Magnetic moments in odd-*A* Cd isotopes and coupling of particles with zero-point vibrations

S. Mishev*
Joint Institute for Nuclear Research, 6 Joliot-Curie Street, Dubna 141980, Russia and Institute for Advanced Physical Studies, New Bulgarian University, 21 Montevideo Street, Sofia 1618, Bulgaria

V. V. Voronov
Joint Institute for Nuclear Research, 6 Joliot-Curie Street, Dubna 141980, Russia

(Received 23 July 2015; published 28 October 2015)

Background: The coupling of the last nucleon with configurations in the ground state of the even-even core is known to augment the single quasiparticle fragmentation pattern. In a recent experimental study by Yordanov *et al.* the values of the magnetic dipole and electric quadrupole moments of the $11/2^-$ state in a long chain of Cd isotopes were found to follow a simple trend which we try to explain by means of incorporating long-range correlations in the ground state.
Purpose: Our purpose is to study the influence of ground-state correlations (GSCs) on the magnetic moments and compare our results with the data for the odd-*A* Cd isotopes.
Method: In order to evaluate if the additional correlations have bearing on the magnetic moments we employ an extension to the quasiparticle-phonon model (QPM) which takes into account quasiparticle⊗phonon configurations in the ground state of the even-even core affecting the structure of the odd-*A* nucleus wave function.
Results: It is shown that the values for the magnetic moments which the applied QPM extension yields deviate further from the Schmidt values. The latter is in agreement with the measured values for the Cd isotopes.
Conclusions: The GSCs exert significant influence on the magnetic dipole moments and reveal a potential for reproducing the experimental values for the studied cadmium isotopes.

DOI: 10.1103/PhysRevC.92.044329 PACS number(s): 21.60.Jz, 21.60.Ev, 21.60.Cs, 27.60.+j

I. INTRODUCTION

The important role that the particle-vibration interaction plays in explaining the deviations of nuclear magnetic moments from the Schmidt values was first acknowledged in Refs. [1,2]. The strength of this interaction depends mainly on the state that the participating nucleon occupies as well as on the distribution of multiparticle-multihole configurations constituting the vibrating core. Intuitively clear, and still supported by the experimental data, is the finding that this interaction is weak near the magic nuclei and becomes substantial in the open-shell regions. For example, performing calculations relying on the first- and second-order perturbation theory, as shown in Ref. [3], this interaction manifests itself as capable of explaining the differences between the magnetic moments in the near magic odd-*A* Tl isotopes. In the open-shell regions, however, there are a large number of nuclei, referred to as transitional, in which the diversity of configurations contributing to the vibration grows rapidly, and the terms which could be neglected near the magic configurations are no longer small. In this respect the data and analysis by Yordanov *et al.* [4], which we interpret in this work, are important for at least two reasons. In the first place, the long chain of cadmium isotopes, which is explored in Ref. [4], enters the transitional region with respect to the neutron subsystem. Second, the measured quantities, namely, the magnetic dipole and electric quadrupole moments, not only triggered the invention of the nuclear shell model and the collective model [5,6] but still are a major tool to test the validity of modern nuclear theories.

The theoretical interpretations of the data concerning the quadrupole moments, reported in Ref. [4], were previously carried out by using two kinds of pairing models: the seniority scheme [7] and the BCS approximation [8]. Decent agreement with the experimental data is reached by the authors of Ref. [4] using the seniority model at the cost of introducing an effective neutron charge and by neglecting configurations with seniorities greater than 1. A more robust study, which does not compromise the use of effective charges, is performed in Ref. [9] by employing the BCS approximation to account for the pairing of nucleons. The important result of this study is the estimation that the contribution from a polarized core with 40 protons to the quadrupole moment is as large as the contribution from the valence protons in the $g_{9/2}$ orbital. The authors of Ref. [10] elaborated on the influence of the density dependence of the effective pairing interaction on the quadrupole moments in odd-*N* Sn and Pb nuclei, which they found to be notable and produce deviations between 10% and 50% in different isotopes. The electric quadrupole moments in odd-*A* Sn nuclei have been successfully reproduced in Ref. [11] using the nucleon-pair approximation of the shell model, and also the magnetic moments were found to be very close to their single-particle estimates.

If the pairing correlations seem sufficient to describe the trends in the behavior of the quadrupole moments, the magnetic moments of the low-lying states require further efforts mainly due to the role of the 1^+ magnetic excitations and the nucleon correlations in the ground state induced by

*mishev@theor.jinr.ru

the long-range part of the residual nucleon-nucleon force. Although the contribution to the wave function coming from configurations owing to the coupling with the magnetic giant dipole resonance are small, their influence on the magnetic moments is significant because of the strong $M1$ transition to the ground state. In Ref. [12] a systematic theoretical analysis of experimental data on magnetic moments in different nuclei is performed utilizing the theory of finite Fermi systems. The effects of the *M1* giant resonance on the magnetic moments (the Arima-Horie effect) were the main focus of many research studies while the contribution from other modes seem to be a less explored territory. The configuration mixing generated by such modes can be taken into account in the calculations of the magnetic moments by either introducing effective two-body operators [13,14] or evaluating the average values of the single-particle operator in multiparticle configurations.

The influence on the magnetic moments coming from the coupling of the last nucleon with the low-lying collective quadrupole and octupole core excitations is studied in Ref. [15]. The generated admixtures from these interactions were found to give important contributions in most of the studied isotopes but are most considerable in odd-Z nuclei.

In the present development we account for the above effects by relying on the concepts and instruments of the quasiparticle phonon model (QPM) [16] without making use of two-body operators for the magnetic moment. Of special interest for our research is the evaluation of the role of the quasiparticle and the quasiparticle⊗phonon configurations residing in the ground state of the even-even core on the magnetic moments. This problem, which has not been explored so far, is approached by allowing nonzero values for the amplitudes in the wave function of an odd-A nucleus corresponding to the above configurations [17–20]. Our previous investigation on this topic showed considerable fragmentation of the low-energy single-quasiparticle states due to such long-range correlations. The latter is a crux in reliably applying the perturbation theory, which otherwise yields rather tractable results [1–3,21]. In applying this, idea using the so-called backward-going amplitudes in the odd-A nucleus wave function to calculate the expectation value of the magnetic operator, we registered considerable shifts in the magnetic moments. The subtle effects owing to the account of the ground state correlations is what distinguishes our work from the studies performed in Ref. [15].

The meson exchange currents between the nucleons inside a nucleus, which modify the single-particle nature of the magnetic moment operator [22], are taken into account by introducing effective g_s-factor values.

This paper proceeds as follows. In Sec. II, we outline the basics of the approach that we utilize to estimate the magnetic moments. Numerics and physical interpretations are the subject of Sec. III. The results of this work are summarized in Sec. IV.

II. THEORETICAL FRAMEWORK

In the approach that we follow, the properties of the nuclear states are interpreted as a result of the interaction between two types of fictitious particles: quasiparticles and phonons represented in spherical basis by operators denoted by α_{jm} and $Q_{\lambda\mu i}$, respectively [16]. In this framework the odd-even nuclei are formed by the interaction of the last quasiparticle with the ground and excited states of the even-even core. The possibilities for the last particle to couple with different states of the even-even core are accounted for by constructing a wave function as a mixture of one quasiparticle and quasiparticle-phonon pure states [17,18]:

$$\Psi_\nu(JM) = O^\dagger_{JM\nu}|\rangle \tag{1}$$

with

$$\begin{aligned} O^\dagger_{JM\nu} = {} & C_{J\nu}\alpha^\dagger_{JM} + \sum_{j\lambda i} D_{j\lambda i}(J\nu)P^\dagger_{j\lambda i}(JM) \\ & - E_{J\nu}\tilde{\alpha}_{JM} - \sum_{j\lambda i} F_{j\lambda i}(J\nu)\tilde{P}_{j\lambda i}(JM), \end{aligned} \tag{2}$$

where $|\rangle$ denotes the ground state of the even-even core, $P^\dagger(JM) = [\alpha^\dagger_j Q^\dagger_{\lambda i}]_{JM}$ is the quasiparticle⊗phonon creation operator, and $\tilde{}$ stands for time conjugation according to the convention $\tilde{a}_{jm} = (-1)^{j-m}a_{j-m}$. The last terms of this equation address the nonzero probabilities for the last quasiparticle to interact with quasiparticle and quasiparticle⊗phonon configurations residing in the ground state of the even-even core. The importance of these terms to the magnetic and electric moments is discussed in detail in Sec. III.

The dynamics of the physical setting described in this way is governed by the following Hamiltonian:

$$H = H_{\rm MF} + H_{\rm PAIR} + H_{\rm RES}, \tag{3}$$

which includes parts representing the integral effect from the mean-field generating forces of the nucleon-nucleon interaction, the monopole pairing field, and the residual central long-range interaction between the spatial and spin degrees of freedom:

$$H_{\rm RES} = H_M + H_{\rm SM}. \tag{4}$$

Assuming this part of the interaction to be a separable form, we expand it by multipoles and spin multipoles:

$$H_M = -\frac{1}{2}\sum_{\substack{\lambda \\ \rho=\pm 1}} \left(\kappa_0^{(\lambda)} + \rho\kappa_1^{(\lambda)}\right) \sum_{\substack{\mu \\ \tau=n,p}} M^\dagger_{\lambda\mu}(\tau)M_{\lambda\mu}(\rho\tau), \tag{5}$$

$$\begin{aligned} H_{SM} = {} & -\frac{1}{2}\sum_{\substack{\lambda \\ L=\lambda,\lambda\pm 1 \\ \rho=\pm 1}} \left(\kappa_0^{(\lambda L)} + \rho\kappa_1^{(\lambda L)}\right) \\ & \times \sum_{\substack{M \\ \tau=n,p}} \left(S^\lambda_{\rm LM}\right)^\dagger(\tau)S^\lambda_{\rm LM}(\rho\tau), \end{aligned} \tag{6}$$

where τ enumerates the neutron (n) and proton (p) subsystems,

$$M^\dagger_{\lambda\mu} = \sum_{jj'mm'} \langle jm|i^\lambda R_\lambda(r)Y_{\lambda\mu}|j'm'\rangle a^\dagger_{j'm'}a_{jm}, \tag{7}$$

and

$$\left(S^{\lambda}_{\mathrm{LM}}\right)^{\dagger} = \sum_{jj'mm'} \langle jm|i^{\lambda}R_{\lambda}(r)[\sigma Y_{\lambda}]_{\mathrm{LM}}|j'm'\rangle a^{\dagger}_{j'm'}a_{jm} \quad (8)$$

are the single-particle multipole and spin-multipole operators [16]. From the sum over L in Eq. (6) we include only the terms with $L = \lambda - 1$. In the phonon space, the eigenstates of this part of the interaction are of unnatural parity $(-1)^{L-1}$. Of particular interest to our present research are the 1^+ states which in QPM accounts for the dipole core spin polarization [15], induced by the $\sigma\sigma$ forces. The reduced matrix elements related to equations (7) and (8) are denoted by $f^{(\lambda)}_{jj'}$ and $f^{(\lambda L)}_{jj'}$ respectively.

We obtain the eigenstates of the system, defined by the Hamiltonian (4), using an approximate step-by-step diagonalization procedure in which initially the first two terms are diagonalized using the canonical Bogoliubov transformation

$$a_{jm} = u_j\alpha_{jm} + (-)^{j-m}v_j\alpha^{\dagger}_{j-m}. \quad (9)$$

The term of the Hamiltonian (H_{RES}), which contains nondiagonal elements in the quasiparticle basis after the first step of this procedure, couples different quasiparticles to form mixed states which in the QPM are understood using the concept of phonons,

$$Q^{\dagger}_{\lambda\mu i} = \frac{1}{2}\sum_{jj'}\left[\psi^{\lambda i}_{jj'}A^{\dagger}(jj';\lambda\mu) - (-1)^{\lambda-\mu}\varphi^{\lambda i}_{jj'}A(jj';\lambda-\mu)\right], \quad (10)$$

where $A^{\dagger}$ (and its inverse) stand for the bifermion quasiparticle operator:

$$A^{\dagger}(jj';\lambda\mu) = \sum_{mm'}\langle jmj'm' \mid \lambda\mu\rangle\alpha^{\dagger}_{jm}\alpha^{\dagger}_{j'm'}. \quad (11)$$

In order to determine the structure of even-even nuclei, this part of the interaction is diagonalized in a space spanned by one-phonon wave functions where it is assumed that the ground state of the even-even core is a vacuum for the phonon operators, i.e., $Q_{\lambda\mu i}|\rangle = 0$. Of special interest is the fact that the core's ground state from Eq. (1) contains additional correlations that are not included in the phonon vacuum state. The latter are incorporated by equating numbers rather than wave functions as theorized in the equation-of-motion method [23], which we apply in the following form:

$$\langle|\{\delta O_{JM\nu}, H, O^{\dagger}_{JM\nu}\}|\rangle = \eta_{J\nu}\langle|\{\delta O_{JM}, O^{\dagger}_{JM}\}|\rangle. \quad (12)$$

This method allows us to harness the already obtained phonon vacuum state for calculating the average values of the Hamiltonian. In Eq. (12) $\{\cdot,\cdot,\cdot\}$ stands for the double commutator and $\eta_{J\nu}$ is the energy of the νth eigenstate with angular momentum J. Despite lowering of the particle rank, the double commutator yields two-body operators whose average values still depend on the ground-state correlations. In this work, we evaluate the operators' average values using the random phase approximation (RPA) with corrections for certain three-quasiparticle configurations affected by the Pauli exclusion principle [24].

Having determined the structural composition of the odd-even nucleus, estimates for the observable quantities of interest are obtained by evaluating the average values of the corresponding operators. For the magnetic dipole and electric quadrupole moments they are defined as

$$\mu_1(J\nu) = \sqrt{\frac{4\pi}{3}}\langle JJ\nu|\mathcal{M}(M;10)|JJ\nu\rangle, \quad (13)$$

$$Q_2(J\nu) = \sqrt{\frac{16\pi}{5}}\langle JJ\nu|\mathcal{M}(E;20)|JJ\nu\rangle, \quad (14)$$

where the electric and magnetic multipole operators are expressed as

$$\mathcal{M}(X;\lambda\mu) = \frac{1}{\pi_{\lambda}}\sum_{\substack{j_1m_1\\ j_2m_2}}(-1)^{j_2-m_2}\mathcal{F}^{(\lambda)}_{j_1j_2} \times \langle j_1m_1, j_2-m_2|\lambda\mu\rangle a^{+}_{j_1m_1}a_{j_2m_2}.$$

Hereafter $\pi_{\lambda} = \sqrt{2\lambda+1}$. $\mathcal{F}^{(\lambda)}_{j_1j_2}$ are the reduced single particle matrix elements:

$$\mathcal{F}^{(\lambda)}_{j_1j_2} = \begin{cases} e\langle j_2||r^{\lambda}i^{\lambda}Y_{\lambda\mu}||j_1\rangle & \text{for electric transitions,}\\ \mu_0\left(g_s\langle j_2||s\cdot\nabla(r^{\lambda}Y_{\lambda\mu})||j_1\rangle + g_l\frac{2}{\lambda+1}\langle j_2||l\cdot\nabla(r^{\lambda}Y_{\lambda\mu})||j_1\rangle\right) & \text{for magnetic triansitions.}\end{cases} \quad (15)$$

Here e and μ_0 are the electron charge and nuclear magneton, respectively.

The matrix elements of the electromagnetic operator (15) in nuclear wave functions derived by using the independent-particle approximation can be decomposed into two parts [25]: one evaluating its expectation value in the even-even core and the other representing the matrix element of this operator between the corresponding single-particle states. For the magnetic moments, only the second term gives a contribution. Depending on the degree of correlation of the core in its ground state this simple picture gets modified by correcting these terms and also by considering additional terms which vanish in the single-particle model. To take such corrections into account, we represent all respective quantities in terms of quasiparticles (9) and phonons (10). In that way the matrix elements in Eqs. (13) and (14) are obtained in a form which can be derived from the following formulas:

$$\langle J_2M_2\nu_2|\mathcal{M}(X;\lambda\mu)|J_1M_1\nu_1\rangle = \frac{\langle J_1-M_1\lambda-\mu|J_2-M_2\rangle}{\pi_{J_2}}(x_{qp\text{-}qp} + x_{qp\text{-}ph} + x_{ph\text{-}ph}), \quad (16)$$

where $x_{qp\text{-}qp}$ gives the transition amplitude between two quasiparticle states:

$$x_{qp\text{-}qp} = \langle|\{(C_{J_2\nu_2}\alpha_{J_2M_2} - E_{J_2\nu_2}\tilde{\alpha}^{\dagger}_{J_2M_2}),[\mathcal{M}(X;\lambda\mu),C_{J_1\nu_1}\alpha^{\dagger}_{J_1M_1} - E_{J_1\nu_1}\tilde{\alpha}_{J_1M_1}]\}|\rangle = (-C_{J_1\nu_1}C_{J_2\nu_2} + E_{J_1\nu_1}E_{J_2\nu_2})\mathcal{F}^{\lambda}_{J_1J_2}v^{\pm}_{J_1J_2}; \quad (17)$$

$x_{qp\text{-}ph}$ evaluates the transition amplitudes between quasiparticle and quasiparticle⊗phonon states:

$$x_{qp\text{-}ph} = \frac{1}{2}\frac{1}{\pi_\lambda}\sum_i\left[\pi_{J_1}\left(-C_{J_2\nu_2}D^{\lambda i}_{J_2}(J_1\nu_1) + E_{J_2\nu_2}F^{\lambda i}_{J_2}(J_1\nu_1)\right) - (-1)^{J_1+J_2+\lambda}\pi_{J_2}\left(C_{J_1\nu_1}D^{\lambda i}_{J_1}(J_2\nu_2) - E_{J_1\nu_1}F^{\lambda i}_{J_1}(J_2\nu_2)\right)\right] \times\left[\sum_{12}^{n}\mathcal{F}^{(\lambda)}_{j_1j_2}u^{\mp}_{j_1j_2}\left(\psi^{\lambda i}_{j_1j_2}\mp\varphi^{\lambda i}_{j_1j_2}\right) + \sum_{12}^{p}\mathcal{F}^{(\lambda)}_{j_1j_2}u^{\mp}_{j_1j_2}\left(\psi^{\lambda i}_{j_1j_2}\mp\varphi^{\lambda i}_{j_1j_2}\right)\right]; \quad (18)$$

and $x_{ph\text{-}ph}$ corresponds to the transition amplitudes between two quasiparticle⊗phonon states:

$$x_{ph\text{-}ph} = \pi_{J_1}\pi_{J_2}\sum_{j_1j_2\lambda' i'}(-)^{j_1+J_2+\lambda'}\left(D^{\lambda' i'}_{j_1}(J_2\nu_2)D^{\lambda' i'}_{j_2}(J_1\nu_1) - F^{\lambda' i'}_{j_1}(J_2\nu_2)F^{\lambda' i'}_{j_2}(J_1\nu_1)\right)\begin{Bmatrix} J_1 & \lambda & J_2 \\ j_1 & \lambda' & j_2\end{Bmatrix}\mathcal{F}^{(\lambda)}_{j_1j_2}v^{\pm}_{j_1j_2}. \quad (19)$$

In formulas (17)–(19), the plus and minus signs in ± and ∓ apply to the magnetic and the electric moments, respectively. If the residual interaction is switched off, then the terms (18) and (19) disappear and the term (17) gives the well-known Schmidt values. The terms (18) and (19) have the following important difference: while the term (18) involves properties of the phonons whose angular momenta coincide with the multipolarity of the transition operator, the summation in Eq. (19), in contrast, runs over all angular momenta $\lambda = 1,2,3\ldots$.

III. RESULTS

For the evaluation of the electromagnetic moments it is of great importance to know the values of the structural coefficients entering into Eqs. (17), (18), and (19). In performing calculations for determining these coefficients we retain only the quadrupole term from the multipole expansion of the residual interaction between the nucleons' spatial degrees of freedom and only the dipole term from the part describing the residual interaction between the spacial (5) and spin (6) degrees of freedom. In our investigation the strengths of these interactions are free parameters that are fixed by fitting on the experimental data. The mean field is approximated, for simplicity, by the potential well of Woods-Saxon form with the parameters determined by reproducing the nuclear binding energies. The monopole pairing strengths G_τ are obtained to match the odd-even mass differences in neighboring nuclei, as detailed in Ref. [17]. The strength of the isoscalar quadrupole-quadrupole interaction $\kappa_0^{(2)}$ is adjusted so as to reproduce the experimental spectrum of the low-lying states of each individual odd-even nucleus. The dependence of the magnetic moment on this parameter are shown in Fig. 2, varying it in a broad range of values. The parameter $\kappa_1^{(2)}$ is calculated by using the relation (cf. Ref. [26])

$$\kappa_1^{(\lambda)} = -0.2(2\lambda+3)\kappa_0^{(\lambda)}. \quad (20)$$

The isovector spin-multipole–spin-multipole interaction strength $\kappa_1^{(10)}$ is determined by the centroid of the giant dipole magnetic resonance while the strength of the isoscalar spin-multipole–spin-multipole interaction $\kappa_0^{(10)}$ plays a negligible role for the structure of the 1^+ giant resonance and is set to 0.

The structural compositions of the wave functions of the $11/2^-_1$ states in the studied isotopes is a result of the interplay between the neutron subshell $1h_{11/2}$ and the quasiparticle⊗phonon configurations in the ground and in the excited states. The calculated components of these wave functions in versions of the model including backward amplitudes (FRW+BCW for short) and disregarding them (abbreviated by FRW) are listed in Table I. As seen in this table, by allowing quasiparticle⊗phonon configurations in the ground state, one obtains an increased fragmentation of the $1h_{11/2}$ quasiparticle strength (cf. Refs. [17,20]) which causes a reduction in the contribution $x_{qp\text{-}qp}$ [see Eq. (16)] to the quantities of interest. In

TABLE I. Major components of the $11/2^-_1$ state in ^{121}Cd, ^{123}Cd, ^{125}Cd, and ^{127}Cd calculated using both forward (FRW) and forward+backward (FRW+BCW) amplitudes in the wave functions.

Isotope	FRW+BCW			FRW	
	$C(D)$	$E(F)$	Component	$C(D)$	Component
^{121}Cd	0.88	0.02	$\nu 1h_{11/2}$	0.98	$\nu 1h_{11/2}$
	−0.01	−0.42	$\nu 1h_{11/2}\otimes 2^+_1$	0.08	$\nu 2f_{7/2}\otimes 2^+_1$
	0.14	−0.11	$\nu 2f_{7/2}\otimes 2^+_1$	0.08	$\nu 1h_{11/2}\otimes 1^+_8$
	0.08	0.00	$\nu 1h_{11/2}\otimes 1^+_8$	0.07	$\nu 1h_{11/2}\otimes 1^+_3$
^{123}Cd	0.87	−0.01	$\nu 1h_{11/2}$	0.98	$\nu 1h_{11/2}$
	−0.08	−0.41	$\nu 1h_{11/2}\otimes 2^+_1$	0.10	$\nu 1h_{11/2}\otimes 1^+_8$
	0.14	−0.11	$\nu 2f_{7/2}\otimes 2^+_1$	0.07	$\nu 2f_{7/2}\otimes 2^+_1$
	0.1	0.00	$\nu 1h_{11/2}\otimes 1^+_8$	−0.06	$\nu 1h_{9/2}\otimes 1^+_8$
^{125}Cd	0.91	−0.03	$\nu 1h_{11/2}$	0.98	$\nu 1h_{11/2}$
	−0.15	−0.30	$\nu 1h_{11/2}\otimes 2^+_1$	−0.13	$\nu 1h_{11/2}\otimes 2^+_1$
	0.09	−0.10	$\nu 2f_{7/2}\otimes 2^+_1$	0.10	$\nu 1h_{11/2}\otimes 1^+_8$
	0.09	0.00	$\nu 1h_{11/2}\otimes 1^+_8$	0.07	$\nu 2f_{7/2}\otimes 2^+_1$
^{127}Cd	0.92	−0.05	$\nu 1h_{11/2}$	0.97	$\nu 1h_{11/2}$
	−0.22	−0.23	$\nu 1h_{11/2}\otimes 2^+_1$	−0.20	$\nu 1h_{11/2}\otimes 2^+_1$
	0.10	0.00	$\nu 1h_{11/2}\otimes 1^+_8$	0.10	$\nu 1h_{11/2}\otimes 1^+_8$
	0.06	−0.08	$\nu 2f_{7/2}\otimes 2^+_1$	0.05	$\nu 2f_{7/2}\otimes 2^+_1$

the FRW+BCW model version the largest part of this strength is transferred to the $\nu 1h_{11/2} \otimes 2_1^+$ admixture in the ground state of the even-even core which interacts most intensely with the last quasiparticle. The main part of the strength of this interaction is given by

$$W(Jj\lambda 1) = \langle|\alpha^\dagger_{JM} H P^\dagger_{j\lambda i}(JM)|\rangle = -\frac{1}{4}\frac{\pi_\lambda}{\pi_J}\sum_{\tau_0}\mathcal{A}_{\tau_0}(\lambda i 1)\varphi^{\lambda 1}_{Jj}, \quad (21)$$

where $\varphi^{\lambda i}_{Jj}$ are the phonons' backward amplitudes and $\mathcal{A}_{\tau_0}(\lambda i i')$ is defined in [17].

As seen from Eq. (21), the only two-quasiparticle state which influences this interaction strength is the one which bears nucleons from shells designated by the quantum numbers of the participating noncollectivized quasiparticles, namely J and j. The high amplitude of $\varphi^{\lambda i}_{Jj}$ (between 0.3 and 0.5 in different isotopes) from annihilating the two-quasineutron state $[\nu 1h_{11/2} \otimes \nu 1h_{11/2}]_2$ in the ground state for the formation of the 2_1^+ phonon explains the enhanced magnitude of the interaction between the $\nu 1h_{11/2}$ and $\nu 1h_{11/2} \otimes 2_1^+$ states, which varies from −1.5 MeV to −2.5 MeV along the isotope chain as in Fig. 1. The contribution from the configuration $[\nu 2f_{7/2} \otimes 2_1^+]_{11/2}$ to the structure of the $11/2_1^-$ state is the second largest. The reason for such a high rank of this component is the considerable (of the order of 0.13) 2_1^+ phonon amplitude from annihilating the $[\nu 1h_{11/2} \otimes \nu 2f_{7/2}]_2$ state residing in the ground state of the neighboring even-even nucleus.

On the other hand, if the particle-vibration interaction in the ground state is not taken into account, then the structure of the $11/2_1^-$ states changes significantly from one nucleus to

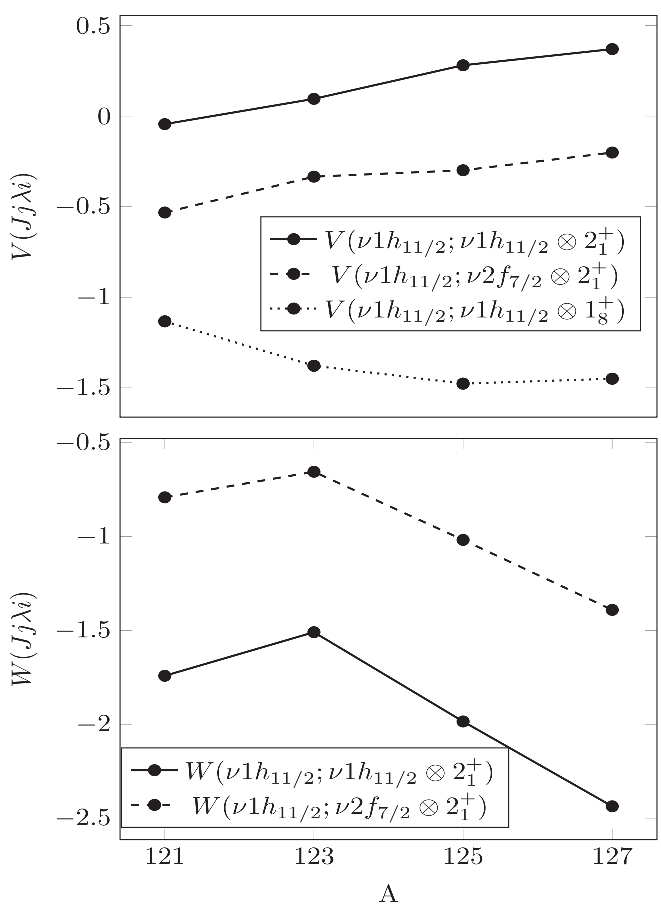

FIG. 1. Evolution of select interaction vertices in the forward $V(Jj\lambda i)$ and backward $W(Jj\lambda i)$ directions with the mass number A.

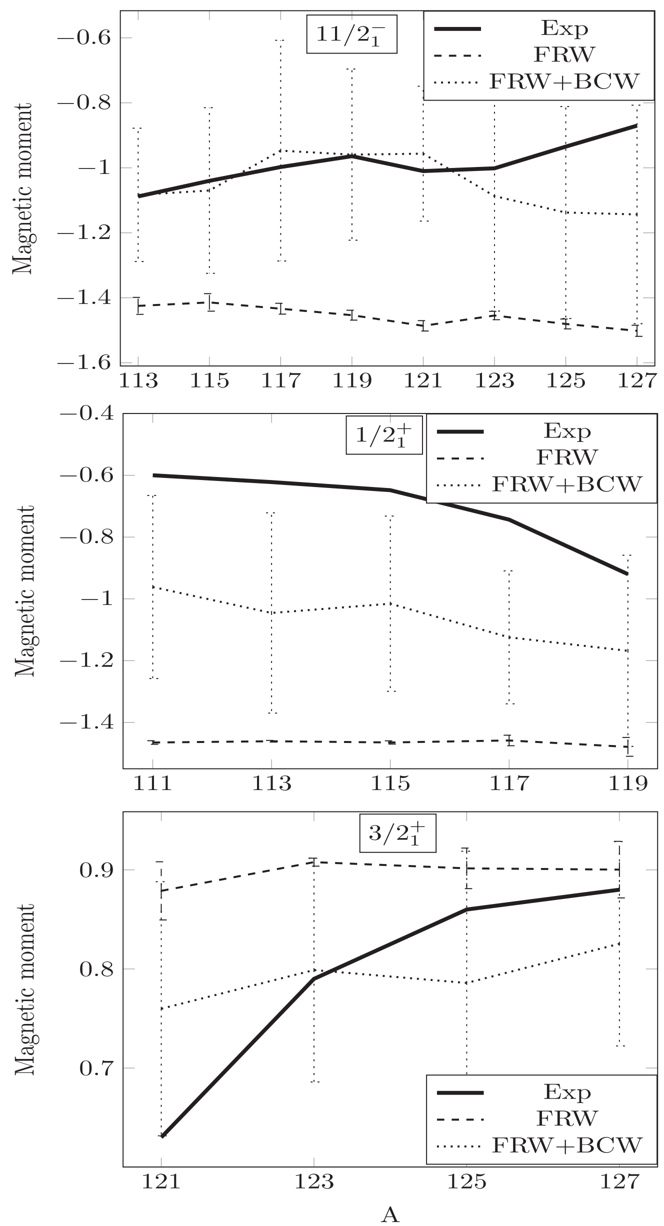

FIG. 2. Magnetic moments in units μ_0 of the first $11/2^-$, $3/2^+$, and $1/2^+$ states in the chain $^{111-127}$Cd for which experimental data is available [4,27]. The calculations are performed within the BCW+FRW (dotted line) and FRW (dashed line) versions of the model. The solid line represents the experimental values. The error bars give the uncertainty in evaluating the magnetic moments when varying the strength of the quadrupole-quadrupole interaction in a wide range of values. See text for details.

another, as seen from Table I. The reason for the reordering of the components is the interaction between the last particle and different excited quasiparticles⊗phonon configurations, which is quantified by the vertex:

$$V(Jj\lambda i) = -\langle|\alpha_{JM} H P^{+}_{j\lambda i}(JM)|\rangle = \begin{cases} \frac{1}{\sqrt{2}} \frac{\pi_\lambda}{\pi_J} \frac{f^{(\lambda)}_{Jj} v^{(-)}_{Jj}}{\sqrt{Y^{\lambda i}}} & \text{for } \lambda^\pi = 2^+, \\ \frac{1}{\sqrt{2}} \frac{\pi_\lambda}{\pi_J} \frac{f^{(\lambda L)}_{Jj} v^{(+)}_{Jj}}{\sqrt{Y^{\lambda i}}} & \text{for } \lambda^\pi = 1^+, \end{cases} \quad (22)$$

where $v^{(\pm)}_{Jj} = u_J u_j \pm v_J v_j$ with u_j and v_j being the pairing occupation numbers for the level j.

The trends for these interaction vertices, imposed by the relationship in Eq. (22), when changing the neutron number are complex because of their dependence on both the pairing properties of the noncollectivized particles and the degree of collectivity of the participating phonon. For instance, the key to understanding the trends in $V(\nu 1h_{11/2}; \nu 1h_{11/2} \otimes 2^+_1)$, plotted in Fig. 1, are the changes in the values of $v^{(-)}_{\nu 1h_{11/2},\nu 1h_{11/2}}$ which drop from 0.4619 in ^{117}Cd, pass through 0.05 in ^{121}Cd, and reach −0.530 in ^{127}Cd. Analogously, the line for the vertex $V(\nu 1h_{11/2}; \nu 2f_{7/2} \otimes 2^+_1)$ is explained by the attenuation of the quantity $v^{-}_{\nu 1h_{11/2},\nu 2f_{7/2}}$ from 0.61 in ^{121}Cd to 0.36 in ^{127}Cd. In contrast, the vertex $V(\nu 1h_{11/2}; \nu 1h_{11/2} \otimes 1^+_8)$ does not depend on the pairing directly since $v^{(+)}_{\nu 1h_{11/2},\nu 1h_{11/2}} = 1$ and its evolution over the isotope chain is driven by the collective properties of the 1^+_8 state in the respective even-even core.

The first-order core polarization is manifested by the significant contribution to the magnetic moment of the quasiparticle ⊗ 1^+-phonon states. The structure of the 1^+ states has a simple form and is represented by a mixture of coupled quasiparticle states belonging to spin-orbit doublets. Although the contribution to the odd-even nucleus wave function coming from the configurations including 1^+ states is small, its importance to describing the magnetic moments is vital, as will be discussed in the following section. The 1^+ state which mostly contributes to the structure of the first 11/2- states in the odd-even cadmium isotopes is the one acquiring the highest degree of collectivity and having an energy in the region of 13 MeV. Its structure is dominated by the configuration $[\nu 1h_{11/2} \otimes \nu 1h_{9/2}]_1$. If the higher order correction, given by Eq. (19), could be neglected, then the coupling of the last nucleon with the 1^+ states might be treated by following the classical approaches by Blin-Stoyle [1] and Arima and Horie [2], which are based on the smallness of the mixing coefficients. In our calculations we confirm that the perturbing configuration from Ref. [1],—which, applied to the $11/2^-$ state in the cadmium isotopes, is $[[\nu 1h_{11/2} \otimes \nu 1h_{9/2}]_1 \otimes \nu 1h_{11/2}]_{11/2}$—indeed contributes the most to the magnetic moment if the backward amplitudes are not taken into account. However, the inclusion of a single complex configuration to the wave function is not adequate for describing the magnetic moments because the coupling with excitation modes of the even-even core having higher angular momenta is crucial to gain a more realistic picture of the wave function.

A. Magnetic moment

The particularities in the structure of the odd-A isotopes discussed in the previous section determine the deviations of the magnetic moment of the $11/2^+_1$, $3/2^+_1$, and $1/2^+_1$ states from their single-particle estimates. The results from the calculations performed by using the featured QPM versions are plotted in Fig. 2 and are compared to the experimental values. The dotted and dashed lines in this figure depict the most important achievement of this work: the finding that the interaction between the last quasiparticle and the ground-state phonon admixtures produces configurations which contribute significantly to the magnetic moment of odd-A nuclei

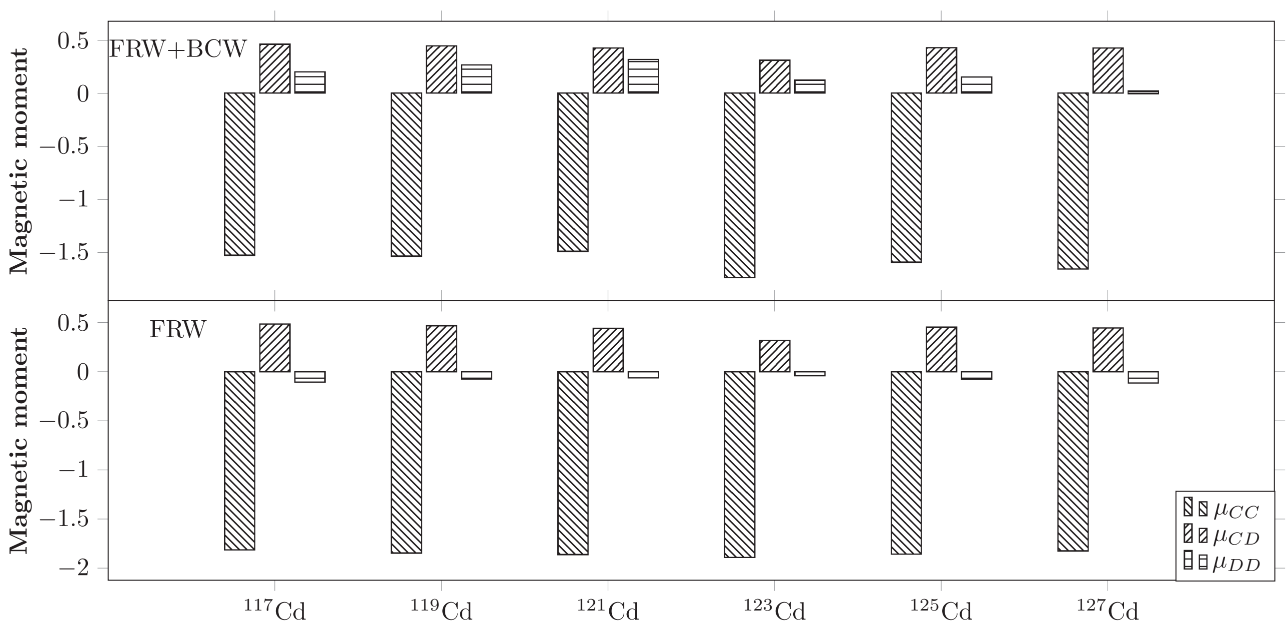

FIG. 3. Contribution to the magnetic moment of the $11/2^-$ state in the series of isotopes $^{117\text{–}127}$Cd.

and reveal a potential for reproducing their experimental values, which proves impossible if this interaction is neglected. The importance of each contribution from Eqs. (17)–(19) to the magnetic dipole moment is visualized in Fig. 3. The enhanced fragmentation due to the quasiparticle-phonon interaction in the ground state leads to systematically shrunk values of the single-quasiparticle contribution $\mu_{qp\text{-}qp}$ and to an increase in the quasiparticle-phonon contribution $\mu_{qp\text{-}ph}$ leading to an overall decrease in the magnitude of the magnetic moment. The escalation of the magnetic transitions between different quasiparticle⊗phonon configurations, given by $\mu_{ph\text{-}ph}$, is due to configurations involving a quadrupole phonon, of which $\nu 1h_{11/2} \otimes 2_1^+$ plays the most important role. It is worth noting that, because of the weakened coupling between the quasiparticles and the quadrupole phonons in the core's ground state near the neutron shell closure, the quantity $\mu_{ph\text{-}ph}$ tends to diminish while $\mu_{qp\text{-}qp}$ remain almost unchanged along the isotope chain. This interaction, however, leaves the first-order core polarization term $\mu_{qp\text{-}ph}$, describing the magnetic transitions between quasiparticle and quasiparticle⊗1^+-phonon states, virtually unaffected because the latter configurations represent a negligible part in the $11/2_1^-$ state mixture.

However, despite its capacity of reaching the experimental values, this theoretical development suffers from the shortcoming (cf. [17,20]) that the residual interaction strength, which yields results that are of sound agreement with the odd-*A* experimental data, generates substantially less collective 2_1^+ states in the even-even cores than the ones implied from the data for the neighboring even-even nuclei. The origin of this inconsistency is the set of approximation techniques embedded in the considered QPM versions, namely, the BCS and RPA, which tend to overestimate the degree of correlations in the ground state for open-shell nuclei. One path to solving this problem is to apply the more consistent extended RPA [28], or use a tractable method based on the variational principle for the ground state, as in [29].

IV. CONCLUSIONS

The magnetic dipole moments of the low-lying states in odd-*A* Cd nuclei are found to be significantly affected by the correlations in the ground state. The obtained corrections allow one to reproduce the experimental values in open-shell nuclei, which proves impossible if the existence of the quasiparticle⊗phonon configurations in the ground states of even-even nuclei is ignored. Despite the reported improvements, the calculations based on this version of the model exhibit a very high sensitivity to interaction parameters which limit its predictive power, and pertinent work in this direction is ongoing.

ACKNOWLEDGMENTS

S. Mishev would like to thank Dr. D. Yordanov for useful remarks and discussion. S. Mishev acknowledges the financial support from the Bulgarian National Science Fund under Contract No. DFNI-E02/6.

[1] R. J. Blin-Stoyle, Proc. Phys. Soc. A **66**, 1158 (1953); R. J. Blin-Stoyle and M. A. Perks, *ibid.* **67**, 885 (1954).
[2] A. Arima and H. Horie, Prog. Theor. Phys. **11**, 509 (1954); **12**, 623 (1954).
[3] A. Arima and H. Sagawa, Phys. Lett. B **173**, 351 (1986).
[4] D. T. Yordanov *et al.*, Phys. Rev. Lett. **110**, 192501 (2013).
[5] A. Bohr, Rev. Mod. Phys. **48**, 365 (1976).
[6] M. G. Mayer, Phys. Rev. **78**, 16 (1950).
[7] A. de-Shalit and I. Talmi, *Nuclear Shell Theory* (Academic Press, New York, 1963).
[8] J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. **108**, 1175 (1957).
[9] P. W. Zhao, S. Q. Zhang, and J. Meng, Phys. Rev. C **89**, 011301 (2014).
[10] S. V. Tolokonnikov, S. Kamerdzhiev, D. Voitenkov, S. Krewald, and E. E. Saperstein, Phys. Rev. C **84**, 064324 (2011).
[11] H. Jiang, Y. Lei, C. Qi, R. Liotta, R. Wyss, and Y. M. Zhao, Phys. Rev. C **89**, 014320 (2014).
[12] I. N. Borzov, E. E. Saperstein, and S. V. Tolokonnikov, Phys. At. Nucl. **71**, 469 (2008).
[13] L. Zamick, Phys. Lett. B **34**, 472 (1971).
[14] M. Nomura, Phys. Lett. B **40**, 522 (1972).
[15] A. I. Levon, S. N. Fedotkin, and A. I. Vdovin, Phys. At. Nucl. **43**, 1416 (1986).
[16] V. G. Soloviev, *Theory of Atomic Nuclei: Quasiparticles and Phonons* (Institute of Physics, Bristol, 1992).
[17] S. Mishev and V. V. Voronov, Phys. Rev. C **78**, 024310 (2008).
[18] S. Mishev and V. V. Voronov, Phys. Rev. C **82**, 064312 (2010).
[19] T. T. S. Kuo, E. U. Baranger, and M. Baranger, Nucl. Phys. A **79**, 513 (1965).
[20] V. V. der Sluys, D. V. Neck, M. Waroquier, and J. Ryckebusch, Nucl. Phys. A **551**, 210 (1993).
[21] N. B. de Takacsy, Phys. Rev. C **89**, 034301 (2014).
[22] H. Miyazawa, Progr. Theor. Phys. **6**, 801 (1951).
[23] D. Rowe, *Nuclear Collective Motion* (Menthuen, London, 1970).
[24] C. Z. Khuong, V. G. Soloviev, and V. V. J. Voronov, Phys. G: Nucl. Phys. **7**, 151 (1981).
[25] P. Ring and P. Schuck, *The Nuclear Many-Body Problem* (Springer-Verlag, New York, 1980).
[26] A. I. Vdovin and V. G. Soloviev, Fiz. Elem. Chastits At. Yadra **14**, 237 (1983).
[27] N. J. Stone, At. Data Nucl. Data Tables **90**, 75 (2005).
[28] K. Hara, Progr. Theor. Phys. **32**, 88 (1964); R. V. Jolos and W. Rybarska, Z. Phys. A **296**, 73 (1980); D. Karadjov, V. V. Voronov, and F. Catara, Phys. Lett. B **306**, 197 (1993).
[29] S. Mishev, Phys. Rev. C **87**, 064310 (2013).