

## New low-energy $0^+$ state and shape coexistence in $^{70}\text{Ni}$

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In recent models, the neutron-rich Ni isotopes around  $N = 40$  are predicted to exhibit multiple low-energy excited  $0^+$  states attributed to neutron and proton excitations across both the  $N = 40$  and  $Z = 28$  shell gaps. In  $^{68}\text{Ni}$ , the three observed  $0^+$  states have been interpreted in terms of triple shape coexistence between spherical, oblate, and prolate deformed shapes. In the present work a new ( $0_2^+$ ) state at an energy of 1567 keV has been discovered in  $^{70}\text{Ni}$  by using  $\beta$ -delayed,  $\gamma$ -ray spectroscopy following the decay of  $^{70}\text{Co}$ . The precipitous drop in the energy of the prolate-deformed  $0^+$  level between  $^{68}\text{Ni}$  and  $^{70}\text{Ni}$  with the addition of two neutrons compares favorably with results of Monte Carlo shell-model calculations carried out in the large  $fpg_{9/2}d_{5/2}$  model space, which predict a  $0_2^+$  state at 1525 keV in  $^{70}\text{Ni}$ . The result extends the shape-coexistence picture in the region to  $^{70}\text{Ni}$  and confirms the importance of the role of the tensor component of the monopole interaction in describing the structure of neutron-rich nuclei.

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Atomic nuclei display regular patterns as either the proton  $Z$  or neutron  $N$  number changes. Examples include properties such as the energy necessary to remove a pair of nucleons (protons or neutrons) from a nucleus and the excitation energy of the first  $2^+$  state in nuclei with even numbers of neutrons and protons. Such regularities eventually led to the establishment of the nuclear shell model wherein nucleons fill separate sets of single-particle states. The latter states cluster together in energy with large gaps between groups at characteristic nucleon numbers [1]: the so-called magic numbers. This nuclear shell structure is analogous to that observed in atomic systems responsible for the regular behavior of the chemical elements; e.g., the chemical inertness of the noble gases [2].

The shape of the nucleus, described by Bohr and Mottelson [3], also exhibits regularity with respect to the location of the shell gaps; near the gaps, nuclei are predominately spherical and more deformed shapes occur as a progression is made toward the middle of a shell. Transitions between spherical and deformed nuclei can occur rapidly as a function of neutron or proton number and have sometimes been used to infer the collapse of predicted shell closures [4] and the development of new ones [5].

Changes in shape can also occur within a single nucleus as a function of excitation energy, based on a redistribution of nucleons across a shell gap that can drive the nucleus toward deformation. Naively, the energy cost to promote a nucleon across the energy gap should be prohibitive, but residual proton-neutron interactions can provide an energy stabilization to offset the cost. When the energy gain obtained from the residual proton-neutron interactions is comparable to the magnitude of the shell gap, the probability of exciting an  $n$ -particle  $n$ -hole configuration increases and this excitation mode can drive the nucleus towards a deformed shape. Near a shell gap, in nuclei with even numbers of protons and neutrons, such excitations can give rise to multiple, low-energy  $0^+$  states often taken as a hallmark of shape coexistence. Shape coexistence occurs when two or more states with different underlying configurations of protons and/or neutrons associated with differing intrinsic shapes coexist at similar excitation energies [6]. Prominent examples of this type of shape coexistence can be found in both the Hg [7–9] and Sn [10–13] regions. In particular,  $^{186}\text{Pb}$  exhibits triple shape coexistence between spherical, prolate, and oblate configurations, all located below 700 keV excitation energy [14].

The region around  $^{68}\text{Ni}$  has recently been studied extensively, both experimentally and theoretically, and an overall picture of shape coexistence is progressively emerging. Three  $0^+$  states in  $^{68}_{28}\text{Ni}_{40}$ , at 0, 1604 [15,16], and 2511 keV [17] associated with multiple particle-hole excitations across  $Z = 28$  and  $N = 40$  [18,19], have been interpreted in terms of spherical, oblate, and prolate shapes based on comparisons

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with Monte Carlo shell-model (MCSM) calculations [15,20]. The presence of spherical-prolate shape coexistence in the lighter  $^{66}\text{Ni}$  [21], as well as in  $^{68}\text{Ni}$  [21,22], is also expected based on mean-field calculations. However, the mean-field and shell-model calculations have qualitatively different expectations for the presence of shape coexistence beyond  $N = 40$  in  $^{70}\text{Ni}$ , with the former finding none and the latter suggesting that the prolate excitation occurs at lower excitation energy in  $^{70}\text{Ni}$  than in  $^{68}\text{Ni}$ . The key difference between the two sets of calculations can be traced to the role of the tensor component of the monopole interaction [23,24]. As neutrons are added from  $^{68}\text{Ni}$  to  $^{70}\text{Ni}$ , the occupancy of the  $g_{9/2}$  orbital increases and alters the energy gap at  $Z = 28$ , increasing the probability of proton particle-hole excitations across the gap. These proton excitations drive the nucleus toward a deformed shape, further enhancing the occupancy of the neutron  $g_{9/2}$  orbital. As a result, when progressing to more neutron-rich Ni isotopes, only the more recent shell-model calculations have predicted an increase in the depth of the prolate potential well and a decrease in the energy of the associated  $0^+$  state [20]. Identification of a low-energy excited ( $0^+$ ) state in  $^{70}\text{Ni}$  elevates the experimental evidence of shape coexistence in the Ni isotopes from its isolation to a single nucleus,  $^{68}\text{Ni}$ , to a more general characteristic of the region and validates the importance of the tensor interaction.

In the present manuscript, a new ( $0^+$ ) state in  $^{70}\text{Ni}$  at 1567 keV is identified and is in agreement with recent theoretical predictions, expanding the picture of shape coexistence in the region. Excited states in  $^{70}\text{Ni}$  were populated through the  $\beta$ -decay of  $^{70}\text{Co}$  at the National Superconducting Cyclotron Laboratory (NSCL). Ions of  $^{70}\text{Co}$  were produced via projectile fragmentation on a  $^9\text{Be}$  target of a  $^{76}\text{Ge}$  primary beam at 130 MeV/ $A$ . Fragments of interest were separated from other reaction products using the A1900 fragment separator [25] and transmitted to the experimental end station. This  $\beta$ -decay station consisted of a series of three silicon PIN detectors located approximately 1 m upstream of a central implantation detector. All incident ions were deposited 1 mm deep into a planar germanium double-sided strip detector (GeDSSD) [26] and identified event by event using standard  $\Delta E$ -TOF techniques. The GeDSSD is electrically segmented into 16 5-mm strips on one side and 16 5-mm orthogonal strips on the other for a total of 256 pixels. The position and time of arrival of each ion was recorded and subsequent  $\beta$ -decay electrons were correlated with previously implanted ions using both spatial and temporal information. The GeDSSD was surrounded by 16 detectors of the segmented germanium array (SeGA) [27] arranged into two concentric rings of eight detectors each to record the  $\beta$ -delayed  $\gamma$  rays. All detectors were read out by using the NSCL digital data-acquisition system [28]. Absolute  $\gamma$ -ray efficiencies were determined by using a NIST-calibrated  $^{154,155}\text{Eu}$  source and GEANT4 simulations [29] of the detection system.

The  $\beta$ -delayed,  $\gamma$ -ray spectra observed within two correlation windows of 0 to 500 and 500 to 1000 ms of the implantation of a  $^{70}\text{Co}$  ion are given in Fig. 1. Many of the transitions labeled as belonging to  $^{70}\text{Ni}$  have been seen previously in multinucleon-transfer reactions [30], in-beam  $\gamma$ -ray spectroscopy following secondary fragmentation [30],

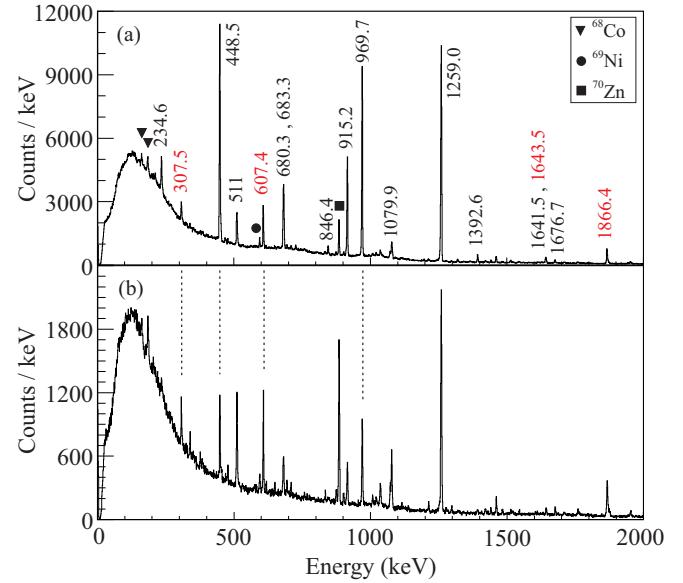


FIG. 1. (Color online) Spectrum of  $\beta$ -delayed  $\gamma$  rays observed within (a) 0 to 500 ms and (b) 500 to 1000 ms of the arrival of a  $^{70}\text{Co}$  ion at the experimental station. Gamma rays with black energy labels, such as the 448.5- and 969.7-keV  $\gamma$  rays, follow the  $\beta$  decay of the high-spin, short-lived  $^{70}\text{Co}$  isomeric state, while  $\gamma$  rays with red energy labels, such as those at 307.5, 607.4, and 1866.4 keV, are associated with the decay of the low-spin, long-lived  $^{70}\text{Co}$  isomer. For all transitions shown in red, their intensities relative to the 1259.0-keV  $\gamma$  ray increase from panel (a) to panel (b). Additional  $\gamma$  rays associated with  $^{69}\text{Co}$ ,  $^{69}\text{Ni}$ , and  $^{70}\text{Zn}$  are also observed and are indicated by black triangles, circles, and squares, respectively.

$\beta$  decay [31,32], and isomeric decay studies [33]. Additional transitions were placed in  $^{70}\text{Ni}$  based on correlated  $\beta$ - $\gamma$ - $\gamma$  coincidence relationships. Some additional  $\gamma$  rays associated with  $^{69}\text{Co}$ ,  $^{69}\text{Ni}$ , and  $^{70}\text{Zn}$  are also observed in Fig. 1, indicated by black triangles, circles, and squares, respectively, due to the long correlation time taken for the present analysis. However,  $\gamma$  rays from these other implanted nuclei do not impact the present results.

There are two known isomeric states in  $^{70}\text{Co}$ ; a low-spin, long-lived state [31] and a high-spin, short-lived state [31,32,34,35]. The contributions of both isomeric decays are observed in Figs. 1(a) and 1(b) and assignment of  $\gamma$  rays to either the long- or short-lived  $^{70}\text{Co}$  isomeric state is based on  $\gamma$ -gated  $\beta$ -decay curves. The 607.4- and 1866.4-keV  $\gamma$  rays are known from previous studies to be exclusive to the  $\beta$  decay of the low-spin, long-half-life isomeric state [31]. Many transitions are associated with the deexcitation of the high-spin, short-half-life isomeric state and the rapid decrease in peak area at longer correlation times is observed in Fig. 1(b). The 448.5- and 969.7-keV  $\gamma$  rays are marked in Fig. 1 as examples. The 1259.0-keV  $\gamma$  ray is common to both decays.

The 307.5-keV  $\gamma$  ray, associated with the decay of the long-lived  $^{70}\text{Co}$  isomer, has not been observed in previous  $^{70}\text{Ni}$  studies and is placed in  $^{70}\text{Ni}$  based on the  $\gamma$ - $\gamma$  coincidence relationships. Figure 2 presents a series of  $\gamma$ - $\gamma$  coincidence

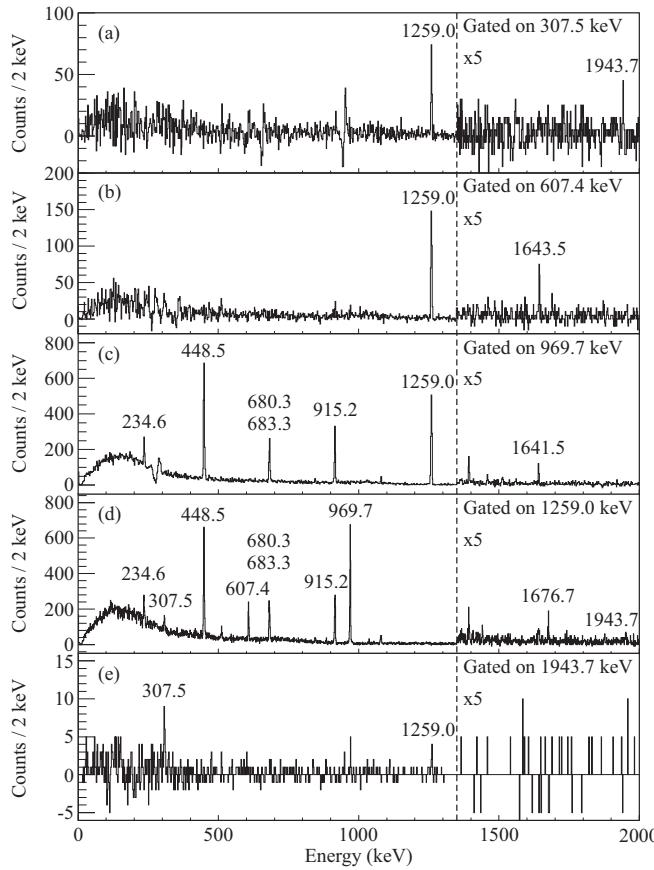


FIG. 2. Background-subtracted  $\gamma$ - $\gamma$  coincidence spectra following the  $\beta$  decay of  $^{70}\text{Co}$  within 2000 ms of the arrival of the ion. Spectra are gated on the (a) 307.5-, (b) 607.4-, (c) 969.7-, (d) 1259.0-, and (e) 1943.7-keV  $\gamma$  rays.

spectra gated on the  $\gamma$  rays at (a) 307.5, (b) 607.4, (c) 969.7, (d) 1259.0, and (e) 1943.7 keV.

The  $\gamma$  rays in coincidence with the  $2_1^+ \rightarrow 0_1^+$ , 1259.0-keV  $\gamma$  ray are seen in Fig. 2(d). Strong coincidence relationships are observed at 969.7, 448.5, 607.4, 915.2, and 234.6 keV, which correspond to the  $4_1^+ \rightarrow 2_1^+$ ,  $6_1^+ \rightarrow 4_1^+$ ,  $2_2^+ \rightarrow 2_1^+$ ,  $(6^-_1) \rightarrow 6_1^+$ , and  $(5^-_1) \rightarrow 6_1^+$  transitions, respectively, where the spins and parities are adopted from the most recent  $^{70}\text{Ni}$  level scheme by Chiara *et al.* [30]. A number of other  $\gamma$  rays are observed in Fig. 1 which depopulate higher-energy levels and will be detailed in a subsequent publication. However, the coincident  $\gamma$  rays at 307.5 and 1943.7 keV will be discussed separately below. The 640-keV  $\gamma$  ray observed previously [30] was not detected in coincidence with either the 1259.0- or the 607.4-keV,  $2_2^+ \rightarrow 2_1^+$  transition [Fig. 2(b)] in the present work, suggesting that the known  $(4_2^+)$  state was not populated in the  $^{70}\text{Co}$   $\beta$  decay and could indicate that the long-lived  $\beta$ -decaying  $^{70}\text{Co}$  isomer has a spin lower than three. Likewise, the known  $8_1^+$  isomer at 2861 keV [33] was not observed following  $^{70}\text{Co}$   $\beta$  decay, in agreement with results of earlier investigations [31,32]. Lastly, the 1259.0-keV  $\gamma$  ray was not observed to be self-coincident, as suggested in Ref. [31].

Gating on the 969.7-keV,  $4_1^+ \rightarrow 2_1^+$  transition of the ground-state band leads to the coincidence spectrum of Fig. 2(c).

Based on the number of counts in the 969.7-keV peak in the singles spectrum, the known  $^{70}\text{Ni}$  level scheme, and the detector efficiency at 1259.0 keV, a total of  $1158 \pm 12$  counts are expected at 1259.0 keV. The measured number of counts is  $1130 \pm 50$ . All other previously known coincident  $\gamma$  rays measured in the present analysis were checked in a similar manner and were found to be consistent with the known level scheme. This exercise confirms both our efficiency determination and the validity of the main portions of the  $^{70}\text{Ni}$ , low-energy level scheme.

The strong 307.5-keV  $\gamma$  ray present in Fig. 1(a), and in the coincidence spectrum gated by the 1259.0-keV  $\gamma$  ray [Fig. 2(d)], is new to the current work. In Fig. 2(a), the gate on this 307.5-keV  $\gamma$  ray indicates a coincidence with a strong 1259.0-keV line and a weaker 1943.7-keV  $\gamma$  ray. The 1943.7-keV coincidence spectrum [Fig. 2(e)] also contains the 307.5- and 1259.0-keV  $\gamma$  rays. The former is not observed in coincidence with any of the other  $\gamma$  rays assigned to the decay of  $^{70}\text{Co}$  in Fig. 1(a), suggesting that this 307.5-keV  $\gamma$  ray directly populates the 1259.0-keV,  $2_1^+$  state from a level at 1567 keV. There is no direct  $\gamma$ -ray emission from the latter level to the ground state, based on the absence of a detectable  $\gamma$  ray in the appropriate region of the  $\beta$ -delayed,  $\gamma$ -ray spectrum in Fig. 1, and on the lack of a 1567-keV  $\gamma$  ray in Fig. 2(e). The low-energy level scheme of  $^{70}\text{Ni}$ , highlighting the decay of the long-lived  $^{70}\text{Co}$  isomeric state, is shown in Fig. 3 (left). Gamma-ray intensities are relative to the 1259-keV transition.

The relative intensities of the 307.5- and 1943.7-keV transitions strongly suggest the placement of the 1943.7-keV transition feeding the 1567-keV level. If the order of the 307.5- and 1943.7-keV transitions were reversed, a resulting 3203-keV state would have seven times more intensity feeding it than depopulating it. Furthermore, no  $\gamma$  rays are observed that could correspond to the deexcitation of such a 3203-keV state. The 3203-keV state could not be assigned a  $0^+$  spin and parity due to the absence of 511-keV  $\gamma$  rays coincident with the 307.5-keV transition. Therefore, the 1943.7-keV transition is placed as feeding the firmly established 1567-keV state in the revised  $^{70}\text{Ni}$  level scheme of Fig. 3. The 1567-keV state is tentatively assigned a  $0^+$  spin and parity based on its association with the decay of the long-lived, low-spin isomeric state, its nonobservation in multinucleon transfer reactions populating yrast states [30], and the lack of a  $\gamma$  ray at 1567 keV in the  $\beta$ -delayed,  $\gamma$ -ray spectrum. In addition, an upper limit of 100 ns is placed on the lifetime of the 1567-keV state based on analysis of the time-difference spectra between  $\beta$ -decay electrons detected in the planar GeDSSD and the 307.5-keV  $\gamma$  rays detected in SeGA.

Low-energy  $0^+$  states are now known in both  $^{68,70}\text{Ni}$  and the evolution of these states beyond  $N = 40$  can be investigated. The two low-energy  $0_2^+$  and  $0_3^+$  states in  $^{68}\text{Ni}$  are attributed to configurations associated predominately with two-particle, two-hole excitations across the  $N = 40$  and  $Z = 28$  gaps. The low-energy level structure of  $^{68}\text{Ni}$  has been predicted numerous times (see Ref. [15] and references therein). However, only theoretical calculations utilizing the full  $fpg_{9/2}d_{5/2}$  model space for both protons and neutrons, thereby allowing excitations across their respective energy gaps, can account for

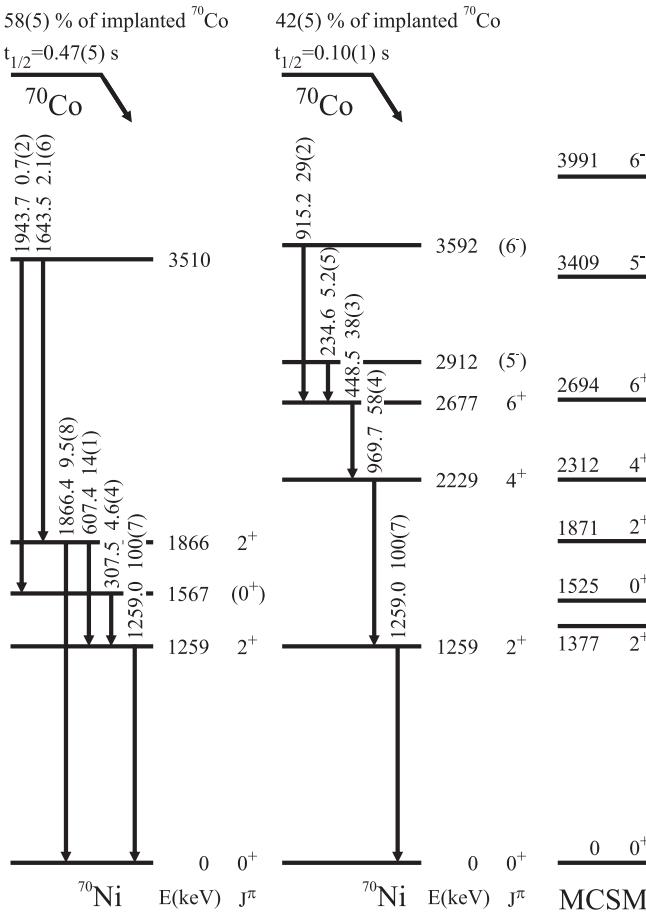


FIG. 3. Simplified low-energy level scheme of  $^{70}\text{Ni}$  populated in the decay of  $^{70}\text{Co}$ , highlighting the decay of the long-lived  $^{70}\text{Co}$  isomeric state (left). Experimentally observed transitions are labeled with their energy and intensity relative to the 1259-keV transition. Experimentally observed levels are labeled with excitation energies along with spins and parities and are compared to the predictions of the MCSM shown on the extreme right [30].

the presence of three  $0^+$  states in  $^{68}\text{Ni}$  [18,20] at low excitation energy.

Other shell-model calculations with more restrictive model spaces that do not explicitly allow for proton excitations out of the  $\pi f_{7/2}$  single-particle state fail to reproduce the energy of the  $0_3^+$  state [36–38]. A fourth  $0^+$  level at 2202 keV was briefly proposed experimentally [39], but subsequent investigations employing similar techniques have so far failed to support the claim [17] and such a fourth  $0^+$  state in  $^{68}\text{Ni}$  is not suggested by theoretical calculations, either.

The MCSM [15] calculations with the A3DA [40] interaction further predict that the three  $0^+$  states in  $^{68}\text{Ni}$  are characterized by different intrinsic deformations with the lowest-energy state being spherical, the  $0_2^+$  level having a slight oblate deformation, and the  $0_3^+$  state being associated with a large prolate deformation. The energy [15,16] and  $E0$  decay [15] of the  $0_2^+$  state at 1604 keV in  $^{68}\text{Ni}$  are consistent with the theoretical predictions of a slight oblate deformation.

The  $0_3^+$  state in  $^{68}\text{Ni}$  was confirmed by angular-correlation measurements at 2511 keV [17]. However, the  $E0$  branching ratios from the prolate  $0_3^+$  to either the  $0_2^+$  or  $0_1^+$  states have not been directly observed, but the upper limit for the sum intensity has been placed at 4% [41].

The MCSM predictions for  $^{70}\text{Ni}$  are included in Fig. 3 (right). Transitioning from  $^{68}\text{Ni}$  to  $^{70}\text{Ni}$ , the potential well which confines the prolate-deformed two-particle two-hole proton excitation is predicted to increase in depth [20], resulting in a concomitant drop in the energy of the predicted prolate  $0^+$  state from 2511 keV in  $^{68}\text{Ni}$  to 1525 keV in  $^{70}\text{Ni}$ . The energies of the  $2_2^+$  and  $4_2^+$  states, also associated with the prolate potential well, have been observed to drop for  $^{70}\text{Ni}$  compared to  $^{68}\text{Ni}$  [30]. This is explained by the increased occupancy of the  $\nu g_{9/2}$  orbital in  $^{70}\text{Ni}$  compared to  $^{68}\text{Ni}$ . The attractive  $\nu g_{9/2} - \pi f_{5/2}$  and repulsive  $\nu g_{9/2} - \pi f_{7/2}$  monopole interactions of the tensor force alter the effective single-particle energies of the  $\pi f_{7/2}$  and  $\pi f_{5/2}$  single-particle states, thereby increasing the likelihood of excitations into the  $\pi f_{5/2}$  state, the dominant proton excitation in the prolate-deformed  $0^+$  states in  $^{68,70}\text{Ni}$  [20,30].

The energy of the tentative  $0_3^+$  state in  $^{70}\text{Ni}$  of 1567 keV agrees with the theoretically predicted value, as observed in Fig. 3, further supporting its  $0^+$  assignment. The  $2_2^+$  level at 1866 keV and the  $4_2^+$  one at 2508 keV (not observed in the present work, but proposed in Ref. [30]) have already been suggested as members of a band built on the  $0_2^+$  state [30], but the  $0_2^+$  state itself had not yet been identified. Unfortunately, it was not possible to observe the  $2_2^+ \rightarrow 0_2^+$  branch due to strong competition from the higher-energy  $2_2^+ \rightarrow 0_1^+$  transition. Based on the predicted ratio of  $B(E2, 2_2^+ \rightarrow 0_2^+)/B(E2, 2_2^+ \rightarrow 0_1^+)$  of 400 [30], the expected branching ratio of the  $2_2^+ \rightarrow 0_2^+$  transition would be unobservable in the present experiment in the singles spectrum or in coincidence with the 1643.5-keV  $\gamma$  ray.

In conclusion, a new ( $0_2^+$ ) state in  $^{70}\text{Ni}$ , located at 1567 keV, has been discovered through  $\beta$ -decay spectroscopy at NSCL. The present experimental results are in good agreement with theoretical predictions by the MCSM with the A3DA interaction, which successfully reproduces the low-energy level scheme of  $^{68}\text{Ni}$ . The predicted deepening of the prolate potential well from  $^{68}\text{Ni}$  to  $^{70}\text{Ni}$  is borne out experimentally based on the drop in energy of the excited ( $0^+$ ) states. The observations support a picture of shape coexistence in the neutron-rich Ni isotopes, based on proton excitations across the  $Z = 28$  shell gap.

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