Derivation of breakup probabilities of weakly bound nuclei from experimental elastic and quasi-elastic scattering angular distributions

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We present a simple method to derive breakup probabilities of weakly bound nuclei by measuring only elastic (or quasi-elastic) scattering for the system under investigation and a similar tightly bound system. When transfer followed by breakup is an important process, one can derive only the sum of breakup and transfer probabilities.

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I. INTRODUCTION

The complex reaction mechanisms and scattering in collisions involving weakly bound nuclei, both stable and radioactive, have been the subject of intense theoretical and experimental investigations in recent years. These systems have special features, particularly the very specific characteristics of halo nuclei. Some comprehensive review papers have been published on this subject [1–6].

Several competing reaction processes may occur when weakly bound nuclei are involved, at energies near the Coulomb barrier, in addition to the usual processes which are present when tightly bound nuclei interact (inelastic excitations, direct transfer of nucleons or clusters of nucleons, fusion). If at least one of the colliding nuclei has small breakup threshold energy, typically smaller than 3 MeV, this nucleus may break up in the field of the partner nucleus and different processes may occur, such as sequential complete fusion (when all fragments fuse), incomplete fusion (when some but not all fragments fuse), and noncapture breakup (when neither fragment fuses). As recently observed [7,8], at sub-barrier energies the breakup following direct transfer of nucleons of stable weakly bound nuclei (^{6}Li, ^{7}Li, ^{9}Be) predominates over the direct breakup of these nuclei.

Among the most important questions on this subject one finds the following: Does the breakup enhance or suppress the complete fusion cross sections? How large are the noncapture breakup cross sections, compared with the fusion cross sections? The answers to these questions depend on the energy regime (above or below the barrier), the target mass or charge, and if the projectile has halo characteristics. The effect of breakup on the fusion cross section has been intensively studied and some qualitative trends on the systematic behavior of complete fusion suppression, at energies above the barrier, have been found [1,9–16], although the subject is still far from being fully understood. In particular, the study of the fusion reactions involving nuclei close to the drip lines so far has led to the contradictory results.

Concerning the measurement of noncapture breakup, this is a very difficult task. It requires very accurate exclusive experiments with coincidences between the fragments and then conversion of the events in integrated cross sections. A clear identification of the processes, including sequential breakup (breakup following transfer), may be possible through the Q values of the reactions [7]. If one is interested in the investigation of the effect of breakup on the fusion cross section, it is of fundamental importance to have indications on the time scale of the breakup. If the breakup occurs when the projectile approaches the target, which is called prompt breakup, it may affect fusion. Otherwise, if the breakup occurs when the projectile is already far from the target and moving away from it, which is called delayed breakup, the process cannot affect fusion. The breakup probability is an important quantity but almost impossible to be measured, because it involves several reaction mechanisms (depending on what happens with the fragments after the breakup) and also may be triggered by transfer reactions where the breakup occurs after the direct transfer of nucleons. From the theory side, there is no model which provides all the cross sections related to breakup simultaneously.

We believe that the establishing relationships between various reaction observables is useful for a quantitative understanding of the reaction processes, providing a comprehensive picture of the physics of low-energy heavy-ion collisions. As an alternative to derive breakup and capture probabilities in a simpler way than to measure these processes directly, the use of elastic and quasi-elastic backscattering data has recently been suggested [12,13,17], since the elastic and quasi-elastic experiments are usually not so complicated as the fusion (capture) and breakup measurements. Particularly, for the derivation of breakup probabilities at backward angle $\theta = 180^\circ$, Sargsyan *et al.* [13] have proposed the use of elastic (quasi-elastic) backscattering data. However, experiments cannot be peformed at $\theta = 180^\circ$, but rather backscattering experiments are usually performed in the range $\theta = 130^\circ$–170°. Then, the data must be corrected and theoretically transformed to $\theta = 180^\circ$ by introducing an effective energy that is obtained from the relation between elastic scattering energy and scattering angle. In the present paper we propose one even simpler method to obtain breakup probabilities. Now, the extraction of breakup probabilities can be obtained from elastic (quasi-elastic) scattering data obtained at any arbitrary angle. Furthermore, now we take into account the effects of nucleon or cluster transfer.

The results of the present method are similar to the previous one using backscattering data. However, when transfer channels are important, as for instance when transfer Q values are positive, the present method works much better.

In Secs. II and III we derive the relationships between probabilities of various reaction channels. In Sec. IV, using these relationships, we suggest the method for extracting breakup probabilities from the experimental elastic (quasi-elastic) scattering data. The results of calculations and conclusions are given in Secs. V and VI.

II. ELASTIC SCATTERING

The total elastic scattering amplitude can be written as [18]

$$f(\theta) = f_n(\theta) + f_C(\theta) = \frac{i}{2k}\sum_J (2J+1)(1-S_J)P_J(\cos\theta) = \sum_J f_J(\theta), \quad (1)$$

where θ is the scattering angle in the center-of-mass system and $k = \sqrt{2\mu E/\hbar^2}$ denotes the asymptotic wave number in the elastic channel, μ and E being the reduced mass and the bombarding energy in the center-of-mass system, respectively. In Eq. (1), $f_n(\theta) = \sum_J f_J^n(\theta)$ is the Coulomb modified nuclear amplitude obtained by subtracting the Coulomb (or the Rutherford) amplitude $f_C(\theta) = \sum_J f_J^C(\theta)$ from $f(\theta)$. The S_J is the elastic partial wave S matrix, given by

$$S_J = S_J^C S_J^n = |S_J^n| e^{2i(\sigma_J^C + \sigma_J^n)}, \quad (2)$$

where S_J^n (S_J^C) represents the effect of the short-range nuclear (the long-range Coulomb) interaction and σ_J^n (σ_J^C) is the scattering phase shift associated with S_J^n (S_J^C).

For heavy systems, one usually considers the ratio $\sigma_{\rm el}(\theta)/\sigma_C(\theta)$, where $\sigma_{\rm el}(\theta) = |f(\theta)|^2 = \sum_{J,J'} \sigma_{\rm el}(\theta,J,J') = \sum_{J,J'} f_J(\theta) f_{J'}^*(\theta)$ is the differential elastic-scattering cross section and $\sigma_C(\theta) = |f_C(\theta)|^2 = \sum_{J,J'} f_J^C(\theta) f_{J'}^{C*}(\theta)$ is the Rutherford cross section.

Note that we are using the notation that when σ_i is written with an argument θ it corresponds to a differential cross section; otherwise it corresponds to an integrated cross section.

From Eq. (1), one can write

$$\sigma_R(\theta) = \sigma_C(\theta) - \sigma_{\rm el}(\theta) - \sigma_{fg}(\theta), \quad (3)$$

where

$$\sigma_R(\theta) = \frac{1}{4k^2}\sum_{J,J'} (2J+1)(2J'+1) S_J^C S_{J'}^{C*}\left(1 - S_J^n S_{J'}^{n*}\right) \times P_J(\cos\theta) P_{J'}(\cos\theta) \quad (4)$$

is the differential reaction cross section and the last term in the right-hand side of Eq. (3)

$$\sigma_{fg}(\theta) = \sum_J \sigma_{fg}(\theta,J) = -\frac{2}{k}\,{\rm Im}[f_n(\theta)]\delta(1-\cos\theta) \quad (5)$$

is the differential forward nuclear glory scattering cross section, which is zero everywhere except at $\theta = 0$, due to the δ function $\delta(1-\cos\theta)$.

So, the reaction cross section is the difference between Coulomb and elastic cross sections. Equation (3) has been formally derived by Hussein *et al.* [19,20], by using the optical theorem and forward glory effects in heavy-ion scattering.

At the angular range $0 < \theta \leqslant \pi$, one can rewrite Eq. (3) in the following way:

$$\frac{\sigma_R(\theta)}{\sigma_C(\theta)} + \frac{\sigma_{\rm el}(\theta)}{\sigma_C(\theta)} = P_R(\theta) + P_{\rm el}(\theta) = 1, \quad (6)$$

where $P_i(\theta) = \sigma_i(\theta)/\sigma_C(\theta)$ refer to the elastic scattering ($i = el$) and reaction ($i = R$) probabilities at a given scattering angle θ. Owing to the fact that the differential cross sections are positive and the ratios $\sigma_i(\theta)/\sigma_C(\theta)$ can be interpreted as probabilities, the conditions $0 \leqslant P_i(\theta) \leqslant 1$ are fulfilled. One can observe that there is the unique relation (6) among different channels.

III. QUASI-ELASTIC SCATTERING

In several scattering experiments, it is not possible to resolve elastic and inelastic scattering, and only quasi-elastic scattering can be measured, where *quasi-elastic* means the sum of elastic and inelastic scattering; that is, the differential quasi-elastic scattering cross section is defined as

$$\sigma_{\rm qe}(\theta) = \sigma_{\rm el}(\theta) + \sigma_{in}(\theta). \quad (7)$$

The differential reaction cross section can be written as

$$\sigma_R(\theta) = \sigma_{\rm cap}(\theta) + \sigma_{in}(\theta) + \sigma_{\rm tr}(\theta) + \sigma_{BU}(\theta) + \sigma_{DIC}(\theta), \quad (8)$$

that is, the sum of differential cross sections of capture $\sigma_{\rm cap}(\theta)$ (a sum of evaporation-residue formation, fusion-fission, and quasi-fission cross sections), inelastic excitations, few-nucleon transfer $\sigma_{\rm tr}(\theta)$, breakup $\sigma_{BU}(\theta)$, and deep inelastic collisions $\sigma_{DIC}(\theta)$. Substituting Eqs. (7) and (8) into Eq. (3) one gets

$$\sigma_{\rm qe}(\theta) + \sigma_{\rm cap}(\theta) + \sigma_{BU}(\theta) + \sigma_{\rm tr}(\theta) + \sigma_{DIC}(\theta) + \sigma_{fg}(\theta) = \sigma_C(\theta). \quad (9)$$

Employing Eq. (9) at $0 < \theta \leqslant \pi$, one obtains

$$P_{\rm cap}(\theta) + P_{\rm qe}(\theta) + P_{BU}(\theta) + P_{\rm tr}(\theta) + P_{DIC}(\theta) = 1, \quad (10)$$

where $P_i(\theta) = \sigma_i(\theta)/\sigma_C(\theta)$ refer to different channel probabilities at a given scattering angle θ. For systems involving only tightly bound nuclei, $P_{BU}(\theta)$ is negligible.

IV. DERIVATION METHOD OF BREAKUP PROBABILITIES

From Eq. (6) one observes the direct relationship between the elastic scattering and the other reaction processes, since any loss of flux from the elastic scattering channel contributes directly to other channels. One may write

$$P_{\rm el}(\theta) + P_R(\theta) = P_{\rm el}(\theta) + P_{BU}(\theta) + P_{\rm rest}(\theta) = 1, \quad (11)$$

where $P_{\rm rest}(\theta)$ stands for all reaction channels apart from the breakup.

Dividing all terms of Eq. (11) by $1 - P_{BU}(\theta)$, Eq. (11) is rewritten as

$$\frac{P_{\rm el}(\theta)}{1 - P_{BU}(\theta)} + \frac{P_{\rm rest}(\theta)}{1 - P_{BU}(\theta)} = P_{\rm el}^{{\rm no}BU}(\theta) + P_{\rm rest}^{{\rm no}BU}(\theta) = 1, \tag{12}$$

where

$$P_{\rm el}^{{\rm no}BU}(\theta) = \frac{P_{\rm el}(\theta)}{1 - P_{BU}(\theta)} \tag{13}$$

and

$$P_{\rm rest}^{{\rm no}BU}(\theta) = \frac{P_{\rm rest}(\theta)}{1 - P_{BU}(\theta)} \tag{14}$$

are the elastic scattering and other channels probabilities, respectively, in the absence of the breakup process. This is the situation when the colliding nuclei are tightly bound. From these expressions we obtain the useful formulas

$$\frac{P_{\rm el}(\theta)}{P_{\rm rest}(\theta)} = \frac{P_{\rm el}^{{\rm no}BU}(\theta)}{P_{\rm rest}^{{\rm no}BU}(\theta)} = \frac{P_{\rm el}^{{\rm no}BU}(\theta)}{1 - P_{\rm el}^{{\rm no}BU}(\theta)}. \tag{15}$$

Using Eq. (13), one can find the relationship between the breakup and elastic scattering processes:

$$P_{BU}(\theta) = 1 - \frac{P_{\rm el}(\theta)}{P_{\rm el}^{{\rm no}BU}(\theta)}. \tag{16}$$

In Ref. [13], a similar equation was derived for the partial breakup and elastic scattering probabilities with $J = 0$ ($\theta = 180^\circ$), for backscattering.

Equation (6) is also rewritten as

$$\frac{P_{\rm el}(\theta)}{1 - P_{BU}(\theta) - P_{\rm tr}(\theta)} + \frac{P_{\rm rest'}(\theta)}{1 - P_{BU}(\theta) - P_{\rm tr}(\theta)} = P_{\rm el}^{{\rm no}BU+{\rm notr}}(\theta) + P_{\rm rest'}^{{\rm no}BU+{\rm notr}}(\theta) = 1, \tag{17}$$

where

$$P_{\rm el}^{{\rm no}BU+{\rm notr}}(\theta) = \frac{P_{\rm el}(\theta)}{1 - P_{BU}(\theta) - P_{\rm tr}(\theta)} \tag{18}$$

and

$$P_{\rm rest'}^{{\rm no}BU+{\rm notr}}(\theta) = \frac{P_{\rm rest'}(\theta)}{1 - P_{BU}(\theta) - P_{\rm tr}(\theta)} \tag{19}$$

are the elastic scattering and other channels probabilities, respectively, in the absence of the breakup and transfer processes. Employing Eq. (18), one can find the relationship between the breakup, transfer, and elastic scattering processes:

$$P_{BU}(\theta) = 1 - P_{\rm tr}(\theta) - \frac{P_{\rm el}(\theta)}{P_{\rm el}^{{\rm no}BU+{\rm notr}}(\theta)}. \tag{20}$$

One can also obtain the following useful relations:

$$\frac{P_{\rm el}(\theta)}{P_{\rm rest'}(\theta)} = \frac{P_{\rm el}^{{\rm no}BU+{\rm notr}}(\theta)}{P_{\rm rest'}^{{\rm no}BU+{\rm notr}}(\theta)} = \frac{P_{\rm el}^{{\rm no}BU+{\rm notr}}(\theta)}{1 - P_{\rm el}^{{\rm no}BU+{\rm notr}}(\theta)}. \tag{21}$$

As already mentioned, in several experiments it is not possible to separate elastic and inelastic scattering, and therefore it is important to consider the derivation of the expressions similar to Eqs. (16) and (20) for quasi-elastic scattering. Using Eq. (10), we derive the following relations:

$$P_{BU}(\theta) = 1 - \frac{P_{\rm qe}(\theta)}{P_{\rm qe}^{noBU}(\theta)} \tag{22}$$

and

$$P_{BU}(\theta) = 1 - P_{\rm tr}(\theta) - \frac{P_{\rm qe}(\theta)}{P_{\rm qe}^{{\rm no}BU+{\rm notr}}(\theta)}, \tag{23}$$

where $P_{\rm qe}$, $P_{\rm qe}^{noBU}$, and $P_{\rm qe}^{{\rm no}BU+{\rm notr}}$ are the quasi-elastic scattering probabilities with and without the breakup process, and without the breakup and transfer processes, respectively. At sub-barrier energies, $P_{\rm qe}^{{\rm no}BU+{\rm notr}}(\theta) \approx 1$, and from Eq. (23) we obtain

$$P_{BU}(\theta) \approx 1 - P_{\rm tr}(\theta) - P_{\rm qe}(\theta). \tag{24}$$

By measuring elastic or quasi-elastic scattering angular distributions with weakly bound and tightly bound nuclei with similar Coulomb barriers and bombarding energies, and using Eq. (20) or Eq. (23), one can derive the breakup probability of the weakly bound nuclei. For example, using Eq. (20) or Eq. (23) at a given angle and assuming approximate equality [21]

$$P_{\rm el,qe}^{{\rm no}BU+{\rm notr}}[^6{\rm He} + {}^{A-2}X] \approx P_{\rm el,qe}[^4{\rm He} + {}^{A}X]$$

at the condition

$$V_b(^4{\rm He} + {}^{A}X) - E_{\rm c.m.}(^4{\rm He} + {}^{A}X) = V_b(^6{\rm He} + {}^{A-2}X) - E_{\rm c.m.}(^6{\rm He} + {}^{A-2}X)$$

for the corresponding Coulomb barriers and bombarding energies, using the experimental $P_{\rm el,qe}[^4{\rm He} + {}^{A}X]$ of the $^4{\rm He} + {}^{A}X$ reaction with tightly bound nuclei (without breakup and transfer), and the experimental $P_{\rm el,qe}[^6{\rm He} + {}^{A-2}X]$ and $P_{\rm tr}[^6{\rm He} + {}^{A-2}X]$ of the $^6{\rm He} + {}^{A-2}X$ reaction with weakly bound projectile (with breakup and transfer), one can extract the breakup probability of ^{6}He:

$$P_{BU}(\theta) = 1 - P_{\rm tr}(\theta)[^6{\rm He} + {}^{A-2}X] - \frac{P_{\rm el,qe}(\theta)[^6{\rm He} + {}^{A-2}X]}{P_{\rm el,qe}(\theta)[^4{\rm He} + {}^{A}X]}. \tag{25}$$

So, to extract the pure $P_{BU}(\theta)$, one should measure the $P_{\rm tr}(\theta)[^6{\rm He} + {}^{A-2}X]$ for the weakly bound system together with $P_{\rm el,qe}(\theta)[^6{\rm He} + {}^{A-2}X]$ and $P_{\rm el,qe}(\theta)[^4{\rm He} + {}^{A}X]$. Alternatively, instead of forming the same compound nucleus, one could use scattering data of weakly and tightly bound isotopes with the same target. At deep sub-barrier energies, $P_{\rm el,qe}[^4{\rm He} + {}^{A}X] \approx 1$ and from Eq. (25) one obtains

$$P_{BU}(\theta) \approx 1 - P_{\rm tr}(\theta)[^6{\rm He} + {}^{A-2}X] - P_{\rm el,qe}(\theta)[^6{\rm He} + {}^{A-2}X]. \tag{26}$$

To extract the pure $P_{BU}(\theta)$ in this case, one should measure only the $P_{\rm tr}(\theta)[^6{\rm He} + {}^{A-2}X]$ and $P_{\rm el,qe}(\theta)[^6{\rm He} + {}^{A-2}X]$ for the weakly bound system.

It should be stressed that in the case of a direct transfer of nucleons followed by breakup, one should use Eq. (16) or Eq. (22).

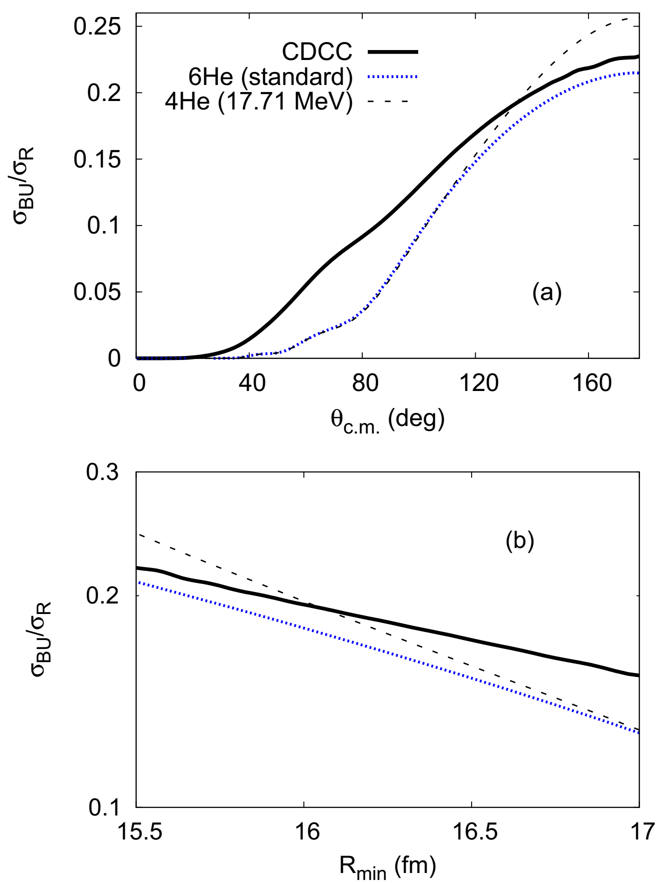

FIG. 1. (Color online) (a) The breakup probability of formula (16) (standard) is compared with the CDCC results (solid line) for ^{6}He + ^{208}Pb at the sub-barrier energy $E_{\text{lab}} = 16$ MeV. The *nonbreakup* elastic case of ^{6}He has been replaced with the ^{4}He + ^{210}Pb reaction at $E_{\text{lab}} = 17.71$ MeV (dashed line). (b) The breakup probability as a function of the distance of closest approach for previous curves at angles larger than 110°. See text for further details.

V. RESULTS OF CALCULATIONS

As an example, Fig. 1(a) shows the performance of formula (16) using the results for $P_{BU}(\theta)$, $P_{\text{el}}(\theta)$, and $P_{\text{el}}^{noBU}(\theta)$ obtained with four-body continuum-discretized coupled-channels (CDCC) calculations for ^{6}He + ^{208}Pb at $E_{\text{lab}} = 16$ MeV. It is assumed that the ^{6}He halo projectile has a three-body cluster structure ($\alpha + n + n$), while the ^{208}Pb target is considered inert. The ground state and continuum states of ^{6}He along with all the continuum couplings are explicitly included. The impact of other reaction channels (fusion, transfer, target excitations, etc.) on the ^{6}He elastic scattering is effectively treated through optical potentials between the ^{6}He fragments and the target. This CDCC model describes elastic breakup only, and the converged CDCC calculations used in this work were published in detail in Ref. [22]. At the studied sub-barrier energy, it is observed that $P_{BU}(\theta)$ of formula (16) (dotted line) compares with direct $P_{BU}(\theta)$ of CDCC calculations (thick solid line) fairly well. In Fig. 1(a), an additional calculation is presented, in which the ^{6}He *nonbreakup* elastic case is replaced [as in Eq. (25)] with a 17.71-MeV beam of ^{4}He on the ^{210}Pb target (dashed line), so the $V_b - E_{\text{c.m.}}$ for ^{4}He + ^{210}Pb is the same as that for ^{6}He + ^{208}Pb. These results of formula (16) are in satisfactory agreement with the CDCC outcomes (thick solid line). One can observe that the backscattering region is the dominant one. Figure 1(b) shows the breakup probability as a function of the distance of minimal approach, assuming the Rutherford trajectory for the ^{6}He + ^{208}Pb and ^{4}He + ^{210}Pb collisions at the sub-barrier energies studied. This breakup function at backward angles shows up a linear behavior in logarithmic scale. This function is a critical input of the PLATYPUS code for calculating the cross sections at above-barrier energies [23]. The results are sensitive to the slope of the breakup function [24] that determines the radial location (relative to the target) of the projectile breakup [23].

VI. CONCLUSIONS

We established the simple and interesting relationship between the breakup and elastic (quasi-elastic) scattering probabilities. With this relationship and the experimental quasi-elastic (elastic) scattering data at a given angle in the reactions with a weakly bound projectile and a tightly bound isotope and the same or almost the same compound nucleus, the breakup probability, which is important and difficult to be measured quantity, may be satisfactorily extracted. As shown, the elastic (quasi-elastic) scattering technique could be a very useful tool in the study of the breakup probability as a function of scattering angle. Our method may be useful for current experimental activities in the field, as it puts together different measurable processes.

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