

## Baryon-strangeness correlation in quark combination models

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The baryon-strangeness correlation in the hadronization of quark matter is studied within the quark combination mechanism. We calculate the correlation coefficient  $C_{BS} = -3(\langle BS \rangle - \langle B \rangle \langle S \rangle) / (\langle S^2 \rangle - \langle S \rangle^2)$  of initial hadrons produced from the deconfined free quark system with  $C_{BS}^{(q)} = 1$ . The competition of the production of baryons against that of mesons is the key dynamics that is most relevant to the change of baryon-strangeness correlation during system hadronization. Results of quark combination under the Poisson statistics agree with the statistical model predictions for a hadron resonance gas at vanishing chemical potential but differ from those at relatively large chemical potentials. Results beyond Poisson statistics are also obtained and are compared with calculations of lattice QCD in the quark-hadron phase boundary. We predict the dependence of the  $C_{BS}$  of the hadron system on the baryon chemical potential and strangeness. These predictions are expected to be tested by future lattice QCD calculations at nonzero chemical potentials and/or by the Beam Energy Scan experiment of the STAR Collaboration at RHIC.

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### I. INTRODUCTION

The baryon-strangeness correlation is an effective diagnostic tool for the relevant degrees of freedom of the hot nuclear matter produced in relativistic heavy-ion collisions [1]. It has been extensively investigated by various phenomenological models of high-energy collisions [2–7] and first-principles calculations in lattice QCD [8–10]. The correlation is usually quantified by [1]

$$C_{BS} \equiv -3 \frac{\langle BS \rangle - \langle B \rangle \langle S \rangle}{\langle S^2 \rangle - \langle S \rangle^2} = -3 \frac{\langle BS \rangle}{\langle S^2 \rangle}. \quad (1)$$

Here, angle brackets denote the event or ensemble average. In the second step, strangeness neutrality  $\langle S \rangle = 0$  is applied to relativistic heavy-ion collisions. For a deconfined system consisting of free quarks and antiquarks,  $C_{BS}^{(q)} = 1$  because strangeness is carried only by the strange (anti)quarks which carry the baryon number in strict proportion to their strangeness with the coefficient  $-1/3$ , i.e.,  $B_s = -\frac{1}{3}S_s$ . In contrast, the relation between baryon number and strangeness in a hadron system is multiple, e.g., baryon number 1 for a strange baryon but 0 for a strange meson.  $C_{BS}$  of the hadron system is usually smaller than 1 (at low baryon number density) due to the fact that most strange quarks, at hadronization, will come into mesons instead of baryons, which unlocks the intimate baryon-strangeness correlation existing previously in quarks. Statistical model estimation of  $C_{BS}$  for a hadron resonance gas at zero baryon number density is about 0.66 [1]. Calculations of lattice QCD show that the  $C_{BS}$  of the strongly interacting system at temperature above the phase transition temperature  $T_c$  tends to 1, while near and below  $T_c$  the system  $C_{BS}$  decreases rapidly [8,9].

Hadronization refers to the process of the formation of hadrons out of quarks and/or gluons, accompanied

by the change of the correlation between baryon number and strangeness. Phenomenological models describing the hadronization should reproduce this change of baryon-strangeness correlation due to the transformation of the degrees of freedom in the system. Quark combination is one of the effective mechanisms for the hadronization of the hot quark matter produced in relativistic heavy-ion collisions and has explained many experimental phenomena in heavy-ion collisions at SPS, RHIC, and recently LHC; see, e.g., Refs. [11–16]. The change of baryon-strangeness correlation during quark combination hadronization is intuitive. A strange quark (antiquark) combines with a light antiquark (quark) to form a strange meson, which completely destroys the original baryon-strangeness correlation carried by the strange (anti)quark, i.e.,  $B_{(s\bar{q})} = 0 \times S_{(s\bar{q})}$ , but inherits the strangeness losslessly. Occasionally, a strange quark combines with a strange antiquark to form a hidden-strange meson, which inherits nothing from original quarks. On the contrary, in the baryon formation a strange quark combines with two light quarks to form a baryon, which alters the baryon-strangeness correlation coefficient  $B_{(sqq)} = -S_{(sqq)}$ . The combination of two strange quarks with one light quark also alters the correlation coefficient  $B_{(ssq)} = -\frac{1}{2}S_{(ssq)}$ . Obviously, the  $C_{BS}$  of the system depends on the relative proportion of the produced baryons to mesons.

In this paper, we study the change of system  $C_{BS}$  caused by quark combination hadronization. We discuss in details how the dynamics of baryon-meson production competition at hadronization dominates the  $C_{BS}$  of hadron system. In addition, we study the dependence of  $C_{BS}$  on the strangeness content and the baryon number density of the system, which is related to the hot quark system produced in relativistic heavy-ion collisions at different collisional energies. We compare our results with the calculations of lattice QCD [8,9] and the prediction of the statistical model for a hadron resonance gas [1].

The paper is organized as follows. In Sec. II, we present a working model in the quark combination mechanism (QCM)

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for the yields of hadrons. In Sec. III, using the model we explain the experimental data of the mid-rapidity yields of strange hadrons that are most relevant to  $C_{BS}$  calculations in central Pb+Pb collisions at  $\sqrt{s_{NN}} = 2.76$  TeV. In Sec. IV, we calculate the  $C_{BS}$  of the hadron system in which two assumptions are made for initially produced hadrons. One assumption is that different kinds of hadrons are uncorrelated, which can be expected in the case in which the quark system existing previously is made up of free quarks and antiquarks ( $C_{BS}^{(q)} = 1$ ). We only consider this kind of quark system in this paper since the effects of hadronization on system  $C_{BS}$  are already addressed properly. The other assumption is Poisson statistics for yield fluctuations of hadrons. In Sec. V, we present a different approach of calculating  $C_{BS}$  which is beyond Poisson statistics and can also reflect intuitively the essence of baryon-strangeness correlation. Summary is finally given in Sec. VI.

## II. A WORKING MODEL OF HADRON YIELDS

In this section, we present a working model of the yields of hadrons that are produced from the deconfined quark phase after hadronization in the framework of quark combination mechanism, for the facility of the subsequent correlation studies. As discussed in the introduction, baryon-strangeness correlation  $C_{BS}^{(h)}$  is strongly dependent on the relative production of baryons to mesons, which requires our model to well address this point. Therefore, we introduce the working model according to the following strategy: (1) Give the global properties for the global production of all mesons, all baryons, and all antibaryons, in which the production competition of baryons against mesons is properly addressed. (2) On the basis of (1), give the yield formulas of various identified hadrons.

In previous work [17], we have studied the properties of the global production of all mesons, baryons, and antibaryons, and obtained their yield formulas utilizing only the basic ideas of QCM. We start from a deconfined system consisting of  $N_q$  constituent quarks and  $N_{\bar{q}}$  antiquarks. Here, the gluon contribution to the system on the threshold of hadronization is replaced by quark-antiquark pairs. After hadronization, the system changes the basic degrees of the freedom to become the hadronic system and produces *on average*  $B(N_q, N_{\bar{q}})$  baryons,  $\bar{B}(N_q, N_{\bar{q}})$  antibaryons, and  $M(N_q, N_{\bar{q}})$  mesons. Four properties of hadron production from general principles are used to constrain the behavior of their yield formulas.

(1) Charge conjugation symmetry of the hadron yields,

$$\begin{aligned} B(N_q, N_{\bar{q}}) &= \bar{B}(N_{\bar{q}}, N_q), \\ M(N_q, N_{\bar{q}}) &= M(N_{\bar{q}}, N_q). \end{aligned} \quad (2)$$

(2) Unitarity of the hadronization, i.e., production of mesons, baryons, and antibaryons, should exhaust all quarks and antiquarks of the system existing previously,

$$\begin{aligned} M(N_q, N_{\bar{q}}) + 3B(N_q, N_{\bar{q}}) &= N_q, \\ M(N_q, N_{\bar{q}}) + 3\bar{B}(N_q, N_{\bar{q}}) &= N_{\bar{q}}. \end{aligned} \quad (3)$$

(3) Boundary condition, i.e.,

$$\begin{aligned} \bar{B} &= 0, \quad B = \frac{N_q}{3}, \quad M = 0 \quad \text{if } N_{\bar{q}} = 0, \\ \bar{B} &= \frac{N_{\bar{q}}}{3}, \quad B = 0, \quad M = 0 \quad \text{if } N_q = 0. \end{aligned} \quad (4)$$

(4) Linear response of meson and baryon yields to quark antiquark numbers,

$$\begin{aligned} M(\lambda N_q, \lambda N_{\bar{q}}) &= \lambda M(N_q, N_{\bar{q}}), \\ B(\lambda N_q, \lambda N_{\bar{q}}) &= \lambda B(N_q, N_{\bar{q}}), \\ \bar{B}(\lambda N_q, \lambda N_{\bar{q}}) &= \lambda \bar{B}(N_q, N_{\bar{q}}). \end{aligned} \quad (5)$$

Using these properties, we obtained in Ref. [17] the yield formulas of baryons, antibaryons, and mesons,

$$\begin{aligned} M(x, z) &= \frac{x}{2} \left\{ 1 - z \frac{(1+z)^a + (1-z)^a}{(1+z)^a - (1-z)^a} \right\}, \\ B(x, z) &= \frac{xz}{3} \frac{(1+z)^a}{(1+z)^a - (1-z)^a}, \\ \bar{B}(x, z) &= \frac{xz}{3} \frac{(1-z)^a}{(1+z)^a - (1-z)^a}. \end{aligned} \quad (6)$$

Here, we have rewritten  $x = N_q + N_{\bar{q}}$  which characterizes the bulk property of the system related to the system size or energy and rewritten  $z = (N_q - N_{\bar{q}})/x$  which depicts the asymmetry between quarks and antiquarks in the system ( $|z| \leq 1$ ) and is a measurement of the baryon number density of the system.

The production competition of baryons against mesons is often quantified by the yield ratios  $R_{B/M}(z) = B(x, z)/M(x, z)$  and  $R_{\bar{B}/M}(z) = \bar{B}(x, z)/M(x, z)$ , which are the function of only  $z$ . The factor  $a$  in Eq. (6) represents the degree of the baryon-meson competition by the relation  $a = \frac{1}{3R_{B/M}(0)} + 1$ . Studies in Ref. [16] show that  $R_{B/M}(0)$  of value about 1/12, i.e.,  $a \approx 5$ , can well describe yield ratios of various baryons to mesons in heavy-ion collisions at LHC energy. We note that such a competition as well as yield formulas Eq. (6) can be properly addressed by a phenomenological combination rule in the quark combination model developed by the Shandong group (SDQCM) [14]. In addition, using vested  $R_{B/M}(0)$ , we have successfully explained the yield ratios of various antihadrons to hadrons at the nonzero  $z$  region in relativistic heavy-ion collisions [16–18], i.e., the data of these yield ratios at different collision energies and at different rapidities. We emphasize that through the dependence of  $C_{BS}^{(h)}$  on the factor  $a$  or  $R_{B/M}(0)$  we can address the effects of hadronization on the baryon-strangeness correlation of the system.

To study  $C_{BS}^{(h)}$ , we have to obtain the yield formulas of identified hadrons. Given the total yield of baryons, that of antibaryons, and that of mesons, the inclusive/averaged yields of identified hadrons  $M_i(q_1\bar{q}_2)$ ,  $B_j(q_1q_2q_3)$  are calculated by their individual production weights,

$$N_{M_i} = P_{M_i} M(x, z) = C_{M_i} P_{q_1\bar{q}_2, M} M(x, z), \quad (7)$$

$$N_{B_j} = P_{B_j} B(x, z) = C_{B_j} P_{q_1q_2q_3, B} B(x, z). \quad (8)$$

Here,  $P_{q_1\bar{q}_2, M}$  denotes the probability that, as a meson is known to be produced, the flavor content of this meson is  $q_1\bar{q}_2$ .  $C_{M_i}$

further denotes the branch ratio of this meson with given flavor composition  $q_1\bar{q}_2$  to a specific meson state  $M_i$ .  $C_{M_i}P_{q_1\bar{q}_2,M}$  thus gives the emerging probability of a specific meson,  $P_{M_i}$ , when a meson is known to be formed. Similarly,  $P_{q_1q_2q_3,B}$  denotes the probability that, as a baryon is known to be produced, the flavor content of this baryon is  $q_1q_2q_3$ , and  $C_{B_j}$  denotes the branch ratio of this baryon with given flavor composition  $q_1q_2q_3$  to a specific baryon state  $B_j$ , and  $C_{B_j}P_{q_1q_2q_3,B}$  thus gives the emerging probability of a specific baryon,  $P_{B_j}$ , when a baryon is known to be formed. A similar formula holds for antibaryons.

The probability  $P_{q_1\bar{q}_2,M}$  is posteriorly evaluated by the proportion of  $q_1\bar{q}_2$  pairs in all quark-antiquark pairs in the system,  $P_{q_1\bar{q}_2,M} = N_{q_1\bar{q}_2}/N_{q\bar{q}}$ , by considering the fact that every quark/antiquark has both the probability of entering into a meson and the probability of entering a baryon/antibaryon. Here,  $N_q = \sum_i N_{q_i}$  and  $N_{\bar{q}} = \sum_i N_{\bar{q}_i}$  are total number of quarks and antiquarks in the system, respectively. We have  $N_{q_1\bar{q}_2} = N_{q_1}N_{\bar{q}_2}$  and  $N_{q\bar{q}} = N_qN_{\bar{q}}$ . Similarly, the probability  $P_{q_1q_2q_3,B}$  is evaluated by the proportion of  $q_1q_2q_3$  combinations to all three-quark combinations in the system,  $P_{q_1q_2q_3,B} = N_{\text{iter}}N_{q_1q_2q_3}/N_{qqq}$ .  $N_{q_1q_2q_3}$  is the number of  $q_1q_2q_3$  combinations which satisfies  $N_{q_1q_2q_3} = N_{q_1}N_{q_2}N_{q_3}$  for  $q_1 \neq q_2 \neq q_3$ ,  $N_{q_1q_2q_3} = N_{q_1}(N_{q_1} - 1)N_{q_3}$  for  $q_1 = q_2 \neq q_3$ , and  $N_{q_1q_2q_3} = N_{q_1}(N_{q_1} - 1)(N_{q_1} - 2)$  for  $q_1 = q_2 = q_3$ .  $N_{qqq} = N_q(N_q - 1)(N_q - 2)$  is the number of all  $qqq$  combinations in the system. In the large quark number limit we can apply  $N_{qqq} \approx N_q^3$  and  $N_{q_1q_2q_3} \approx N_{q_1}N_{q_2}N_{q_3}$  always.  $N_{\text{iter}}$  stands for the number of possible iterations of  $q_1q_2q_3$ , which is 1, 3, and 6 for the cases of three identical flavors, two different flavors, and three different flavors, respectively.

$C_{M_j}$  and/or  $C_{B_j}$  denote the branch ratio of a given flavor composition (e.g.,  $u\bar{s}$ ) to a specific hadron state (e.g.,  $K^+$ ) under the condition that they are known to form a hadron. In the case in which the ground state  $J^P = 0^-$  and  $1^-$  mesons and  $J^P = \frac{1}{2}^+$  and  $\frac{3}{2}^+$  baryons are considered only, we have, for mesons,

$$C_{M_j} = \begin{cases} 1/(1 + R_{V/P}) & \text{for } J^P = 0^- \text{ mesons,} \\ R_{V/P}/(1 + R_{V/P}) & \text{for } J^P = 1^- \text{ mesons,} \end{cases}$$

where  $R_{V/P}$  represents the ratio of the  $J^P = 1^-$  vector mesons to the  $J^P = 0^-$  pseudoscalar mesons of the same flavor composition; and for baryons,

$$C_{B_j} = \begin{cases} R_{O/D}/(1 + R_{O/D}) & \text{for } J^P = (1/2)^+ \text{ baryons,} \\ 1/(1 + R_{O/D}) & \text{for } J^P = (3/2)^+ \text{ baryons,} \end{cases}$$

except that  $C_\Lambda = C_{\Sigma^0} = 2R_{O/D}/(1 + 2R_{O/D})$ ,  $C_{\Sigma^{*0}} = 1/(1 + 2R_{O/D})$ ,  $C_{\Delta^{++}} = C_{\Delta^-} = C_{\Omega^-} = 1$ . Here,  $R_{O/D}$  stands for the ratio of the  $J^P = (1/2)^+$  octet to the  $J^P = (3/2)^+$  decuplet baryons of the same flavor composition. The two parameters  $R_{V/P}$  and  $R_{O/D}$  can be determined using the data from different high-energy reactions [14,19], and they are taken to be 3 and 2 in this paper, respectively.

### III. YIELDS OF STRANGE HADRONS AT LHC

With the above working model, we can conveniently predict yields of various identified hadrons. As an illustration, we now explain the experimental data of mid-rapidity yields of strange

hadrons  $K$ ,  $\Lambda$ ,  $\Xi$ , and  $\Omega^-$  in Pb+Pb collisions at  $\sqrt{s_{NN}} = 2.76$  TeV. These hadrons are most relevant to  $C_{BS}$  calculation.

As applying yield formulas in Sec. II to the finite rapidity window in heavy-ion collisions, we note that Eqs. (2), (4), and (5) still hold generally and Eq. (3) also approximately holds with good numerical accuracy due to the locality of the hadronization, and therefore no conceptual issues exist for our mid-rapidity predictions. In addition, our formulas in Sec. II give the averaged yields of hadrons at the given quark system with the fixed quark numbers while the experimental data of hadronic yields are event-averaged quantities. The produced quark system in heavy-ion collisions at a given collision energy is varied in size event-by-event and thus the number of quarks and that of antiquarks should follow a certain distribution around the averaged quark number  $\langle N_{q_i} \rangle$  and antiquark numbers  $\langle N_{\bar{q}_i} \rangle$  (where  $i = u, d, s$  considered in this paper). Our final predictions of hadron yields should be the average over this distribution. In general such averages depend on the precise form of the distribution. Here, we approximate these averages by taking the corresponding values of the quantities at the event averages  $\langle N_{q_i} \rangle$  and  $\langle N_{\bar{q}_i} \rangle$ , i.e.,

$$\langle N_{M_i} \rangle = C_{M_i} \bar{P}_{q_1\bar{q}_2,M} M(\langle x \rangle, \langle z \rangle), \quad (9)$$

$$\langle N_{B_j} \rangle = C_{B_j} \bar{P}_{q_1q_2q_3,B} B(\langle x \rangle, \langle z \rangle), \quad (10)$$

where  $\langle x \rangle = \langle N_q \rangle + \langle N_{\bar{q}} \rangle$ ,  $\langle x \rangle \langle z \rangle = \langle N_q \rangle - \langle N_{\bar{q}} \rangle$ , and  $\bar{P}$  is also calculated with the event-averaged quark numbers. Such approximation can be expected in considering that properties Eqs. (2)–(5) in our derivation of hadron yields are also apparently satisfied even for the event-averaged quantities.

For convenience, we use factors  $\lambda_s = \langle N_{\bar{s}} \rangle / \langle N_{\bar{u}} \rangle = \langle N_{\bar{s}} \rangle / \langle N_{\bar{d}} \rangle$  and associated  $\lambda'_s = \langle N_s \rangle / \langle N_u \rangle = \langle N_s \rangle / \langle N_d \rangle$  to denote the suppression of strange antiquarks relative to light antiquarks and that of strange quarks to light quarks, respectively, and simply the yield formulas of hadrons. Here, isospin symmetry between (anti-)up-quarks and (anti-)down-quarks is applied.

Considering the decay contribution from the short-lived resonances, we obtain the yields of final-state hadrons

$$\langle N_{h_j}^{(f)} \rangle = \langle N_{h_j} \rangle + \sum_k Br(h_k \rightarrow h_j) \langle N_{h_k} \rangle, \quad (11)$$

where we use the superscript ( $f$ ) to denote the results for the final hadrons to differentiate them from those for the directly produced hadrons. The data of the decay branch ratios are taken from the Particle Data Group [20].

With  $\lambda_s = \lambda'_s$  at LHC ( $\langle z \rangle = 0$  approximation), we obtain yields of these strange hadrons in the final state:

$$\begin{aligned} \langle N_{K^\pm}^{(f)} \rangle &= \langle N_{K^+} \rangle + \langle N_{K^{*+}} \rangle Br(K^{*+} \rightarrow K^+) \\ &\quad + \langle N_{K^{*0}} \rangle Br(K^{*0} \rightarrow K^+) \\ &= \left( 1 + 0.493 \frac{R_{V/P}}{1 + R_{V/P}} \lambda_s \right) \frac{\lambda_s}{(2 + \lambda_s)^2} M(\langle x \rangle, 0), \quad (12) \\ \langle N_{\Lambda}^{(f)} \rangle &= \langle N_{\Lambda} \rangle + \langle N_{\Sigma^0} \rangle Br(\Sigma^0 \rightarrow \Lambda) \\ &\quad + \langle N_{\Sigma^{*+}} \rangle Br(\Sigma^{*+} \rightarrow \Lambda) \end{aligned}$$

$$\begin{aligned}
& + \langle N_{\Sigma^{*0}} \rangle Br(\Sigma^{*0} \rightarrow \Lambda) + \langle N_{\Sigma^{*-}} \rangle Br(\Sigma^{*-} \rightarrow \Lambda) \\
& = \frac{7.736\lambda_s}{(2 + \lambda_s)^3} B(\langle x \rangle, 0), \quad (13)
\end{aligned}$$

$$\begin{aligned}
N_{\Xi^-}^{(f)} & = \langle N_{\Xi^-} \rangle + \langle N_{\Xi^{*0}} \rangle Br(\Xi^{*0} \rightarrow \Xi^-) \\
& \quad + \langle N_{\Xi^{*-}} \rangle Br(\Xi^{*-} \rightarrow \Xi^-) \\
& = 3 \frac{\lambda_s^2}{(2 + \lambda_s)^3} B(\langle x \rangle, 0), \quad (14)
\end{aligned}$$

$$\langle N_{\Omega^-}^{(f)} \rangle = \langle N_{\Omega^-} \rangle = \frac{\lambda_s^3}{(2 + \lambda_s)^3} B(\langle x \rangle, 0). \quad (15)$$

Here, we have only taken into account the strong and electric-magnetic (S&EM) decays of short-lived resonances.

Absolute yields of these hadrons are dependent on the total quark number  $\langle x \rangle$  of the system by  $M(\langle x \rangle, 0) = 2\langle x \rangle/5$  and  $B(\langle x \rangle, 0) = \langle x \rangle/30$  with fixed  $R_{B/M}(0) = 1/12$ . To eliminate this  $\langle x \rangle$  dependence, we consider the relative production of these hadrons to pions, i.e., the yield ratios of these hadrons to pions, which finally rely only on the strangeness of the system, besides the baryon-meson competition in hadronization. Since the decay contributions to pion are complex, we here give directly the numerical result of the yield of  $\pi^+$ , i.e.,  $\langle N_{\pi^+}^{(f)} \rangle = 0.213\langle x \rangle$  under S&EM decays, instead of the detailed compositions such as Eqs. (12)–(15). In Fig. 1, we use Eqs. (12)–(15) to explain the experimental data of the mid-rapidity yield ratios  $K^+/\pi^+$ ,  $\Lambda/\pi^+$ ,  $\Xi^-/\pi^+$ , and  $\Omega^-/\pi^+$  in central Pb+Pb collisions at  $\sqrt{s_{NN}} = 2.76$  TeV [36–38]. To incorporate the change of  $\lambda_s$  at different collision centralities, a varied strangeness  $\lambda_s(N_{\text{part}}) = (0.43 \pm 0.02)/(1 + 10.5N_{\text{part}}^{-1.3})$  is used. We can see that the hierarchy properties in yields of these strange hadrons and their  $N_{\text{part}}$  dependence can be systematically described by our formulas Eqs. (12)–(15). Here, we would like to emphasize that the yield difference between  $K$  and those hyperons is mainly due to the baryon-meson competition at hadronization while the hierarchy structures among  $\Lambda$ ,  $\Xi^-$ , and  $\Omega^-$  are mainly strangeness relevant.

Applying our yield formulas to other hadrons at LHC and those at RHIC energies with nonzero baryon number densities, we also find a good agreement with available experimental data. This is not surprise. In fact, QCM has already shown its effectiveness in explaining the data of hadronic yields and longitudinal rapidity distributions in relativistic heavy-ion collisions at different collisional energies [14,16,21–25]. The related low- $p_T$  issues of QCM such as entropy conservation and pion production have been properly addressed [13,21,26–30]. There are also many successful applications of QCM in correlation studies, e.g., multihadron yield correlations [16–18,22,23,31], baryon-meson correlated emission [32,33], and the charge balance function [34,35].

#### IV. $C_{BS}$ OF HADRONS UNDER POISSON FLUCTUATIONS

For initial hadrons produced by the hadronization of deconfined quark systems, the baryon number of the system is  $B = \sum_{\alpha} Q_{\alpha,B} N_{\alpha}$  and the strangeness  $S = \sum_{\alpha} Q_{\alpha,S} N_{\alpha}$ , where the species  $\alpha$  has baryon number  $Q_{\alpha,B}$  and strangeness  $Q_{\alpha,S}$ .

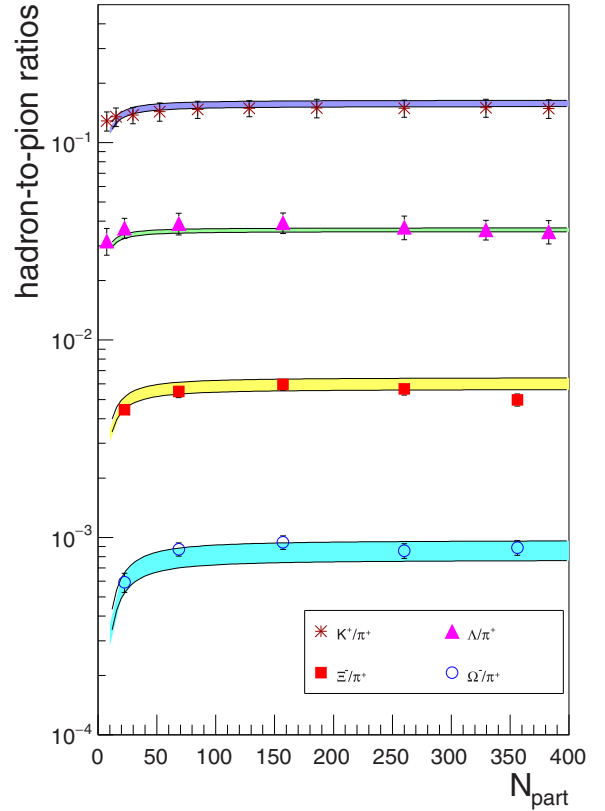


FIG. 1. (Color online) Yield ratios  $K^+/\pi^+$ ,  $\Lambda/\pi^+$ ,  $\Xi^-/\pi^+$ , and  $\Omega^-/\pi^+$  at mid-rapidity as a function of nuclear participants  $N_{\text{part}}$  in central Pb+Pb collisions at  $\sqrt{s_{NN}} = 2.76$  TeV. Symbols are experimental data from Refs. [36–38]. Shadow regions are our results with an  $N_{\text{part}}$ -dependent strangeness  $\lambda_s(N_{\text{part}}) = (0.43 \pm 0.02)/(1 + 10.5N_{\text{part}}^{-1.3})$ .

By definition Eq. (1), the  $C_{BS}$  of hadrons is

$$C_{BS}^{(h)} = -3 \frac{\sum_{\alpha,\beta} Q_{\alpha,B} Q_{\beta,S} C_{\alpha\beta}}{\sum_{\alpha,\beta} Q_{\alpha,S} Q_{\beta,S} C_{\alpha\beta}}, \quad (16)$$

where the covariance  $C_{\alpha\beta} = \langle N_{\alpha} N_{\beta} \rangle - \langle N_{\alpha} \rangle \langle N_{\beta} \rangle$  describes the correlation between hadron  $\alpha$  and hadron  $\beta$ . A general calculation of  $C_{\alpha\beta}$  in QCM is still unavailable in the current progress of hadronization phenomenology since there still are many unsolved dynamics in hadronization due to its nonperturbative feature. In all “on market” combination models, there are a few that can give the calculation of  $C_{\alpha\beta}$  with their own specific model details or assumptions. Such specific calculations are not the purpose of this paper since we intend to analyze the hadronization effects in a general and transparent way. Here, we consider a simple case in which after hadronization different kinds of produced hadrons are uncorrelated, i.e.,  $C_{\alpha\beta} = \delta_{\alpha\beta} \sigma_{\alpha}^2$ . This can be expected if the quark system existing previously is made up of free quarks and antiquarks (i.e.,  $C_{BS}^{(q)} = 1$ ). We note that, above  $T_c$ , the  $C_{BS}$  of strongly interacting system calculated by lattice QCD [9] indeed tends to 1 and the off-diagonal flavor susceptibilities also tend to be relatively small. We only consider this kind of



quark system in this paper because the effects of hadronization on system  $C_{BS}$  are already addressed clearly.

In this section, we first calculate the  $C_{BS}$  of the initial hadron system, assuming the Poisson statistics  $\sigma_\alpha^2 \approx \langle N_\alpha \rangle$  for the yield distribution of identified hadrons. Now we have

$$C_{BS}^{(h)} = -3 \frac{\langle BS \rangle}{\langle S^2 \rangle} \approx -3 \frac{\sum_\alpha Q_{\alpha,B} Q_{\alpha,S} \langle N_\alpha \rangle}{\sum_\alpha Q_{\alpha,S}^2 \langle N_\alpha \rangle}. \quad (17)$$

and

$$\begin{aligned} \langle S^2 \rangle &= \{\langle N_\Lambda \rangle + \langle N_{\Sigma^{\pm,0}} \rangle + \langle N_{\Sigma^{*0}} \rangle\} + 4\{\langle N_{\Xi^{0,-}} \rangle + \langle N_{\Xi^{*0,-}} \rangle\} + 9\langle N_{\Omega^-} \rangle + \text{antihyperons} + \{\langle K^\pm \rangle + \langle K^{*\pm} \rangle + \langle K^0 \rangle \\ &\quad + \langle K^{*0} \rangle + \langle \bar{K}^0 \rangle + \langle \bar{K}^{*0} \rangle\} \\ &= \frac{12\lambda'_s + 24\lambda_s'^2 + 9\lambda_s'^3}{(2 + \lambda_s')^3} B(\langle x \rangle, \langle z \rangle) + \frac{12\lambda_s + 24\lambda_s^2 + 9\lambda_s^3}{(2 + \lambda_s)^3} \bar{B}(\langle x \rangle, \langle z \rangle) + \frac{2\lambda_s + 2\lambda'_s}{(2 + \lambda_s)(2 + \lambda_s')} M(\langle x \rangle, \langle z \rangle). \end{aligned} \quad (19)$$

We note that the above correlations are independent of the parameters  $R_{O/D}$  and  $R_{V/P}$  and thus they are unaffected by S&EM decays. Substituting them into Eq. (17), we obtain

$$C_{BS}^{(h)} = 3 \frac{\frac{3\lambda'_s}{2+\lambda'_s} R_{B/M}(\langle z \rangle) + \frac{3\lambda_s}{2+\lambda_s} R_{\bar{B}/M}(\langle z \rangle)}{\frac{3\lambda'_s(3\lambda'_s+2)}{(2+\lambda'_s)^2} R_{B/M}(\langle z \rangle) + \frac{3\lambda_s(3\lambda_s+2)}{(2+\lambda_s)^2} R_{\bar{B}/M}(\langle z \rangle) + \frac{2\lambda'_s+2\lambda_s}{(2+\lambda'_s)(2+\lambda_s)}}, \quad (20)$$

which gives the dependence of  $C_{BS}^{(h)}$  on baryon-meson competition factor  $R_{B/M}(0)$ , strangeness  $\lambda_s$ , and the baryon number density of the system.

We first consider the situation of zero baryon number density  $\langle z \rangle = 0$  to study the dependence of  $C_{BS}^{(h)}$  on  $R_{B/M}(0)$  and  $\lambda_s$ . With  $\lambda'_s = \lambda_s$  and  $R_{B/M}(0) = R_{\bar{B}/M}(0)$ , we get

$$C_{BS}^{(h)} = 3 \frac{(2 + \lambda_s) R_{B/M}(0)}{(3\lambda_s + 2) R_{B/M}(0) + 2/3}. \quad (21)$$

Figure 2(a) shows the dependence of  $C_{BS}^{(h)}$  on the strangeness  $\lambda_s$ , as the  $R_{B/M}(0)$  is taken to 1/12. One can see that  $C_{BS}^{(h)}$  is insensitive to the change of the strangeness; i.e.,  $C_{BS}^{(h)}$  only increases about 5% as  $\lambda_s$  increases from 0.3 (the rough value in  $pp$  reactions) to 0.7 (almost maximum value occurred in heavy-ion collisions). In addition, we see that  $C_{BS}^{(h)}$  of the initial hadron system is obviously smaller than 1 (the value of the ideal quark system) because the produced strange mesons significantly outnumber the strange baryons in the current baryon-meson competition  $R_{B/M}(0) = 1/12$ . Figure 2(b) shows the dependence of  $C_{BS}^{(h)}$  on baryon-meson competition factor  $R_{B/M}(0)$ , as the strangeness  $\lambda_s$  is taken to the saturated value 0.43 in relativistic heavy-ion collisions. Here, the saturated  $\lambda_s$  is extracted from the fit of yields of strange hadrons in Sec. III and in our previous work [16], and we note that this value is consistent with calculations of lattice QCD in the quark-hadron phase boundary [8,39]. Clearly, we see that the increase of  $R_{B/M}(0)$  will enhance the yields of baryons against mesons and the  $C_{BS}^{(h)}$  increases rapidly. With preferred values  $R_{B/M}(0) = 1/12$  and  $\lambda_s = 0.43$ ,  $C_{BS}^{(h)}$  is about 0.65, much smaller than  $C_{BS}^{(q)} = 1$ . To reach the unit correlation

With Eqs. (9) and (10), we are ready to calculate  $C_{BS}$  of hadrons via Eq. (17),

$$\begin{aligned} \langle BS \rangle &= -\{\langle N_\Lambda \rangle + \langle N_{\Sigma^{\pm,0}} \rangle + \langle N_{\Sigma^{*0}} \rangle\} \\ &\quad - 2\{\langle N_{\Xi^{0,-}} \rangle + \langle N_{\Xi^{*0,-}} \rangle\} - 3\langle N_{\Omega^-} \rangle - \text{antihyperons} \\ &= -\frac{12\lambda'_s + 12\lambda_s'^2 + 3\lambda_s'^3}{(2 + \lambda_s')^3} B(\langle x \rangle, \\ &\quad \langle z \rangle) - \frac{12\lambda_s + 12\lambda_s^2 + 3\lambda_s^3}{(2 + \lambda_s)^3} \bar{B}(\langle x \rangle, \langle z \rangle) \end{aligned} \quad (18)$$

for hadrons, an extremely high baryon-meson competition factor  $R_{B/M}(0) \approx 1/5$  is needed, which is completely unable to reproduce yields of strange mesons and baryons.

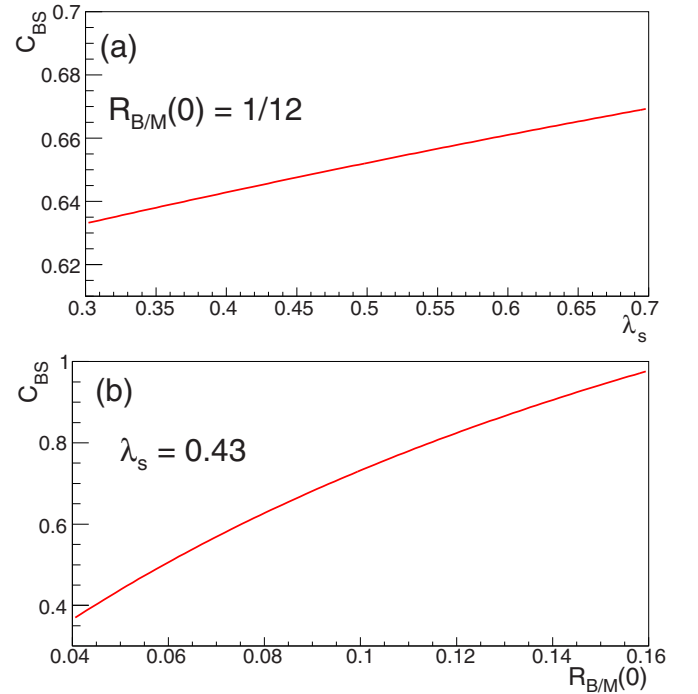


FIG. 2. (Color online) (a) Dependence of  $C_{BS}^{(h)}$  on strangeness suppression factor  $\lambda_s$ ; (b) on baryon-meson competition factor  $R_{B/M}(0)$ .

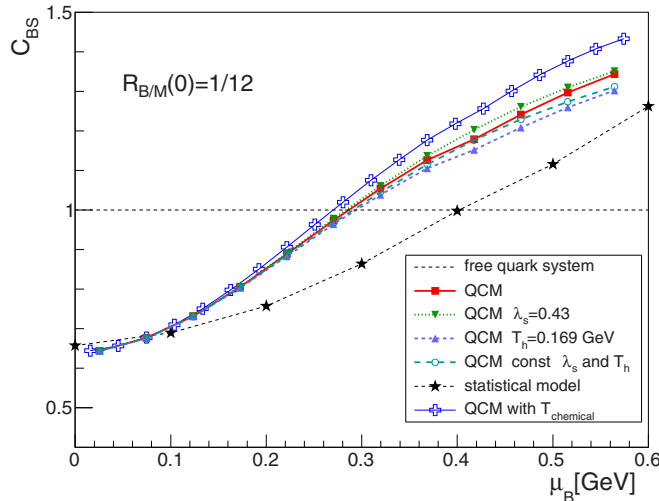


FIG. 3. (Color online) The dependence of  $C_{BS}^{(h)}$  on the chemical potential  $\mu_B$  of the system. The lines with filled squares, down-triangles, up-triangles, and open circles are our results at both varied  $\lambda_s$  and  $T_h$ , at constant  $\lambda_s = 0.43$ , at constant  $T_h = 0.169$  GeV, and at both constant  $\lambda_s = 0.43$  and  $T_h = 0.169$  GeV, respectively. The solid line with open cross symbols is our results using the chemical freeze-out temperature. They are compared with the prediction of statistical model for hadron resonance gas [1], the dashed line with stars.

Subsequently, we study the dependence of  $C_{BS}^{(h)}$  on the baryon number density of the system. In previous discussions, we use the quark-antiquark asymmetry  $\langle z \rangle$  to characterize the baryon number density of the system. To compare our results with existing predictions of statistical models, we alternatively use the chemical potential  $\mu_B$  which relates  $\langle z \rangle$  via

$$\langle z \rangle = \frac{2 \sinh\left(\frac{\mu_B}{3T_h}\right)}{2 \cosh\left(\frac{\mu_B}{3T_h}\right) + \lambda_s \exp\left(-\frac{\mu_B}{3T_h}\right)}, \quad (22)$$

under the assumption of the Boltzmann distribution for thermalized quarks and antiquarks. Here, strangeness neutrality  $N_s = N_{\bar{s}}$  is applied.  $T_h$  is the temperature of the quark system at hadronization.

Figure 3 shows our predictions of the  $C_{BS}^{(h)}$  of initial hadrons as a function of  $\mu_B$ . Here, we have taken into account the fact that both  $T_h$  and  $\lambda_s$  are varied with the  $\mu_B$  for the hot quark matter produced in relativistic heavy-ion collisions. For the  $\mu_B$  dependence of  $T_h$ , we apply the calculation of lattice QCD by Endrődi *et al.* [40] for strange deconfinement temperature, i.e.,  $T_h(\mu_B) = T_0(1 - 0.0089\mu_B^2/T_0^2)$  with  $T_0$  being 0.169 GeV, the transition temperature at  $\mu_B = 0$ . For the  $\mu_B$  dependence of  $\lambda_s$ , we use our previous extractions at mid-rapidity in heavy-ion collisions at different collision energies [14,25], i.e.,  $\lambda_s = (0.43, 0.43, 0.44, 0.44, 0.48, 0.5, 0.57, 0.8, 0.7)$  as  $\sqrt{s_{NN}} = (2760, 200, 130, 62.4, 17.3, 12.3, 8.7, 7.6, 6.3)$  GeV, and convert the energy dependence into  $\mu_B$  dependence by the formula  $\sqrt{s_{NN}} = (1.308 \text{ GeV}/\mu_B - 1)/0.273 \text{ GeV}$  in Refs. [41,42]. The solid line with filled square symbols is our result. We also calculate  $C_{BS}^{(h)}$  at fixed  $\lambda_s = 0.43$  (dotted line with down-triangles), at fixed  $T_h = 0.169$  GeV (dashed

line with up-triangles), and at both fixed  $T_h = 0.169$  GeV and fixed  $\lambda_s = 0.43$  (dashed line with open circles).

A striking behavior of  $C_{BS}^{(h)}$  of initial hadrons is that it increases with the increasing  $\mu_B$  and in the large- $\mu_B$  region ( $\mu_B \gtrsim 0.3$  GeV) it surpasses the unity and become higher at larger  $\mu_B$ . This is in sharp contrary to the free quark system existing previously where the correlation coefficient remains strictly unity at all temperatures and chemical potentials. This is because as the  $\mu_B$  increases, the relative production of baryons to mesons, i.e.,  $R_{B/M}(z)$  in our approach, becomes large, and then the weight of this item in the  $C_{BS}$  formula Eq. (20) increases and thus  $C_{BS}$  increases correspondingly. Comparing the result of  $C_{BS}$  at fixed  $T_h$  with those with varied  $T_h$ , we see that the change (decrease) of  $T_h$  at large chemical potential increases the  $C_{BS}$  of the system by several percent. Comparing the result of  $C_{BS}$  at fixed  $\lambda_s$  with those with varied  $\lambda_s$ , we find that the change of the strangeness also slightly influences the  $C_{BS}$  of the system.

We also compare our results with the early prediction of Koch *et al.* [1] (the dotted line with star symbols in Fig. 3) for a hadron resonance gas in the statistical treatment, considering that (1) the chemical freeze-out temperature in the statistical hadronization model is close to (or can be) the hadronization temperature in relativistic heavy-ion collisions due to the very rapid longitudinal and transverse expansion in the late stage of system evolution [43,44], and (2) Poisson fluctuation is the common feature in statistical model. A  $\mu_B$ -dependent chemical freeze-out temperature,  $T_{\text{chemical}}(\mu_B) \approx 0.17 - 0.13\mu_B^2 - 0.05\mu_B^4$  in Ref. [45], is used in their prediction, which falls obviously below the transition temperature we used above at intermediate and large  $\mu_B$ . For comparison, we also calculate the  $C_{BS}$  of initial hadrons using their chemical freeze-out temperature and the result is presented in Fig. 3 as the solid line with open cross symbols. We find that our result is close to the prediction of the statistical model at small  $\mu_B$  but at the intermediate and large  $\mu_B$  region our result is about 15%–20% larger than that of the statistical model. We note that recently Becattini *et al.* [46] have reconstructed the original hadro-chemical equilibrium temperature after considering the effects of the final hadron expansion phase and the results have closely followed the phase transition boundary line predicted by lattice QCD [40].

## V. $C_{BS}$ OF HADRONS BEYOND POISSON STATISTICS

In the above calculations of  $C_{BS}^{(h)}$ , Poisson statistics for hadronic yields is applied as an open approximation. In this section, we calculate the  $C_{BS}^{(h)}$  of the initial hadron system by another approach which is independent of the Poisson statistics assumption and also reflects intuitively the essence of the baryon-strangeness correlation.

To probe the intrinsic correlation between the baryon number and strangeness for a hadron system, we suppose that the system has a change by stochastically emitting a small amount of strange hadrons and then the system strangeness changes  $\delta S^{(h)}$  and the baryon number also changes  $\delta B^{(h)}$ . Regarding the baryon number of the system  $B^{(h)}$  as a function of the strangeness  $S^{(h)}$ , the change of baryon number due to

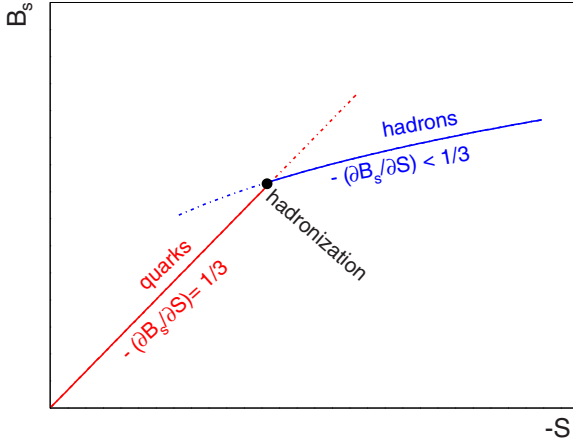


FIG. 4. (Color online) Schematic picture for the ideal quark phase and the hadron phase in  $B$ - $S$  plane.

the perturbation of strangeness can be calculated by

$$\delta B^{(h)} = \left( \frac{\partial B_s^{(h)}}{\partial S^{(h)}} \right) \delta S^{(h)} + \mathcal{O}((\delta S^{(h)})^2), \quad (23)$$

With yield formulas Eqs. (7) and (8), we have

$$\begin{aligned} B_s^{(h)} &= N_\Lambda + N_{\Sigma^{\pm,0}} + N_{\Sigma^{* \pm,0}} + N_{\Xi^{0,-}} + N_{\Xi^{*0,-}} + N_{\Omega^-} - \text{antihyperons} \\ &= \frac{6N_{uds} + 3N_{uus} + 3N_{dds} + 3N_{uss} + 3N_{dss} + N_{sss}}{N_{qqq}} B(x, z) - \frac{6N_{\bar{u}\bar{d}\bar{s}} + 3N_{\bar{u}\bar{u}\bar{s}} + 3N_{\bar{d}\bar{d}\bar{s}} + 3N_{\bar{u}\bar{s}\bar{s}} + 3N_{\bar{d}\bar{s}\bar{s}} + N_{\bar{s}\bar{s}\bar{s}}}{N_{\bar{q}\bar{q}\bar{q}}} \bar{B}(x, z), \end{aligned} \quad (25)$$

and the strangeness of initial hadrons  $S^{(h)} = \sum_\alpha Q_{\alpha,S} N_\alpha = -(N_s - N_{\bar{s}})$  because all strange quarks and antiquarks are combined into hadrons after hadronization. Then, we have

$$\begin{aligned} C_{BS}^{(h)} &= 3 \times \left\{ \frac{24 B(1, \langle z \rangle)}{(2 + \lambda'_s)^3 (1 + \langle z \rangle)} + \frac{24 \bar{B}(1, \langle z \rangle)}{(2 + \lambda_s)^3 (1 - \langle z \rangle)} + \frac{12\lambda'_s + 6\lambda_s^2 + 6\lambda_s^3}{(2 + \lambda'_s)^3} (1 + \langle z \rangle)^{a-1} \right. \\ &\quad \times \frac{[(1 - \langle z \rangle)^{a-1} (\langle z \rangle^2 - 2a\langle z \rangle - 1) + (1 + \langle z \rangle)^{a+1}]}{3[(1 + \langle z \rangle)^a - (1 - \langle z \rangle)^a]^2} \\ &\quad \left. + \frac{12\lambda_s + 6\lambda_s^2 + 6\lambda_s^3}{(2 + \lambda_s)^3} (1 - \langle z \rangle)^{a-1} \frac{[(1 + \langle z \rangle)^{a-1} (\langle z \rangle^2 + 2a\langle z \rangle - 1) + (1 - \langle z \rangle)^{a+1}]}{3[(1 + \langle z \rangle)^a - (1 - \langle z \rangle)^a]^2} \right\} \end{aligned} \quad (26)$$

for the initial hadron system. This result shows a complex dependence on baryon-meson competition factor  $R_{B/M}(0)$  by the factor  $a = \frac{1}{3R_{B/M}(0)} + 1$ , strangeness  $\lambda_s$ , and baryon number density of the system  $z$ .

In the case of baryon number density  $z = 0$ , we have  $\lambda'_s = \lambda_s$  and  $B(1, 0) = \bar{B}(1, 0) = 1/6a$  and simplify Eq. (26),

$$C_{BS}^{(h)} = 1 + 8 \frac{6R_{B/M}(0) - 1}{(2 + \lambda_s)^3 [1 + 3R_{B/M}(0)]}. \quad (27)$$

Figure 5(a) shows the dependence of  $C_{BS}^{(h)}$  on the strangeness  $\lambda_s$ , as the  $R_{B/M}(0)$  is taken to 1/12. We see that  $C_{BS}^{(h)}$  of initial hadrons in the current approach is also insensitive to the strangeness of the system, but the result is about 20% larger than the previous prediction in Fig. 2(a) in Poisson

where  $B_s^{(h)}$  is the baryon number carried by strange hadrons. To second-order fluctuation of strangeness we get

$$C_{BS}^{(h)} = -3 \frac{\langle \delta B^{(h)} \delta S^{(h)} \rangle}{\langle \delta S^{(h)2} \rangle} \approx -3 \left. \frac{\partial B_s^{(h)}}{\partial S^{(h)}} \right|_{\langle N_h \rangle}. \quad (24)$$

The partial derivative is evaluated at the event average values of hadron yields  $\langle N_h \rangle$ . Note that here we do not consider the correlations between light and strange hadrons induced by the interactions among hadrons; i.e., we consider an ideal hadron gas system after hadronization.

The same philosophy can be applied to the quark system before hadronization. Considering that the quark system is made up of free quarks and antiquarks with three flavors, the strangeness of the system  $S^{(q)} = -(N_s - N_{\bar{s}})$  and the baryon number carried by these strange quarks and antiquarks  $B_s^{(q)} = \frac{1}{3}(N_s - N_{\bar{s}})$ , and we get  $C_{BS}^{(q)} = 1$  exactly. The difference of  $C_{BS}$  between quarks and initial hadrons can be illustrated by the different slopes of two phases in the  $B - S$  plane, as shown in our schematic Fig. 4. The cross point between two phases stands for the hadronization that changes the basic degrees of freedom of the system and also stands for the global charge conservations during hadronization.

fluctuation. Figure 5(b) shows the dependence of  $C_{BS}$  on the baryon-meson competition factor  $R_{B/M}(0)$ . We see that  $C_{BS}^{(h)}$  is strongly dependent on the  $R_{B/M}(0)$ , which is also qualitatively consistent with the result in Fig. 2(b).

The qualitative consistency between Fig. 5 and Fig. 2 can be naturally understood via the definition  $C_{BS} = -3\langle BS \rangle / \langle S^2 \rangle$ . Because both the numerator  $\langle BS \rangle$  and denominator  $\langle S^2 \rangle$  are dependent on the strangeness mainly by the single-strange hadrons, the strangeness dependence is partially offset in their ratio and we observe a weak dependence of  $C_{BS}^{(h)}$  on  $\lambda_s$ . On the other hand, since the numerator  $\langle BS \rangle$  receives larger contributions from multi-strange baryons than the denominator  $\langle S^2 \rangle$  which is always dominated by single-strange mesons, we would observe a slightly increase of  $C_{BS}^{(h)}$  with the increasing  $\lambda_s$ . In addition, the increase of  $R_{B/M}(0)$  enhances the production

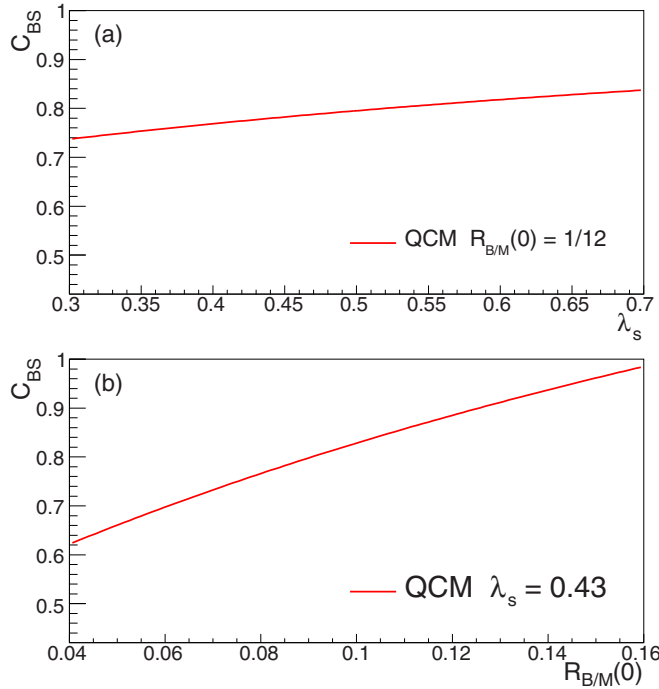


FIG. 5. (Color online) The dependence of  $C_{BS}^{(h)}$  on strangeness suppression factor  $\lambda_s$  (a) and on baryon-meson competition factor  $R_{B/M}(0)$  (b).

weights of (strange) baryons against mesons and contributes largely to the numerator  $\langle BS \rangle$  and we observe a rapid increase of  $C_{BS}^{(h)}$  with the increasing  $R_{B/M}(0)$ .

With the preferred parameter values  $R_{B/M}(0) = 1/12$  and  $\lambda_s = 0.43$ , our calculated  $C_{BS}^{(h)}$  by Eq. (27) is 0.777 at vanishing quark-antiquark asymmetry. In Fig. 6, we compare this result (the dashed line) with calculations of lattice QCD [9,10] at  $\mu_B = 0$  in the quark-hadron phase boundary. The cross point between the result of the microscopic QCM and lattice QCD calculations at finite temperature locates a characteristic

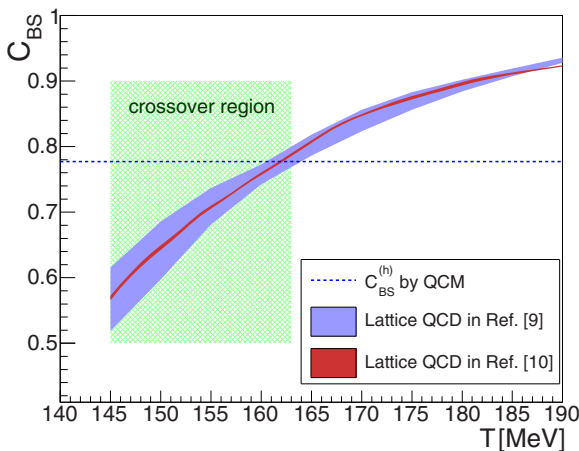


FIG. 6. (Color online) Comparison of hadronic  $C_{BS}$  calculated by QCM with lattice QCD calculations in the quark-hadron phase boundary [9,10].

temperature of hadron production at hadronization. The validity of QCM can be tested by comparing this characteristic temperature with other theory or model results. We see that the cross point is located at 162 MeV with small uncertainty about 0.2 MeV constrained by the latest data of the HotQCD Collaboration [10] and a relatively large uncertainty about 2 MeV constrained by the data of the Wuppertal-Budapest Lattice QCD Collaboration [9].

This temperature can be comparable with the chemical freeze-out temperature  $T_{ch}$  in the statistical hadronization model because (1)  $T_{ch}$  is close to (or can be) the hadronization temperature in relativistic heavy-ion collisions due to very rapid longitudinal and transverse expansion in system evolution [43,44] and (2) our model parameters are chosen by comparing with experimental hadronic yields. Therefore, 162 MeV is our estimation of  $T_{ch}$  for strange hadrons at  $\mu_B = 0$ . This value is within the range of current  $T_{ch}$  estimation ( $\sim 140$ – $170$  MeV) [46–56]. We note that  $T_{ch}$  extracted from strange hadrons is usually above 160 MeV [36,38,48], closer to our estimation.

Comparing this temperature with the strangeness deconfinement temperature calculated in lattice QCD, we also find consistency. This is compatible with the discovery of Ref. [57] that strangeness deconfinement sets in around the chiral crossover temperature  $T_c = 154 \pm 9$  MeV. For our result hitting the upper boundary of the  $T_c$  region, it may be related with the flavor hierarchy in the deconfinement transition suggested by Ref. [58] which shows a subtle difference of about 15 MeV in light and strange sectors.

We further give our final prediction of  $C_{BS}^{(h)}$  at nonzero chemical potential  $\mu_B$ . The results are shown in Fig. 7. Here, the transformation of variable  $z$  to  $\mu_B$  is by Eq. (22) and the  $\mu_B$  dependence of  $\lambda_s$  is the same as that in Sec. IV. For the  $\mu_B$  dependence of hadronization temperature  $T_h$ , we also

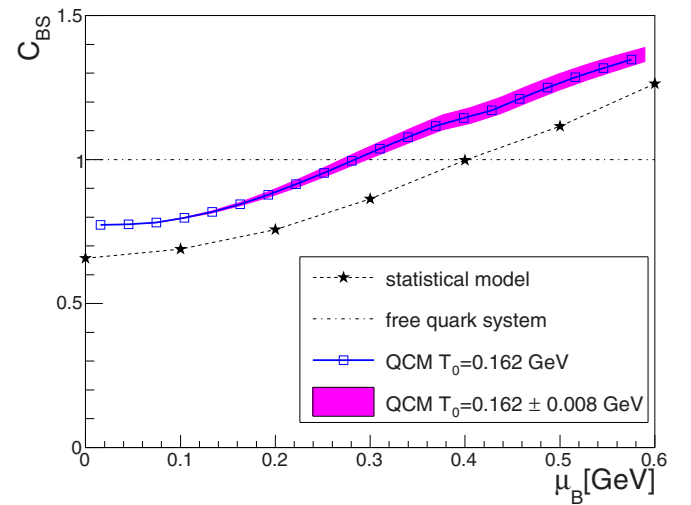


FIG. 7. (Color online) The dependence of hadronic  $C_{BS}$  on the chemical potential  $\mu_B$  of the system. The open squares with solid line are our results at both varied  $\lambda_s$  and  $T_h$  with  $T_0 = 0.162$  GeV. The band area shows the uncertainties due to  $T_0 = 0.162 \pm 0.008$  GeV. They are compared with the prediction of the statistical model for a hadron resonance gas [1].



apply the form  $T_h(\mu_B) = T_0(1 - \kappa\mu_B^2/T_0^2)$  for the strangeness deconfinement in Ref. [40]. The temperature  $T_0$  at zero  $\mu_B$ , as discussed above, still has a relatively large uncertainty due to the current precision of lattice QCD calculations and inconclusive chemical freeze-out temperature extracted in relativistic heavy-ion collisions. We take  $T_0 = 162 \pm 8$  MeV and the resulting  $C_{BS}^{(h)}$  is shown as the band area in Fig. 7. The choice of center value 162 MeV is based on the above discussions of  $C_{BS}^{(h)}$  at  $\mu_B = 0$  and the lower/upper limit 154/170 MeV is available at the chiral or strangeness phase transition temperature in lattice QCD [40,59]. For the curvature parameter  $\kappa$ , we take 0.0089 calculated in Ref. [40], considering that this curvature can be reproduced by the statistical model fitting of hadron yields in relativistic heavy-ion collisions [46]. We also note that the newest calculations [60] with some different definitions and approaches in lattice QCD give a stronger  $\kappa = 0.0149$  for  $\mu_b \leq 300$  MeV. In this small- $\mu_B$  region, however, the increase of  $\kappa$  causes few changes for our prediction of  $C_{BS}^{(h)}$ . In summary, the uncertainty of  $T_h$  shows slight influence on  $C_{BS}^{(h)}$  prediction at small  $\mu_B$  and weak influence (less than about 5%) at intermediate and high  $\mu_B$ .

Comparing our results with the early predictions of the statistical model [1] (dashed line with stars) for the hadron resonance gas, we find that our results are about 15% higher than the predictions of the statistical model in all  $\mu_B$  regions. This leads to a different prediction for the position at which the  $C_{BS}$  of hadrons reaches 1. QCM predicts the position at  $\mu_B \approx 0.29$  GeV while the statistical model predicts the position at  $\mu_B \approx 0.4$  GeV. Interestingly, it has been shown recently [10] that by supplementing the hadron list with additional, experimentally uncharted strange hadrons but predicted by the quark model and observed in the lattice QCD spectrum in the conventional statistical model calculations, the  $C_{BS}^{(h)}$  can increase to about 0.8 at  $\mu_B = 0$ . This is very close to our result at  $\mu_B = 0$  (within 3% difference). With the expectation of the same increase magnitude at nonzero  $\mu_B$ , the statistical model predictions will closely follow our results, yielding the nearly same prediction of  $C_{BS}^{(h)} = 1$  at  $\mu_B \approx 0.3$  GeV. The Beam Energy Scan program of the STAR Collaboration can test our model results and the above subtle effect in the statistical model by precise measurement at collisional energy 9–12 GeV.

## VI. SUMMARY

Hadronization describes the process of the formation of hadrons out of quarks and/or gluons. In the meantime, the correlation properties between conservative charges of the system change also due to the transformation of basic degrees of freedom of the system. Phenomenological models of the hadronization should reproduce these changes of charge correlation properties. In this paper, we have studied the

change of the baryon-strangeness correlation caused by quark combination hadronization. We found that this correlation is a good quantity for studying the hadronization because of its sensitivity to the dynamics of production competition between baryons and mesons. We calculated the correlation coefficient  $C_{BS} = -3(\langle BS \rangle - \langle B \rangle \langle S \rangle) / \langle S^2 \rangle$  of initial hadrons produced from the deconfined free quark system with  $C_{BS}^{(q)} = 1$ . The calculated  $C_{BS}^{(h)}$  of initial hadrons under Poisson fluctuation is about 0.65 at zero baryon chemical potential, which is consistent with the prediction of statistical model for the hadron resonance gas. Beyond Poisson statistics, we calculated  $C_{BS}^{(h)}$  of initial hadrons by  $C_{BS}^{(h)} = -3\partial B_s^{(h)} / \partial S^{(h)}$ . The resulting  $C_{BS}^{(h)}$  at zero baryon chemical potential is about 0.777. This value is consistent with calculations of lattice QCD in the quark-hadron phase boundary by noticing that our estimation of the characteristic temperature of hadronization is within the region of strangeness deconfinement in available lattice QCD calculations and is also close to the chemical freeze-out temperature extracted from the yield data of strange hadrons in relativistic heavy-ion collisions. This suggests that the quark combination is able to describe the change of conservative charge correlations at hadronization, revealing certain basic dynamics of the realistic hadronization process. We also predicted the correlation coefficient of hadrons at different baryon chemical potentials and compared them with the existing calculations of statistical method. These predictions are expected to be tested by the Beam Energy Scan experiment of the STAR Collaboration at RHIC and/or by the future lattice QCD calculations at nonzero chemical potentials.

Finally we would like to comment on the measurement of  $C_{BS}$  directly by the definition Eq. (1) either in realistic experiments or in model simulations using an event generator. To measure the  $C_{BS}$  in general we usually have to select a finite accept window, e.g., a mid-rapidity region  $|y| < 0.5$ . Selecting a small window size will increase the Poisson statistical fluctuations while selecting a large window size will inevitably involve the global conservation effect of baryon number and in particular the strangeness. Amending such a finite window effect is usually complex. These considerations prompt us to calculate the  $C_{BS}$  of the system in Sec. V by the response of the baryon number of the system with respect to the change of the system strangeness. This method is less relevant to the accept window size and thus can closely reflect the intrinsic baryon-strangeness correlation of the hadron system.

## ACKNOWLEDGMENTS

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