Gamow-Teller transitions and magnetic moments using various interactions

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(Received 14 July 2015; published 22 September 2015)

In a single j-shell calculation we consider the effects of several different interactions on Gamow-Teller $B(\mathrm{GT})$ values and magnetic moments. The interactions used are MBZE, $J=0$ pairing, $J_{\max}$ pairing, and half and half. Care is taken when there are isospin crossings and/or degeneracies.

DOI: 10.1103/PhysRevC.92.034322 PACS number(s): 21.60.Cs, 23.40.−s, 24.10.Cn

I. INTRODUCTION

In examining the spectrum of a system of a neutron and a proton beyond a closed shell one sees that not only the $J=0$ $T=1$ but also $J=1 T=0$ and $J=J_{\max}=2j$ lie low. For example in ^{42}Sc the matrix elements taken from the experiment by Escuderos *et al.* [1] are shown in Table I. This calculation has updated $T=0$ two-body matrix elements which are different from those of the earlier works by Bayman *et al.* [2], McCullen *et al.* [3], and Ginocchio *et al.* [4].

In this work we will consider the above interaction which we call MBZE, as well as some extreme interactions:

(a) $J=0$ pairing, the eight matrix elements: $-1,0,0,0,0,0,0,0$,

(b) $J_{\max}$ pairing: $0,0,0,0,0,0,0,-1$,

(c) Half and half: $-1,0,0,0,0,0,0,-1$.

We will study how Gamow-Teller $B(\mathrm{GT})$ values and magnetic moments in the $f_{7/2}$ shell respond to these different interactions.

II. GAMOW-TELLER $B(\mathrm{GT})$ VALUES

We start with the well-known formula for the case where the Fermi matrix element vanishes.

$$ft = 6177/[B(F) + 1.583/B(\mathrm{GT})].$$

In an allowed Fermi transition neither the total angular momentum nor the isospin can change. We will only consider cases where one or both change so that $B(F)=0$. We then obtain

$$ft = 3902.0846497/B(\mathrm{GT}),$$

$$\log(ft) = 3.591266854 - \log[B(\mathrm{GT})].$$

We will be using bare operators throughout. As an orientation we note that for a free neutron $B(\mathrm{GT})=3$.

With the interactions mentioned in the Introduction we can go to more complex systems and obtain wave functions that are represented by amplitudes $D^I(J_p,J_n)$. The square of this amplitude is the probability that in a state I the protons couple to J_p and the neutrons to J_n.

We first consider a simple case where we do not require the amplitude of the transition $^{42}\mathrm{Sc}(I=7^+)\rightarrow{}^{42}\mathrm{Ca}(I=6^+)$. The initial state has isospin $T=0$ and the final $T=1$.

The experimental value is $B(\mathrm{GT})=0.2699$, while the theoretical value, assuming a configuration $(f_{7/2})^2$ for both the initial and final states, is 0.2743. Thus, to agree with experiment, one needs a quenching factor of 0.992 for the GT operator. In Ref. [2] this quenching factor was used. However, in this third work we will stick with the bare operator. It is worth mentioning that in this case we have a proton changing into a neutron inside the nucleus, and a positron and neutrino escaping.

We now show results in Table II which do depend on the amplitudes. The expression for $B(\mathrm{GT})$ is given in two previous publications and is here repeated.

$$X_1 = \sum_{J_p,J_n} D^f(J_p,J_n)D^i(J_p,J_n)U(1J_pI_fJ_n;J_pI_i) \times \sqrt{J_p(J_p+1)},$$

$$X_2 = \sum_{J_p,J_n} D^f(J_p,J_n)D^i(J_p,J_n)U(1J_nI_fJ_p;J_nI_i) \times \sqrt{J_n(J_n+1)},$$

$$B(\mathrm{GT}) = \frac{1}{2}\frac{2I_f+1}{2I_i+1}f(j)^2\left[\frac{\langle 1T_i1M_{T_i}|T_fM_{T_f}\rangle}{\langle 1T_i0M_{T_i}|T_fM_{T_i}\rangle}\right]^2 \times [X_1-(-1)^{I_f-I_i}X_2]^2,$$

where

$$f(j) = \begin{cases} \frac{1}{j} & j=l+\frac{1}{2}, \\ \frac{-1}{j+1} & j=l-\frac{1}{2}. \end{cases}$$

TABLE I. Experimental two-body matrix elements.

$T=1$ J	E	$T=0$ J	E
0	0.0000	1	0.6111
2	1.5865	3	1.4904
4	2.8135	5	1.5101
6	3.2420	7	0.6163

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TABLE II. B(GT) values

Transition	I_i	I_f	MBZE	$J=0$	Half	$J=7$	Expt.
^{43}Sc → ^{43}Ca	3.5	2.5	0.1181	0	0.0592	0.2434	0.039
^{43}Sc → ^{43}Ca	3.5	3.5	0.1682	0.5713	0.2747	0.0397	0.049
^{43}Sc → ^{43}Ca	3.5	4.5	8.31×10^{-6}	0	3.29×10^{-4}	0.00136	
^{44}Sc → ^{44}Ca	2	2	0.0505	0.0613	0.0142	0.0259	0.01962
^{45}Sc → ^{45}Ca	3.5	2.5	0.0094	0	0.0094	2.32×10^{-5}	
^{45}Ca → ^{45}Sc	3.5	3.5	0.0552	0.4571	0.1423	4.49×10^{-4}	
^{45}Sc → ^{45}Ca	3.5	4.5	1.64×10^{-4}	0	3.16×10^{-4}	1.03×10^{-5}	
^{45}Ti → ^{45}Sc	3.5	3.5	0.1466	0.1499	0.1732	5.89×10^{-4}	0.0980
^{46}Ti → ^{46}V	4	4	0.0065	0.0166	0.2898	2.03×10^{-4}	0.0025
^{46}Ti → ^{46}V	4*	4	0.0058	0.5458	0.0018	6.36×10^{-4}	0.0025
^{46}Ti → ^{46}V	1	0	0.0789	0	0.0367	0.2332	0.0273
^{46}Ti → ^{46}V	1*	0	0.0184	0.1523	6.73×10^{-4}	0	

If $T_f \neq T_i$ or $I_f \neq I_i$, we find that $X_1 = -(-1)^{I_f - I_i} X_2$. We then get a simplified formula for B(GT):

$$B(\mathrm{GT}) = 2\frac{2I_f+1}{2I_i+1} f(j)^2 \left[\frac{\langle 1T_i 1M_{T_i}|T_f M_{T_f}\rangle}{\langle 1T_i 0M_{T_i}|T_f M_{T_i}\rangle}\right]^2 (X_1)^2.$$

This formula does not apply to the case of neutron decay because in that case, $I_f = I_i$ and $T_f = T_i$.

Consider first the behavior in going from $J=0$ pairing to $J=7$ pairing via half and half. For the case ^{43}Sc ($I = 7/2$ $T=1/2$) → ^{43}Ca($T=3/2$) we find that when I_f is 5/2 or 9/2, B(GT) vanishes for $J=0$ pairing. For this interaction, seniority v is a good quantum number. We can classify the states by (v,T,t) where t is the reduced isospin. The initial $I = 7/2$ state has $v=1$ and the final states have $v=3$. The reduced isospins are also different, $t=1/2$ and $t=3/2$ respectively. It is not correct to say that seniority must be conserved—that is not the case. As discussed by Harper and Zamick [5,6], with a $J=0$ pairing interaction one cannot have both the seniority and reduced isospin change at the same time.

As we go from $J=0$ pairing to $J=7$ pairing we get a steady increase in B(GT) in the $7/2 \to 9/2$ and $7/2 \to 5/2$ cases. The former values are (0, 3.29×10^{-4}, 0.00136) while for $7/2 \to 5/2$ the values are (0, 0.0592, 0.2434). We next consider $7/2 \to 7/2$ in ^{43}Sc. Now we have an opposite behavior. The $J=0$ case yields the largest value for B(GT).

In ^{45}Sc we have two examples of nonmonotonic behavior for the cases $7/2 \to 9/2$ and $7/2 \to 5/2$. The three values are (0, 3.16×10^{-4}, 1.03×10^{-5}) and (0, 9.4×10^{-3}, 2.32×10^{-5}) respectively. In general, the values of B(GT) in ^{45}Sc are smaller than in ^{43}Sc. It should be mentioned that systematics of B(GT) in the $f_{7/2}$ region can be explained by the Lawson K selection rule [7].

We next carefully discuss the case $I=1^+ \to I=0^+$ in ^{46}Ti. This was discussed by Harper and Zamick [6] but in the context of an $M1$ transition $B(M1)$. However, that makes no difference because it was shown that B(GT) and the corresponding $B(M1)$ were proportional. There is, nonetheless, an apparent difference in the behavior as we go from $J_{\max}$ pairing to $J=0$ pairing. Harper *et al.* [6] state that there is nonmonotonic behavior $-J=7$ is relatively large, half and half small, and $J=0$ pairing large again. But in the second last row of the present work we get a monotonic decrease as we go from $J=7$ to $J=0$.

The difference is that Harper *et al.* [6] always chose the state of lowest energy, while in the present work we take the state of lowest energy for a fixed isospin. As we go to the $J=0$ pairing limit, the $T=2$ $J=1^+$ state in the ^{46}Ti state starts coming below a $T=1$ $J=1^+$ state. The B(GT) [or $B(M1)$] to the $T=2$ state is relatively large and this explains why the value of B(GT), which first decreases in going from $J=7$ to half and half, suddenly increases. If, as we do in this work, we constrain the isospin to be unchanged, we get the simpler monotonic behavior. To get the Harper *et al.* result [6] we take the $J=7$ pairing and the half value from the second last row, 0.0307, and the $J=0$ result from the last row, 0.1532. The $I=1^+$ state in this last row has isospin $T=2$, whereas in the second last row the 1^+ state is the lowest with $T=1$.

For B(GT) ^{46}Ti 4 to 4 we have to take care since for $J=0$ pairing, the lowest 4^+ $T=1$ states are degenerate. We therefore slightly remove the degeneracy by considering an interaction 0.9 $J=0$ pairing and 0.1 $J=7$ pairing. We see that one of the B(GT)'s is small and the other large. With

TABLE III. Log(ft) values.

Transition	I_i	I_f	MBZE	$J=0$	Half	$J=7$	Expt.
^{43}Sc → ^{43}Ca	3.5	2.5	4.519	∞	4.819	4.205	5.0
^{43}Sc → ^{43}Ca	3.5	3.5	4.365	3.834	4.152	4.992	4.9
^{43}Sc → ^{43}Ca	3.5	4.5	8.672	∞	7.074	6.458	
^{44}Sc → ^{44}Ca	2	2	4.888	4.804	5.440	5.178	5.3
^{45}Sc → ^{45}Ca	3.5	2.5	5.619	∞	5.619	8.226	
^{45}Ca → ^{45}Sc	3.5	3.5	4.849	3.931	4.438	7.948	
^{45}Sc → ^{45}Ca	3.5	4.5	7.376	∞	7.092	8.578	
^{45}Ti → ^{45}Sc	3.5	3.5	4.425	4.415	4.353	6.821	4.6
^{46}Ti → ^{46}V	4	4	5.779	5.370	4.130	7.284	6.2
^{46}Ti → ^{46}V	4*	4	5.828	3.854	6.336	6.788	6.2
^{46}Ti → ^{46}V	1	0	4.694	∞	5.027	4.224	5.16
^{46}Ti → ^{46}V	1*	0	5.326	4.409	6.763	∞	

TABLE IV. Magnetic moments

Nucleus	Spin	MBZE	$J = 0$	Half	$J = 7$	Experiment
^{43}Sc	3.5	4.324	3.614	4.204	4.328	+4.62
^{44}Sc	2	1.990	0.592	1.779	2.268	+2.56
^{45}Sc	3.5	4.646	4.468	4.703	4.158	+4.76
^{45}Ti	2.5	−0.764	0.041	−0.905	−0.751	−0.133
^{45}Ti	3.5	−0.604	−0.891	−0.779	−0.377	0.095
^{46}Ti	2	0.991	1.990	1.152	0.613	−0.98

MBZE the B(GT)'s to the lowest two $I = 4^+$ states are both small.

We next compare the "realistic" MBZE results with experiment. Although things are in the right ballpark, there are significant deviations, indicating the need for configuration mixing.

III. MAGNETIC MOMENTS

In Table IV we show a corresponding study of magnetic moments. It should be noted that since 1964 a new magnetic moment has been measured experimentally, that of ^{45}Ti. The value is 0.095, but the sign is undetermined. All our interactions yield negative magnetic moments. The closest is the case of $J_{\max}$ pairing which gives −0.377, still a big discrepancy.

We lastly note that there has been considerable activity with the (^{3}He ,t) reaction by Fujita *et al.* [8–10].The targets in these reactions include ^{44}Ca [8] and ^{42}Ca [9,10] and ^{54}Fe [10]. We also note the theoretical work of Bai *et al.* [11] where GT transitions are calculated with "the isoscalar spin-triplet pairing interaction included in QRPA on top of the isovector spin-singlet one in the HFB method." The B(GT) value in Table II from ^{46}Ti to ^{46}V (0.0273) was inferred from the work of F. Molina *et al.* [12] assuming that in the transition from ^{46}Cr to ^{46}V the spin 1 state at 3806.7 keV has isospin one.

ACKNOWLEDGMENTS

R.G. acknowledges support from the US National Science Foundation through Grant No. PHY-1263280, and he thanks the REU Physics Program at Rutgers University for its support.

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