Isotopic trends in capture reactions with radioactive and stable potassium beams

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The isotopic dependence of the capture cross section is analyzed in the reactions $^{37,39,41,43,45,46,47}\mathrm{K}+{}^{124}\mathrm{Sn}, {}^{208}\mathrm{Pb}$ with stable and radioactive beams. A comparison between the reactions $^{46}\mathrm{K}+{}^{124}\mathrm{Sn}, {}^{208}\mathrm{Pb}$ and $^{48}\mathrm{Ca}+{}^{124}\mathrm{Sn}, {}^{208}\mathrm{Pb}$ is performed. The sub-barrier capture cross sections are larger in the reactions with a stable beam at fixed $E_{\mathrm{c.m.}}$-V_b.

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I. INTRODUCTION

The new generation of radioactive ion beam facilities will provide exotic beams with rather high intensity. One of the most interesting studies with these beams will be the complete fusion reactions [1,2] in which new isotopes of existing elements are synthesized and studied. The central issue is whether the capture and fusion cross sections are enhanced in the reactions with neutron-rich or neutron-deficient projectile nucleus. However, discussing the reactions with radioactive ion beams, one should bear in mind the smaller intensities of these beams in comparison with those of stable beams.

The goal of the present article is to compare the capture of stable ^{39,41}K and radioactive ^{37,43,45,46,47}K beams by the same target, ^{208}Pb or ^{124}Sn, in order to study the effects of the neutron excess (isospin) on the capture process. The systems $^{37-45}\mathrm{K}+{}^{124}\mathrm{Sn}, {}^{208}\mathrm{Pb}$ have positive two-neutron transfer Q values while [3] the reactions $^{46,47}\mathrm{K}+{}^{124}\mathrm{Sn}, {}^{208}\mathrm{Pb}$ display negative Q values for two-neutron transfer. In the present paper we will demonstrate the neutron transfer effect on the capture process. The calculated capture cross sections in the reactions $^{46}\mathrm{K}+{}^{124}\mathrm{Sn}, {}^{208}\mathrm{Pb}$ in comparison to the reactions $^{48}\mathrm{Ca}+{}^{124}\mathrm{Sn}, {}^{208}\mathrm{Pb}$ will show an advantage or disadvantage of radioactive beams in the production of new isotopes. Such experiments are currently in the planning stage [1]. In Sec. II, we describe the capture model. In Sec. III, the isospin dependence of the calculated capture cross section is revealed. The conclusions are given in Sec. IV.

II. MODEL

The quantum diffusion approach [4,5] is applied to study the capture process. With this approach many heavy-ion capture reactions at energies above and well below the Coulomb barrier have been successfully described [4–6]. Since the details of our theoretical treatment were already published in Refs. [4–6], the model will be only briefly described.

In the quantum diffusion approach [4,5] the collisions of nuclei are described with a single relevant collective variable: the relative distance between the colliding nuclei. This approach takes into consideration the fluctuation and dissipation effects [7] in collisions of heavy ions which model the coupling with various channels (for example, coupling of the relative motion with low-lying collective modes such as dynamical quadrupole and octupole modes of the target and projectile [8,9]). We have to mention that many quantum-mechanical and non-Markovian effects accompanying the passage through the Coulomb barrier are taken into consideration in our formalism [4–6]. The diffusion models, which include the quantum statistical effects, were also proposed in Refs. [10]. The nuclear deformation effects are taken into account through the dependence of the nucleus-nucleus potential on the deformations and mutual orientations of the colliding nuclei. To calculate the nucleus-nucleus interaction potential $V(R)$, we use the procedure presented in Ref. [5]. For the nuclear part of the nucleus-nucleus potential, the double-folding formalism with a Skyrme-type density-dependent effective nucleon-nucleon interaction is used [11]. The nucleon densities of the projectile and target nuclei are specified in the form of the Woods-Saxon parametrization, where the nuclear radius parameter is $r_0 = 1.15$ fm and the diffuseness parameter takes the value $a = 0.55$ fm for all nuclei. We assume that the unknown quadrupole deformation parameter of the ^{A}K nucleus coincides with the quadrupole deformation parameter of the $^{A+1}$Ca nucleus with the same number of neutrons. For the isotopes of Ca, the absolute values of the quadrupole deformation parameters β_2 in the excited (ground) state were taken from Ref. [12] ([13]). So, $\beta_2 = 0$ in the ground state of ^{A}K. For the ^{208,206}Pb and ^{122,124}Sn nuclei in the ground state, we set $\beta_2 = 0$ and $\beta_2 = 0.1$, respectively.

The capture cross section is the sum of the partial capture cross sections [4,5]

$$\sigma_{\mathrm{cap}}(E_{\mathrm{c.m.}}) = \sum_J \sigma_{\mathrm{cap}}(E_{\mathrm{c.m.}},J) = \pi\lambda\!\!\!^{-2} \sum_J (2J+1) \int_0^{\pi/2} d\theta_1 \sin\theta_1 \int_0^{\pi/2} d\theta_2 \times \sin\theta_2 P_{\mathrm{cap}}(E_{\mathrm{c.m.}},J,\theta_1,\theta_2), \quad (1)$$

where $\lambda\!\!\!^{-2} = \hbar^2/(2\mu E_{\mathrm{c.m.}})$ is the reduced de Broglie wavelength, $\mu = m_0 A_1 A_2/(A_1 + A_2)$ is the reduced mass (m_0 is the nucleon mass), and the summation is over the possible values of the angular momentum J at a given bombarding

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energy $E_{\text{c.m.}}$. Knowing the potential of the interacting nuclei for each orientation with the angles $\theta_i (i = 1,2)$, one can obtain the partial capture probability P_{cap} which is defined by the probability to penetrate the potential barrier in the relative distance coordinate R at a given J. The value of P_{cap} is obtained by integrating the propagator G from the initial state (R_0, P_0) at time $t = 0$ to the final state (R, P) at time t (P is the momentum):

$$P_{\text{cap}} = \lim_{t\to\infty} \int_{-\infty}^{r_{\text{in}}} dR \int_{-\infty}^{\infty} dP\, G(R,P,t|R_0,P_0,0) = \lim_{t\to\infty} \frac{1}{2} \text{erfc}\left[\frac{-r_{\text{in}} + \overline{R(t)}}{\sqrt{\Sigma_{RR}(t)}}\right]. \quad (2)$$

Here, r_{in} is an internal turning point. With the nucleus-nucleus potential used the inner turning points correspond to R larger than those for touching nuclei. So, the possible correction from the neck degree of freedom is expected to be small. The second line in Eq. (2) is obtained by using the propagator $G = \pi^{-1} |\det \Sigma^{-1}|^{1/2} \exp(-\mathbf{q}^T \Sigma^{-1} \boldsymbol{q})$ ($\mathbf{q}^T = [q_R, q_P]$, $q_R(t) = R - \overline{R(t)}$, $q_P(t) = P - \overline{P(t)}$, $\overline{R(t=0)} = R_0$, $\overline{P(t=0)} = P_0$, $\Sigma_{kk'}(t) = 2\overline{q_k(t) q_{k'}(t)}$, $\Sigma_{kk'}(t=0) = 0$, $k,k' = R,P$) calculated for an inverted oscillator which approximates the nucleus-nucleus potential V in the variable R as follows. At given $E_{\text{c.m.}}$ and J, the classical action is calculated for the realistic nucleus-nucleus potential with the WKB approximation. Then the realistic nucleus-nucleus potential is replaced by an inverted oscillator which has the same barrier height and classical action. So, the frequency $\omega(E_{\text{c.m.}}, J)$ of this oscillator is set to obtain an equality of the classical actions in the approximated and realistic potentials. Usually in the literature the parabolic approximation with $E_{\text{c.m.}}$-independent ω is employed that is unsuitable at the deep sub-barrier energies. Our approximation takes the features of the realistic potential into account and is well tested for the reactions and energy range considered here [4,5]. However, the microscopic justification is desirable. Note that the capture probability is calculated with the quantum stochastic equation modeling the coupling with various channels.

We assume that the sub-barrier capture mainly depends on the two-neutron transfer with positive Q value. Our assumption is that just before the projectile is captured by the target nucleus (just before the crossing of the Coulomb barrier), the transfer occurs and leads to the population of the first excited collective state in the recipient nucleus [14]. So, the motion to the N/Z equilibrium starts in the system before the capture because it is energetically favorable in the dinuclear system in the vicinity of the Coulomb barrier. For the reactions under consideration, the average change of mass asymmetry is connected to the two-neutron transfer. Since after the transfer, the isotopic composition and the deformation parameters of the interacting nuclei, and, correspondingly, the height $V_b = V(R_b)$ and shape of the Coulomb barrier are changed, one can expect an enhancement or suppression of the capture. If after the neutron transfer the deformations of the interacting nuclei increase (decrease), the capture probability increases (decreases). When the isotopic dependence of the nucleus-nucleus potential is weak and after the transfer the deformations of the interacting nuclei do not change, there is no effect of the neutron transfer on the capture. In comparison with Ref. [15], we assume that the negative transfer Q values do not play a visible role in the capture process. Our scenario was verified in the description of many reactions [5].

The primary neutron-rich products of the complete fusion reactions ^{A}K + ^{124}Sn of interest are excited and transformed into the secondary products with a smaller number of neutrons. Since neutron emission is the dominant de-excitation channel in the neutron-rich isotopes of interest, the production cross sections of the secondary nuclei are the same as those of the corresponding primary nuclei. This seems to be evident without special treatment.

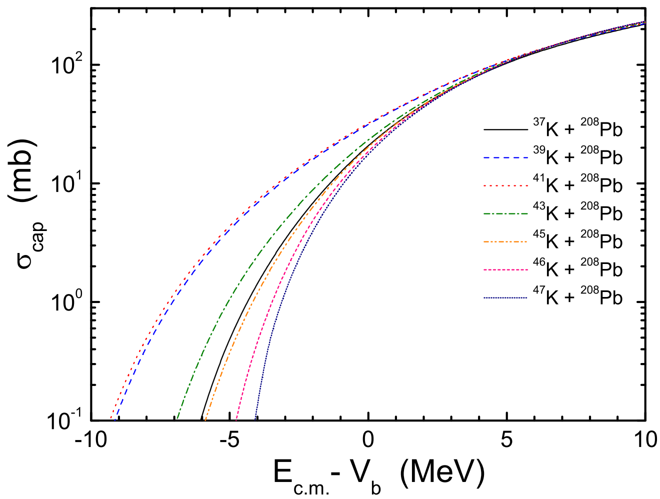

FIG. 1. (Color online) Calculated dependence of capture cross section σ_{cap} on $E_{\text{c.m.}} - V_b$ for the indicated reactions ^{A}K + ^{208}Pb.

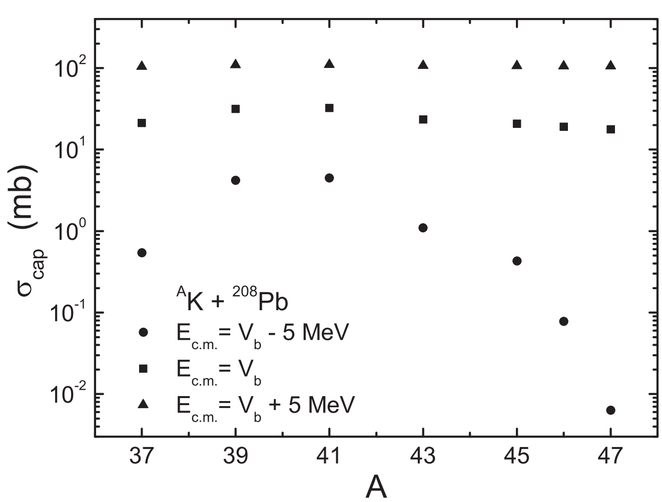

FIG. 2. Calculated dependence of capture cross section σ_{cap} on A for the reactions ^{A}K + ^{208}Pb at fixed bombarding energies $E_{\text{c.m.}} = V_b + 5$ MeV (triangles), V_b (squares), $V_b - 5$ MeV (circles).

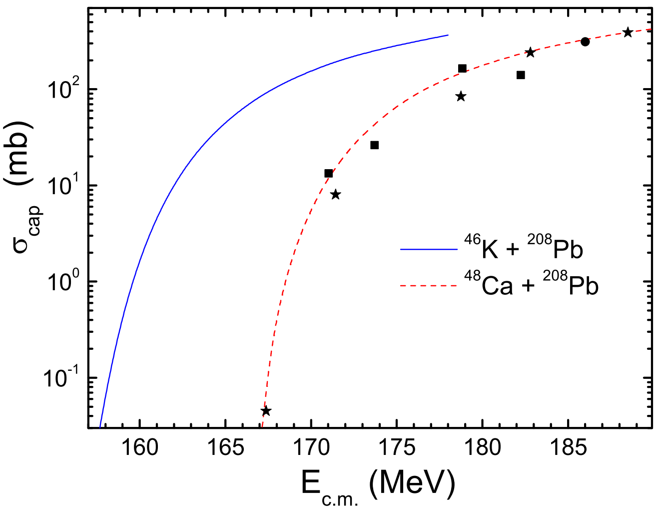

FIG. 3. (Color online) Calculated dependence of capture cross section σ on $E_{\text{c.m.}}$ for the reactions $^{46}\text{K}+{}^{208}\text{Pb}$ (solid line) and $^{48}\text{Ca}+{}^{208}\text{Pb}$ (dashed line). Experimental data (symbols) for the $^{48}\text{Ca}+{}^{208}\text{Pb}$ reaction are from Refs. [17–19].

III. RESULTS OF THE CALCULATIONS

A. Reactions $^{A}\text{K}+{}^{208}\text{Pb}$

In Fig. 1, one can see the comparison of the calculated capture cross sections for the reactions $^{37,39,41,43,45,46,47}\text{K}+{}^{208}\text{Pb}$ with stable and radioactive beams. The sub-barrier cross sections for the reactions $^{37,39,41,43,45}\text{K}+{}^{208}\text{Pb}$ with two-neutron transfer (positive Q_{2n} values) are larger than those for the reactions $^{46,47}\text{K}+{}^{208}\text{Pb}$, where the neutron transfer is suppressed (negative Q_{2n} values). Because after two-neutron transfer the mass numbers and the deformation parameters of the interacting nuclei are changed and the height of the Coulomb barrier decreases, one can expect an enhancement of the capture. For example, after the neutron transfer in the reaction $^{41}\text{K}(\beta_2=0)+{}^{208}\text{Pb}(\beta_2=0)\rightarrow{}^{43}\text{K}(\beta_2=0.25)+{}^{206}\text{Pb}(\beta_2=0)$ [$Q_{2n}=3.1$ MeV], the deformation of the projectile-nucleus increases and the mass asymmetry of the system decreases, and thus the value of the Coulomb barrier decreases and the capture cross section becomes larger (Fig. 1). We observe the same behavior in the reactions with the projectiles ^{37,39,43,45}K. Thus, at sub-barrier energies the complete fusion (capture) cross section is larger in the reactions with two-neutron transfer. As shown in Refs. [4,5], the role of two-neutron transfer in capture is rather complicated. At positive Q_{2n} values the capture is only enhanced if the deformations of colliding nuclei increase after two-neutron transfer.

The capture cross sections for the reactions $^{37,39,41,43,45,46,47}\text{K}+{}^{208}\text{Pb}$ are presented in Fig. 2 at different bombarding energies. The isotopic dependency is rather weak at energies of 5 MeV above the corresponding Coulomb barriers. At sub-barrier energies (5 MeV below the Coulomb barriers) the behavior of the cross section in Fig. 2

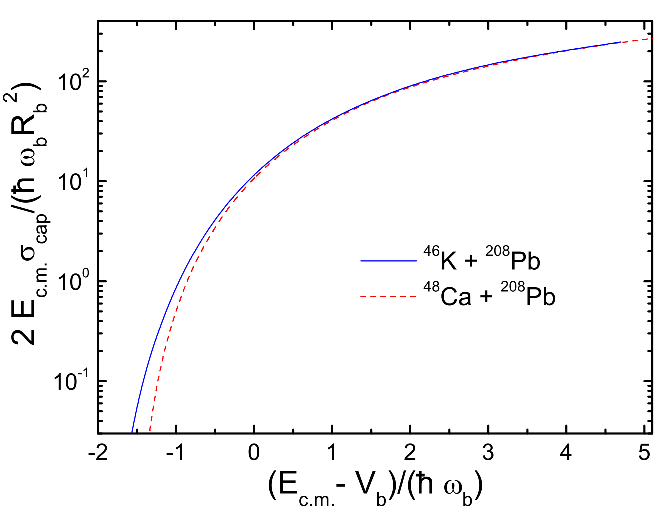

FIG. 4. (Color online) Calculated dependencies of $F(x)=\frac{2E_{\text{c.m.}}}{\hbar\omega_b R_b^2}\sigma$ on $x=\frac{E_{\text{c.m.}}-V_b}{\hbar\omega_b}$ for the reactions $^{46}\text{K}+{}^{208}\text{Pb}$ (solid line) and $^{48}\text{Ca}+{}^{208}\text{Pb}$ (dashed line).

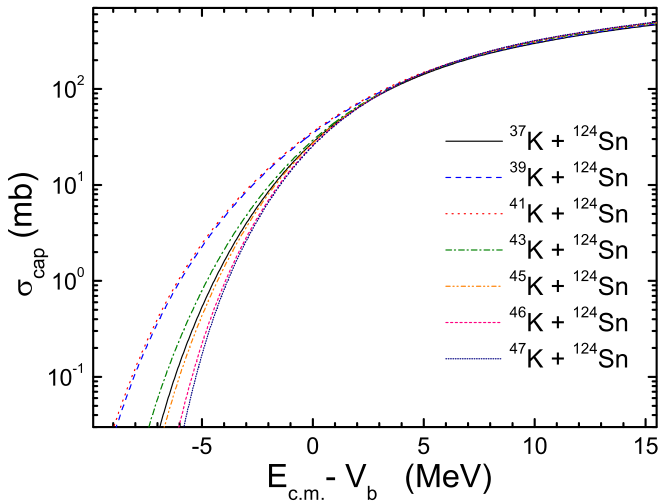

FIG. 5. (Color online) The same as in Fig. 1, but for the indicated reactions $^{A}\text{K}+{}^{124}\text{Sn}$.

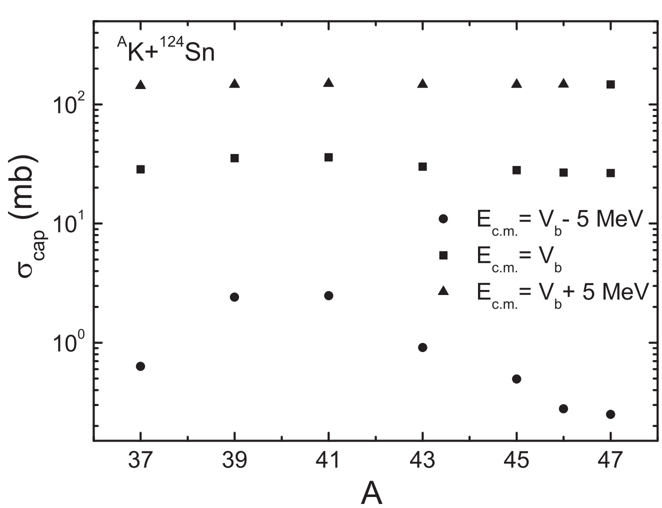

FIG. 6. The same as in Fig. 2, but for the indicated reactions $^{A}\text{K}+{}^{124}\text{Sn}$.

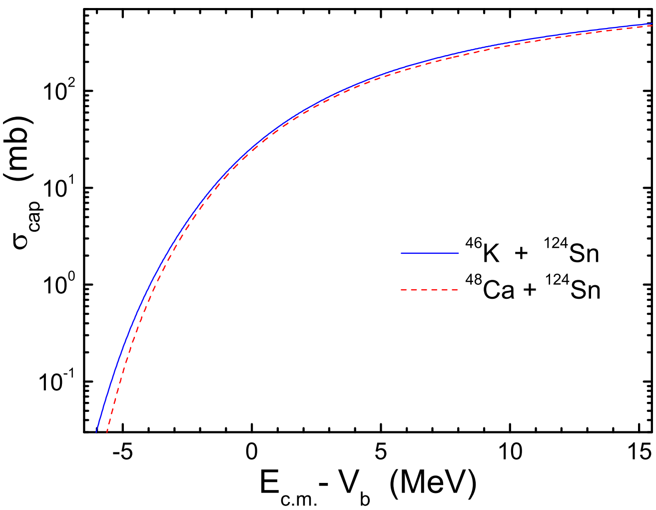

FIG. 7. (Color online) Calculated dependence of capture cross section $\sigma_{\rm cap}$ on $E_{\rm c.m.} - V_b$ for the indicated reactions ^{46}K + ^{124}Sn (solid line) and ^{48}Ca + ^{124}Sn (dashed line).

is determined by the neutron transfer effect. The capture cross section increases by about one order of magnitude with increasing mass number A of the projectile from $A = 37$ ($N = 18$) up to $A = 41$ ($N = 22$) and decreases by about three orders of magnitude from $A = 41$ to $A = 47$ (magic $N = 28$). The reactions with nuclei ^{46,47}K have the smallest cross sections. At the Coulomb barrier energies the cross section changes in a similar way but the curve is much flatter.

The calculated cross sections are compared in Figs. 3 and 4 for the reactions ^{46}K + ^{208}Pb and ^{48}Ca + ^{208}Pb. One can see in Fig. 3 that the slopes of the curves in both reactions are similar and there is only the energy shift between them due to the difference of the heights of their Coulomb barriers. To analyze the isotopic trend of the fusion cross section, it is useful to use the universal fusion function $F(x)$ representation [16]. The advantage of this representation appears clearly when one wants to compare fusion cross sections for systems with different Coulomb barrier heights and positions. In Fig. 4, one can see the comparison of the calculated functions $F(x)[x = (E_{\rm c.m.} - V_b)/(\hbar\omega_b)]$ for the reactions ^{46}K + ^{208}Pb and ^{48}Ca + ^{208}Pb with radioactive and stable beams.

B. Reactions ^{A}K + ^{124}Sn

In Fig. 5 we present the calculated capture cross sections for the reactions ^{37,39,41,43,45,46,47}K + ^{124}Sn at energies above and below the corresponding Coulomb barriers. The sub-barrier cross sections for the reactions ^{37,39,41,43,45}K + ^{124}Sn with positive Q_{2n} values of the two-neutron transfer are larger than those for the reactions ^{46,47}K + ^{124}Sn with the negative Q_{2n} values. We predict the largest (smallest) cross sections for the reactions ^{39,41}K + ^{124}Sn (^{46,47}K + ^{124}Sn). The isotopic dependency is rather weak at energies above and near the corresponding Coulomb barriers (Fig. 6). At sub-barrier energies, the capture cross section increases by about 5 times with increasing mass number A of the projectile from $A = 37$ up to $A = 41$ and decreases by about one order of magnitude from $A = 41$ to $A = 47$ (Fig. 6). In Fig. 7, the calculated capture cross sections as functions of $E_{\rm c.m.} - V_b$ have similar behavior with the same slopes for the reactions ^{46}K + ^{124}Sn and ^{48}Ca + ^{124}Sn.

IV. SUMMARY

The isospin dependence of the capture cross section was found to be strong at sub-barrier energies in the reactions ^{37,39,41,43,45}K + ^{124}Sn , ^{208}Pb. At fixed sub-barrier energy, the cross section increases with mass number of the projectile-nucleus from $A = 37$ up to $A = 41$ and decreases with increasing A from $A = 41$ to $A = 47$. The capture cross sections for the reactions ^{37,39,41,43,45}K + ^{124}Sn , ^{208}Pb with neutron transfer are larger than those for the reactions ^{46,47}K + ^{208}Pb without neutron transfer. At the same $E_{\rm c.m.} - V_b < 0$ the capture cross sections are larger in the reactions with stable ^{39,41}K. We demonstrated the similarity of the reactions ^{46}K + ^{208}Pb (^{46}K + ^{124}Sn) and ^{48}Ca + ^{208}Pb (^{48}Ca + ^{124}Sn). The present calculations provide new insight into the role of isospin and closed shell structures in the entrance channel for the production of new isotopes in fusion reactions.

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