38(1)$ and $\beta_{\rm S} = -0.23(2)$. These parameters reproduce experimental transition strengths well except for the $4_1^+ \rightarrow 2_1^+$ transition, which suggests a smaller D-band deformation for $J \geqslant 4$.

DOI: 10.1103/PhysRevC.93.014315

Neutron-rich even-even isotopes of Sr and Zr around neutron number 60 display a sudden change in their ground-state behavior. Below $N = 60$ the ground-state configurations appear to be near spherical with $E(2_1^+)$ at 800–1200 keV, $B(E2; 2_1^+ \rightarrow 0_1^+)$ values around 10 Weisskopf units (W.u.), and an energy ratio of the $4_1^+$ to $2_1^+$ state ($R_{42}$) of 1.5–2.2. With the addition of the 30th pair of neutrons, the system assumes a quadrupole deformed shape in the ground state with $E(2_1^+) < 220$ keV, $B(E2; 2_1^+ \rightarrow 0_1^+) \approx 100$ W.u., and $R_{42} > 2.6$. The drastic change in the structure of the low-lying states is further evidenced in the two-neutron separation energies [1] and in the mean-square charge radii measured through isotope shifts [2–6]. This phenomenon is interpreted as the intrusion of the deformed $0^+$ state in the Sr isotope chain (see Fig. 1) and the subsequent hierarchy inversion with the less-deformed $0^+$ state occurring at $N = 60$.

The presence of excited $0^+$ states at low excitation energies in even-even nuclei has been discussed in the context of shape coexistence [13] due to the possibility of significant shape mixing at near-degenerate energies. In particular, the $0_2^+$ state in $^{98}$Sr has the lowest $E(0_2^+)$ among all known even-even nuclei at 215 keV. This prompted an investigation of its electric monopole ($E0$) transition strength $\rho^2(E0; 0_2^+ \rightarrow 0_1^+)$ to gauge the shape-mixing amplitude between the ground state and the $0_2^+$ state. The determination of $\rho^2(E0)$ requires the knowledge of both the half-life of the $0_2^+$ state and the branching ratio of the transitions to either the $2_1^+$ state with $E(2_1^+) = 144$ keV or the ground state. In this case the half-life has been measured independently in two experimental studies [14,15]. The branching ratio has been measured only once in the study by Schussler *et al.* [14]. In this work we report an independent measurement of the branching ratio to yield a new value of $\rho^2(E0)$.

The $\gamma$-ray spectroscopy and the internal conversion electron (ICE) spectroscopy of $^{98}$Sr were performed at the TRIUMF-ISAC facility [16]. A neutron-rich $^{98}$Rb beam was produced from uranium fission reactions induced by a 500-MeV proton beam from the main cyclotron. The beam intensity of $^{98}$Rb was around $1 \times 10^6$ particles per second with some additional isobar contamination. This radioactive beam was then implanted on a movable tape at the central focus of the detection system, which would later undergo $\beta$ decay into the excited states of $^{98}$Sr. The $8\pi$ spectrometer consisting of 20 Compton-suppressed high-purity germanium (HPGe) detectors was used for $\gamma$-ray measurements, surrounding the vacuum chamber containing ten plastic scintillators and five 200-cm$^2$ lithium-drifted silicon [Si(Li)] detectors named PACES (pentagonal array for conversion electron spectroscopy, [17]) for ICE detection. Data were collected in a cycling mode: 3 s of background, 15 s of beam implantation, 15 s without beam implantation, and 1 s of tape roll (no data collection during this phase) for a single cycle period of 34 s. Unwanted long-lived daughter activity was effectively removed from the sight of

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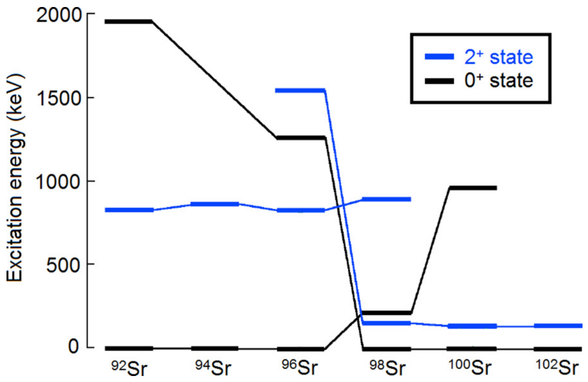


FIG. 1. Systematic of low-lying $0^+$ and $2^+$ states in Sr isotopes around $N = 60$. A sudden deformation in the ground state of $^{98}$Sr occurs as the two $0^+$ states with very different shapes undergo order inversion at $N = 60$. The data for these nuclei are taken from Refs. [7–12].

the detectors by rolling the implanted tape region out of the vacuum chamber. The master trigger condition allowed the collection of only coincidence events of the form $\beta$-$\gamma$, $\beta$-ICE, $\gamma$-$\gamma$, ICE-ICE, and $\gamma$-ICE. The data were collected for about 10 h. The HPGe detectors were calibrated for energy and efficiency using the standard radioactive sources $^{133}$Ba, $^{152}$Eu, $^{60}$Co, and $^{56}$Co. PACES was calibrated for energy and efficiency in the energy range from 35 to 273 keV using the K-shell ICEs from well-known transitions in $^{98}$Sr and $^{98}$Y assuming theoretical internal conversion (IC) coefficients from Ref. [18]. The evaluated half-life has been used in the calculation of $\rho^2(E0)$ in this work; because of problems with the time-to-digital converter modules, an independent measurement of the half-life was impossible.

Figure 2 shows the background-subtracted energy spectrum of ICEs detected in PACES, in coincidence with the $(2_2^+) \rightarrow 0_2^+$ 656-keV $\gamma$ ray detected in one of the HPGe detectors of the $8\pi$ spectrometer. $E0$ transitions depopulating the $0_2^+$ state at 215 keV are clearly visible, in addition to the $E2$ transitions in the low-lying states of $^{98}$Sr. By gating from above, side feeding into the 144-keV $2_1^+$ state was eliminated and the intensities of the 71-keV and the 144-keV $E2$ transitions were

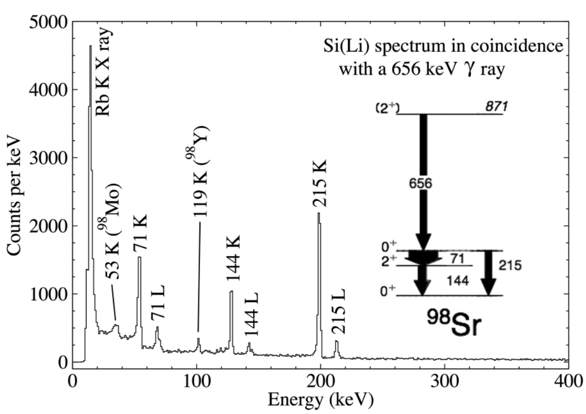


FIG. 2. Energy spectrum of PACES after gating on the 656-keV $\gamma$ ray detected in the $8\pi$ spectrometer. The insert shows a partial level scheme of $^{98}$Sr.

TABLE I. Transition energies, ICE energies, IC coefficients, and electronic factors in $^{98}$Sr used in this work.

| $\Delta E$ (keV) | Shell | $E_e$ (keV) | $\alpha$ | $\Omega$ (s$^{-1}$) |
|---|---|---|---|---|
| 71.2 | | | 3.55(5) | |
| | K | 55.0 | 2.86(4) | |
| 144.2 | | | 0.267(4) | |
| | K | 128.0 | 0.229(4) | |
| 215.4 | K | 199.2 | | $2.160 \times 10^8$ |
| | L1 | 213.1 | | $2.343 \times 10^7$ |
| | L2 | 213.3 | | $5.774 \times 10^4$ |

consistent with each other. Counts in the K-shell ICE peaks of the 71- and 144-keV transitions were corrected for detector efficiency and scaled by the ratio $(1 + \alpha_{\text{tot}})/\alpha_{\text{K}}$ to determine absolute intensities of the two $E2$ transitions, where $\alpha$ is the IC coefficient [18].

The experimental electric monopole transition strength can be obtained from the partial decay rate $\lambda(E0)$ of the $0_2^+$ state and the electronic factors $\Omega_{\text{K,L1,L2...}}$ of the $E0$ transition, given by the atomic theory. The half-life of the 215-keV $0_2^+$ state has been previously measured [14,15] with a weighted average of 22.8(19) ns [10]. The weighted average of the intensities of the 71- and 144-keV transitions was used for $I(E2)$, and the branching ratio $I(E0)/I(E2) = 0.72(6)$ was used in the calculation of $\lambda(E0)$. The values for $\alpha$ and $\Omega$ used in this work are shown in Table I and were taken from Refs. [18,19]. For $\Omega$, contributions from higher shells were negligible. Without uncertainties on the theoretical ratio $\Omega_{\text{L}}/\Omega_{\text{K}} = 0.11$, it is impossible to determine whether it is consistent with the experimental ratio of $I_{\text{L}}/I_{\text{K}} = 0.14(1)$ for the $E0$ transition.

Combining the results together and following the derivation given in Ref. [14], the electric monopole transition strength in $^{98}$Sr was calculated to be $\rho^2(E0) = 0.053(5)$. This is consistent with the previous measured value of $\rho^2 = 0.051(5)$ [14,20]. An alternative formalism provided by Kibédi and Spear [20] utilizes the intensity of ICEs from only the K shell. Using this method, $\rho^2(E0) = 0.049(7)$ was obtained.

To interpret the significance of the $\rho^2(E0)$ value, the shape coexistence scenario can be explored further using a simple two-state mixing model under the assumption that each of the two configurations has a well-determined rigid deformation. Schussler *et al.* [14] proposed a mixing scenario where both the $0_{1,2}^+$ and the $2_{1,2}^+$ state pairs were admixtures of pure vibrational (S) and rotational (D) configurations. Using the formalism given in Ref. [14], the mixing is characterized by parameters $a_{0,2}$ (major component) and $b_{0,2}$ (minor component) in the following way:

$$|0_1^+\rangle = a_0|0_{\text{D}}^+\rangle + b_0|0_{\text{S}}^+\rangle, \tag{1}$$

$$|0_2^+\rangle = a_0|0_{\text{S}}^+\rangle - b_0|0_{\text{D}}^+\rangle, \quad \text{with } a_0^2 + b_0^2 = 1, \tag{2}$$

$$|2_1^+\rangle = a_2|2_{\text{D}}^+\rangle + b_2|2_{\text{S}}^+\rangle, \tag{3}$$

$$|2_2^+\rangle = a_2|2_{\text{S}}^+\rangle - b_2|2_{\text{D}}^+\rangle, \text{with } a_2^2 + b_2^2 = 1. \tag{4}$$

Because the half-life of the 871-keV $2_2^+$ state of the S band remains unknown, the mixing and the deformation parameters cannot be determined unambiguously. Here, the 434-keV $4^+$

state in the D band was assumed to be pure ($|4_1^+\rangle = |4_D^+\rangle$). The known $B(E2;4_1^+ \to 2_1^+)$ value was then used to complete the following set of equations involving $a$, $b$, and $\beta$:

$$B(E2;2_1^+ \to 0_1^+)/\mathrm{W.u.} = |\langle 2020|00\rangle|^2 \left(\frac{5Z}{2\sqrt{\pi}}\right)^2 [a_0a_2\beta_D(1+0.36\beta_D) + b_0b_2\beta_S(1+0.36\beta_S)]^2, \quad (5)$$

$$B(E2;0_2^+ \to 2_1^+)/\mathrm{W.u.} = \left(\frac{5Z}{2\sqrt{\pi}}\right)^2 [a_0b_2\beta_S(1+0.36\beta_S) - a_2b_0\beta_D(1+0.36\beta_D)]^2, \quad (6)$$

$$B(E2;4_1^+ \to 2_1^+)/\mathrm{W.u.} = |\langle 4020|20\rangle|^2 \left(\frac{5Z}{2\sqrt{\pi}}\right)^2 [a_2\beta_D(1+0.36\beta_D)]^2, \quad (7)$$

$$\rho^2(E0;0_2^+ \to 0_1^+) = \left(\frac{3Z}{4\pi}\right)^2 [a_0b_0(\beta_D^2 - \beta_S^2)]^2. \quad (8)$$

The experimental $B(E2)$ and $\rho^2(E0)$ values are listed in Table II, where $R = 1.2A^{1/3}$ fm was used as the nuclear charge radius. The derivation follows Eqs. (2)–(5) of Ref. [21] and the arguments therein. $|\langle J_i020|J_f0\rangle|^2$ are squares of Clebsch-Gordan coefficients, where $|\langle 2020|00\rangle|^2 = 1/5$ and $|\langle 4020|20\rangle|^2 = 2/7$.

In principle, the four equations with four known transition strengths lead to unique solutions for $b_{0,2}$ and $\beta_{D,S}$. An exact solution set ($\beta_D = 0.57$, $\beta_S = 0.21$, $b_0^2 = 0.008$, $b_2^2 = 0.68$) is not physically valid because the $2^+$ state mixing amplitude $b_2^2 > 50\%$ implies incorrectly that the $2_1^+$ state is predominantly an S-band state. In this study, several alternative methods based on the systematics in this region were applied to determine reasonable mixing amplitudes and deformation parameters. We then explore the sensitivity of the transition strengths on these mixing amplitudes and deformation parameters. Finally we report the most probable values based on the empirical transition strengths within the reasonable systematic constraints of the region.

First, the systematics of $B(E2;2_1^+ \to 0_1^+)$ values for $Z \simeq 38$ isotopes provided rough estimates for $\beta_D$ and $\beta_S$ via Eq. (5), where $b_0 = b_2 = 0$ was assumed. Figure 3 shows the deformation parameters obtained from $B(E2;2_1^+ \to 0_1^+)$ of even-even Kr, Sr, Zr, and Mo isotopes with $50 \leqslant N \leqslant 64$. Note that the $B(E2)$ for $N < 60$ provides a suitable estimate for the soft deformation parameter $\beta_S$; $|\beta_S| = 0.14(4)$ for $^{98}$Sr was estimated from a quadratic fit of $\beta$ as a function of $N$, whose trend resembles those of Kr and Mo isotopes. There is an ambiguity in the sign of $\beta_S$ from this approach, whereas the large deformation is generally understood as a prolate deformation in this mass region. On the other hand, the $B(E2)$ for $N \geqslant 60$ provides an estimate for $\beta_D$. Compared to Kr and Mo isotopes, the rapid onset of deformation is clearly visible at $^{98}$Sr. Conversion of $B(E2)$ directly into $\beta_D = 0.36(1)$ assuming a pure D-band transition is only an approximation, but the magnitude compares well with theoretical predictions of $\beta_D = 0.32$–$0.40$ [22,23].

Second, restrictions on $b_2^2$ and $\beta_D$ were provided from the two-state mixing model and $B(E2;4_1^+ \to 2_1^+)$. Energy considerations suggest that the mixing of the $2^+$ states is weaker than that of the $0^+$ states. Taking the minimum unperturbed energy difference between the two $2^+$ states to be $871 - 144 - 215 = 512$ keV (215 keV is the energy difference between the two $0^+$ states), the mixing ratio of $R = \Delta E/V = 512/108 = 4.7$ was used in Eq. (1.8) derived in Casten's work [40] to obtain $b_2^2 < 0.04$. The nonzero transition intensity of the $2_2^+$ state to the D-band $2_1^+$ state indicates $b_2^2 > 0$, but there is insufficient experimental information to improve the lower limit. Using Eq. (7) with the upper estimate of the $B(E2)$ at 148 W.u., $\beta_D < 0.38$. On the other hand, taking the lower limits $b_2^2 = 0$ and $B(E2) = 108$ W.u., $\beta_D > 0.33$ was obtained. These bounds are consistent with the first approximation from the $B(E2;2_1^+ \to 0_1^+)$ value.

Third, the unperturbed bandhead energy $E(0_D)$ and the $2^+$ state mixing energy $\Delta E_2 = E(2_D) - E(2_1^+)$ were estimated

TABLE II. Experimental $B(E2)$ and $\rho^2(E0)$ values used in the analysis; the asterisk denotes the new values measured in the present work.

| Measurement | Value | Ref. |
|---|---|---|
| $B(E2;2_1^+\to 0_1^+)$/W.u. | 96(3) | [10] |
| $B(E2;0_2^+\to 2_1^+)$/W.u. | 66(6) | * |
| $B(E2;4_1^+\to 2_1^+)$/W.u. | 127(10) | [10] |
| $\rho^2(E0;0_2^+\to 0_1^+)$ | 0.053(5) | * |

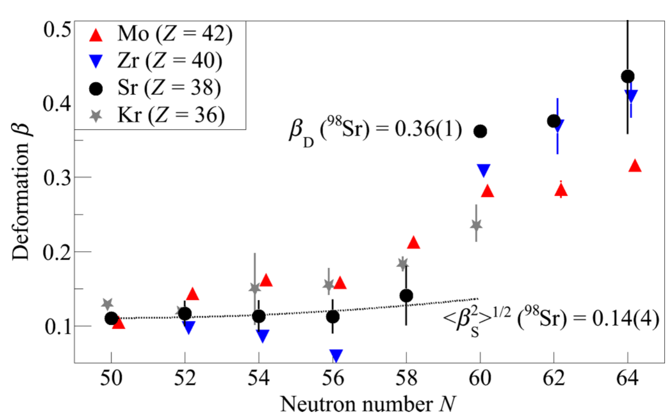


FIG. 3. Deformation parameters obtained from experimental $B(E2;2_1^+ \to 0_1^+)$ of nuclei around $^{98}$Sr, assuming zero shape mixing. A quadratic fit of $\beta$ was performed on Sr isotopes with $N < 60$ to estimate the root-mean-square value of the S-band deformation. The data are taken from Refs. [7,10–12,24–39].

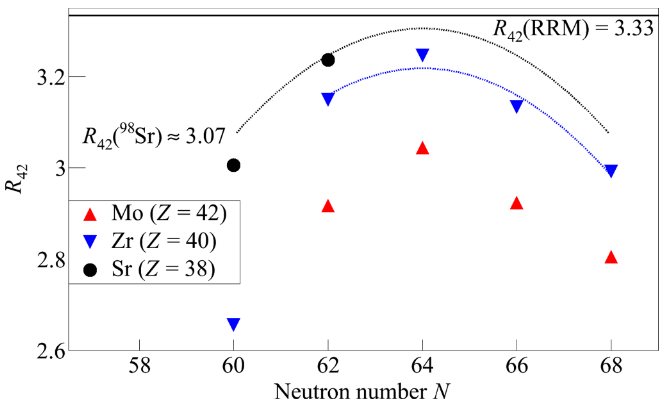


FIG. 4. Experimental $R_{42}$ of well-deformed $N \geqslant 60$ isotopes of Sr, Zr, and Mo. A quadratic fit on $R_{42}$ of Zr isotopes was applied, and the trend was shifted by the $R_{42}$ difference at $N = 62$ to estimate the unperturbed $R_{42}$ for $^{98}$Sr to be used in the bandhead $E(0_\mathrm{D})$ and $\Delta E_2$ estimation. The data are taken from Refs. [10–12,35–39,41–43].

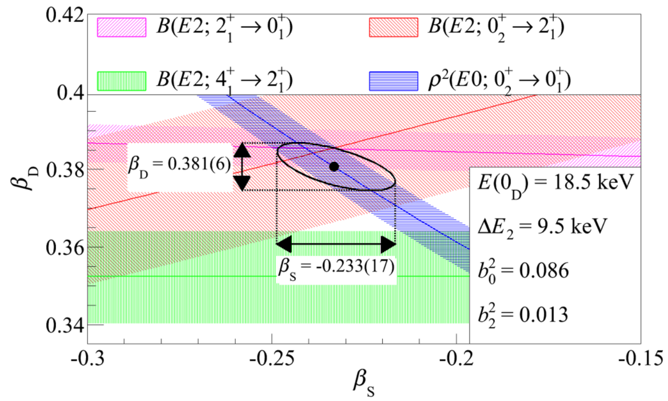


FIG. 5. $1\sigma$ confidence interval bands of the four experimental quantities as functions of $\beta_\mathrm{D}$ and $\beta_\mathrm{S}$, at given mixing energies and amplitudes. A $\chi^2$ minimization was applied to obtain $\beta_\mathrm{D} = 0.381(6)$ and $\beta_\mathrm{S} = -0.233(17)$, whose uncertainties are bounds of the $1\sigma$ contour from the fit. The large discrepancy in $\beta_\mathrm{D}$ with the experimental $B(E2; 4^+_1 \rightarrow 2^+_1)$ value suggests a shape reconfiguration into a smaller deformation of the D-band at higher spins.

through a systematic study of the D-band energies in $N > 60$ Sr and Zr isotopes. One motivation behind this is the remarkable fact that $E(J^+_1, ^{98}\mathrm{Sr})$–$E(J^+_1, ^{100}\mathrm{Sr})$ are between 13.9 and 15.5 keV for even spins up to $J = 10$. Assuming there is very little mixing of the $0^+$ states in $^{100}$Sr, this energy difference alone is a good estimate of $E(0_\mathrm{D})$ for $^{98}$Sr. In addition, a systematic study of the $R_{42}$ values suggests an unperturbed $R_{42}$ value of 3.07 as shown in Fig. 4. Rather than choosing $E(0_\mathrm{D})$ and $\Delta E_2$ to fix $R_{42} = 3.33$ as was done by Schussler *et al.* [14] in using the rigid rotor model (RRM), the nucleus was assumed in this work to be more fluid.

Several sets of trial mixing energies and their corresponding mixing amplitudes are shown in Table III to demonstrate the sensitivity of the mixing amplitudes, deformation parameters, and transition strengths on these quantities. The transition strengths shown in Table III were determined for each set of $b_0^2$, $b_2^2$, $\beta_\mathrm{D}$, and $\beta_\mathrm{S}$; after fixing the mixing amplitudes, the deformation parameters were scanned over the confidence intervals given by the experimental transition strengths as shown in Fig. 5. The uncertainties of $\beta_\mathrm{D}$ and $\beta_\mathrm{S}$ were given by the range of the maximum and minimum values at which $(\chi^2 - \chi^2_\mathrm{min})/\mathrm{ndf} = 1$. Then, standard error analysis on Eqs. (5)– (8) was applied to determine the uncertainties of calculated transition strengths.

The D-band deformation remained robust at $\beta_\mathrm{D} = 0.38$, while the magnitude of the S-band deformation was highly correlated with the mixing amplitude. For the calculated transition strengths, the $B(E2; 0^+_2 \rightarrow 2^+_1)$ was the most sensitive. In all cases, the fits resulted in $\beta_\mathrm{S} < 0$. The oblate deformation was also predicted using the self-consistent mean-field approximation based on the Gogny-D1S interaction (see Fig. 4 in Ref. [45]).

Using the experimental quantities and their uncertainties in Table II, a $\chi^2$ fit was applied to determine the most likely deformation parameters. The minimum $\chi^2$ was achieved for $b_0^2 = 8.6\%$, $b_2^2 = 1.3\%$, $\beta_\mathrm{D} = 0.381(6)$, and $\beta_\mathrm{S} = -0.233(17)$ with $E(0_\mathrm{D}) = 18.5$ keV and $\Delta E_2 = 9.5$ keV. The bandhead energy and the $2^+$ state mixing energy were adjusted while keeping the $R_{42}$ ratio at 3.07. The magnitude of $\beta_\mathrm{S}$ from this procedure is much greater than the initial estimate of 0.14(4) given in Fig. 3, which was an extrapolation of a slow quadratic increase from $N = 50$. The calculated transition strengths are similar to the work by Mach *et al.* [21], whose result was based on zero $2^+$ state mixing and thus had an arbitrary sign for $\beta_\mathrm{S}$. An oblate deformation for the S band was already suggested in Ref. [44] with parameters that reproduce $B(E2; 4^+_1 \rightarrow 2^+_1)$ very well, but the $B(E2; 2^+_1 \rightarrow 0^+_1)$ is underestimated by more than $6\sigma$ of the current experimental value. That work cites a historical $B(E2; 2^+_1 \rightarrow 0^+_1) = 67(19)$ W.u. from Refs. [14,46].

A consistent overestimation of the $B(E2; 4^+_1 \rightarrow 2^+_1)$ for all mixing and deformation parameters is an open question; mixing of the $4^+_1$ state is likely to be minimal, because the lowest possible $(4^+)$ state is at 1681.5 keV. In addition, any mixing with possible $|4_\mathrm{S}\rangle$ states will only increase the $B(E2)$ value unless the phase of the S-band mixing parameter is destructive. An independent half-life measurement of this state to compare with $T_{1/2}(4^+_1) = 80(6)$ ps [21] may clarify the situation. On the other hand, the $B(E2)$ values of the yrast $12^+$, $10^+$, and $8^+$ D-band states [47] were used to deduce a weighted average of $\beta_\mathrm{D} = 0.32(1)$. The decrease in the deformation appears to be correlated with spin increase; a precision half-life measurement of the $6^+_1$ state is needed for $\beta_\mathrm{D}(6^+)$ to investigate the reduced collectivity and deformation.

In summary, the electric monopole transition strength $\rho^2(E0)$ between the $0^+_2$ state at 215.4 keV and the ground state in $^{98}$Sr has been determined in a $\beta$-decay study using the $8\pi$ spectrometer and PACES at TRIUMF-ISAC. The $\rho^2(E0)$ value of 0.053(5) is consistent with the previously measured value, and it provides the strongest constraint on the relationship between the two deformation parameters $\beta_\mathrm{D}$ and $\beta_\mathrm{S}$ of the D and S bands. From a simple two-state mixing model, the mixing amplitudes and the deformation parameters were determined from the systematic study of $N \sim 60$ isotopes of Kr, Sr, Zr, and Mo. Mixing of the two low-lying $0^+$ states is approximately 9%, while it is about 1.3% for the $2^+$ states. The deformed ground state band has a robust $\beta_\mathrm{D}$ of 0.38, while the

TABLE III. Calculated values of mixing amplitudes, deformation parameters, and corresponding experimental quantities by Eqs. (5)–(8). In the first four columns, a set of $E(0_D)$ and $\Delta E_2$ values are trialed while keeping the $R_{42} = 3.07$ fixed to show the sensitivity of the deformation parameters and the calculated transition strengths. In the fifth column the deformation parameters, mixing amplitudes, and mixing energies are fitted to the experimental transition strengths from this work and Ref. [10]. The final three columns compare to previous studies.

| Parameter/ | Value | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| measurement | Sensitivity test of mixing energies | | | | This work | Ref. [21] | Ref. [44] | Ref. [14] |
| $E(0_D)$, keV | 25.0 | 20.0 | 15.0 | 10.0 | 18.5 | 23.3[a] | 24.0 | 39.7 |
| $\Delta E_2$, keV | 13.8 | 10.5 | 7.1 | 3.7 | 9.5 | 0[a] | 2.5 | 13.7 |
| $b_0^2$ | 0.12 | 0.093 | 0.070 | 0.046 | 0.086 | 0.11(1) | 0.11 | 0.18 |
| $b_2^2$ | 0.019 | 0.014 | 0.010 | 0.005 | 0.013 | 0 | 0.004 | 0.019 |
| $\beta_D$ | 0.38(1) | 0.38(1) | 0.38(1) | 0.38(1) | 0.38(1) | 0.38(1) | 0.35 | 0.37 |
| $\beta_S$ | −0.26(2) | −0.24(2) | −0.22(2) | −0.17(2) | −0.23(2) | 0.25(2)[b] | −0.20 | 0.20 |
| $B(E2;2_1^+\to0_1^+)$/W.u. | 89(3) | 93(3) | 96(3) | 100(4) | 94(3) | 95(6) | 77 | 86 |
| $B(E2;0_2^+\to2_1^+)$/W.u. | 90(3) | 70(2) | 50(2) | 31(1) | 64(2) | 59(6) | 58 | 66 |
| $B(E2;4_1^+\to2_1^+)$/W.u. | 152(5) | 152(5) | 152(5) | 151(5) | 152(5) | 153(9) | 127 | 142 |
| $\rho^2(E0;0_2^+\to0_1^+)$ | 0.055(12) | 0.054(11) | 0.051(10) | 0.047(7) | 0.053(11) | 0.053(17) | 0.055 | 0.116 |

[a]These mixing energies result in $R_{42} = 3.39 > 3.33$ of the RRM.
[b]Absolute value. The sign is ambiguous due to $b_2^2 = 0$.

S band has $\beta_S$ between −0.22 and −0.24. These parameters reproduce experimental $B(E2)$ and $\rho^2$ values well for $J \leqslant 2$, but the $B(E2;4_1^+ \to 2_1^+)$ is consistently overestimated by more than $2\sigma$. To accurately determine the $2^+$ state mixing, half-life measurements of the $2_2^+$ state will be needed. A precision half-life measurement of the $6_1^+$ state will also enlighten the dependence of quadrupole collectivity on spin in the D band of $^{98}$Sr.

The authors would like to thank the personnel in charge of targets, beam delivery, and operators at TRIUMF-ISAC for providing the radioactive beam. This work was supported in part by the Natural Sciences and Engineering Research Council of Canada (NSERC). TRIUMF receives federal funding via a contribution agreement through the Natural Research Council Canada (NRC).

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