Preserving local gauge invariance with t-channel Regge exchange

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Considering single-meson photo- and electroproduction off a baryon, it is shown how to restore *local* gauge invariance that was broken by replacing standard Feynman-type meson exchange in the t channel by exchange of a Regge trajectory. This is achieved by constructing a contact current whose four-divergence cancels the gauge-invariance-violating contributions resulting from all states above the base state on the Regge trajectory. To illustrate the procedure, modifications necessary for the process $\gamma + p \to K^+ + \Sigma^{*0}$ are discussed in some detail. We also provide the general expression for the contact current for an arbitrary reaction.

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I. INTRODUCTION

Photo- and electroproduction of mesons off baryons provide arguably the most direct routes to information about hadronic structure. At high energies, where multimeson production abounds, such processes can be described economically in terms of pomeron and Regge-trajectory exchanges [1–3]. At lower energies, single-meson production provides a direct avenue for baryon spectroscopy [4], with theoretical descriptions that attempt to model the contributing mechanisms as detailed as possible in terms of Feynman-type exchange processes.

The present work is concerned with an intermediate-energy transition region, where one starts within the Feynman-type picture and replaces some exchanges by Regge trajectories in an attempt to bring the economic features of the high-energy Regge approach to bear in the more traditional Feynman framework. Specifically, we will apply such a hybrid framework to the generic electromagnetic production process depicted in Fig. 1 of a meson (m) off an initial baryon (b) going over into a final baryon (b'), i.e.,

$$\gamma(k) + b(p) \to m(q) + b'(p'), \tag{1}$$

where arguments denote the corresponding four-momenta.

For this single-meson production process, it is argued that replacing the t-channel single-meson exchange (third diagram in Fig. 1) by the exchange of an entire Regge trajectory would lead to a better, simpler description of the dynamics of the process in question, in particular, if it is dominated by small-momentum transfers [5–19]. However, the good success of such hybrid approaches notwithstanding, it is well known that this replacement destroys gauge invariance even if the underlying Feynman formulation was gauge invariant to start with.

One widely used recipe for restoring gauge invariance is the method of Ref. [5] which basically uses the residual function of the base state of the t-channel Regge trajectory as a common suppression function for all terms of the tree-level current. Current conservation—i.e., $k_\mu M^\mu = 0$, where M^μ denotes the current—is achieved in this method because one starts from a conserved tree-level current, and multiplication by a common suppression function does not destroy this property. Even though the method is quite successful in providing good descriptions of data in many applications (see, for example, Refs. [5–7,11–19]), there is no dynamical foundation for it.

We point out in this context that the current-conservation condition, $k_\mu M^\mu = 0$, only implies *global* gauge invariance which is little more than charge conservation. *Local* gauge invariance, i.e., the requirement that the physical observables are invariant under local U(1) transformations of the fields, on the other hand, implies the very existence of the electromagnetic field [20]. Because global gauge invariance follows from local gauge invariance, but not the other way around, imposing current conservation by itself to find ways of repairing a current that was damaged by approximations, therefore, does not imply that the damage done to the underlying electromagnetic field is repaired as well.

We will show here how *local* gauge invariance can be restored when the t-channel single-meson exchange is replaced by the exchange of an entire mesonic Regge trajectory. The method as such is not restricted to the t channel and could also be applied to a u-channel description in terms of baryon Regge trajectories in a straightforward manner.[1] The proposed mechanism is based on the necessary and sufficient conditions for local gauge invariance formulated as generalized Ward-Takahashi identities for the production current [21,22]. These are *off-shell* conditions that automatically reduce to the familiar current-conservation relation, $k_\mu M^\mu = 0$, when taken

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[1] Technically, it could also be used for s-channel Reggeization, but because the s-channel contribution for a given experiment is a constant, without any angular dependence, it seems doubtful that there would be much point in doing so, even if one ignores duality issues between s- and t-channel processes [1–3].

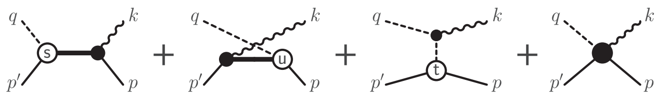

FIG. 1. Generic diagrams with external four-momenta of the photoproduction process of Eq. (1) satisfying $q + p' = k + p$. Labels s, u, and t at the hadronic $b \to m + b'$ vertices refer to Mandelstam variables of the respective exchanged intermediate particles. Summations over all intermediate states compatible with initial and final states are implied. The right-most diagram depicts the contact-type interaction current. Time runs from right to left.

on shell. The implementation of these conditions results in contact-type interaction currents [23–25] as minimal additions to a given current to restore local gauge invariance. The method is well established within the usual Feynman picture and it was applied successfully to a variety of photoprocesses [26–36]. The extension given here to include exchanges of Regge trajectories is straightforward.

The paper is organized as follows. In the subsequent Sec. II, we will recapitulate basic details of meson photoproduction within the general field-theory approach of Haberzettl [22] and discuss, in particular, how the set of generalized Ward-Takahashi identities ensures the local gauge invariance of the production current. The Regge treatment of t-channel meson exchanges considered in Sec. III is then immediately seen to violate these conditions thus leading to a current that is not conserved. The reason for this violation can be traced to the fact that higher-lying mass states above the base state of the Regge trajectory have the wrong coupling to the electromagnetic field. Using the residual function from the pole at the base state of the Regge trajectory, we show then how to construct a contact current that restores validity of the full set of generalized Ward-Takahashi identities and therefore ensures local gauge invariance. As an illustration of the relevant details, we treat in Sec. IV the example of the strangeness-production reaction $\gamma + p \to K^+ + \Sigma^{*0}$. In Sec. V, we will provide a summarizing discussion of the present approach. Finally, in the Appendix, we write out the generic expressions applicable to any single-meson production process that allow one to construct the minimal contact currents necessary to maintain local gauge invariance.

II. PHOTOPRODUCTION BASICS

The following description is based on the field-theoretical approach of Haberzettl [22] originally developed for pion photoproduction off the nucleon. This formalism, however, is quite generic and can be readily applied to meson-production processes off any baryon.

The basic topological structure of the single-pion production current M^μ was given a long time ago [37] as arising from how the photon can couple to the underlying hadronic πNN vertex. The resulting structure depicted in Fig. 1 is generic and applies to all photo- and electroproduction processes of a single meson off a baryon. The full current M^μ, therefore, can be written generically as

$$M^\mu = M_s^\mu + M_u^\mu + M_t^\mu + M_{\text{int}}^\mu, \tag{2}$$

as indicated in Fig. 1, where the indices s, u, and t here refer to the Mandelstam variables of the respective exchanged intermediate off-shell particle. This structure is based on topology alone and therefore independent of the details of the individual current contributions. The first three (polar) contributions are relatively simple; the real complication of the problem lies in how complex the reaction mechanisms are that are taken into account in the interaction current M_{int}^μ because in principle M_{int}^μ subsumes all mechanisms that do not have s-, u-, or t-channel poles, and this comprises *all* final-state interactions and therefore necessarily all effects that arise from the coupling of various reaction channels [22,25,36].

Here, we will ignore all of these reaction-dynamical complications and treat the interaction current M_{int}^μ simply as a "black box" that must satisfy certain four-divergence constraints [22]. If needed, one may add the manifestly transverse contributions of the more complete treatment [24,25] to the minimal explicit structure discussed here.

We emphasize that the particles explicitly entering all expressions here must be physical particles. In other words, the Regge-specific implementation of the formalism does not apply to bare particles. The corresponding propagators here must describe physical particles, with poles at the respective physical masses, but their structure is not limited otherwise, i.e., they may contain explicit dressing functions or they can be simple Feynman-type propagators, however, with physical masses, with the dressing mechanisms that gave them their physical masses hidden in form factors. In other words, the diagrams of Fig. 1 must be taken as representing the solution of the meson-production problem and not as the Born-type bare input for a Bethe-Salpeter- or Dyson-Schwinger-type reaction equation.

Also, for the purpose of gauge invariance, the only relevant intermediate states in the s-, u-, and t-channel diagrams of Fig. 1 are those where the photon does not initiate a transition (because transition currents are transverse), i.e., where the states before and after the photon interacts are the same particle with nonzero charge. Thus, for the present purpose, without lack of generality, we may ignore all diagrams and intermediate states that do not contribute to the four-divergence of the production current M^μ.

As a consequence, with this understanding, all three hadronic vertices in Fig. 1 describe the same three-point vertex $b \to m + b'$, for which we will use the notation $F(p_{b'}, p_b)$, where the arguments here are the incoming and outgoing baryon momenta, as depicted in Fig. 2. The vertex notation F subsumes all coupling operators and isospin dependence,

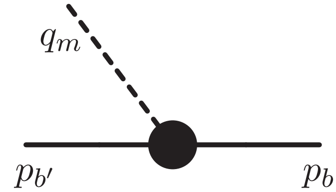

FIG. 2. Generic vertex $F(p_{b'}, p_b)$ for $b \to m + b'$ with associated momenta. The meson momentum $q_m = p_b - p_{b'}$ is given by four-momentum conservation across the vertex.

etc., and depending on the specific reaction, it may also carry Lorentz indices [see Eq. (11) below, and also the example in Sec. IV.] The three kinematic situations in which this vertex appears in Fig. 1 are then uniquely identified by the Mandelstam variables of the exchanged intermediate hadron,

$$s = (p+k)^2 = (p'+q)^2, \tag{3a}$$

$$u = (p'-k)^2 = (p-q)^2, \tag{3b}$$

$$t = (q-k)^2 = (p-p')^2, \tag{3c}$$

and we will use

$$F_t = F(p',p), \quad F_u = F(p'-k,p), \quad F_s = F(p',p+k) \tag{4}$$

to abbreviate the corresponding vertices, and generically write F_x for $x = s,u,t$.

A. Generalized Ward-Takahashi identities

First, to set the stage for the Regge treatment, we will recapitulate how the local gauge-invariance requirements differ from mere current conservation, i.e., global gauge invariance.

To preserve *local* gauge invariance for the photoprocess (1) the following set of *off-shell* four-divergence relations need to be satisfied [22,24,25]. At the very base are the Ward-Takahashi identities (WTI) [38,39] for the individual electromagnetic currents J^μ of mesons (index m) and baryons (indices b' or b),

$$k_\mu J_m^\mu(q,q-k) = \Delta_m^{-1}(q)Q_m - Q_m\Delta_m^{-1}(q-k), \tag{5a}$$

$$k_\mu J_{b'}^\mu(p',p'-k) = S_{b'}^{-1}(p')Q_{b'} - Q_{b'}S_{b'}^{-1}(p'-k), \tag{5b}$$

$$k_\mu J_b^\mu(p+k,p) = S_b^{-1}(p+k)Q_b - Q_bS_b^{-1}(p), \tag{5c}$$

where $\Delta_m(q)$, $S_{b'}(p')$, and $S_b(p)$ are the respective propagators for the meson and baryons, with arguments providing their four-momenta, and Q_m, $Q_{b'}$, and Q_b denoting their associated charge operators. The photoproducton current M^μ of Eq. (1) must satisfy the generalized WTI (gWTI) [21,22],

$$\begin{aligned} k_\mu M^\mu = {} & \Delta_m^{-1}(q)Q_m\Delta_m(q-k)\,F_t \\ & + S_{b'}^{-1}(p')Q_{b'}S_{b'}(p'-k)F_u \\ & - F_s\,S_b(p+k)Q_bS_b^{-1}(p), \end{aligned} \tag{6}$$

and, finally, the interaction current M_{int}^μ needs to satisfy the condition,

$$k_\mu M_{\text{int}}^\mu = Q_m\,F_t + Q_{b'}\,F_u - F_s\,Q_b. \tag{7}$$

In view of the isospin dependence of the vertices, charge operators and vertices do not commute. Note that the right-hand side vanishes here identically if all vertices are replaced by simple coupling constants for we have then $F_x \to g\boldsymbol{\tau}$, where g is the coupling constant and $\boldsymbol{\tau}$ generically denotes the isospin operator of the vertex, and hence $Q_m\boldsymbol{\tau} + Q_{b'}\boldsymbol{\tau} - \boldsymbol{\tau}Q_b \equiv 0$ provides charge conservation across the photoprocess [22]. In a manner of speaking, therefore, Eq. (7) amounts to the formulation of the effective charge difference across the reaction in the presence of hadronic vertices with structure.

It is of paramount importance here that all three four-divergence equations are off-shell relations, and that the off-shellness is a necessary requirement for local gauge invariance [22] because it ensures that the (off-shell) current M^μ provides the correct, consistent contributions to gauge invariance even if it is embedded as an off-shell subprocess in a larger process (for example, electromagnetic production of two or more mesons [28,40]).

With the off-shell WTIs (5) and (6) given, global gauge invariance follows trivially by taking the respective on-shell matrix elements, with the inverse propagators in the four-divergences (5) and (6) then ensuring that the four-divergences vanish; in particular,

$$k_\mu M^\mu = 0 \quad \text{(on shell)}. \tag{8}$$

To be sure, this is a necessary condition the physical production current needs to satisfy that follows trivially from local gauge invariance, however, this on-shell restriction by itself contains no information that allows one to meaningfully "guess" at a nontrivial structure for M^μ. Thus, it is ill-suited to be used as a starting point for restoring gauge invariance destroyed by approximations.

The proper starting point should be the set of off-shell equations (5), (6), and (7). One easily sees here that only two—any two—of these conditions are necessary to ensure the validity of the respective third equation. For the practical purpose of restoring gauge invariance, it is easiest to work with Eqs. (5) and (7). In any microscopic formulation of photoprocesses, the single-hadron WTIs (5) are a given from the start. Therefore, to obtain the gWTI (6) for the full production current M^μ and thus ensure the preservation of local gauge invariance, one needs to construct an interaction current M_{int}^μ that satisfies Eq. (7). Note, in particular, that the structure of this equation does not change even if the external hadrons are on shell, and thus—quite in contrast to the current-conservation condition (8)—*even its on-shell limit provides a nontrivial constraint* that ensures that the on-shell result (8) is a consequence of *local* gauge invariance and not just mere global gauge invariance.

1. *t-Channel contribution*

To see what needs to be done to restore gauge invariance in the Regge case, let us first look at how the usual t-channel term as depicted by the third diagram in Fig. 1 contributes to upholding local gauge invariance.

Using the momenta of the diagram and stripped of all unnecessary factors, it reads

$$M_t^\mu = J_m^\mu(q,q-k)\Delta_m(q-k)F_t, \tag{9}$$

and its four-divergence is given by

$$k_\mu M_t^\mu = \Delta_m^{-1}(q)Q_m\Delta_m(q-k)F_t - Q_mF_t. \tag{10}$$

The first term on the right-hand side is precisely the first term appearing on the right-hand side of the gWTI (6); the second term involving only the vertex, but no propagator, is canceled by the first term on the right-hand side of the interaction-current condition (7). Similar cancellations happen for the respective contributions from all three polar current contributions and this cancellation mechanism ensures the validity of the full gWTI—and thus of local gauge invariance—once Eqs. (5) and (7) are satisfied.

It is this cancellation mechanism that will be exploited in the subsequent Regge treatment.

III. GAUGE-INVARIANT REGGE TREATMENT

First, let us write the hadronic $b \to m + b'$ vertex (see Fig. 2) as[2]

$$F(p_{b'},p_b) = \boldsymbol{G}(q_m)\boldsymbol{\tau}\, f\big(q_m^2,p_{b'}^2,p_b^2\big), \tag{11}$$

where the outgoing meson four-momentum q_m is given by $q_m = p_b - p_{b'}$ in terms of the incoming and outgoing baryon momenta. The operator $\boldsymbol{G}$ describing the coupling structure of the vertex subsumes as necessary all strength parameters, masses, signs, gamma matrices, etc.; in the simplest case it is just a constant, but in more complicated cases it contains derivatives of the outgoing meson field which lead to the q_m dependence. The extended structure of the vertex is given by the scalar form factor f normalized as

$$f\big(M_m^2,M_{b'}^2,M_b^2\big) = 1, \tag{12}$$

where the squared momenta of (11) sit on there respective mass shells. The operator $\boldsymbol{\tau}$ summarily describes the isospin dependence of the vertex, with relevant indices suppressed. Combined with the respective charge operators Q for the three legs of the vertex, one obtains [22]

$$Q_m\boldsymbol{\tau} = e_m, \quad Q_{b'}\boldsymbol{\tau} = e_{b'}, \quad \boldsymbol{\tau} Q_b = e_b, \tag{13}$$

where

$$e_m + e_{b'} - e_b = 0 \tag{14}$$

provides charge conservation across the reaction. Taken in an appropriate isospin basis, the charge-isospin operators e_m, $e_{b'}$, and e_b are equal to the respective charges of the individual legs.

We will need only on-shell kinematics here where all external hadron legs of Fig. 1 sit on their respective mass shells. The form factors associated with the vertices F_x, $x = s,u,t$, for these cases are

$$f_s(s) = f\big(M_m^2,M_{b'}^2,s\big), \tag{15a}$$

$$f_u(u) = f\big(M_m^2,u,M_b^2\big), \tag{15b}$$

$$f_t(t) = f\big(t,M_{b'}^2,M_b^2\big), \tag{15c}$$

where the Mandelstam variables (3) are used. The t-channel vertex, in particular, then reads

$$F_t = F(p',p) = \boldsymbol{G}(q-k)\,\boldsymbol{\tau}\, f_t(t), \tag{16}$$

and for the corresponding meson-exchange propagator, we may write without lack of generality,

$$\Delta_m(q-k) = \frac{N_m(q-k)}{t-M_m^2}, \tag{17}$$

where the pole at $t = M_m^2$ was pulled out explicitly and the residual numerator $N_m(q-k)$ defined by this relation may describe dressing effects and/or the coupling structure of the propagator. In the simplest cases, N_m equals unity for pseudoscalar mesons and $-g^{\beta\alpha}$ for vector mesons (in Feynman gauge), for example.

Standard Reggeization of the t-channel meson exchange corresponds to the replacement [5–12],

$$\frac{1}{t-M_m^2}\, f_t(t) \to \mathcal{P}_m(t), \tag{18}$$

where $\mathcal{P}_m(t)$ is the Regge-trajectory propagator appropriate for this particular meson exchange. By construction (see details Sec. III A), it contains poles at higher-lying meson masses along this particular trajectory, in addition to the primary pole at the base of the trajectory at $t = M_m^2$ of (17). Moreover, the residue at this primary pole,

$$\lim_{t\to M_m^2}\big(t-M_m^2\big)\mathcal{P}_m(t) = 1, \tag{19}$$

is exactly the same as that of the left-hand side of (18). The residual function,[3]

$$\mathcal{F}_t(t) = \big(t-M_m^2\big)\,\mathcal{P}_m(t), \tag{20}$$

thus, is finite and normalized to unity at $t = M_m^2$, just like the usual t-channel form factor $f_t(t)$. Details of $\mathcal{F}_t$ will be given in the subsequent Sec. III A.

The Reggeized t-channel current now reads

$$M_t^\mu \to \mathcal{M}_t^\mu = J_m^\mu(q,q-k)\,\Delta_m(q-k)\,F_{\mathrm{R},t}(p',p), \tag{21}$$

where

$$F_{\mathrm{R},t}(p',p) = \boldsymbol{G}(q-k)\,\boldsymbol{\tau}\,\mathcal{F}_t(t) \tag{22}$$

describes the Reggeized vertex, with the corresponding four-divergence given by

$$k_\mu\mathcal{M}_t^\mu = \Delta_m^{-1}(q)Q_m\;\Delta_m(q-k)\,F_{\mathrm{R},t} - Q_m\,F_{\mathrm{R},t}. \tag{23}$$

The first term on the right-hand side with the inverse meson propagator depending on the external (outgoing) meson momentum vanishes on-shell and thus provides an acceptable contribution for gWTI in analogy to Eq. (6). The second term, however, has no counterpart in the four-divergence (7) and thus violates local gauge invariance (and therefore obviously also global gauge invariance).

This violation comes about because in the Regge treatment *all* particles on the trajectory are taken to couple to the electromagnetic field with the *same* current J_m^μ as the primary base state, whereas if one were to incorporate these contributions via Feynman-type exchange mechanisms, each of the higher-lying states would couple transversely to the photon because the corresponding currents are transition currents for the transition from intermediate higher-mass states to the lower-mass primary base state, which is the final meson state of the reaction, and such transverse transition currents would not contribute to the four-divergence.

[2]For a more general description of the vertex, see discussion in the Appendix.

[3]It is this residual function that was used in Ref. [5] as an overall multiplicative function for their gauge-invariance-restoring recipe.

Clearly, to restore local gauge invariance, Regge treatment of the t channel by itself is not enough—one *must* also Reggeize the interaction current $M^{\mu}_{\rm int}$ so that its four-divergence will provide the necessary cancellation of the offending contribution in (23), thus in essence restoring the transversality of these contributions with higher mass. In other words, to preserve local gauge invariance, one must apply the Reggeization process consistently across all elements of the production current M^{μ}. Because t-channel-type exchanges also contribute (as off-shell processes) to the internal mechanisms of $M^{\mu}_{\rm int}$, an appropriate Reggeization of such internal exchanges will then provide the cancellation for the offending term in Eq. (23).

Obviously then, treating Regge consistently with local gauge invariance simply entails consistently replacing the usual t-channel vertex F_t by the Reggeized vertex $F_{{\rm R},t}$ everywhere. In addition to the Reggeized t-channel current (21), this also requires modification of the contact current,

$$F_t \to F_{{\rm R},t}: \quad M^{\mu}_{\rm int} \to \mathcal{M}^{\mu}_{\rm int}, \tag{24}$$

such that the Reggeized contribution from the corresponding four-divergence,

$$k_\mu \mathcal{M}^{\mu}_{\rm int} = Q_m\, F_{{\rm R},t} + Q_{b'}\, F_u - F_s\, Q_b, \tag{25}$$

now cancels the previously gauge-invariance-violating term from (23).

The resulting Reggeized photoamplitude,

$$M^{\mu} \to \mathcal{M}^{\mu} = M^{\mu}_s + M^{\mu}_u + \mathcal{M}^{\mu}_t + \mathcal{M}^{\mu}_{\rm int}, \tag{26}$$

then, by construction, satisfies the appropriate gWTI,

$$\begin{aligned} k_\mu \mathcal{M}^{\mu} &= \Delta_m^{-1}(q) Q_m \Delta_m(q-k)\, F_{{\rm R},t} \\ &\quad + S_{b'}^{-1}(p') Q_{b'} S_{b'}(p'-k) F_u \\ &\quad - F_s\, S_b(p+k) Q_b S_b^{-1}(p), \end{aligned} \tag{27}$$

and thus is fully consistent with local gauge invariance.

The construction of the Reggeized contact current $\mathcal{M}^{\mu}_{\rm int}$ that produces the correct four-divergence (25) from the Reggeized vertex $F_{{\rm R},t}$ follows exactly along the same lines as those given for un-Reggeized contact currents $M^{\mu}_{\rm int}$ [24]. The procedure is straightforward, and we provide the corresponding generic expressions for the minimal interaction current that restores local gauge invariance in the Appendix. However, to understand how it works, it might be more illuminating to consider an example. To this end, we discuss in Sec. IV a strangeness-production process with a Kroll-Ruderman-type [41] bare contact current.

A. Regge residual function

To provide explicit expressions for the residual function (20), it is convenient to rewrite the standard expressions for positive- and negative-signature Regge propagators given in Refs. [1,5] to obtain the unified form,

$$\mathcal{F}_t(t) = \left(\frac{s}{s_{\rm sc}}\right)^{\alpha_\lambda(t)} \frac{N[\alpha_\lambda(t);\eta]}{\Gamma(1+\alpha_\lambda(t))} \frac{\pi\alpha_\lambda(t)}{\sin(\pi\alpha_\lambda(t))}, \tag{28}$$

where the functions,

$$\alpha_\lambda(t) = \alpha'_\lambda\big(t - M_\lambda^2\big), \quad \text{for } \lambda = 0,1, \tag{29}$$

are related to the usual Regge trajectories by

$$\alpha_\zeta(t) = \begin{cases} \alpha_0(t), & \text{for } \zeta = +1, \\ 1+\alpha_1(t), & \text{for } \zeta = -1. \end{cases} \tag{30}$$

Here, the signature for pseudoscalar mesons is $\zeta = +1$ (corresponding to $\lambda = 0$) and $\zeta = -1$ (corresponding to $\lambda = 1$) for vector mesons. The masses M_λ here are the lowest masses at the bases of the respective trajectories, with their slopes given by α'_λ. For these base states, at $t = M_\lambda^2$, the residual function thus is given by a manageable $0/0$ situation.

Even though $\mathcal{F}_t$ is also s dependent analytically through the scale factor $(s/s_{\rm sc})^{\alpha_\lambda(t)}$, this is irrelevant for our purposes because for a given experiment, s is fixed, and we may consider $\mathcal{F}_t$ as a function of t for fixed s. The exponential scale factor suppresses the Regge contribution for $s > s_{\rm sc}$ for (negative) physical values of t; the scale parameter $s_{\rm sc}$ usually is chosen as $s_{\rm sc} = 1$ GeV^2.

The signature function N appears here as

$$N[\alpha_\lambda(t);\eta] = \eta + (1-\eta)e^{-i\pi\alpha_\lambda(t)}, \tag{31}$$

where η is a real parameter whose three standard values are

$$\eta = \begin{cases} \frac{1}{2}, & \text{pure-signature trajectories,} \\ 0, & \text{add trajectories: rotating phase,} \\ 1, & \text{subtract trajectories: constant phase.} \end{cases} \tag{32}$$

In the pure-signature case ($\eta = 1/2$), N vanishes for every odd integer value of $\alpha_\lambda(t)$, thus leaving only the even integer values to produce poles in (28). This corresponds to even and odd angular momenta,

$$\alpha_+ = 0,2,4,\ldots \quad \text{and} \quad \alpha_- = 1,3,5,\ldots, \tag{33}$$

associated with the states along the respective positive- or negative-signature trajectories. Equation (31) also subsumes treatment of strongly degenerate trajectories [1,5], where the rotating phase ($\eta = 0$) results from adding degenerate trajectories and the constant phase ($\eta = 1$) arises from subtracting them. Which case applies is largely determined semiphenomenologically by G-parity arguments [5].

Going beyond these standard cases, because the signature function is largely phenomenological anyway, one may consider η as a convenient interpolating fit parameter for optimizing the description of data for the value range $0 \leqslant \eta \leqslant 1$. Note that $\exp(-i\pi\alpha_\lambda)$ in (31) is $+1$ at the poles of the primary trajectory and -1 at the poles of the added or subtracted secondary (degenerate) trajectory. Hence, taking into account the minus sign arising from the negative slope of the denominator sine function in (28) at those secondary poles, this effectively changes the coupling strength for the latter exchange by the factor $(1-2\eta)$ that can vary between $+1$ and -1; it is positive or negative depending on whether its degeneracy effect is more additive or subtractive, respectively. The coupling strength of the primary trajectory remains unchanged. Clearly, if the strong-degeneracy hypothesis is warranted for a particular application, fitted values of η should come out close to either 0 or 1.

At the base of the trajectories, one has

$$N\big[\alpha_\lambda\big(M_\lambda^2\big);\eta\big] = 1, \tag{34}$$

for any value of η, thus ensuring the validity of the necessary condition,

$$\mathcal{F}_t(M_\lambda^2) = 1, \tag{35}$$

for both $\lambda = 0,1$ for the residue of the corresponding Regge propagators. The fact that the Regge residue function $\mathcal{F}_t$ thus preserves the normalization of the standard form factor f_t is crucial for the construction of the gauge-invariance-preserving contact current, as will be seen explicitly in the following example.

IV. EXAMPLE: $\gamma + p \to K^+ + \Sigma^{*0}$

In this reaction only the incoming proton and the outgoing kaon carry charge. Hence, extracting the isospin operators from the respective vertices, the relevant charge parameters are (in an appropriate isospin basis)

$$Q_{b'}\boldsymbol{\tau} \to e_\Sigma = 0, \quad Q_m\boldsymbol{\tau} \to e_K = e, \quad \boldsymbol{\tau} Q_b \to e_p = e, \tag{36}$$

where e is the fundamental charge unit, and charge conservation obviously reads

$$e_\Sigma + e_K = e_p \quad \text{or} \quad e_K = e_p. \tag{37}$$

Hence, as far as gauge invariance is concerned, only s and t channels and a contact term contribute. It suffices to consider this as an on-shell process if the corresponding un-Reggeized amplitude is constructed already such that it obeys the appropriate gWTI. Moreover, we can ignore all possible resonance contributions and other meson exchanges because they do not contribute to the four-divergence (for a more complete discussion; see Ref. [17]). The only relevant exchange particles are the proton (with mass M_N) in the s channel and the kaon K^+ (with mass M_K) in the t channel.

The $p \to K^+\Sigma^{*0}$ vertices for the s- and t-channel terms are given by [17]

$$F_s \to F_s^\nu = g\boldsymbol{\tau}\, q^\nu f_s(s), \tag{38a}$$

$$F_t \to F_t^\nu = g\boldsymbol{\tau}\,(q-k)^\nu f_t(t), \tag{38b}$$

with scalar form factors f_x $(x = s,t)$ normalized as

$$f_s(M_N^2) = 1 \quad \text{and} \quad f_t(M_K^2) = 1. \tag{39}$$

The constant g subsumes all coupling constants, mass factors, signs, etc., $\boldsymbol{\tau}$ generically describes the isospin dependence, and q^ν and $(q-k)^\nu$ are the operators for s and t channels, respectively, providing coupling to the spin-3/2 Rarita-Schwinger spinor of the outgoing Σ^{*0} baryon.

The resulting current reads

$$M^{\nu\mu} = M_s^{\nu\mu} + M_t^{\nu\mu} + M_{\text{int}}^{\nu\mu}, \tag{40}$$

where the Lorentz indices μ and ν connect to the incoming photon state and the outgoing Rarita-Schwinger spinor, respectively. Assuming validity of the single-particle WTI for the proton and the kaon (which are trivially true), $M^{\nu\mu}$ is locally gauge invariant, according to (7), if the interaction current satisfies

$$k_\mu M_{\text{int}}^{\nu\mu} = Q_K F_t^\nu - F_s^\nu Q_p = e_K g\,(q-k)^\nu f_t - e_p g\, q^\nu f_s. \tag{41}$$

Then, explicitly writing out the t-channel contribution,

$$\begin{aligned} M_t^{\nu\mu} &= \frac{(2q-k)^\mu Q_K}{t - M_K^2}\, F_t^\nu \\ &= e_K g\, \frac{(2q-k)^\mu}{t-M_K^2}(q-k)^\nu f_t, \end{aligned} \tag{42}$$

we see that its (on-shell) four-divergence contribution,

$$k_\mu M_t^{\nu\mu} = -Q_K F_t^\nu = -e_K g\,(q-k)^\nu f_t, \tag{43}$$

is canceled by the t-channel term in (41). A similar finding for the s channel shows that the validity of (41) is both necessary and sufficient for making the current $M^{\nu\mu}$ locally gauge invariant.

In the structureless limit, when all form factors are unity, the bare interaction current $m_c^{\nu\mu}$ also must satisfy the analog of (41), i.e.,

$$k_\mu m_c^{\nu\mu} = e_K g\,(q-k)^\nu - e_p g\, q^\nu = k_\mu(-e_K g\, g^{\nu\mu}), \tag{44}$$

which shows that the minimal interaction current is given by

$$m_c^{\nu\mu} = -e_K g\, g^{\nu\mu}. \tag{45}$$

This is precisely the contact current resulting from the usual four-point contact Lagrangian for the present process. This result is seen here to be an immediate consequence of local gauge invariance.

To construct the corresponding minimal interaction current, we adapt the generic expression (A1) provided in the Appendix to the present case and obtain

$$M_{\text{int}}^{\nu\mu} = -e_K g\, g^{\nu\mu} f_t(t) + g\, q^\nu C^\mu. \tag{46}$$

The auxiliary contact current,

$$\begin{aligned} C^\mu &= -e_K(2q-k)^\mu \frac{f_t - 1}{t - M_K^2} f_s - e_p(2p+k)^\mu \frac{f_s - 1}{s - M_N^2} f_t \\ &\quad + \hat{A}(s,t)(1-f_t)(1-f_s) \\ &\quad \times \left[e_K \frac{(2q-k)^\mu}{t-M_K^2} + e_p \frac{(2p+k)^\mu}{s-M_N^2} \right], \end{aligned} \tag{47}$$

follows from Eq. (A2). It was derived from imposing local gauge-invariance requirements in the presence of vertices with structure [22,24]. In view of the normalizations (39), this current is manifestly nonsingular. The function $\hat{A}(s,t)$ in front of the manifestly transverse term here is a phenomenological (complex) function that must vanish at high energies, but otherwise can be freely chosen to improve fits to the data.

It is now a trivial exercise to show that

$$k_\mu C^\mu = e_K f_t - e_p f_s, \tag{48}$$

and thus the interaction current (46) indeed provides the correct four-divergence (41) to ensure local gauge invariance.

We emphasize in this context that the contact-type interaction current constructed here provides only the *minimal* structure necessary for maintaining local gauge invariance. If the physics of the problem should make it necessary to consider additional current contributions, they can only arise from additional manifestly transverse currents and thus do not contribute when taking the four-divergence of the current.

1. Regge-trajectory exchange

To Reggeize the K^+ exchange of the present example, the explicit expression in analogy to (18) reads [5]

$$\frac{f_t}{t-M_K^2} \to \frac{\mathcal{F}_t}{t-M_K^2}, \tag{49}$$

with the residual function given by (28) for $\lambda = 0$, where

$$\alpha_0(t) = \frac{t-M_K^2}{\Delta t_K} \tag{50}$$

is the kaonic Regge trajectory, with slope

$$\alpha_0' = \frac{1}{\Delta t_K} = 0.7\,\text{GeV}^{-2}, \tag{51}$$

which puts the Regge states at

$$t \to t_n = M_K^2 + n\Delta t_K, \quad \text{for } n = 0,1,2,\ldots \tag{52}$$

For pure pseudoscalar signature ($\zeta = +1 \Rightarrow \eta = 1/2$), only even values are realized on the trajectory; for all other values of η, all states contribute.

The Reggeized t-channel current reads now

$$M_t^{\nu\mu} \to \mathcal{M}_t^{\nu\mu} = e_K g \frac{(2q-k)^\mu}{t-M_K^2}(q-k)^\nu \mathcal{F}_t, \tag{53}$$

with the associated modified interaction current,

$$M_{\text{int}}^{\nu\mu} \to \mathcal{M}_{\text{int}}^{\nu\mu} = -e_K g\, g^{\nu\mu}\mathcal{F}_t + g\, q^\nu \mathcal{C}^\mu, \tag{54}$$

and modified auxiliary current,

$$\begin{aligned} C^\mu \to \mathcal{C}^\mu = &-e_K(2q-k)^\mu \frac{\mathcal{F}_t - 1}{t-M_K^2} f_s \\ &- e_p(2p+k)^\mu \frac{f_s-1}{s-M_N^2}\mathcal{F}_t \\ &+ \hat{A}(s,t)(1-f_t)(1-f_s) \\ &\times \left[e_K \frac{(2q-k)^\mu}{t-M_K^2} + e_p \frac{(2p+k)^\mu}{s-M_N^2} \right]. \end{aligned} \tag{55}$$

Despite the Reggeization of the t-channel form factor, because of the limit (35), this current is still nonsingular as far as the primary propagator singularities here are concerned. Note in this respect that there is no reason to replace f_t by $\mathcal{F}_t$ in the last term because this current piece is manifestly transverse and does not contribute to the four-divergence. However, no harm would result if one did replace it because the difference can be absorbed in redefining $\hat{A}$.

The auxiliary current $\mathcal{C}^\mu$ now does have higher-mass singularities at unphysical $t > 0$ from the Regge trajectory but those are necessary to compensate the corresponding higher-mass contributions from the t-channel exchange which have the wrong electromagnetic coupling that led to the violation of gauge invariance.

It is obvious now that the Reggeized production current for this process,

$$M^{\nu\mu} \to \mathcal{M}^{\nu\mu} = M_s^{\nu\mu} + \mathcal{M}_t^{\nu\mu} + \mathcal{M}_{\text{int}}^{\nu\mu}, \tag{56}$$

by construction does indeed satisfy the generalized Ward-Takahashi identity for this process and thus provides a conserved current,

$$k_\mu \mathcal{M}^{\nu\mu} = 0 \quad \text{(on shell)}, \tag{57}$$

as a matter of course.

V. SUMMARY AND DISCUSSION

We have considered here a mechanism to repair gauge invariance broken by Reggeization of t-channel meson exchanges in single-meson photoproduction off a baryon. Consistent with the underlying field-theoretical foundations of such processes [22], we have argued that this must be done by constructing contact-type interaction currents whose four-divergence compensates for the wrong coupling to the electromagnetic field of higher-mass contributions of the Regge trajectory that is responsible for the violation of gauge invariance. The construction principle was based on the underlying generalized Ward-Takahashi identities whose validity ensure local gauge invariance.

We emphasize once more in this respect that mere (on-shell) current conservation, $k_\mu M^\mu = 0$, is not very helpful as a starting point for repairing gauge-invariance violations. As argued, the goal of any repair mechanism must be the construction of an interaction current M_{int}^μ that satisfies the crucial four-divergence condition (7) for this interaction current. The resulting local gauge-invariance property will then automatically ensure a conserved on-shell current M^μ.

The present way of maintaining local gauge invariance in terms of a Regge form factor $\mathcal{F}_t$ to replace the usual t-channel cutoff function f_t shows that when viewed from the Feynman perspective, the Regge approach basically can be understood as a prescription for the functional form of the t-channel form factor. Numerical tests show that at (negative) physical t (and fixed s), the main features of $\mathcal{F}_t$ that survive are the exponential scale factor and the phase function,

$$S_t(t) = \left(\frac{s}{s_{\text{sc}}}\right)^{\alpha_\lambda(t)} N[\alpha_\lambda(t);\eta]. \tag{58}$$

This exponential function falls off faster than any power-law form factor and thus compared to a conventional phenomenological form factor drastically cuts out the high-$|t|$ (i.e., backward-angle) scattering contributions.

The onset of the “Regge regime” is oftentimes very much under debate in practical applications, in particular, if Regge exchanges are employed at intermediate-energy ranges within hybrid approaches as discussed here that mix Regge with the traditional Feynman picture. In this situation, it seems natural to consider mechanisms for smooth transitions into that regime [10]. An interpolating mechanism like $\mathcal{F}_{\text{R},t} = \mathcal{F}_t\, R + f_t\,(1-R)$, for example, that determines an effective t-channel form factor $\mathcal{F}_{\text{R},t}$ somewhere between its non-Regge (f_t) and Regge ($\mathcal{F}_t$) limits in terms of an (s- and t-dependent) interpolating function R can be fine-tuned to the requirements of particular applications [10,13]. Hence, fitting the interpolation parameters to experimental data lets the data “decide” to what extent Regge exchanges should be necessary for a particular process at a particular photon energy. Because this would take much of the contention out of the debate, we strongly advocate employing such interpolation schemes. This may be especially advisable for energy ranges where details of baryon-resonance structure may still play a role. Clearly, the procedure outlined here is not affected by such an interpolation scheme because $\mathcal{F}_{\mathrm{R},t}$ is normalized to unity by construction and may thus be used for building a contact current, just like f_t or $\mathcal{F}_t$.

A similarly useful interpolation procedure is provided by the η dependence of the signature function $N[\alpha_\lambda(t);\eta]$ of Eq. (31) that allows for the smooth transition from the pure-signature case to the two limiting cases of adding or subtracting degenerate trajectories and thus, again, lets the data decide which description is better suited for a given application.

One should point out that fixing local gauge invariance as presented here does not imply that the resulting expressions will automatically provide good results for the problem at hand. It merely means that whatever is missing for a good description will not be because of a violation of local gauge invariance. In other words, anything that should be found lacking in this respect would necessarily be resulting from manifestly transverse current mechanisms not relevant for local gauge invariance.

The locally gauge-invariant Reggeization procedure outlined here is currently being applied to describe Jefferson Lab data [42] for $\gamma + n \to K^+ + \Sigma^*(1385)^-$ at photon energies between 1.5 and 2.5 GeV. The preliminary results are encouraging; the full report will be published elsewhere [43].

Finally, we mention once more that the procedure given here can also be used for the Reggeization of the u channel in terms of baryonic Regge trajectories. With the details given here, it should be quite obvious how to implement this for the u channel in a locally gauge-invariant manner (see also footnote 1).

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APPENDIX: GENERIC MINIMAL INTERACTION CURRENT

To make the present paper self-contained, we provide in this Appendix the generic expression for the *minimal* interaction current necessary for preserving local gauge invariance in the process (1). We largely follow here Ref. [24], but we provide additional clarification about constraints on the parameter $\hat{h}$ introduced in Ref. [24]. Also, as it is sufficient for the present purpose, we assume on-shell kinematics (for the off-shell case; see Ref. [24]). The variables used in the following are those of Fig. 1.

The minimal interaction current appropriate for the hadronic vertex of the form (11) reads [24]

$$M^\mu_{\mathrm{int}} = m^\mu_c\, f_t(t) + \boldsymbol{G}(q)\, C^\mu, \tag{A1}$$

where m^μ_c is a Kroll-Ruderman-type bare contact current resulting from an elementary four-point Lagrangian appropriate for the reaction under consideration. The effect of this current is to make the photoprocess of Fig. 1 locally gauge invariant if all scalar form factors f_x are put to unity. The auxiliary current C^μ is given by [24]

$$\begin{aligned} C^\mu = {}& -e_m(2q-k)^\mu \frac{f_t - 1}{t - M_m^2}(\delta_s f_s + \delta_u f_u - \delta_s\delta_u f_s f_u) \\ & - e_{b'}(2p'-k)^\mu \frac{f_u - 1}{u - M_{b'}^2}(\delta_t f_t + \delta_s f_s - \delta_t\delta_s f_t f_s) \\ & - e_b(2p+k)^\mu \frac{f_s - 1}{s - M_b^2}(\delta_u f_u + \delta_t f_t - \delta_u\delta_t f_u f_t) \\ & + \hat{A}(s,u,t)(1-\delta_s f_s)(1-\delta_u f_u)(1-\delta_t f_t) \\ & \times \left[e_m \frac{(2q-k)^\mu}{t - M_m^2} + e_{b'}\frac{(2p'-k)^\mu}{u - M_{b'}^2} + e_b \frac{(2p+k)^\mu}{s - M_b^2} \right], \end{aligned} \tag{A2}$$

where the factors δ_x ($x = s,u,t$) are unity if the corresponding channel contributes to the reaction in question, and zero otherwise. This contact current is manifestly nonsingular because the form factors become unity at the respective poles thus providing well-defined 0/0 situations. The function $\hat{A}(s,u,t)$ is an arbitrary (complex) phenomenological function, possibly subject to crossing symmetry constraints, that must vanish at high energies. The expression here follows from Eq. (31) of Ref. [24] choosing the function $\hat{h}$ appearing there as $\hat{h} = 1 - \hat{A}$. The vanishing high-energy limit of $\hat{A}$ is necessary to prevent the "violation of scaling behavior" noted in Ref. [44] if $\hat{h}$ is different from unity at high energies.

The $\hat{A}$-dependent term in Eq. (A2) is easily seen to be manifestly transverse in view of the charge-conservation relation (14) and therefore not necessary for preserving local gauge invariance. However, it provides added flexibility when fitting data. In principle, of course, any transverse (nonsingular) current may be added to the right-hand side of (A1) without affecting gauge invariance.

The four-divergence of C^μ evaluates to

$$k_\mu C^\mu = e_m\, f_t + e_{b'}\, f_u - e_b\, f_s. \tag{A3}$$

In deriving this result repeated use was made of the charge-conservation relation (14). This is the scalar form of the generalized Ward-Takahashi identity (7) for the interaction current. The right-hand side here vanishes for structureless particles where all form factors are replaced by unity.

For the entire interaction current (A1) one then finds

$$k_\mu M^\mu_{\mathrm{int}} = \left[k_\mu m^\mu_c + e_m\,\boldsymbol{G}(q)\right] f_t + e_{b'}\,\boldsymbol{G}(q) f_u - e_b\,\boldsymbol{G}(q) f_s. \tag{A4}$$

Because the structureless contact current m^μ_c also must satisfy the gWTI (7) with all form factors replaced by unity, we have

$$\begin{aligned} k_\mu m^\mu_c &= e_m \boldsymbol{G}(q-k) + e_{b'}\boldsymbol{G}(q) - e_b \boldsymbol{G}(q) \\ &= e_m \boldsymbol{G}(q-k) - e_m \boldsymbol{G}(q). \end{aligned} \tag{A5}$$

Equation (A4) then reads

$$k_\mu M^\mu_{\mathrm{int}} = e_m\,\boldsymbol{G}(q-k) f_t + e_{b'}\,\boldsymbol{G}(q) f_u - e_b\,\boldsymbol{G}(q) f_s, \tag{A6}$$

which is the full gWTI (7) with structure for the vertex of the form (11).

1. Beyond model treatment

We mention without going into much detail here that one may generalize the vertex (11) by employing an expansion of the form,

$$F(p_{b'},p_b) = \boldsymbol{\tau} \sum_i \lambda_i \boldsymbol{G}_i(q_m) f_i\big(q_m^2, p_{b'}^2, p_b^2\big), \tag{A7}$$

where the sum extends over all possible coupling operators $\boldsymbol{G}_i$, each with its own (normalized) form factor f_i such that the mixing parameters λ_i add up to unity, $\sum_i \lambda_i = 1$. In principle, one may even consider a formulation where the scalar functions f_i are no longer phenomenological suppression functions, but are determined within a consistent dynamical framework. Depending on the sophistication of this framework, the expansion (A7) then may be made arbitrarily close to a complete dynamical description for the three-point vertex $b \to m + b'$.

To accommodate the combination vertex (A7) with more than one coupling operators $\boldsymbol{G}_i$, one needs bare currents $m^\mu_{c,i}$ for each one satisfying a separate gWTI like (A5) and leading to, in particular,

$$k_\mu m^\mu_{c,i} = e_m \boldsymbol{G}_i(q-k) - e_m \boldsymbol{G}_i(q). \tag{A8}$$

The analog of the interaction current ansatz (A1) then is

$$M^\mu_{\rm int} = \sum_i \lambda_i \big[m^\mu_{c,i} f_{i,t} + \boldsymbol{G}_i(q) C_i^\mu\big], \tag{A9}$$

where $f_{i,t}$ is the form factor f_i in the t channel and C_i^μ is like (A2) using f_i alone. Everything then goes through as before, and so this current obviously satisfies the correct gWTI for the vertex (A7) and thus preserves local gauge invariance by construction.

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