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Consistent description of the cluster-decay phenomenon in transactinide nuclei

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Systematic investigation of the known even-even transactinide cluster emitters has been carried out by considering the cluster as a point particle and using the exact quantum mechanical treatment of the decay process. It is shown that the cluster decay phenomenon can be described reasonably well using a simple Woods-Saxon mean field. Sensitivity of the half-lives on various aspects of the mean field has been investigated in detail.

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I. INTRODUCTION

The nuclear many-body problem is one of the most complex and intriguing problems in nature. The availability of sophisticated and powerful detection techniques has led to the discovery of a number of unusual phenomena, including halo nuclei and the existence of super- and hyperdeformed bands. It was theoretically predicted at the end of the 1970s that transactinide nuclei would decay spontaneously by emitting heavy clusters [1]. This was later confirmed experimentally [2] by measuring the spontaneous emission of ^{14}C from the ^{223}Ra mother nucleus. In the following years, several experimental investigations (see, e.g., [3–5]) reported more such decays. The lightest known cluster emitting nucleus is ^{114}Ba, and the heaviest is ^{242}Cm [6], which decays by emission of ^{34}Si. These decays are difficult to detect, since their typical branching ratios are very small as compared to α emission, thereby making the corresponding half-lives very large (see, e.g., [6]). It is therefore important to have a reliable and simple method to perform an estimation of the expected cluster decay rate.

Theoretically, the cluster decay mode is quite difficult to describe consistently due to the apparent complexity of i) cluster-daughter interaction and ii) sensitive dependence of the half-lives on structures of the cluster and daughter involved. A number of investigations of the cluster decay phenomenon have been reported in the literature. These investigations principally belong to a few wide classes, namely, i) determination of half-lives using a universal formula [7,8], ii) superasymmetric fission approach [9,10], iii) Coulomb and proximity potential model [11], iv) cluster models [12], and v) semiclassical approaches based on the WKB approximation (see, e.g., [13]). It has been found that these different approaches do describe the decay half-lives reasonably well.

A full microscopic quantum mechanical treatment of cluster decay is extremely difficult. Moreover, rather rough approximations have to be made in order to be able to perform the calculations [14].

In the present work, we propose to describe the cluster as a point particle moving in a Gamow state under the interaction potential (nuclear plus Coulomb). This approach was pursued in the past and is still been followed [15]. The new feature of the formalism presented here is that it gives the exact cluster decay width from a given state of the potential well [16].

Relevant details of the theoretical framework are presented and discussed in Sec. II. The calculational aspects are detailed in Sec. III, along with the results. Summary and conclusions are contained in Sec. IV.

II. FORMALISM

As in, e.g., Ref. [12], we will treat the cluster as a point particle with well-defined attributes like mass and total angular momentum. The point particle is assumed to move in an external potential, which is determined from the interaction between the cluster and the daughter nucleus. We will use as mean field a Woods-Saxon potential of the standard form.

In principle, one needs to take the finite size of cluster into account. This leads to further complications, since one needs to build a fully antisymmetric wave function for the cluster. Even if one assumes that the cluster is moving in a mean field induced by the cluster-daughter interaction, constructing the cluster wave function with internal structure is a very difficult task. The model presented here is intended to be a simple effective model, and therefore we consider the cluster to be a point particle. The size of the external potential is considered to be equal to $r_o A^{1/3}$, where A is the mass of the parent nucleus, and r_o is a parameter (see the discussion in Sec. III). This is expected to take care of the finite size effects of the cluster into account, at least to some extent, which is apparent from the quality of agreement achieved between experiment and the present calculations.

The parent nuclei and a number of clusters are known to be deformed in their ground states, whereas all the daughter nuclei turn out to be spherical or nearly spherical. In the present calculations, the explicit deformation effects have been ignored. This might change the half-lives by a factor which can be rather large for superdeformed nuclei, but for deformations up to $\beta = 0.3$ the influence of the deformation amount to a factor of at most three [17]. Moreover, it has been demonstrated that the double folded cluster-daughter potential computed by using the $L = 0$ projected densities describes the cluster decay half-lives rather well within the framework of the WKB approximation [13], supporting the approximation made in the present calculation.

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It has been shown by Gherghescu *et al.* that the emission process is strongly influenced by the shell effects of the participants [18,19]. We will asses whether our approach considers shell effects effectively by its adequacy to reproduce the experimental half-lives.

The radial wave function corresponding to the outgoing particle in a certain state (in our case, s state) is just the linear combination of regular (F) and irregular (G) Coulomb functions [16]:

$$r\psi_{00}^{\text{out}}(r) = \mathcal{N}_{00}[G_0(r) + iF_0]. \tag{1}$$

Here, $\mathcal{N}$ is the normalisation constant. Since we shall be considering s waves only, we suppress the subscript 00 for the sake of brevity. In the limit as $r \to \infty$, it is

$$\lim_{r\to\infty} \left|r\psi^{\text{out}}(r)\right|^2 = \mathcal{N}^2. \tag{2}$$

The normalization constant is determined by a matching condition, where we demand that $\psi^{\text{out}}(r)$ is equal to the internal Gamow wave function $\phi(r)$ at some matching radius, R. That is,

$$|\mathcal{N}|^2 = \frac{R^2|\phi(R)|^2}{F^2(kR) + G^2(kR)}, \tag{3}$$

where k is wave number of the outgoing particle. Physically, this quantity represents the decay probability per unit length at infinity. Therefore, it follows that the quantity $v|\mathcal{N}|^2$ is the decay probability per second. Here, v is the velocity of the outgoing particle, and is given by $v = \hbar k/\mu$, where μ is the reduced mass of the cluster-daughter binary system. Hence, if $T_{1/2}$ is the decay half-life, then the decay width is [16]

$$\Gamma = \frac{\hbar}{T_{1/2}} = \frac{\hbar^2 k}{\mu}\frac{R^2|\phi(R)|^2}{F^2(kR) + G^2(kR)}. \tag{4}$$

This quantity is an exact quantum mechanical expression for the decay width. Clearly, this identification is meaningful, provided the quantity on the right-hand side is independent of R, as we shall see below in an example.

III. CALCULATIONAL DETAILS AND RESULTS

In the calculations to be presented below we shall evaluate the outgoing cluster wave function by using the computer code GAMOW [20].

As stated earlier, in the present work the mean field is assumed to be of the usual Woods-Saxon form, with three adjustable parameters, depth (V_o), half-density radius ($R_{1/2} = r_o A^{1/3}$, A being the mass number of parent nucleus) and diffusivity (a). We shall focus on even-even clusters emitted by even even parent nuclei. In particular, we shall consider cluster emission from $^{222-226}$Ra, $^{228-232}$Th, $^{230-236}$U, ^{236,238}Pu, and ^{242}Cm. The cluster decays from these parent nuclei have Q values ranging from 28 MeV to 96 MeV. In our model, the cluster is assumed to be a point particle, moving in a state with energy equal to the Q value. In the calculations, we keep the Q value fixed, and adjust the depth of the potential to get the desired state. Since we are looking at states induced by heavy particles and lying very high up in the continuum, the potential well results to be quite deep. It has been found that it is adequate to search for the depth around −200 MeV. This initial value of the depth is used in all the calculations.

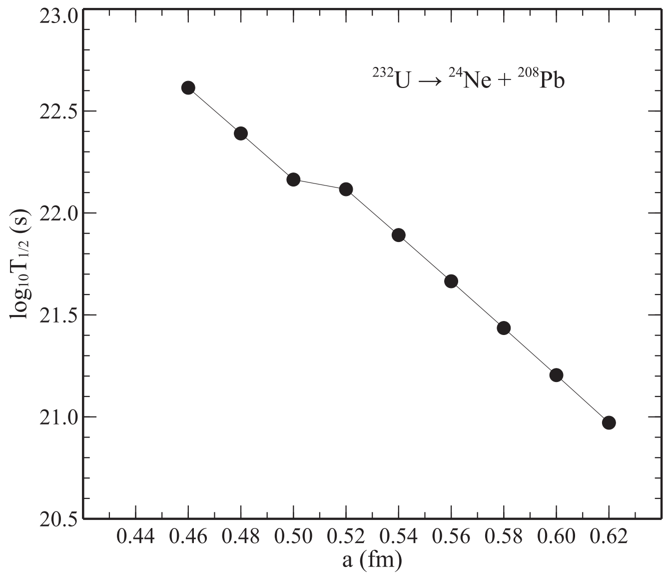

FIG. 1. Variation of $\log_{10} T_{1/2}$ as a function of a for the decay process ^{232}U → ^{24}Ne + ^{208}Pb.

Next we discuss the diffusivity parameter, a. We focus on only one decay process: ^{232}U → ^{24}Ne + ^{208}Pb. It has been explicitly verified that the conclusions drawn in this case are valid for all the decay processes studied here. The diffusivity parameter is usually assumed to be in the range 0.5 fm to 0.6 fm. Pinning down the diffusivity precisely is quite difficult. Therefore, we first investigate the sensitivity of half-lives on a. For this purpose, we choose $r_o = 1.31$ fm (see the discussion below). The calculated half-lives as function of a for the ^{24}Ne emission from ^{232}U have been plotted in Fig. 1. It is seen that variation in a from 0.46 fm to 0.62 fm leads to a change of only 1.5 orders of magnitude in the resulting half-lives, implying that the half-lives do not depend strongly on the parameter a. Moreover, it is worthwhile to notice that around $a = 0.5$ fm, $T_{1/2}$ is practically a constant.

In order to understand this behavior, we plot the sum of nuclear and Coulomb potentials for the ^{24}Ne-^{208}Pb system for different values of a in Fig. 2. The Q value for the decay process has been indicated as horizontal line. Considering that the half-lives depend on the width of the barrier that the particle encounters, it is clear from the figure that the change in diffusivity does not change the barrier width much, explaining why the half-lives are not very sensitive to a. We therefore set the value of a to 0.54 fm.

Next we investigate the behavior of half-lives with respect to the parameter r_o. In order to quantify this, we consider, as before, the decay of ^{232}U by ^{24}Ne emission. The calculated half-lives are plotted as a function of r_o in Fig. 3. It is seen that as r_o changes from 1.25 to 1.41 fm, the half-life decreases by nearly 10 orders of magnitude, which is indeed a dramatic effect. The same feature has also been observed for all the other decays considered here. The horizontal line in the graph stands for the experimental half-life, from which it is apparent that the 'correct' value of r_o should be around 1.31 fm, at least

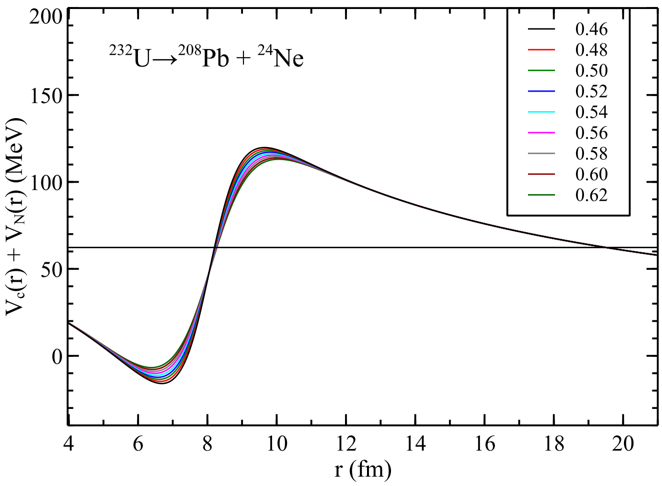

FIG. 2. (Color online) Sum of nuclear and Coulomb potentials for different values of a, for the decay process $^{232}U \rightarrow {}^{24}Ne + {}^{208}Pb$.

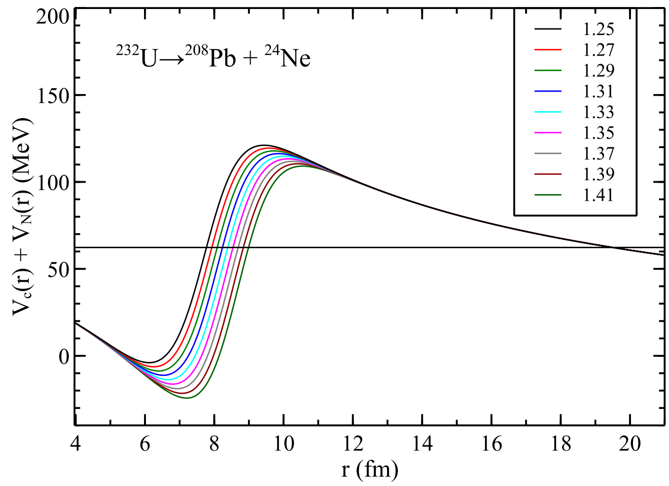

FIG. 4. (Color online) Sum of nuclear and Coulomb potentials for different values of r_o, for the decay process $^{232}U \rightarrow {}^{24}Ne + {}^{208}Pb$.

for this process. The strong variation thus observed at first seems to be counterintuitive, since it is generally believed that such decay processes are governed by the Coulomb potential, and r_o has to do with the nuclear potential. However, this can be understood if one studies the behavior of the total barrier encountered by the particle for different values of r_o, as shown in Fig. 4.

With changing values of r_o, the width and height of the barrier encountered by the particle changes. Since the penetrability (i.e., the Coulomb functions) depends exponentially upon the distance as well as the barrier extension, Fig. 4 reveals the dramatic changes in the half-lives. In other words, the classical turning points for the particle in this case change significantly. This should be compared with Fig. 2, where one sees that the barrier is only weakly dependent (and the turning point virtually independent) upon the diffusivity. If such analysis is carried out for all the cluster decays considered here, it turns out that the value of r_o has a dependence on the Q value. By studying these systematically, we choose, for the sake of simplicity, $r_o = 1.31$ fm for Q values up to 70 MeV, and $r_o = 1.35$ fm for Q values above 70 MeV.

Before presenting the numerical results, one very important test needs to be conducted. Referring to Eq. 3 again, the quantity $\mathcal{N}$ that we compute, should be independent of the matching radius chosen, so far as the matching radius is large enough. In order to check this, we plot the ratio $\mathcal{N}^2$ in Fig. 5.

The quantity $\mathcal{N}^2$ exhibits variation up to a distance of $\sim$10 fm, beyond which it is a constant, thus validating our claim that the normalization constant is a measure of the width. The same observation holds good for all the other decay processes considered here.

We now present the calculated half-lives for the different cluster decay processes considered here, in Table I. For the sake of completeness, we also present there the corresponding

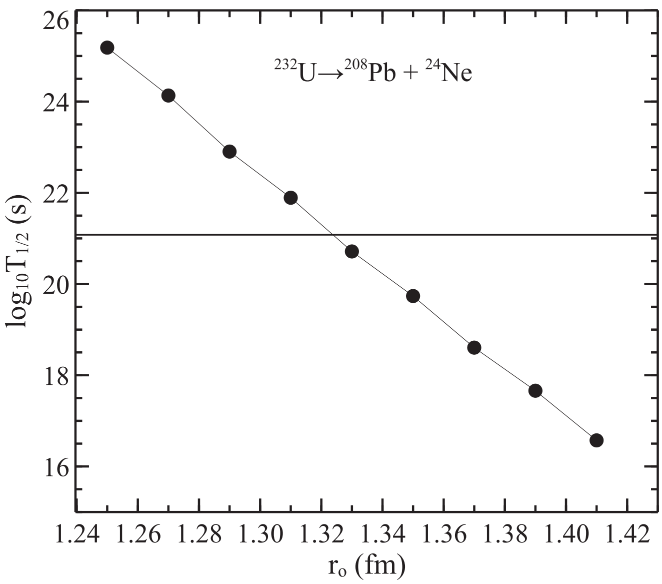

FIG. 3. Variation of $\log_{10} T_{1/2}$ as a function of r_o for the decay process $^{232}U \rightarrow {}^{24}Ne + {}^{208}Pb$.

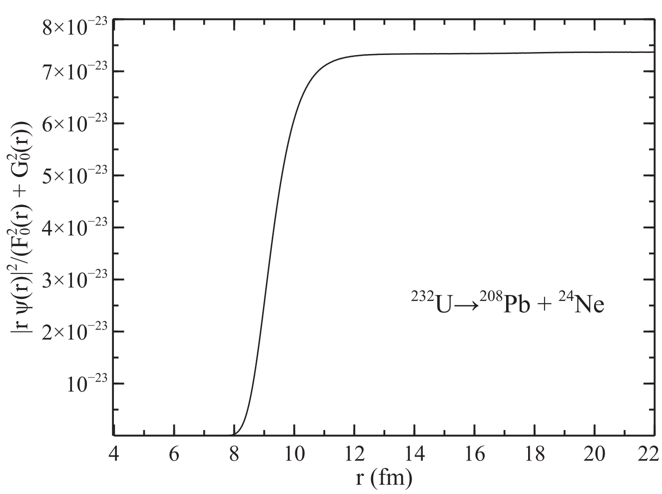

FIG. 5. The modulus squared of the normalisation constant [Eq. (3)] for the decay process $^{232}U \rightarrow {}^{24}Ne + {}^{208}Pb$.

TABLE I. The calculated and experimental values of $\log_{10} T_{1/2}$ for even-even cluster emitters. The corresponding half-lives against α decay are also presented for completeness.

Parent	Daughter	Cluster	Q (MeV)	$\log_{10} T_{1/2}$		Q_α (MeV)	$\log_{10} T_{1/2}$	
				(Calc.)	(Expt.)		(Calc.)	(Expt.)
^{222}Ra	^{208}Pb	^{14}C	33.05	10.44	11.22	6.68	1.82	1.58
^{224}Ra	^{210}Pb	^{14}C	30.54	15.17	15.86	5.79	5.83	5.50
^{226}Ra	^{212}Pb	^{14}C	28.20	20.39	21.34	4.87	11.13	10.70
^{228}Th	^{208}Pb	^{20}O	44.72	21.86	20.72	5.52	8.23	7.78
^{230}Th	^{206}Hg	^{24}Ne	57.76	26.13	24.61	4.77	13.21	12.38
^{232}Th	^{208}Hg	^{24}Ne	54.67	31.19	>29.20	4.08	18.62	17.65
^{232}Th	^{206}Hg	^{26}Ne	55.91	30.76	>29.20			
^{230}U	^{208}Pb	^{22}Ne	61.39	21.77	19.57	5.99	7.01	6.26
^{232}U	^{208}Pb	^{24}Ne	62.31	21.89	21.08	5.41	10.15	9.34
^{232}U	^{204}Hg	^{28}Mg	74.32	25.42	>22.26			
^{234}U	^{210}Pb	^{24}Ne	58.83	27.05	25.92	4.86	13.69	12.89
^{234}U	^{208}Pb	^{26}Ne	59.42	27.75	25.92			
^{234}U	^{206}Hg	^{28}Mg	74.11	25.53	27.54			
^{236}U	^{212}Pb	^{24}Ne	55.95	31.91	>25.90	4.57	15.73	14.87
^{236}U	^{210}Pb	^{26}Ne	56.70	32.40	>25.90			
^{236}U	^{208}Hg	^{28}Mg	70.73	30.35	27.58			
^{236}U	^{206}Hg	^{30}Mg	72.28	29.48	27.58			
^{236}Pu	^{208}Pb	^{28}Mg	79.67	21.07	21.67	5.87	8.57	7.96
^{238}Pu	^{210}Pb	^{28}Mg	75.91	25.95	25.70	5.59	10.06	10.44
^{238}Pu	^{208}Pb	^{30}Mg	76.80	26.05	25.70			
^{238}Pu	^{206}Hg	^{32}Si	91.19	26.66	25.27			
^{240}Pu	^{206}Hg	^{34}Si	91.03	27.71	>25.52	5.26	12.06	11.32
^{242}Cm	^{208}Pb	^{34}Si	96.51	24.23	23.15	6.22	7.66	7.15

α decay half-lives. In all these calculations the experimental Q values were extracted from [21], whereas, the half-lives against cluster decay have been adopted from [6]. The experimental α decay half-lives have been taken from [22]. The calculation of α-decay half-lives has been carried out exactly in the same way as described above, with a potential of the depth of 65 MeV, $r_o = 1.27$ fm and $a = 0.67$ fm. Interestingly, the same set of parameters reproduced all the α decay half-lives from the Sn region to superheavies, indicating that the α particle is more like a real point particle than the heavy clusters, as perhaps expected.

An inspection of Table I reveals that the calculated half-lives against cluster decays are reasonable, and are within a couple of orders of magnitude for most of the cases. In certain cases the experimental half-lives are not known and only lower bounds have been set on them. Our calculated half-lives are found to be above these lower bounds, which is quite satisfactory. The α decay half-lives are reproduced very well, and they are within an order of magnitude of the experimental values. It is worthwhile to point out that the α decay half-lives here are orders of magnitude smaller than the cluster decay half-lives, indicating that the former is a comparatively dominant process, at least in the cases that have been investigated here. Considering the complexity of the cluster decay phenomenon, and the simplicity of the present model, one may conclude that the agreement achieved here is satisfactory.

IV. SUMMARY AND CONCLUSIONS

Systematic calculations of half-lives of even-even trans-actinides have been carried out by evaluating the exact decay width quantum mechanically. The mean field has been assumed to be of the Woods-Saxon form with the standard three parameters, i.e., the depth, the half-density radius, and the diffusivity. It has been shown that in spite of the enormous complexity of the cluster decay phenomenon, it is possible to describe the decay and hence the half-lives reasonably well using a simple mean field potential. The sensitivity of the half lives on the half-density radii has been demonstrated and it has been shown that this is due to the fact that the half-lives depend on the width of the barrier that the cluster encounters, and it is this width that depends sensitively on the half-density radius. This can be readily understood in terms of the classical turning points. We have further shown that the half-lives do not depend very sensitively on the diffusivity parameter. A variation of 35% in the diffusivity parameter changes the half lives by an order of magnitude and a half. This may be compared to changes in the radius parameter r_o. By changing r_o from 1.25 fm to 1.41 fm the half-life changes by about 10 orders of magnitude. This is because variations of the diffusivity do not have an appreciable effect on the classical turning points. However, it is important to stress that the calculated half-lives are independent of the matching radius.

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