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Enhancement factor for two-neutron transfer reactions with a schematic coupled-channels model

K. Hagino[1,2,3] and G. Scamps[1]
[1]*Department of Physics, Tohoku University, Sendai 980-8578, Japan*
[2]*Research Center for Electron Photon Science, Tohoku University, 1-2-1 Mikamine, Sendai 982-0826, Japan*
[3]*National Astronomical Observatory of Japan, 2-21-1 Osawa, Mitaka, Tokyo 181-8588, Japan*
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Probabilities for heavy-ion two-neutron transfer reactions P_{2n} are often discussed in comparison with the square of the corresponding probabilities for the one-neutron transfer process $(P_{1n})^2$, implicitly assuming that $(P_{1n})^2$ provides the probability of two-neutron transfer reactions in the absence of the pairing correlation. We use a schematic coupled-channels model, in which the transfers are treated as effective inelastic channels, and demonstrate that this model leads to $P_{2n} = (P_{1n})^2/4$, rather than $P_{2n} = (P_{1n})^2$, in the pure sequential limit. We argue that a simple model with spin-up and spin-down neutrons in a single-particle orbital also leads to the same conclusion.

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I. INTRODUCTION

It has been well known that the pairing correlation enhances cross sections for the two-neutron transfer process as compared to those in the uncorrelated limit [1–6]. For heavy-ion systems, those cross sections are often converted to the transfer probabilities by dividing them by the Rutherford cross sections, and are plotted as a function of the distance of the closest approach D for the classical Rutherford trajectory. This representation in fact provides a convenient way to discuss the reaction dynamics because the cross sections for different values of incident energies and the scattering angles can be analyzed in a unified way. The enhancement of the two-neutron transfer process has been discussed customarily by taking the ratio between P_{2n} and $(P_{1n})^2$ [3,7–14], where P_{1n} and P_{2n} are the probabilities for the one- and two-neutron transfer processes, respectively. That is, it has been usually believed that the quantity $(P_{1n})^2$ provides a reference probability for the two-neutron transfer process which would be realized in the absence of the pairing correlation [15].

In this paper, we discuss the validity of this assumption for heavy-ion transfer reactions. To this end, we consider the two-neutron transfer probability in the no-correlation limit, where the two-neutron transfer process takes place in a completely sequential manner. This work is partly motivated by a recent result of a time-dependent Hartree-Fock (TDHF) + BCS calculation, which shows that the ratio $P_{2n}/(P_{1n})^2$ in the absence of the pairing correlation is well parametrized as [16]

$$\frac{P_{2n}}{(P_{1n})^2} \sim \frac{N_v - 1}{2N_v} \times \frac{N_f - 1}{N_f}, \tag{1}$$

where N_v and N_f are the number of valence nucleons and the number of available states in the receiver nucleus, respectively. This equation suggests that the ratio $P_{2n}/(P_{1n})^2$ is not unity in general, but is more complex and never exceeds 1/2. In this paper, we employ the coupled-channels approach to investigate this problem from a different perspective. In particular, we use a schematic coupled-channels model for two-neutron transfer, and attempt to understand the result of TDHF.

II. COUPLED-CHANNELS CALCULATIONS

In the coupled-channels approach to transfer reactions, one often treats transfer channels as effective inelastic excitations [17–20]. In this paper, we use the same treatment for the transfer channels and consider the following coupling matrix for a sequential two-neutron transfer reaction [21,22],

$$V = \begin{pmatrix} 0 & F(r) & 0 \\ F(r) & -Q & F(r) \\ 0 & F(r) & -2Q \end{pmatrix}. \tag{2}$$

Here, we have assumed that all the channels have zero angular momentum. We have also neglected the recoil effect, which is expected to be small for heavy-ion transfer reactions. In this equation, $F(r)$ is the form factor for the coupling between the entrance (0n) and the one-neutron (1n) transfer channels, while Q is the Q value for the 1n-transfer reaction. In this coupling scheme, the 0n channel is coupled to the 1n channel, which is sequentially coupled to the two-neutron (2n) channel. The no-correlation limit is simulated by setting the coupling between the 1n and the 2n transfer channels to be the same as that between the 0n and the 1n transfer channels, and also by setting the Q value for the 2n transfer channel to be exactly twice the Q value for the 1n channel. The direct coupling between the 0n and the 2n channels is also set to be zero.

With the coupling potential given by Eq. (2), the coupled-channels equations read

$$\left(-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2} + \frac{l(l+1)\hbar^2}{2\mu r^2} + V_0(r) - E\right)u_i(r) + \sum_j V_{ij}(r)u_j(r) = 0, \tag{3}$$

where μ is the reduced mass, l is the angular momentum of the relative motion between the projectile and the target nuclei, and V_0 is the bare potential. V_{ij} with $i, j = 0$, 1, and 2 is the matrix element of the coupling potential given by Eq. (2). In principle, the reduced mass μ is different for each transfer channel, but for simplicity we neglect the difference and use the same μ for all the channels. We have confirmed that this approximation works well for the ^{40}Ca + ^{96}Zr system studied in this paper. The coupled-channels equations are solved with the scattering boundary condition for $u_i(r)$,

$$u_i(r) \to \frac{i}{2}\left\{H_l^{(+)}(k_i r)\delta_{i,0} - \sqrt{\frac{k_i}{k_0}}\, S_i^l\, H_l^{(-)}(k_i r)\right\}, \quad (4)$$

where $i = 0$ is the entrance channel and S_i^l is the nuclear S matrix. $H_l^{(-)}(k_i r)$ and $H_l^{(+)}(k_i r)$ are the incoming and the outgoing Coulomb wave functions, respectively, in which the channel wave number k_i is given by $\sqrt{2\mu(E+Q_i)/\hbar^2}$ with $-Q_i$ being the diagonal component in Eq. (2). The transfer cross sections are then calculated as [23,24]

$$\frac{d\sigma_i}{d\Omega} = \frac{k_0}{k_i}|f_i(\theta)|^2, \quad (5)$$

with

$$f_i(\theta) = \sum_l e^{i[\sigma_l(E)+\sigma_l(E+Q_i)]}\sqrt{\frac{2l+1}{4\pi}}\,Y_{l0}(\theta)\,\frac{-2i\pi}{k_i k_0}S_i^l, \quad (6)$$

for i =1 and 2, where $\sigma_l(E)$ is the Coulomb phase shift.

We apply this model to the ^{40}Ca + ^{96}Zr reaction, for which the experimental transfer cross sections have been reported in Ref. [12]. To this end, we use a function which asymptotically has an exponential form,

$$F(r) \sim \frac{\beta}{a}e^{-(r-R)/a}, \quad (7)$$

for the coupling form factor $F(r)$, and set the transfer Q value to be Q=0 [20,22]. (In the actual calculations shown below, for a numerical reason, we use a derivative form of the Fermi function with the parameters β, R, and a.) With the Woods-Saxon type for the nuclear potential, with the parameters of V_0 = 140 MeV, r_0 = 1.1 fm, and a_0 = 0.65 fm for the real part and W_0 = 30 MeV, r_W = 1.15 fm, and a_W = 0.1 fm for the imaginary part, we vary the parameters in the coupling form factor, Eq. (7), so that the experimental data for the one-neutron transfer reaction can be reproduced. To this end, the coupled-channels equations are solved using a version of the computer code CCFULL [25]. The resultant values for the parameters are β = 9 MeV fm, $R = 1.15\times(40^{1/3}+96^{1/3})$ fm, and a = 1.3 fm.

Figure 1 shows the transfer probabilities so obtained. Here, the transfer probabilities are defined as the ratio of the transfer cross sections to the Rutherford cross sections, that is, $P_{xn} = (d\sigma_{xn}/d\Omega)/(d\sigma_R/d\Omega)$, where x = 1, 2 is the number of transferred neutron, and $d\sigma_{xn}/d\Omega$ and $d\sigma_R/d\Omega$ are the transfer and the Rutherford cross sections, respectively. This definition is applied both to the experimental data and to the theoretical calculations. The transfer probabilities are plotted as a function of the distance of the closest approach D of the Rutherford trajectory in the entrance channel for the scattering angle of $\theta_{c.m.}$ = 140 degrees in the center-of-mass frame. The dotted and the solid lines denote the transfer probabilities for the 1n and the 2n channels, respectively. While the 1n probabilities are well reproduced, as expected, the 2n probabilities are largely underestimated by this calculation. One can clearly see that the 2n probability P_{2n} is consistent with a quarter of the square of the 1n probability $(P_{1n})^2/4$, which is denoted by the dashed line in the figure.

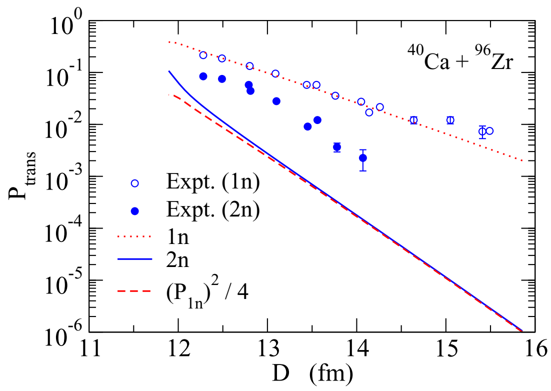

FIG. 1. (Color online) The transfer probabilities, defined as the ratio of the transfer cross sections to the Rutherford cross sections, for the ^{40}Ca + ^{96}Zr reaction. These probabilities are plotted as a function of the distance of the closest approach D of the classical Rutherford trajectory. The dotted and the solid lines denote the one- and two-neutron transfer probabilities, respectively, while the dashed line is a quarter of the square of the one-neutron transfer probability. The experimental data are taken from Ref. [12].

III. INTERPRETATION WITH SEMICLASSICAL APPROXIMATION

The result of the quantal coupled-channels calculation presented in the previous section can be easily understood if one uses the time-dependent perturbation theory based on the semiclassical approximation. In the semiclassical coupled-channels approach, one assumes a classical trajectory $r(t)$ for the relative motion between the colliding nuclei, and solves the time-dependent coupled-channels equations for the intrinsic motion [2,14]. Applying the first- and the second-order perturbation theory, the amplitudes for the one- and the two-neutron transfer processes for the sequential two-neutron transfer coupling, Eq. (2), read

$$a_{1n} = \frac{1}{i\hbar}\int_{-\infty}^{\infty} dt\, e^{-iQt/\hbar}F(r(t)), \quad (8)$$

$$a_{2n} = \left(\frac{1}{i\hbar}\right)^2\int_{-\infty}^{\infty} dt\, e^{-iQt/\hbar}F(r(t))\int_{-\infty}^{t} dt'\, e^{-iQt'/\hbar}F(r(t')) \quad (9)$$

$$= \frac{1}{2}\left[\frac{1}{i\hbar}\int_{-\infty}^{\infty} dt\, e^{-iQt/\hbar}F(r(t))\right]^2, \quad (10)$$

respectively. The last equality is from the property of the pure sequential transfer, that is, $Q_{2n} = 2Q_{1n}$ and F(1n-2n)=F(0n-1n). Notice that, in the no-correlation limit, the nonorthogonality term in the two-neutron transfer amplitude is exactly canceled with the simultaneous term, and only the successive term contributes in Eq. (9) [2]. By squaring these equations, one obtains $P_{2n}/(P_{1n})^2 = |a_{2n}|^2/|a_{1n}|^4 = 1/4$, which is indeed realized in Fig. 1 for large values of D, at which the perturbative treatment is justified.

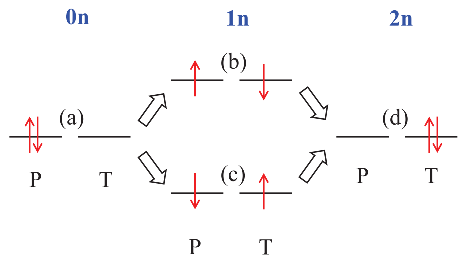

FIG. 2. (Color online) A schematic model for the two-neutron transfer process of spin-up and spin-down neutrons. P and T denote the projectile and the target nuclei, respectively.

IV. SCHEMATIC MODEL

The factor of 1/4 can also be obtained with a more microscopic model, which is illustrated in Fig. 2. Here we consider a transfer of spin-up and spin-down neutrons, which initially occupy a single-particle state in a projectile nucleus [see the state (a) in Fig. 2]. One of those neutrons is initially transferred to a target nucleus [the state (b) or (c), depending on the spin of the transferred neutron], which is followed by a transfer of the other neutron to the target nucleus [the state (d)]. We assume that the matrix elements for the transfer process do not depend on the spin of the transferred neutron, and that the spin flip does not occur during the transfer. We thus have $\langle \uparrow_P |V| \uparrow_T\rangle = \langle \downarrow_P |V| \downarrow_T\rangle \equiv \tilde{F}(r)$ and $\langle \uparrow_P |V| \downarrow_T\rangle = \langle \downarrow_P |V| \uparrow_T\rangle = 0$, where V is the operator which induces the transfer, and P and T denote the projectile and the target nuclei, respectively. We again use the time-dependent perturbation theory to evaluate the transfer probabilities. For the one-neutron transfer probability, there are two distinguishable final states, Figs. 2(b) and 2(c), and one has to add the probabilities for the processes (a)→(b) and (a)→(c). One thus obtains [see Eq. (8)]

$$P_{1\mathrm{n}} = 2 \times \left| \frac{1}{i\hbar} \int_{-\infty}^{\infty} dt\, e^{-iQt/\hbar} \tilde{F}(r(t)) \right|^2. \tag{11}$$

For the two-neutron transfer process, there are two indistinguishable paths, (a)→(b)→(d) and (a)→(c)→(d), to the final state, and one has to add the amplitudes first. This leads to [see Eq. (10)]

$$P_{2\mathrm{n}} = \left| \frac{1}{2} \left[\frac{1}{i\hbar} \int_{-\infty}^{\infty} dt\, e^{-iQt/\hbar} \tilde{F}(r(t)) \right]^2 \times 2 \right|^2. \tag{12}$$

Comparing Eq. (11) with Eq. (12), one again obtains $P_{2\mathrm{n}}/(P_{1\mathrm{n}})^2 = 1/4$. It is easy to confirm that this relation still holds even if one considers the antisymmetrization of each state, e.g., $|a\rangle = (|\uparrow_P\downarrow_P\rangle - |\downarrow_P\uparrow_P\rangle)/\sqrt{2}$ and $|b\rangle = (|\uparrow_P\downarrow_T\rangle - |\downarrow_T\uparrow_P\rangle)/\sqrt{2}$.

As in the multiphonon couplings in the coupled-channels approach [26,27], one can make a relation between the coupled-channels model of Eq. (2) and the schematic model of Fig. 2. That is, by introducing a single effective one-neutron transfer channel defined by $|1n\rangle = (|b\rangle + |c\rangle)/\sqrt{2}$, it is easy to find $\langle 1n|V|a\rangle = \langle d|V|1n\rangle = \sqrt{2}\tilde{F}$, where $|a\rangle, |b\rangle, |c\rangle$, and $|d\rangle$ are the states shown in Fig. 2. Therefore, identifying $F = \sqrt{2}\tilde{F}$, the two models are actually equivalent to each other. Notice that the other combination of the states $|b\rangle$ and $|c\rangle$, that is, $(|b\rangle - |c\rangle)/\sqrt{2}$, couples neither to $|a\rangle$ nor to $|d\rangle$ and is decoupled from the model space.

The factor of 1/4 for the relation between $P_{2\mathrm{n}}$ and $(P_{1\mathrm{n}})^2$ is consistent with the previous result of TDHF, Eq. (1), if one disregards the dependence on N_f. Notice that the N_f dependence in Eq. (1) was obtained by counting the number of possibilities to put nucleons in the final single-particle state [16]. To this end, the probability was assumed to be the same for all the final states with different values of j_z, that is, the z component of the single-particle angular momentum in the receiver nucleus. If one neglects the spin-flip components, however, the formula would become

$$\frac{P_{2\mathrm{n}}}{(P_{1\mathrm{n}})^2} \sim \frac{N_v - 1}{2N_v}, \tag{13}$$

with which one obtains $P_{2\mathrm{n}}/(P_{1\mathrm{n}})^2 = 1/4$ for $N_v = 2$.

V. SUMMARY

In summary, we have investigated the heavy-ion two-neutron transfer reactions in the no-correlation limit. To this end, we have used a schematic coupled-channels model, in which the transfer channels are treated as effective inelastic excitations. We have shown that the probability of the two-neutron transfer process $P_{2\mathrm{n}}$ is approximately given by a quarter of $(P_{1\mathrm{n}})^2$, that is, $P_{2\mathrm{n}}/(P_{1\mathrm{n}})^2 = 1/4$. This result is to some extent consistent with the result of time-dependent Hartree-Fock calculations for two valence neutrons.

The two-neutron transfer probabilities have customarily been compared with $(P_{1\mathrm{n}})^2$, rather than $(P_{1\mathrm{n}})^2/4$. Of course, many experimental data are for inclusive processes, and the enhancement factor for the two-neutron transfer process reflects not only the pairing correlation but also the phase space factor for the intermediate and the final states. Nevertheless, there is no strong reason why the two-neutron transfer probability should be compared with $(P_{1\mathrm{n}})^2$, and we advocate using $(P_{1\mathrm{n}})^2/4$, which has a clearer physical meaning as a reference probability, at least for a core+two-neutron system.

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