Net-proton-number kurtosis and skewness in nuclear collisions: Influence of deuteron formation

Zuzana Fecková,[1,2] Jan Steinheimer,[3] Boris Tomášik,[2,4] and Marcus Bleicher[3,5]

[1]*Univerzita Pavla Jozefa Šafárika, Šrobárova 2, 04001 Košice, Slovakia*

[2]*Univerzita Mateja Bela, Tajovského 40, 97401 Banská Bystrica, Slovakia*

[3]*Frankfurt Institute for Advanced Studies, Johann Wolfgang Goethe-Universität, Ruth-Moufang-Strasse 1,
60438 Frankfurt am Main, Germany*

[4]*České vysoké učení technické v Praze, FJFI, Břehová 7, 11519 Praha 1, Czech Republic*

[5]*Institut für Theoretische Physik, Johann Wolfgang Goethe-Universität, Max-von-Laue-Strasse 1, 60438 Frankfurt am Main, Germany*

We explore the influence of deuteron formation in the late stage of nucleus-nucleus reactions on the fluctuations observed in the final net-proton yields around midrapidity. At each investigated energy, the produced (anti)proton yield at chemical freeze-out is assumed to fluctuate according to a Poisson distribution and in each event the probability for deuteron formation by coalescence is proportional to $(dN_{\mathrm{proton}}/dy)^2$. The protons that are then clustered in deuterons are usually not included in the experimental measurement of the net-proton fluctuations, therefore, we subtract these clustered protons from the final state proton number for the calculation of the net-proton fluctuations (the same is done in the antiproton sector). Due to the nonlinear deuteron formation probability the resulting distribution is not a Skellam distribution, but shows the interesting feature of a decrease in the kurtosis $\kappa\sigma^2$ and a local maximum in the skewness $S\sigma$ observables as the collision energy decreases.

I. INTRODUCTION

The exploration of the properties of ultrahot and superdense nuclear matter is among the most challenging scientific endeavors today. The collisions of heavy ions allow to explore these properties in the laboratory for the first time with unprecedented precision. Fluctuation observables have gained very great interest in recent years as they may be sensitive to the structure of the QCD phase diagram. In particular they can serve as an indicator for the first-order deconfinement phase transition and the associated critical end point [1–4]. Particle number fluctuations were first explored in experiments at the Super Proton Synchrotron (SPS) [5,6]. With the availability of the first lattice QCD data on higher-order susceptibilities in [7] these investigations [4,8–13] have moved to the next level and have been intensively explored with STAR's Beam Energy Scan (BES) at the BNL Relativistic Heavy Ion Collider (RHIC) facility [14]. Recent results from the BES program indicate a nontrivial beam dependence of the higherorder net-baryon number cumulants. This is of particular interest, as these higher-order cumulants are very sensitive to the existence of a second-order phase transition [15,16] and the associated fluctuations and correlations.

While irregular structures have been observed in the experimental data, the interpretation and final experimental validation are still missing today. For example, effects of conservation laws [17–19], final state hadronic interactions [20], repulsive hadronic interactions [21], and acceptance effects [22] as well as efficiency corrections [23] are considered to be very important for the interpretation of experimental results.

It is therefore essential to understand all trivial and nontrivial effects which have an influence on the observed cumulant ratios, in order to make conclusions on the existence and discovery of a phase transition and critical endpoint.

In the following we suggest a further uncertainty, arising from nuclear cluster formation, that needs to be taken into account before final conclusions from these fluctuation observables should be drawn.

II. SET UP

For this exploratory study, we will stay with a simple physical picture to elucidate the main effect of cluster production on the observable fluctuations. It is clear that more elaborate studies will be needed to allow for a detailed and more quantitative investigation of the effect of cluster formation.

The scenario is as follows. We assume that the number of chemically frozen out protons in some midrapidity interval n_i fluctuates according to a Poisson distribution

$$P_i(n_i) = \lambda_p^{n_i} \frac{e^{-\lambda_p}}{n_i!} \,. \tag{1}$$

Here, λ_p is the mean number of these *initial* protons. It is important to realize that after deuteron coalescence some of them will be hidden in deuterons and thus will disappear when counting the observed final state protons. Therefore, λ_p is unknown at this point but will be determined from the *observed* proton and deuteron yields later. At higher collision energies a similar procedure should be done for antiprotons. We will show, however, that for the energy range of SPS and below their influence can be neglected.

Next, we assume that the probability to form a deuteron by coalescence after the kinetic freeze-out is in each event proportional to the actual initial proton yield squared in that event [24–26], i.e.,

$$\lambda_d = B n_i^2 \,, \tag{2}$$

with a coalescence parameter B which depends only on the beam energy. At this point we make the assumption that the neutron yield in a single event is proportional to the proton yield in that event. If the neutron yield was uncorrelated with the proton number the correlation between proton and deuteron production in a single event is weakened. This feature may be experimentally checked by measuring the production yield of deuterons as a function of the proton number, in a specific centrality bin. Further, it is important to note that this assumption is only valid in the absence of volume fluctuations. In a scenario where volume fluctuations dominate the observed proton number fluctuations (due to, e.g., wide centrality bin selections), both the proton and deuteron number fluctuations will scale with the fluctuations of the volume $n_i \propto n_d \propto V$. As a result the measured proton number distribution will be described by a Skellam distribution because proton and deuteron fluctuations can be described with independent Poisson distributions. In the case of vanishing volume fluctuations, as is usually ensured by the tight cuts performed by the experiments, the deuteron number can be considered correlated with the initial proton number, as presented in Eq. (2).

The deuteron yield n_d in an event with initial proton multiplicity n_i then fluctuates according to a Poisson distribution

$$P_d(n_d|n_i) = \lambda_d^{n_d} \frac{e^{-\lambda_d}}{n_d!} = (Bn_i^2)^{n_d} \frac{e^{-Bn_i^2}}{n_d!} . \quad (3)$$

The observed deuteron distribution is obtained by summing over initial proton yields as

$$P_d(n_d) = \sum_{n_i} P_d(n_d|n_i) P_i(n_i) . \quad (4)$$

To get the observed proton number, we have to subtract from the initial protons those which went into deuterons

$$\frac{dN_{\text{proton}}}{dy} = n_p = n_i - n_d . \quad (5)$$

The distribution of n_p then follows from the appropriate convolution of $P_i(n_i)$ with $P_d(n_d|n_i)$:

$$P(n_p) = \sum_{n_i \geqslant n_p} \sum_{n_d} P_i(n_i) P_d(n_d|n_i) \delta_{n_p, n_i - n_d}$$

$$= \sum_{n_i \geqslant n_p} P_i(n_i) P_d(n_i - n_p|n_i) . \quad (6)$$

The two parameters in this model are the mean *initial* proton number λ_p and the coalescence factor B. They can be fixed from the mean *observed* proton multiplicity and the mean observed deuteron multiplicity

$$\langle n_p \rangle = \sum_{n_p} n_p P(n_p), \quad (7)$$

$$\langle n_d \rangle = \sum_{n_d} n_d P_d(n_d), \quad (8)$$

where $P(n_p)$ and $P_d(n_d)$ are determined in Eqs. (6) and (4). Then, the distribution $P(n_p)$ can be used to calculate any moments of the proton distribution.
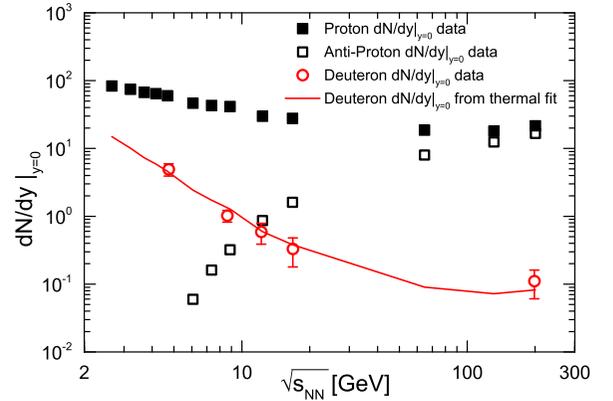

FIG. 1. (Color online) The midrapidity proton, antiproton, and deuteron yields from experimental data (symbols) and the corresponding deuteron yield (red solid line) which was extracted using the thermal fit to the d/p ratio, for central Au + Au reactions.

Deuteron multiplicity has been measured in nuclear collisions at various energies. At low energies the data were abundant and the deuteron-to-proton multiplicity ratio was proposed as a measure of the entropy in the system [27]. From Alternating Gradient Synchrotron (AGS) to top RHIC energies the available data are actually well fitted by the thermal model. The deuteron-to-proton multiplicity ratio at midrapidity can be parametrized as

$$\frac{d}{p} = 0.8 \left[\frac{\sqrt{s_{NN}}}{1 \, \text{GeV}} \right]^{-1.55} + 0.0036 ,$$

as shown in [28]. Figure 1 depicts the midrapidity proton, antiproton, and deuteron yields from experimental data (see [28] and references therein) and the thermal model parametrization of the deuteron yield [29] which serve as input for our calculations.

Using this input we calculate the variance σ, skewness S, and the kurtosis κ from the distribution $P(n_p)$

$$\sigma^2 = \langle (n_p - \langle n_p \rangle)^2 \rangle, \quad (9)$$

$$S = \frac{\langle (n_p - \langle n_p \rangle)^3 \rangle}{\sigma^3}, \quad (10)$$

$$\kappa = \frac{\langle (n_p - \langle n_p \rangle)^4 \rangle}{\sigma^4} - 3 . \quad (11)$$

III. ANALYTIC RESULTS

For illustration, in Fig. 2 we show the scaled skewness $S\sigma$ and in Fig. 3 the scaled kurtosis $\kappa\sigma^2$ of the distribution $P(n_p)$ as functions of the observed mean proton number $\langle n_p \rangle$ and the coalescence parameter B.

Recall that both $S\sigma$ and $\kappa\sigma^2$ would be equal to unity if the underlying observed proton distribution was Poissonian. (There is a deviation from 1 for $S\sigma$ if the distribution was Skellam, but we will scale it out later in the presentation of results.) This is really so in the absence of deuterons, i.e., when $B = 0$ or for small numbers of observed protons.
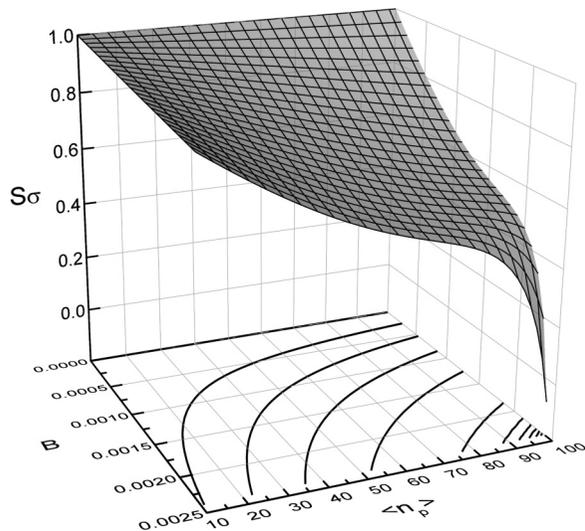
FIG. 2. Scaled skewness $S\sigma$ as a function of the mean observed proton number and the coalescence parameter B in the region of values relevant for AGS and SPS collision energies.

We also see a strong deviation from unity for large values of $\langle n_p \rangle$ and B. In fact the scaled kurtosis may take values which are both smaller or larger than unity.

For a medium value of proton multiplicity and/or coalescence parameter the obtained deuteron number limits the possible observed proton number fluctuation. Since the mean deuteron number scales with the square of proton number, if the proton number becomes large in an event, the number of deuterons makes even larger relative deviation from the mean. This limits the fluctuations of the observed protons mainly on the upper side of the distribution and the tails of the distribution are below a comparable Poisson distribution. When the deuteron number grows further, it starts pushing the observed proton number down [in agreement with Eq. (5)]. This increases the deviations of the proton number from the mean to lower values. The left tail of the distribution is above
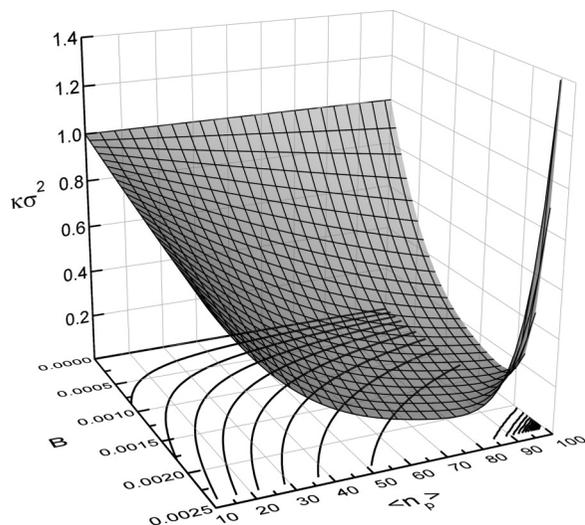

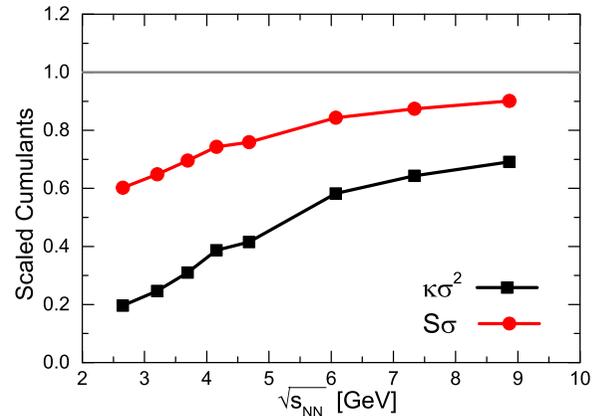FIG. 3. Same as Fig. 2 but for the scaled kurtosis $\kappa\sigma^2$.

FIG. 4. (Color online) Analytic results for the skewness and kurtosis for Au + Au collisions in the energy range where antiproton production is not important.

Poissonian and the kurtosis grows. The skewness becomes clearly smaller.

Figure 4 shows the results for the beam energy dependence of the scaled skewness and kurtosis, using as input measured midrapidity yields of protons and deuterons. We observe a clear deviation from unity for all beam energies under consideration.

IV. RESULTS INCLUDING ANTIPROTONS

In order to extend our investigations to beam energies higher than $\sqrt{s_{NN}} \approx 10$ GeV, we need to include the effects of antiprotons and antideuterons into our study. This extension of our approach is straightforward. As the measured mean proton and antiproton numbers are known we can assume that both the proton and antiproton number separately follow a Poisson distribution. From this assumption, the initial proton and antiproton number can be sampled independently and according to their measured multiplicities, as described in the previous section. Since the fluctuations of both are described by uncorrelated Poisson distributions the fluctuations of the initial net-proton number should be described by a Skellam distribution. Assuming that the coalescence parameter for antideuterons is identical to that of deuteron formation we can numerically determine the final net-proton number in a given event as

$$n_{p-\bar{p}} = (n_i - n_d) - (n_{\bar{i}} - n_{\bar{d}}), \tag{12}$$

as well as the corresponding net-proton number distributions. To obtain the mean final net-proton number and its cumulants we perform a numerical sampling of the initial proton and antiproton number as well as the corresponding (anti)deuteron numbers in each event. To achieve satisfactory statistics we sample 10^9 events per beam energy. The resulting scaled cumulants of the proton number distributions, $S\sigma$ and $\kappa\sigma^2$, are shown in Fig. 5 as functions of energy for central Au + Au reactions over a broad range of energies. We compare our numerical results including antiprotons and antideuterons with the analytical results obtained in the previous section. For the lower SPS energies only small variations are observed.
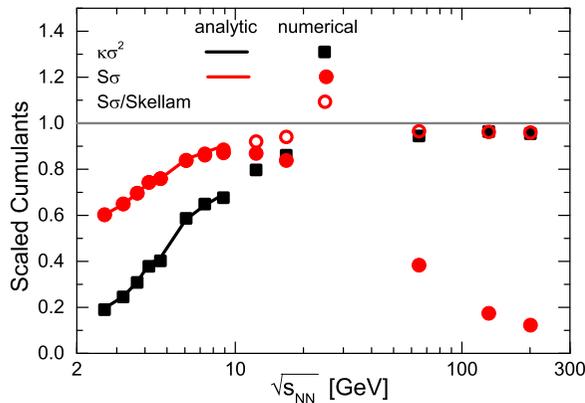
FIG. 5. (Color online) The quantities $S\sigma$ and $\kappa\sigma^2$ as functions of energy for central Au + Au reactions over a broad range of energies. Compared are the numerical and analytic results. We clearly observe a drastic deviation from the expectation of a Poisson or Skellam distribution.

We clearly confirm the drastic deviation from the expectation of a Poisson or Skellam distribution. The $\kappa\sigma^2$ and $S\sigma$ show strong deviations from unity as the energy decreases and the deuteron fraction increases. This indicates that the formation of deuterons after the kinetic freeze-out may lead to substantial modifications of the fluctuation observable considered here.

V. SUMMARY

In light of the ongoing debate on the interpretation of the recently and currently measured data on fluctuations, we have explored the influence of deuteron formation on fluctuation observables. We employed a standard approach and used the deuteron formation probability as proportional to the square of the proton yield. This nonlinear coupling of the proton and deuteron yields results in a substantial modification of the fluctuation patterns towards low energies: a strong reduction of the $\kappa\sigma^2$ and $S\sigma$/Skellam observable has been found.

[1] C. Sasaki, B. Friman, and K. Redlich, Phys. Rev. Lett. **99**, 232301 (2007).

[2] J. Randrup, Phys. Rev. C **79**, 054911 (2009).

[3] J. Steinheimer and J. Randrup, Phys. Rev. Lett. **109**, 212301 (2012).

[4] M. A. Stephanov, K. Rajagopal, and E. V. Shuryak, Phys. Rev. D **60**, 114028 (1999).

[5] C. Alt *et al.* (NA49 Collaboration), Phys. Rev. C **79**, 044910 (2009).

[6] T. Anticic *et al.* (NA49 Collaboration), Phys. Rev. C **79**, 044904 (2009).

[7] C. R. Allton, M. Doring, S. Ejiri, S. J. Hands, O. Kaczmarek, F. Karsch, E. Laermann, and K. Redlich, Phys. Rev. D **71**, 054508 (2005).

[8] M. A. Stephanov, K. Rajagopal, and E. V. Shuryak, Phys. Rev. Lett. **81**, 4816 (1998).

[9] S. Jeon and V. Koch, Phys. Rev. Lett. **85**, 2076 (2000).

[10] M. Asakawa, U. W. Heinz, and B. Muller, Nucl. Phys. A **698**, 519 (2002).

[11] V. Koch, M. Bleicher, and S. Jeon, Nucl. Phys. A **698**, 261 (2002); **702**, 291 (2002).

[12] V. Koch, J. Phys. G **35**, 104030 (2008).

[13] F. Karsch and K. Redlich, Phys. Lett. B **695**, 136 (2011).

[14] M. M. Aggarwal *et al.* (STAR Collaboration), Phys. Rev. Lett. **105**, 022302 (2010).

[15] M. A. Stephanov, Phys. Rev. Lett. **102**, 032301 (2009).

[16] V. Koch, *Relativistic Heavy Ion Physics*, edited by R. Stock, Landolt-Boernstein New Series I, Vol. 23 (Springer, Heidelberg, 2010), pp. 626–652.

[17] M. Nahrgang, T. Schuster, R. Stock, M. Mitrovski, and M. Bleicher, Eur. Phys. J. C **72**, 2143 (2012).

[18] A. Bzdak, V. Koch, and V. Skokov, Phys. Rev. C **87**, 014901 (2013).

[19] M. Sakaida, M. Asakawa, and M. Kitazawa, Phys. Rev. C **90**, 064911 (2014).

[20] M. Kitazawa and M. Asakawa, Phys. Rev. C **85**, 021901(R) (2012).

[21] V. V. Begun, M. Gazdzicki, and M. I. Gorenstein, Phys. Rev. C **88**, 024902 (2013).

[22] A. Bzdak and V. Koch, Phys. Rev. C **86**, 044904 (2012).

[23] A. Bzdak and V. Koch, Phys. Rev. C **91**, 027901 (2015).

[24] S. T. Butler and C. A. Pearson, Phys. Rev. **129**, 836 (1963).

[25] L. P. Csernai and J. I. Kapusta, Phys. Rep. **131**, 223 (1986).

[26] J. L. Nagle, B. S. Kumar, D. Kusnezov, H. Sorge, and R. Mattiello, Phys. Rev. C **53**, 367 (1996).

[27] P. J. Siemens and J. I. Kapusta, Phys. Rev. Lett. **43**, 1486 (1979).

[28] A. Andronic, P. Braun-Munzinger, and J. Stachel, Nucl. Phys. A **772**, 167 (2006).

[29] A. Andronic, P. Braun-Munzinger, J. Stachel, and H. Stocker, Phys. Lett. B **697**, 203 (2011).