

Two-photon exchange amplitude with πN intermediate states: Spin-1/2 and spin-3/2 channels

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We calculate two-photon exchange amplitudes for the elastic electron-proton scattering, and take into account intermediate hadronic states containing a πN system with total angular momentum $1/2$ or $3/2$, which includes eight different channels. This is the improvement of our previous calculation, where only the πN states with quantum numbers of Δ resonance were included. The results show good consistency with recent experimental data. At high Q^2 , newly calculated contributions affect the correction to the measured proton form factor ratio $\mu G_E/G_M$. The total correction becomes somewhat smaller compared to our previous work, but is still significant and grows approximately linearly with Q^2 . Comparing contributions of different channels, we found that larger contributions come from the channels with quantum numbers of lightest resonances.

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I. INTRODUCTION

In our previous work [1], we presented the calculation of the two-photon exchange (TPE) amplitude for the elastic ep scattering with the πN intermediate states with quantum numbers of Δ resonance (P_{33} channel). In the present paper, we report an extension of that calculation, which includes all πN states with spin-parities $J^P = 1/2^+, 1/2^-, 3/2^+, 3/2^-$ and compare the results to recent experimental data.

The only paper that has a similarity to the present work is Ref. [2], where TPE contributions of spin-1/2 and spin-3/2 resonances were studied. However, in Ref. [2] the resonances were assumed to be infinitely narrow, and only one observable, the unpolarized cross section, was considered. In the present work we naturally take into account finite resonance width, realistic resonance shape, and form factors, as well as nonresonant background, and study different observables.

For the details of our method, see Ref. [1]. Here we will briefly describe differences to the previously published material.

II. SOME BACKGROUND

We follow the strategy described in Ref. [1] to calculate the contributions of the πN intermediate states. The πN channel with given (iso)spin and parity can be viewed as a continuum of resonances R_W with those quantum numbers and varying mass W . The TPE contribution of the fictitious resonance R_W can be calculated as usual and the full contribution of the corresponding channel is obtained after the integration over W . States with spin-parity $J^P = 1/2^\pm, 3/2^\pm$ and isospin $I = 1/2, 3/2$ are included, which comprise the following eight channels: $S_{11}, S_{31}, P_{11}, P_{31}, P_{13}, P_{33}, D_{13}, D_{33}$.

The amplitude for the electromagnetic transition between some particle R_W with mass W and the proton with mass M ,

$$R_W(p) \rightarrow \gamma^*(q) + p(p-q) \quad (1)$$

(where the momenta are shown in parentheses) is, for $J = 3/2$

$$\begin{aligned} \langle p | J_\mu | R_W \rangle &= \frac{1}{4M^2 \sqrt{MW}} (g_{\mu\alpha} q_\nu - g_{\nu\alpha} q_\mu) \\ &\times \bar{U} [(\hat{p}\gamma^\nu - p^\nu) F_1 - p^\nu F_2 + q^\nu F_3] \gamma_5^{\frac{1+P}{2}} V^\alpha \quad (2) \end{aligned}$$

and for $J = 1/2$

$$\begin{aligned} \langle p | J_\mu | R_W \rangle &= \frac{1}{2\sqrt{MW}} \bar{U} \left[\left(\gamma_\mu - \frac{q_\mu \hat{q}}{q^2} \right) F_1 \right. \\ &\quad \left. + (\gamma_\mu \hat{q} - q_\mu) \frac{F_2}{2M} \right] \gamma_5^{\frac{1-P}{2}} V, \quad (3) \end{aligned}$$

where U and V are spinors of the proton and the particle R_W , respectively, and $P = \pm 1$ is R_W 's parity. The transition form factors F_i depend on q^2 and W . They are complex, though the overall phase is irrelevant. The relations of the transition form factors F_i with the helicity electroproduction amplitudes A_H ($A_{1/2}, A_{3/2}$, and $S_{1/2}$) are, for $J = 3/2$,

$$\begin{aligned} K F_1 &= [(PW - M)^2 - q^2] (P A_{3/2} + \sqrt{3} A_{1/2}), \\ K F_2 &= [W^2 - M^2 + q^2] (P A_{3/2} - \sqrt{3} A_{1/2}) \\ &\quad + 2q^2 P W \sqrt{6} \frac{S_{1/2}}{|\vec{q}|}, \\ K F_3 &= 2W^2 (P A_{3/2} - \sqrt{3} A_{1/2}) \\ &\quad + [W^2 - M^2 + q^2] P W \sqrt{6} \frac{S_{1/2}}{|\vec{q}|}, \quad (4) \end{aligned}$$

and for $J = 1/2$

$$\begin{aligned} K F_1 &= -\frac{q^2}{2M^2} [(PW - M)^2 - q^2] \left[\frac{A_{1/2}}{\sqrt{2}} + (PW + M) \frac{S_{1/2}}{|\vec{q}|} \right], \\ K F_2 &= \frac{1}{M} [(PW - M)^2 - q^2] \left[(PW + M) \frac{A_{1/2}}{\sqrt{2}} + q^2 \frac{S_{1/2}}{|\vec{q}|} \right], \quad (5) \end{aligned}$$

where K is the same as in Eq. (7) of Ref. [1]:

$$\begin{aligned} K &= \frac{1}{2M^2} [(W + M)^2 - q^2] [(W - M)^2 - q^2]^{3/2} \\ &\quad \times \sqrt{\frac{\pi\alpha}{M(W^2 - M^2)}}. \quad (6) \end{aligned}$$

The above equations supersede Eq. (6) of Ref. [1]. The numerical values of the amplitudes A_H were taken from the MAID model [3].

TABLE I. $\chi^2/\text{d.o.f.}$ for the comparison of VEPP-3 and CLAS experiments with different TPE calculations.

Data	no TPE	elastic [8]	narrow Δ [10]	full P_{33} [1]	this work
VEPP	7.97	2.19	1.86	1.68	1.06
VEPP, renorm.	7.97	3.87	3.37	3.18	2.44
CLAS	1.43	1.24	1.29	1.22	1.21

We need to make a remark on the isospin structure of the amplitudes. For our purposes we need the amplitudes for the transition to states of definite isospin ($I = 1/2$ or $I = 3/2$), they are

$$A(\gamma^* p \rightarrow \pi N|_{I=3/2}) = \frac{\sqrt{2}}{\sqrt{3}} A_{p\pi^0} - \frac{1}{\sqrt{3}} A_{n\pi^+} = \frac{\sqrt{2}}{\sqrt{3}} A^{(3/2)}, \quad (7)$$

$$A(\gamma^* p \rightarrow \pi N|_{I=1/2}) = -\frac{1}{\sqrt{3}} A_{p\pi^0} - \frac{\sqrt{2}}{\sqrt{3}} A_{n\pi^+} = -\sqrt{3} A_p^{(1/2)}, \quad (8)$$

where the amplitudes in the right hand side (r.h.s.) are those defined in Ref. [3].

The calculation is then done in the same way as in Ref. [1], with the updated version of the TPECALC program, which now supports intermediate states with spin-parities $1/2^\pm, 3/2^\pm$ [4].

III. RESULTS

A. R_\pm experiments

There were two recent experiments to search for TPE effects: [5,6]. In both experiments, the positron-to-electron cross-section ratio $R_\pm = \sigma(e^+ p)/\sigma(e^- p)$ was measured. This ratio is strictly equal to 1 in the one photon exchange approximation, thus allowing direct observation of the TPE contribution to the unpolarized cross section.

Results of the VEPP-3 experiment [5] show clear deviation of R_\pm from unity. The assumption of “no TPE” yields bad χ^2 , and is thus inconsistent with data. The authors analyzed several

different models of TPE effects, and had found that their experimental results agree best with Refs. [7,8] and [9] (note however that Ref. [9] estimates the TPE effect from an analysis of experimental data, not from a theoretical calculation).

We have calculated R_\pm for the kinematical conditions of Ref. [5]. The calculation was done using several different “flavors” of our model:

- (i) with the elastic intermediate state only [8],
- (ii) with elastic + Δ resonance with zero width [10],
- (iii) with elastic + πN (P_{33} channel), which includes Δ with realistic width and background [1],
- (iv) with elastic + eight πN channels as in the present work.

There are two approaches to comparison of theory and experiment, used in Ref. [5]. As TPE corrections should vanish at $\epsilon \rightarrow 1$, the experimental points with highest ϵ were used to determine normalization, i.e., R_\pm was assumed to be 1 there. Then the theoretically calculated R_\pm is compared to thus normalized data (first row of Table I and black circles in Fig. 1). The alternative, “renormalization”, approach is to shift data so that R_\pm become equal to the theoretical prediction at that points (second row of Table I and white circles in Fig. 1). We see that, with both approaches, the agreement gets gradually better as we include more intermediate states. The best χ^2 is achieved with the first approach and TPE from the present work—we have $\chi^2/\text{d.o.f.} = 1.06$.

As for the CLAS experiment [6], the results for R_\pm quoted there are not so far from 1 within errors, i.e., the “no TPE” hypothesis is not clearly rejected (the authors mention 2.5σ preference to TPE over “no TPE”). Nevertheless, we see similar gradual improvement of χ^2 with the improvement of the TPE model (Table I). The corresponding curves are plotted in Fig. 2.

B. GEp2 γ experiment

In the GEp2 γ experiment [11] the ϵ dependence of measured proton form factor ratio R at $Q^2 = 2.5 \text{ GeV}^2$ was studied. In the one photon exchange approximation this quantity is equal to $\mu G_E/G_M$, and thus independent of ϵ , so a variation of R with ϵ is (a sign of) the TPE effect. The

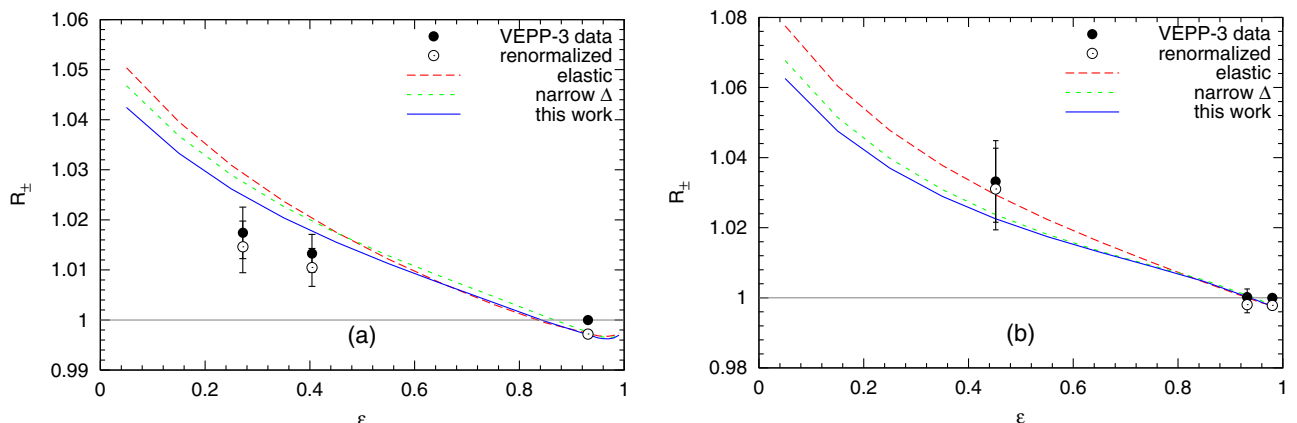


FIG. 1. (Color online) Comparison of VEPP-3 results [5] and our calculations. $E = 1.0 \text{ GeV}$ (a), $E = 1.6 \text{ GeV}$ (b).

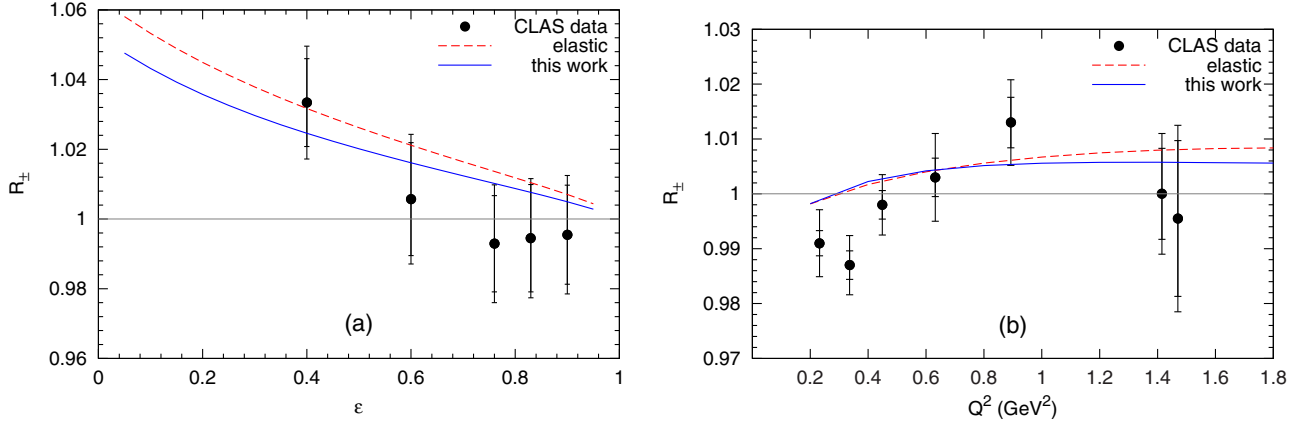


FIG. 2. (Color online) Comparison of CLAS results [6] and our calculations. $Q^2 = 1.45 \text{ GeV}^2$ (a), $\epsilon = 0.88$ (b).

authors conclude that experimental data show no evidence for such a variation.

A comparison of our calculations with the data is shown in Fig. 3, again in two approaches. The dashed lines correspond to the case where the value of R at $\epsilon = 1$, which we need to add to the calculated TPE correction, and is taken as an average of experimental data (dashed lines). In the second case (solid lines) this value is a free parameter, determined by fitting. The agreement of data and theory is evidently better in this case; we see that our results do not conflict with experimental data.

C. High Q^2

The contribution of the newly calculated intermediate states to the *unpolarized cross section* is rather small at high Q^2 ; the elastic contribution dominates in this case.

It was found in Refs. [1,10], that the TPE contribution to the measured *proton form factor ratio* R , coming from either Δ resonance or full $\pi N(P_{33})$ channel, is increasing with Q^2 and far exceeds the elastic contribution. Adding the contributions from the channels considered in the present work, we see that the total effect, though it becomes somewhat smaller, is still

sizeable, and the overall trend persists: δR grows approximately linearly with Q^2 , Fig. 4.

At high Q^2 , other approaches, which exploit quark structure of the nucleon, may be more appropriate [12,13]. However, at present, the correction to electric form factor was not fully calculated in these approaches (because it depends on subleading-order quantities), thus we cannot perform a meaningful comparison with their results.

D. Relative size of the contributions of different channels

As we have calculated the contributions of eight distinct channels, it is interesting to compare them with each other. In Figs. 5 and 6 we plot corresponding contributions to the unpolarized cross section, $\delta\sigma/\sigma$, and measured form factor ratio, δR , versus Q^2 and ϵ .

We see that the dominant contribution among all πN channels always belongs to P_{33} , related to $\Delta(1232)$ resonance. Other significant contributions come from S_{11} , P_{11} (only in $\delta\sigma/\sigma$), P_{13} (only in δR), and in some cases D_{13} , channels. These are the channels where lightest resonances are found— $S_{11}(1535)$, $D_{13}(1520)$, and Roper resonance $P_{11}(1440)$. The findings of Ref. [2] were somewhat different: D_{13} yielded the next largest contribution after P_{33} , and the effect of

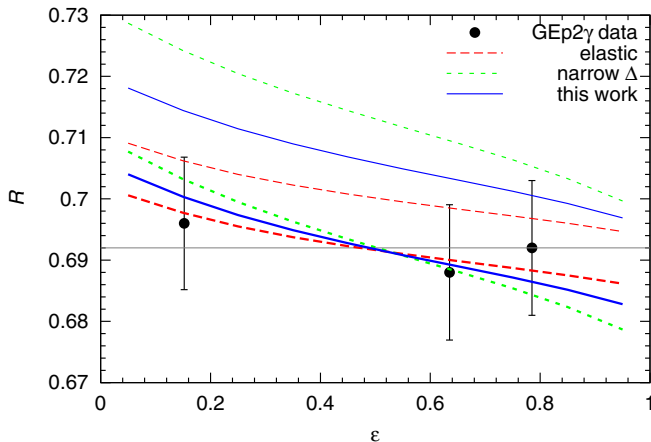


FIG. 3. (Color online) Comparison with GEp2 γ experiment [11]. Thin lines: $R|_{\epsilon=1}$ fixed, thick lines: $R|_{\epsilon=1}$ fitted.

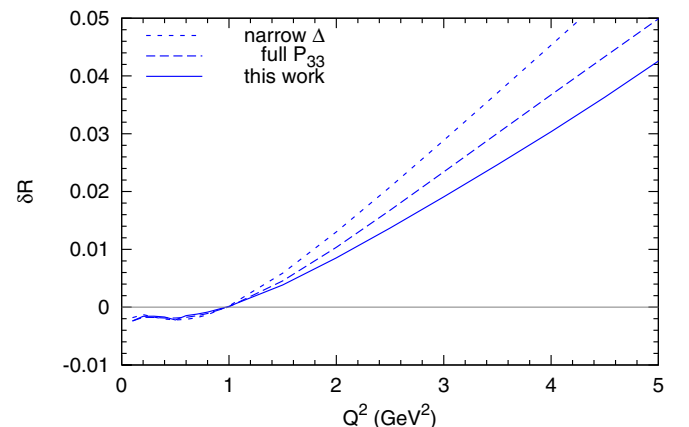


FIG. 4. (Color online) Full TPE correction to measured proton form factor ratio at high Q^2 , fixed $\epsilon = 0.5$.

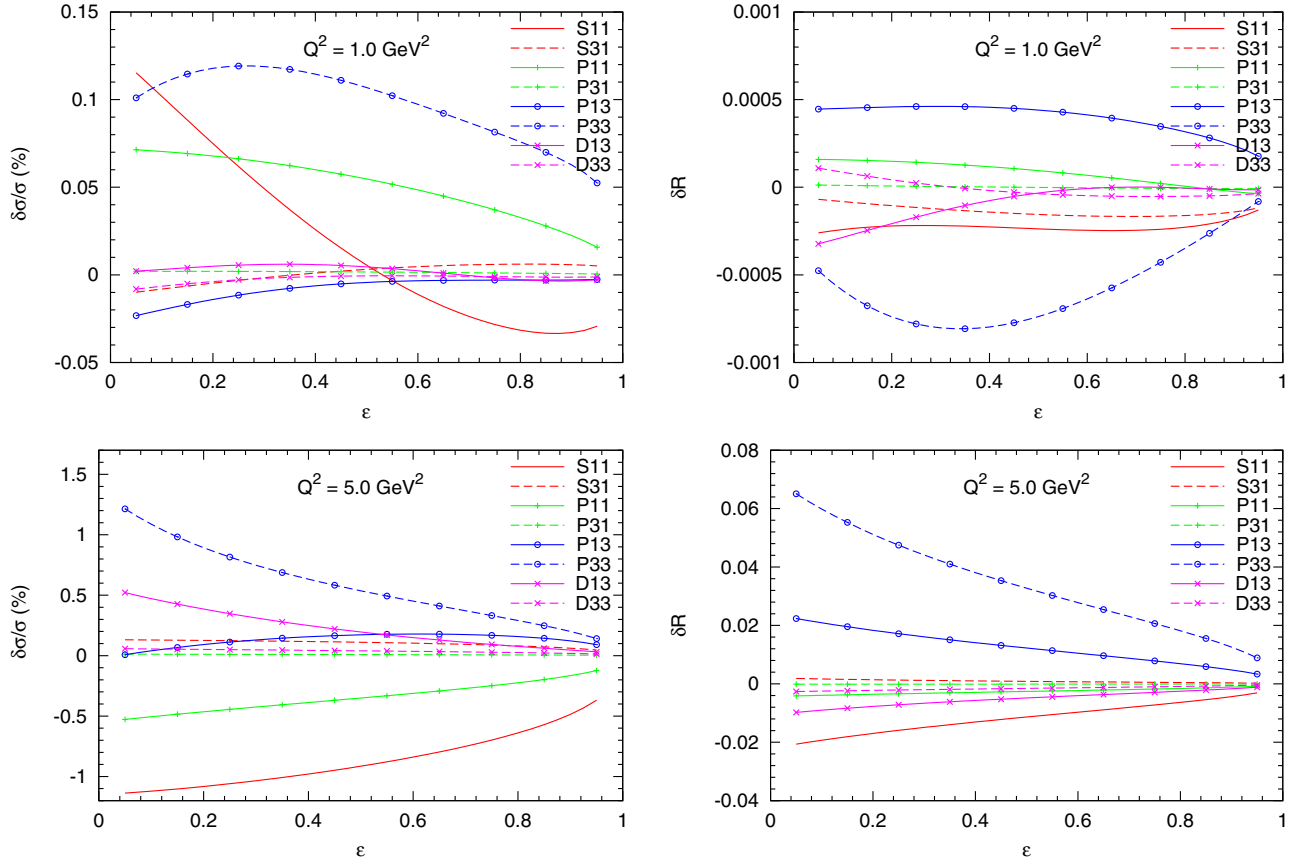


FIG. 5. (Color online) TPE contributions of different channels compared, fixed Q^2 as indicated on the plots.

other resonances was almost negligible. We think this is due to the limitations of the approach of Ref. [2]: the Roper resonance has a large width, and the S_{11} channel has significant nonresonant contribution near threshold, but both are absent in the approximation of Ref. [2].

In qualitative agreement with Ref. [2] (this was also suggested in Ref. [14]), we see that the contributions of different channels tend to cancel each other.

Nevertheless, some doubt remains. The contributions calculated so far can be viewed as the first terms of the (infinite) expansion of the total πN contribution in the intermediate

state's spin J . Whether the series is convergent is not fully clear. Some light may be shed onto this question by calculating contributions of the intermediate states with higher spins ($J \geq 5/2$), which is currently underway.

IV. CONCLUSIONS

We have calculated TPE amplitudes for the elastic electron-proton scattering, taking into account, besides the elastic intermediate state, intermediate states containing a πN system with total angular momentum $1/2$ or $3/2$. This corresponds

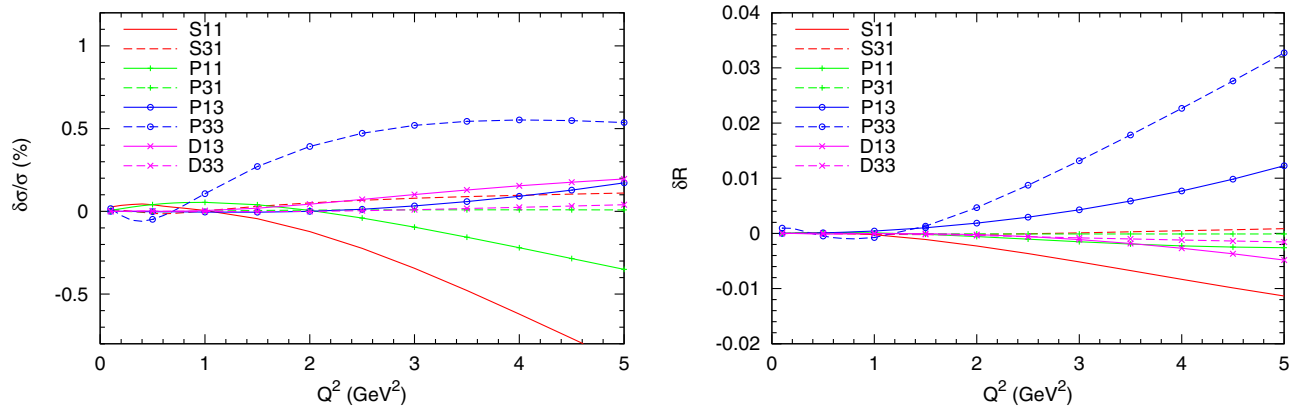


FIG. 6. (Color online) TPE contributions of different channels compared, fixed $\epsilon = 0.5$.

to eight channels (S_{11} , S_{31} , P_{11} , P_{31} , P_{13} , P_{33} , D_{13} , D_{33}). The results agree well with the recent experimental data [5,6]. The agreement is better than with the inclusion of the elastic intermediate state only [8], and better than with the elastic + single P_{33} channel [1], i.e., adding new intermediate states improves the agreement with the experiments.

At high Q^2 , newly calculated contributions affect the correction to the measured proton form factor ratio R . The

total correction, δR , is smaller compared to Ref. [1], but still grows approximately linearly with Q^2 .

Comparing contributions of different channels, we see that larger ones come from the channels with quantum numbers of lightest resonances. It would be interesting to check the contributions of channels with higher spins ($\geq 5/2$), since they also contain prominent resonances, e.g., $F_{15}(1680)$. Such work is currently in progress.

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