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Evidence for the virtual β-γ transition in ^{59}Ni decay

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A novel theory of radiative electron capture in second forbidden non-unique transition is applied to reanalyze experimental data obtained previously for the decay of ^{59}Ni. The measured γ spectrum is shown to be distorted at the high-energy end, presenting the first direct evidence for the virtual β-γ transition through the excited state in ^{59}Co. The complete theoretical predictions, including both the direct and the virtual decay channels, as well as the interference between them, reproduce very well the experimental spectrum. The virtual nuclear component amounts to about 4% of the total γ intensity.

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I. INTRODUCTION

Orbital electron capture by an atomic nucleus (EC) is one of basic nuclear transitions governed by weak interactions [1]. Although known and investigated for a very long time, it is still a subject of interest in the various fields of fundamental research. Examples include neutrino-less double-EC decay [2], the search for sterile neutrinos [3], and testing the Lorentz invariance [4]. A particularly interesting aspect of EC decay, not yet fully explored, is the coupling of electronic and nuclear degrees of freedom, as shown in the decay of highly charged ions [5] or in the searched nuclear excitations by electron capture [6]. Here, we address one such peculiar phenomenon where to account for radiation emitted during the EC decay, both the electronic and nuclear contributions have to be summed coherently. In addition, each contribution depends on the electromagnetic gauge, and in the so-called length gauge employed here, the nuclear radiation is suppressed.

According to quantum mechanics, the evolution of a physical system can be described as proceeding through all possible paths, including those that may temporarily violate the law of energy conservation. The occurrence of such special paths, involving virtual intermediate states, becomes important (and may be observable) when all other possibilities are not allowed or are highly forbidden. An example of such a situation is the nuclear β decay in which the direct transition between the initial and the final states is strongly hindered due to angular momentum and/or parity selection rules, while no other decay channels are present within the available energy window. Then, as noted already long ago by Longmire [7], an alternative decay path may involve a state in the daughter nucleus having energy higher than the initial state. If β transition to such a state is allowed, a combination of this transition with emission of a γ ray may compete with the direct decay. Rose, Perrin, and Foldy discussed such a virtual β-γ emission in more detail and noted that ^{59}Ni is a promising candidate in which to observe this phenomenon [8]. The ground state of ^{59}Ni decays to the ground state of ^{59}Co with a half-life of $(1.08 \pm 0.13) \times 10^5$ yr [9], predominantly by electron capture with a tiny contribution of β^+ [10]. The direct transition between these two states involves the change of spin by two units and no change of parity; thus, it is classified as a second forbidden non-unique (2*nu*) transition [1]. However, in ^{59}Co there exists a state located about 26 keV above the ground state of ^{59}Ni, offering the possibility of the virtual "detour" transition by a combination of allowed (Fermi + Gamow-Teller) electron capture, followed by $E2$ γ emission. Both decay paths and the current knowledge of the involved states are shown in Fig. 1. The direct 2*nu* transition may also proceed with the emission of a photon. Such a second-order radiative electron capture (REC) occurs with the probability of about 1×10^{-4} with respect to the ordinary, non-radiative capture. The dominant source of this radiation is the internal bremsstrahlung (IB) of the captured electron [1]. The REC energy spectrum is continuous because of energy sharing between three bodies in the final state. The maximum photon energy equals the decay energy Q_{EC} minus the binding energy of the captured electron in the daughter atom. In case of the K capture in ^{59}Ni, the REC spectrum extends up to $Q_K = 1064.8$ keV. The virtual detour transitions were predicted to modify the REC spectrum, mainly at the high-energy end [8]. The more extended calculation of this process, including interference effects, was done by Lassila [11].

The REC of ^{59}Ni has already been studied several times. Except for the very first attempt [12], all later experiments were motivated by the search for the virtual β-γ transitions [13–15]. Only Ref. [13] claimed an observation of these transitions. However, as we argue below, results of these papers could not have been conclusive because the applied model of the REC process was either inaccurate or erroneous. We note, in addition, that the virtual transitions were also considered for the case of β^- decay [16,17]. An important motivation of Ref. [17] was the precise calculation of possible background events for the low-energy neutrino detector based on ^{115}In. Draxler *et al.* reported an indirect evidence for the detour transitions in the β^- decay of ^{90}Y, obtained by a polarization measurement [18]. In conclusion, the virtual β-γ transitions have never been observed in a direct and unambiguous way.

In this paper we present clear evidence for the β-γ virtual transition in the γ spectrum of ^{59}Ni. We start by introducing the correct theory of the REC in case of 2*nu* forbidden transitions. Then, we employ this theory to reanalyze experimental results published by Janas *et al.* [15]. The difference between theoretical predictions for the direct 2*nu* transition and the data is interpreted as coming from the virtual transition through the excited state in ^{59}Co. Furthermore,

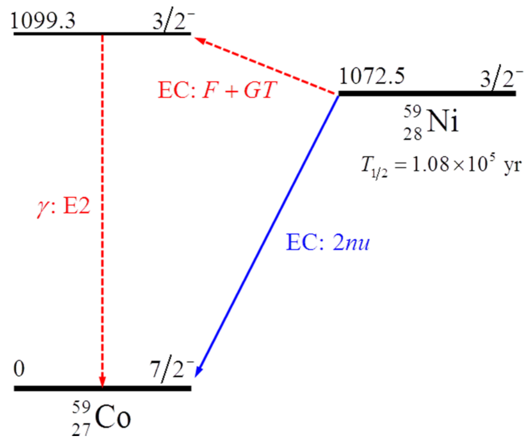

FIG. 1. (Color online) Decay scheme of ^{59}Ni and relevant states in ^{59}Co [10]. The energy levels are labeled by energy in keV, relative to the ground state of ^{59}Co and by spin and parity values. The direct decay of ^{59}Ni proceeds by second forbidden non-unique ($2nu$) electron capture to the ground state of ^{59}Co (solid arrow). The virtual "detour" transition goes by allowed (Fermi and Gamow-Teller) electron capture to the excited state of ^{59}Co combined with γ $E2$ deexcitation (dashed arrows).

we present the first complete theoretical description of the γ spectrum for the specific case of ^{59}Ni which includes the REC part, the detour contribution, as well as the interference between these two channels.

II. NEW MODEL OF REC

Any search for the virtual transitions in EC decays requires good understanding of the radiative capture, and this has a long history that was completed only recently. The first approximation, ignoring the Coulomb field of the nucleus and relativistic effects, valid only for the K capture, was provided by Morrison and Schiff in 1940 [19]. The model was greatly advanced by Glauber and Martin [20,21], who described exactly the electron propagation in the Coulomb field, included relativistic and screening effects, and considered captures from higher shells. This model was found to be in good agreement with a large number of experiments [1]. Nevertheless, it is valid only for allowed decays (spin change $\Delta J = 0,1$, no parity change). The extension of the REC theory to any order of forbiddenness, with inclusion of Coulomb and relativistic corrections, was done by Zon and Rapoport [22,23]. However, measurements of the REC spectra for first-forbidden unique ($1u$, $\Delta J = 2$, parity changes) K captures in ^{41}Ca and ^{204}Tl revealed strong discrepancies with their model [24,25], indicating that some approximations assumed by them are not valid. The $1u$ decay is special as the probability ratio of radiative to non-radiative electron capture does not depend on nuclear matrix elements. The experiment delivers the absolute value of this ratio as a function of the photon energy, which is compared without any normalization to the theoretical prediction which does not have any adjustable parameters. Kalinowski *et al.* [26,27] made an attempt to extend the Zon and Rapoport model by adding the detour transitions via virtual nuclear states following the ideas of Ford and Martin [16], but repeated the same invalid approximations as Zon and Rapoport. Kalinowski *et al.* were able to reproduce the spectrum of ^{41}Ca at the cost of adding a phenomenological parameter [26], but their model failed in the case of ^{204}Tl [25]. Finally, this conundrum was resolved by Pachucki *et al.* [28] with the crucial insight that the description of the electromagnetic radiation in this case is much more advantageous in the *length* gauge in contrast to the Coulomb gauge used in all previous calculations. It allowed one to expose and to circumvent divergent terms which were the source of wrong approximations in the model of Zon and Rapoport for the $\Delta J = 2$ transitions. The problem was caused by neglecting the radiation from the nucleus, which is not justified in the Coulomb gauge. The *length*-gauge model, while reproducing exactly the results of Glauber and Martin for allowed decays, was found to describe very well both the shape and the intensity of experimental spectra of ^{41}Ca and ^{204}Tl [28] and later also the spectrum of $1u$ K capture in ^{81}Kr [29]. It is because the nuclear radiation is strongly suppressed in this gauge and can be neglected.

Following the method described in Ref. [28], here we extend the length-gauge model of REC to the $2nu$ transitions ($\Delta J = 2$, no parity change). Conventionally, the emission probability of a photon with energy k per ordinary non-radiative electron capture and per unit energy is expressed in the form

$$\frac{1}{W_0}\frac{dW_R(k)}{dk} \equiv \frac{\alpha}{\pi\, m^2}\left(1-\frac{k}{Q}\right)^2 k\,\mathcal{R}, \tag{1}$$

where α is the fine structure constant, m is the electron mass, Q is the maximal photon energy, and the units $\hbar = c = 1$ are used. Details of the spectrum are thus represented by the dimensionless shape factor $\mathcal{R}$. We note that in the simplest approximation of Morrison and Schiff, $\mathcal{R} = 1$ [1]. For $2nu$ K capture, the length-gauge model leads to

$$\mathcal{R}_{2nu} = \left(1-\frac{k}{Q}\right)^2 \mathcal{R}^{(1)}(k) + \Lambda\left(\frac{k}{Q}\right)^2 \mathcal{R}^{(2)}(k), \tag{2}$$

where the functions $\mathcal{R}^{(1)}$ and $\mathcal{R}^{(2)}$, which incorporate all Coulomb and relativistic corrections, are derived in Ref. [28]. In this case the spectrum depends on the parameter Λ, which is a combination of nuclear matrix elements relevant for the direct transition between the initial and the final state:

$$\Lambda \equiv \frac{\left|\mathcal{M}^{(4)}_{2nu}\right|^2}{\left|\mathcal{M}^{(2)}_{2nu}\right|^2}, \tag{3}$$

with

$$\mathcal{M}^{(n)}_{2nu} = \left\langle J_f \middle\| r\left[T_{211} - \frac{Z\alpha}{n\sqrt{5}}(\sqrt{2}T_{220} - \lambda\sqrt{3}\gamma^5 T_{221})\right] \middle\| J_i\right\rangle, \tag{4}$$

where Z is the atomic number of the initial atom, J_i and J_f denote the initial and the final states, respectively, and T_{JLS} is a spherical tensor operator. Equation (2) was derived under the approximation $Z\alpha \ll 1$ and essentially represents the photons emitted by the electron in the IB process. The nuclear contribution is strongly suppressed in this model but *only* when there are no close-lying excited states in the daughter or in the parent nucleus [28]. We note that for $\Lambda = 1$ we obtain the shape factor for the K capture in the $1u$ transition, which reproduced so well the experimental data [28,29]. In addition, the shape factor $\mathcal{R}_{2nu}$, Eqs. (2)–(4), has the same form as in the model of Zon and Rapoport [22]; the important difference is in the formula for the function $\mathcal{R}^{(2)}$, which represents electronic transitions through $P_{3/2}$ and $D_{3/2}$ intermediate states. These transitions enter the description of REC for nuclear decays with $\Delta J = 2$. The values of this function in our model, and also of its Coulomb-free limit, differ significantly from the prediction of Zon and Rapoport [28]. This is the main reason why all previous attempts to identify virtual transitions in the decay of ^{59}Ni, which used either the wrong REC model or its inaccurate Coulomb-free approximation, could not have been successful.

III. COMPARISON WITH EXPERIMENT

To test the new REC prediction for $2nu$ decays, we compare it to experimental data published for ^{59}Ni [15]. The K-capture component of the REC spectrum was determined in Ref. [15] by recording γ rays in coincidence with the K x rays of cobalt. The shape factors extracted from two independent measurements, using different detection setups, are shown in Fig. 2. The differences between two data sets reflect inaccuracies of experimental procedures which involved various corrections, the largest being for the Compton scattering and for the absolute efficiency of γ detectors. The prediction of the model is given by Eq. (2), where the functions $\mathcal{R}^{(1)}$ and $\mathcal{R}^{(2)}$ were calculated numerically for the case of ^{59}Ni. Since it is difficult to calculate the value of Λ, it is treated as a free parameter of the model. We found that no value of this parameter can reproduce both the shape and the intensity of the measured shape factor. However, we can adjust this parameter to fit the low-energy part of the spectrum. The least-squares method for the points from both data sets with $k < 600$ keV yields $\Lambda = 1.47 \pm 0.15$ and the resulting shape factor is shown in Fig. 2 by the dashed line.

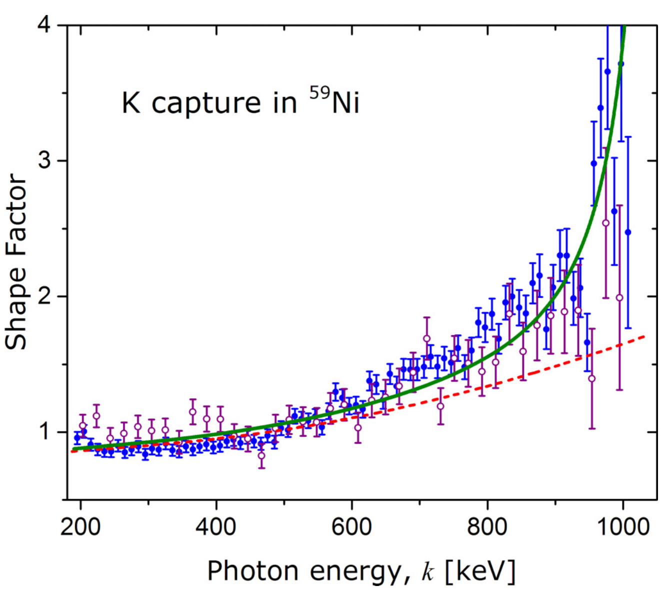

FIG. 2. (Color online) The radiative K-capture shape factor for ^{59}Ni. Experimental points are from Ref. [15]. The dashed line represents the REC prediction, Eq. (2), for the value of the parameter $\Lambda = 1.47$. The solid line shows the best-fitted full model, including the virtual "detour" transitions and the interference term.

In Fig. 2 we see that while the spectrum is well reproduced by the REC model below 600 keV, the experimental intensity increases with energy above the REC prediction, exactly as expected in the early papers of Rose *et al.* [8] and Lassila [11]. We interpret this excess of radiation at the high-energy end as the evidence for the virtual β-γ transition through the excited $3/2^-$ level in ^{59}Co.

IV. VIRTUAL TRANSITIONS

To verify this hypothesis, we calculate the spectrum of photons emitted in the process of virtual transition, marked in Fig. 1 by dashed arrows, using the length-gauge approach. As the result, two additional terms have to be added to the shape factor:

$$\mathcal{R}_{\rm tot} = \mathcal{R}_{2nu} + \mathcal{R}_{\rm vir} \pm \mathcal{R}_{\rm int}, \tag{5}$$

where the contribution from the virtual transition is given by

$$\mathcal{R}_{\rm vir} = \frac{27\pi}{450}\,\frac{m^2 k^4}{Q^2(E^*-k)^2}\,\frac{\mathcal{M}_{E2}^2\left(\mathcal{M}_{\rm F}^2+\mathcal{M}_{\rm GT}^2\right)}{\left|\mathcal{M}_{2nu}^{(2)}\right|^2}, \tag{6}$$

and the interference term reads

$$\mathcal{R}_{\rm int} = \frac{9\sqrt{2\pi}}{25}\,\frac{m\,k^2}{E^*-k}\,\frac{f(E2)}{R\,Q^2}\,\sqrt{\Lambda}\,\frac{\mathcal{M}_{E2}\left(\sqrt{\frac{5}{3}}\mathcal{M}_{\rm F}+\mathcal{M}_{\rm GT}\right)}{\mathcal{M}_{2nu}^{(2)}}. \tag{7}$$

E^* is the excitation energy of the $3/2^-$ level in ^{59}Co, R is the nuclear radius, and $f(E2)$ is expressed in terms of functions defined in Ref. [28]:

$$f(E2) = \frac{F_2^<(R)}{\mathcal{G}(R)}\sqrt{\mathcal{R}_{E2}(S_{1/2}\to D_{3/2}^>)}. \tag{8}$$

The additional nuclear matrix elements which enter the model refer to the electromagnetic $E2$ transition and to Fermi (F) and Gamow-Teller (GT) β transitions:

$$\begin{aligned}\mathcal{M}_{E2} &= \langle J_f \| r^2\, T_{220} \| J_{f'}\rangle,\\ \mathcal{M}_{\rm F} &= \langle J_{f'} \| T_{000} \| J_i\rangle,\\ \mathcal{M}_{\rm GT} &= \langle J_{f'} \| \lambda\,\gamma^5\, T_{101} \| J_i\rangle,\end{aligned} \tag{9}$$

where $J_{f'}$ represents the intermediate $3/2^-$ state.

The formulas above may be simplified in the case of ^{59}Ni by noting that the matrix element for the Fermi transition is negligible. The Fermi transition in question can proceed only due to admixtures of the $T = 5/2$ level, being the isospin analog of the $3/2^-$ state in ^{59}Co, to the ground state of ^{59}Ni. This analog level is located in ^{59}Ni at about 8.4 MeV excitation energy [30], which makes the isospin admixtures very small. Following Ref. [31] we can expect the Fermi matrix element to be of the order of 1×10^{-3}. Therefore, we put $\mathcal{M}_{\rm F} = 0$ and we see that both new terms [Eqs. (6) and (7)] depend on the

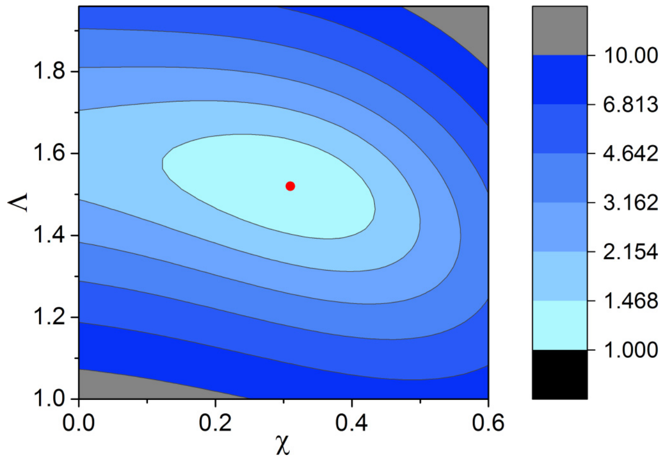

FIG. 3. (Color online) The contour map of the minimizing function. The dot shows the position of the minimum.

single additional combination of nuclear matrix elements:

$$\chi \equiv \frac{\mathcal{M}_{E2}\,\mathcal{M}_{\rm GT}}{\mathcal{M}^{(2)}_{2nu}\,\lambdabar}, \tag{10}$$

where the Compton wavelength of the electron, $\lambdabar = 386.2$ fm, is inserted to ensure that the χ is dimensionless.

Now, treating the Λ and χ as free parameters we fit both data sets in the full energy range. The least-squares minimum is reached for the values

$$\Lambda = 1.52 \pm 0.18, \quad \chi = 0.31^{+0.17}_{-0.34}, \tag{11}$$

and for the positive interference sign. The result, shown in Fig. 2 with the solid line, reproduces the shape factor very well. The value of Λ best fitting the full spectrum is almost equal to the value determined from the analysis of the low-energy part with the pure REC model. This validates our assumption that the virtual contribution does not affect the spectrum at low energy. Figure 3 shows the contour plot of the least-squares function. Due to its rather flat minimum, the values of both parameters cannot be extracted precisely. The resulting total γ emission probability, given by integration of Eq. (1) for $k > 195$ keV, according to prediction, is 7.92×10^{-4}. The integrated virtual and interference components represent 2% and 2.3% of this intensity, respectively.

All matrix elements entering this analysis can be estimated from available experimental information. First, we note that the value of Λ can be determined from the β^+/K ratio for the decay of ^{59}Ni. This ratio was measured in Ref. [15] and yielded the value $\Lambda = 1.0 \pm 0.3$, which agrees within 2σ with the value determined in the present analysis. From the half-life of ^{59}Ni, using the K-capture rate formula given in Ref. [28], we determine that $|\mathcal{M}^{(2)}_{2nu}| = 0.0349$ fm. The $3/2^-$ level in ^{59}Co at 1099.3 keV has a half-life of 3.1(4) ps [30]. From this value the $E2$ transition matrix element can be calculated, yielding $|\mathcal{M}_{E2}| = 21.3$ fm^2. The $\mathcal{M}_{\rm GT}$ element can be estimated from the systematics of the reduced half-life values, ft, for the Gamow-Teller transitions in the region of ^{56}Ni [30]. The measured values suggest that in our case $\log ft = 6$ is a reasonable assumption. Using this value and the relation [32]

$$ft = \frac{6144\,\mathrm{s}}{B_{\rm GT}}, \quad B_{\rm GT} = \frac{1}{2J_i + 1}\mathcal{M}^2_{\rm GT}, \tag{12}$$

we arrive at $|\mathcal{M}_{\rm GT}| = 0.157$. Inserting these values in Eq. (10) leads to $|\chi| = 0.25$, which agrees well with the value determined from the spectrum of ^{59}Ni.

In principle, all these matrix elements can be calculated within the nuclear structure theory. This challenging task remains outside the scope of this paper. We note, however, that those nuclear models that offer precise predictions of spectroscopic quality, like the shell model, usually employ effective or empirical nucleon-nucleon interactions. The theoretical results derived in Ref. [28] and here, based on elementary quantum electrodynamics and weak interaction theory, do not involve any such phenomenological ingredients. Thus, the combinations of matrix elements, Λ and χ, which can be measured with higher precision in the future, in fact can be used to constrain models of nuclear structure.

V. SUMMARY

The electromagnetic radiation accompanying the nuclear EC decay has two components: the electronic and the nuclear. The absolute discrimination between them is meaningless as their relative contribution depends on the adopted gauge of the electromagnetic field. Recently, a long-standing puzzle of REC in $1u$ transitions was resolved by a new theoretical method which utilized the length gauge, which suppresses the nuclear part and thus allows one to neglect it [28]. This suppression, however, is not complete when there are states in the daughter nucleus close to the initial state. Then, an additional contribution of these states to the emitted radiation must be taken into account. We find such a situation in ^{59}Ni which decays by $2nu$ EC to ^{59}Co. In the daughter nucleus there exists an excited state located only 26 keV above the initial state. First, we applied the new model to $2nu$ transitions and compared it with the experimental data published for the decay of ^{59}Ni. This comparison revealed an excess of radiation in the high-energy end of the γ spectrum. We claim that it originates from virtual transitions through this excited state in ^{59}Co. Second, we derived the complete model, which includes the contribution from the virtual process, composed of allowed EC and $E2\,\gamma$ transitions and its interference with the REC effect. It depends on two parameters involving nuclear matrix elements. We showed that when these parameters have values consistent with the estimated values of the relevant matrix elements, the model describes very well the measured spectrum. This finding is the first direct and unambiguous observation of virtual β-γ transitions. The radiation emitted in the EC decay of ^{59}Ni is the coherent sum of both electronic and nuclear components. The latter is responsible for about 4% of the total intensity.

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