Hypernuclei and the hyperon problem in neutron stars

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The likely presence of Λ baryons in dense hadronic matter tends to soften the equation of state to an extent that the observed heaviest neutron stars are difficult to explain. We analyze this "hyperon problem" with a phenomenological approach. First, we review what can be learned about the interaction of Λ particle with dense matter from the observed hypernuclei and extend this phenomenological analysis to asymmetric matter. We add to this the current knowledge on nonstrange dense matter, including its uncertainties, to conclude that the interaction between Λ's and dense matter has to become repulsive at densities below three times the nuclear saturation density.

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I. INTRODUCTION

One of the main motivations for the study of matter at densities in excess of the nuclear saturation density is, besides the application to the physics of neutron stars, the possibility of unveiling new phases of matter. Among these more exotic phases, quark matter is the most sought out as its existence is all but guaranteed by the fact that QCD becomes weakly coupled at arbitrarily high densities. However, even if the transition to quark matter occurs at densities inaccessible to neutron stars, nucleons are not obviously the only relevant degrees of freedom. Of particular interest is the possible existence of Λ particles. Due to a combination of circumstances they are likely the first one to appear as the density of matter increases. First, they are the lightest baryon (besides nucleons). Second, they are neutral so their appearance does not incur in the appearance of an electron and the consequent cost of an electron Fermi energy. Finally, as we will argue below, the phenomenology of hypernuclei strongly suggests that Λ's are attracted to neutron matter at the nuclear saturation density, lowering the energetic cost of Λ's even further.

How can the presence of Λ's be inferred from neutron star observations? For a given (zero temperature) equation of state, general relativity predicts a specific relation between the star mass and radius. Radii are very difficult to measure because of the large systematic uncertainties involved [1,2]. Current constraints on the mass-radius relation from radius measurements are not strong enough to significantly constrain the presence of Λ's. However, one feature of general relativity is particularly helpful for putting bounds on the equation of state. For a given equation of state there is a maximum mass beyond which the star will collapse into a black hole, regardless of its radius. Thus, the discovery of two stars with masses around two solar masses requires fairly stiff equations of state and is in tension with the presence of Λ particles, which soften the equation of state significantly. In fact, the equation of state of matter formed by nucleons only can be softened by having neutron on the top of the Fermi sphere transforming (through weak interactions) into Λ's at rest. As a result, the same baryon density can be achieved with a smaller energy density. A rough estimate of the density for the onset of Λ appearance can be obtained by equating the neutron Fermi energy to the mass difference between neutrons and Λ's. For realistic equations of state this density is around a few times nuclear saturation density, well inside the range relevant for neutron stars. Simple calculations assuming a weak interaction between neutrons and Λ's show that the softening of the equation of state makes it very difficult for an equation of state with Λ degrees of freedom to support a star with a mass around two solar masses as recently observed [3,4]. This apparent contradiction is sometimes referred to as "the hyperon problem" [5–14]. A few ways to solve this paradox immediately come to mind. Simply assuming a harder equation of state for nucleonic matter does not necessarily solve the problem and can actually make it worse. A hard equation of state for the neutrons *lowers* the density at which hyperons appear. Another solution would be to assume that the interaction between Λ's and neutrons is sufficiently repulsive to raise the effective mass of the Λ particle in dense neutron matter, raising the threshold for Λ appearance and postponing the softening of the equation of state to irrelevant densities. However, we know that Λ's are in fact *attracted* to nuclear matter since stable (against strong interactions) bound states of a Λ particle with a variety of nuclei are know (for a review see [15]). Microscopic meson exchange models of the Λ-nucleon interaction also indicates an attraction between Λ and neutrons. They are not sufficient to explain away the "hyperon problem" but the addition of a repulsive enough three-body force (ΛNN) might. In fact, one such a model has been constructed [13]. The difficulty with this approach is that the ΛNN three-body force is very little constrained by either experiment or theory. A lattice QCD calculation of the ΛN and ΛNN interaction, although very difficult, is likely to come out in the near future and, in fact, might be the first reliable calculation of forces between baryon to be accomplished [16–18].

In this paper we will not discuss any microscopic model and, instead, take a radically phenomenological approach. First we will review—and slightly extend—what can be learned about the properties of Λ's in a dense medium from a simple-minded phenomenology of hypernuclei. More precisely, we will review the extraction of the Λ mass shift in *nuclear* matter and discuss a bound on the Λ mass shift in *neutron* matter. This analysis will tell us about the Λ properties in neutron/nuclear matter at nuclear saturation densities. We will then use the existence of the neutron stars with $M \approx 2\text{M}_\odot$ to establish constraints on the change of these properties with density. We will show that, to no surprise, the attraction between Λ's and neutrons at nuclear densities has to quickly into a repulsion and will quantify this statement. We will end by commenting on microscopic mechanisms for this change as well as possible hypernuclei experiments which could help our approach narrow down the range of empirically acceptable equations of state.

II. HYPERNUCLEI AND THE INTERACTION BETWEEN Λ AND NUCLEAR/NEUTRON MATTER

The existence of bound states of one Λ particle to a number of nuclei indicates that the interaction between Λ's and nucleons is mostly attractive. A more quantitative statement statement can be made if we consider the binding energies is some detail. Figure 1 shows the measured binding energies of a Λ particle as a function of the mass number A of the nucleus (from the data compiled in [15,19]). In some cases, more than one hypernucleus with the same value of A appears. In the case of small A they correspond to nuclei with different atomic numbers Z. For the larger values of A they correspond to different excited states of the same nucleus corresponding to different values of the angular momentum l as, for instance, the five states in ^{208}Pb.

The detailed value of these binding energies can only be computed in detailed calculations, either starting from the largely unknown Λ-nucleon and Λ-nucleon-nucleon forces or, more ambitiously, from QCD. Both approaches are in their infancy but, once they succeed one can imagine extend them to the study of hyperon in a neutron medium. However, simple phenomenological methods, akin to the mass formula for nonstrange nuclei, can shed light on the data and have the additional advantage of being immediately extendable to strange dense matter. The basic observation is that the interaction between one Λ and the nucleons is short range; in fact, it is expected to be of even shorter range than nuclear forces on account of the absence of the one-pion exchange component which cannot occur for an isoscalar particle. Thus, a Λ particle inside a hypernucleus interacts with only a few nucleons around it. Since nuclei have a fairly constant density, the main effect of the Λ-nucleon interaction is to provide a spherical constant potential well inside which the Λ particle moves freely. The depth of this well is also expected to be the same for every nucleus, again on account of nuclear saturation. This simple model corresponds to a Λ binding energy given by energy of a Λ particle in a spherical well with radius proportional to $A^{1/3}$ plus a fixed shift:

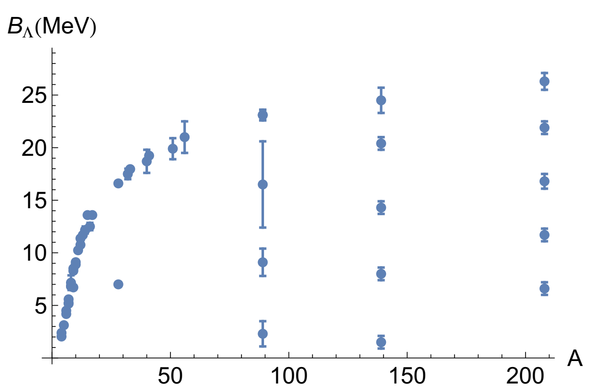

FIG. 1. (Color online) Λ binding energies of known hypernuclei (in MeV). Data taken from [19] and [15].

$$B_\Lambda = \Delta E - c\frac{z_{ln}^2}{A^{2/3}}, \tag{1}$$

where z_{ln} is the nth zero of the lth spherical Bessel function, A is the mass number of the nucleus *not* including the Λ, and ΔE and c are fitting parameters. The values $\Delta E = 24.6$ MeV and $c = 68.7$ MeV were obtained from a fit of the $A \geqslant 8$ nuclei (we exclude very small nuclei where this approach clearly does not make sense). This simple fit does a reasonable job at the qualitative level but leaves a lot of room for improvement. As a way of describing the goodness of the fit we notice that, if we add (in quadratures) a theoretical uncertainty of 10%, the χ^2 per degree of freedom of this fit is about 10.

Two improvements on the model in Eq. (1) make it work much better. One is to use a more precise description of nuclear radii in the kinetic energy term. Nuclear radii, as measured in elastic electron scattering, can be parametrized by $R = r_0(A^{1/3} + 1.565A^{-1/3} - 1.043A^{-1})$ [20]. The parameter r_0 is absorbed in c and is unimportant. This parametrization of the nuclear radius provides a more accurate determination of the kinetic energy of the Λ as a function of A. An alternative procedure would be to let the constants 1.565 and -1.043 to float during the fit but similar results are obtained so, for the sake of brevity, we will not pursue it here. At the boundary of the nucleus the Λ particle potential has to change to zero. Using a smooth shape for the potential as a function of the distance from the center [as opposed to the step function implicit in Eq. (1)] might be more realistic. A similar effect is caused by the thin neutron skin near the boundary. One way of dealing with these corrections is to solve for the Λ energy levels in a Woods-Saxon potential as done in [21]. In order to obtain analytic expressions, we chose to include these effects perturbatively, assuming only that the distance over which the potential transitioned to zero was much smaller than the nucleus radius. A simple first-order perturbative calculation leads to a dependence of the energy of the form $\sim 1/R^3 \sim 1/A$ for this contribution. These two effects combined change Eq. (1) to

$$B_\Lambda = \Delta E - c\frac{z_{ln}^2}{(A^{1/3} + 1.565A^{-1/3} - 1.043A^{-1})^2} + \frac{e}{A}. \tag{2}$$

Fitting the three parameters $\Delta E, c$ and e in Eq. (3) to the binding energy of the Λ to nuclei with $A \geqslant 16$ we obtain

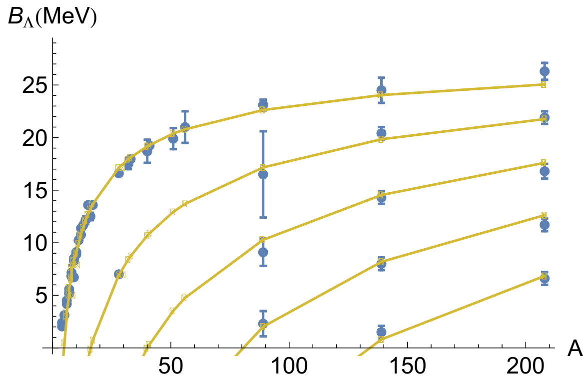

FIG. 2. (Color online) Λ binding energies of known single-Λ hypernuclei and the three parameter fit from Eq. (2). Only nuclei with $A \geqslant 8$ were used in the fit.

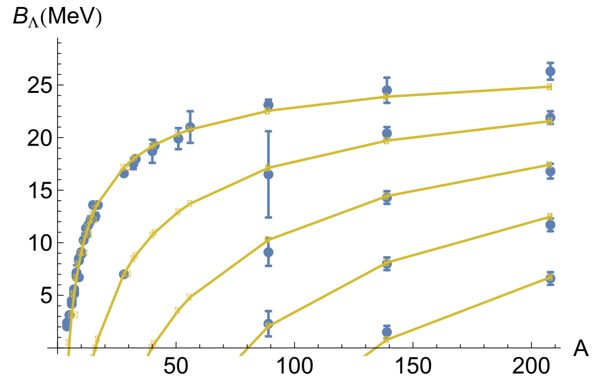

FIG. 4. (Color online) Λ binding energies of known single-Λ hypernuclei and the four parameter fit from Eq. (3).

$\Delta E = 28.5$ MeV, $c = 119$ MeV, and $e = -64.7$ MeV. We again add in quadrature a theoretical uncertainty of 10% to the experimental uncertainty. The plots in Figs. 2 and 3 show this fit compared to the available data. This fit works very well for most nuclei with $A \geqslant 8$, including the excited levels of ^{208}Pb. By varying details of the fit, as the minimum value of A and/or the assumed theoretical error a likely range for $27.5 \text{ MeV} \leqslant \Delta E \leqslant 29.5$ MeV is obtained.

These results indicate that the binding energy of a Λ particle in nuclear matter is about 28 MeV. But in neutron stars, the more relevant quantity is the shift in energy of a Λ particle in *neutron* matter. The data set, however, includes hypernuclei with neutron excess $(A - 2Z)/A$ varying from -0.07 to 0.21 among larger nuclei with $A \geqslant 10$ and it is a natural question whether this spread in neutron excess can be explored in order bound the energy shift in neutron matter.

Starting from the good fit in Eq. (2) (very similar to the one in [21]) we can address the variation of Λ mass with the neutron excess by adding an extra term proportional to the neutron excess $(A - 2Z)/A$ to Eq. (2):

$$B_\Lambda = \Delta E - c\frac{z_{\ln}^2}{(A^{1/3} + 1.565A^{-1/3} - 1.043A^{-1})^2} + \frac{e}{A} + d\left(\frac{A-2Z}{A}\right)^2. \quad (3)$$

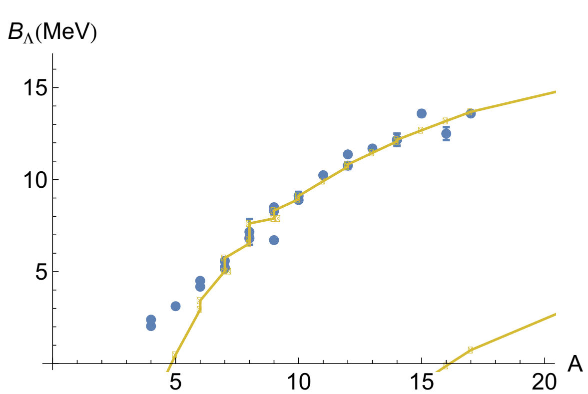

FIG. 3. (Color online) Detail of Fig. 2 for low A.

Terms linear in the neutron excess $(A - 2Z)/A$ are not expected as they are proportional to isospin breaking terms. In fact, a term proportional to $(A - 2Z)/A$ does not improve the fit. A four parameter fit of the $A \geqslant 8$ nuclei gives $\Delta E = 28.5$ MeV, $c = 120$ MeV, $e = -65.1$ MeV, and $d = 4.99$ MeV and is compared to the data in Figs. 4 and 5. This fit is modestly better than the simpler three-parameter fit in Eq. (2) suggesting a small dependence of the binding energies on the neutron excess. In order to estimate the range of acceptable values of d we computed the χ^2 per degree of freedom of a fit of Eq. (3) with fixed value of d. We find that it changes from $\chi^2 = 1.3$ at $d = 4.99$ MeV (the preferred value) to $\chi^2 = 2$ at $d = 20$ MeV or $d = -10$ MeV. Changes in the estimated theoretical uncertainty and minimum value of A used in the fit do not change this result by more than its uncertainty. We take this as evidence that values of d outside the range $-10 \text{ MeV} \lesssim d \lesssim 15$ MeV are disfavored by hypernuclei data. It should be stressed however, that the systematic errors involved in the arbitrary choice of parametrizations are difficult to estimate and are not included in a rigorous way in our estimate. On the experimental side, the observation of further large, neutron rich hypernuclei would help constrain the value of the parameter d.

The small dependence of the Λ binding energy on the neutron excess is expected, at least at small enough densities.

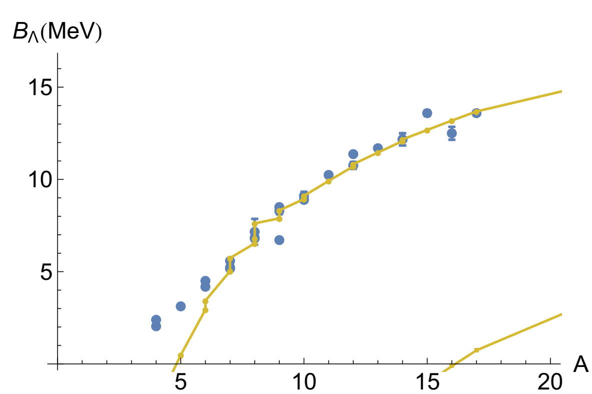

FIG. 5. (Color online) Detail from Fig. 4 for low A. Only nuclei with $A \geqslant 8$ were used in the fit.

In fact, the shift in the energy of a Λ particle at small densities is proportional to the density of the particles in the medium (and proportional to the forward scattering amplitude [22]). But, due to isospin symmetry, the scattering amplitude for the Λ proton and Λ neutron is (approximately) the same. Whether the Λ scatters out of a density n of neutrons or a density xn of protons and $(1-x)n$ of neutrons, the total shift in energy is the same. At high enough densities, the dependence of the energy shift with the density of scatterers is no longer linear and a dependence on the proton fraction appears, even if isospin symmetry was exact.

III. EQUATION OF STATE INCLUDING HYPERONS

We can now use the lessons from the previous section, in special the estimated shift in energy of a single Λ particle in neutron matter, to construct equations of state for matter at the densities relevant for neutron star physics. The first observation is that the presence of too many Λ's softens the equation of state too much to accommodate two solar masses stars. As we will see below, typically, a Λ fraction of no more than about 10% is required. Consequently, the Λ-Λ interaction plays a small role and will be neglected. The energy density is then a linear function of the Λ fraction y. We can write the energy density with baryon number density n and proton fraction x as

$$\epsilon(n,x,y) = \epsilon_N(n,x,y) + \frac{(3\pi^2 yn)^{5/3}}{10\pi^2 M_\Lambda} + yn\left\{M_\Lambda + \left[E_\Lambda + \left(x-\frac{1}{2}\right)^2 S_\Lambda\right] f(n)\right\}, \tag{4}$$

where ϵ_N is the energy of the nucleons, E_Λ is the shift in the Λ energy in nuclear matter (at the saturation density n_0), S_Λ parametrizes the shift of this energy as the proton fraction changes (from nuclear matter with $x = 1/2$ to neutron matter with $x = 0$). Finally, the function $f(n)$ [with $f(n_0) = 1$] parametrizes the change in the Λ energy as the baryon density is changed. Before we discuss the bound on each of these parameters, let us comment on the choice of the form in Eq. (4). Terms linear in $x - 1/2$ (or higher odd powers of $x - 1/2$) are expected to be very small as they result from isospin breaking effects. As commented above, we will be interested in small values of y for which a linear approximation suffices. Finally, if Λ's did not interact with nucleons, the parameters E_Λ and $S_\Lambda = 0$ would vanish and the Λ contribution to the energy density would be given by the free gas term (the term proportional to $y^{5/3}$) plus the contribution of their rest mass.

The information obtained from the study of hypernuclei discussed above sets bounds on the values of E_Λ and S_Λ. The Λ energy shift in nuclear matter ($x = 1/2$), determines $E_\Lambda = \Delta E = 28.5 \pm 2.0$ MeV. This determination is reliable and consistent with the energy shift arising from somewhat different hypernuclei models (for instance, from models including a spin-orbit force [23]). The bounds on S_Λ are looser and follow from our analysis of the previous section:

$$S_\Lambda = 4d \approx 20 \pm 60 \text{ MeV}. \tag{5}$$

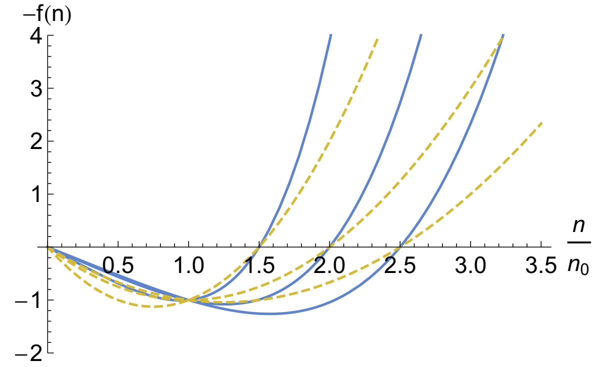

FIG. 6. (Color online) The function $-f(n/n_0)$ for $\Delta = 1.5, 2.0$, and 2.5 and $\delta = 3$ (solid line, blue online) and $\delta = 1$ (dashed, yellow online).

Unfortunately, it does not seem to be currently possible to extract any information about the dependence of the Λ energy away from the saturation density from the phenomenology of hypernuclei. In our parametrization of the equation of state, this dependence is contained in the function $f(n)$ and we will use the existence of two solar mass neutron stars to constrain it. One thing we do know about $f(n)$ is that $f(n) \sim n$ for small values of n (the proportionality constant being related to the forward scattering amplitude [22]). At higher densities, however, the trend of decreasing in-medium Λ mass with the density has to stop. Otherwise, the number of Λ's grows quickly and the equation of state is too soft to support two solar mass stars. We cannot identify the microscopic origin of decreasing in-medium Λ mass, which may come from three-body or higher-order interactions between nucleons and Λ particles. Instead, since we know little about the process leading to the reversal of the trend we will simply parametrize $f(n)$ in terms of two parameters Δ and δ:

$$f(n/n_0) = \frac{n}{n_0}\frac{1}{1-\Delta^{-\delta}}\left[1-\left(\frac{n}{\Delta n_0}\right)^\delta\right]. \tag{6}$$

The parameters Δ fixes the density beyond which the Λ in-medium mass is larger than in the vacuum and it is essential for the plausibility of the equation of state while the parameter δ determines the shape of the mass dependence with density and is of lesser importance. The function $f(n/n_0)$ is plotted in Fig. 6 for several values of Δ and δ.

Now we will discuss the non-strange equation of state $\epsilon_N(n,x,y)$. The highest momentum collisions in neutron matter occur between two back-to-back neutrons at the top of the Fermi surface. At low densities, below $n \lesssim 2n_0$, these momenta are below the inelastic threshold and non-relativistic potential models are adequate to describe them. The force between nucleons can be inferred from the well know measured phase shifts. This program has been carried out and explain well not only nucleon-nucleon scattering but, when phenomenological three-body forces are included, the binding energy of light nuclei. Models with two and three-body forces determined this way were used to study neutron matter (but not nuclear matter) [24–26] using Monte Carlo methods. The dependence on the three-body force, which is less well known than the two-body force, is modest at low densities but becomes more important at higher densities ($n \gtrsim 2n_0$). The same nucleon-nucleon phase shifts can be fit by potentials obtained from a low energy chiral expansion. It has been claimed that the chiral potential, after being evolved according to the renormalization group, can be used perturbatively in calculations of cold neutron matter [27]. The resulting equation of state is very similar, including its uncertainties, as the one obtained by Monte Carlo methods [28,29]. For neutron star applications it is necessary to perform a small extrapolation of the pure neutron matter equation of state to nonzero proton fractions $x \neq 0$. This extrapolation, besides being small ($x \lesssim 6\%$ near saturation), is guided by the empirical values of the symmetry energy and its density dependence. The easiest way to incorporate this information is to parametrize ϵ_N, for instance, in the Skyrme-like form [29,30]:

$$\begin{aligned}\epsilon_N(n,x,y) &= (1-y)nM_N + \frac{3nT_0}{5}(x^{5/3}+(1-x-y)^{5/3})\left(\frac{2n}{n_0}\right)^{2/3}\\ &\quad - T_0[(2\alpha-4\alpha_L)x(1-x-y)+\alpha_L(1-y)^2]\frac{(1-y)^2n^2}{n_0}\\ &\quad + (1-y)nT_0[(2\eta-4\eta_L)x(1-x-y)+\eta_L(1-y)^2]\left(\frac{(1-y)n}{n_0}\right)^{\gamma}, \end{aligned} \tag{7}$$

with $T_0 = (3\pi^2 n_0/2)^{2/3}/2M_N$. When reduced to pure neutron matter ($x = y = 0$), Eq. (7) fits the Monte Carlo results of Refs. [24,26,28,29] very well and it is a convenient manner to parametrize them. Away from $x = 0$ it is the most general even function of $x - (1-y)/2$ (as required by isospin symmetry) up to quadratic order in $x - (1-y)/2$.

The five parameters $\alpha,\alpha_L,\eta,\eta_L$, and γ can be determined by the empirical knowledge of five quantities:

$$\begin{aligned} -B &= \frac{\epsilon_N(n_0,1/2)}{n_0} - \frac{M_N+M_P}{2},\\ p &= n^2\frac{\partial(\epsilon_N/n)}{\partial n}\bigg|_{n=n_0,x=1/2} = 0,\\ K &= 9n_0^2\frac{\partial^2(\epsilon_N/n)}{\partial n^2}\bigg|_{n=n_0,x=1/2},\\ S &= \frac{1}{8n_0}\frac{\partial^2\epsilon_N}{\partial x^2}\bigg|_{n=n_0,x=1/2},\\ L &= \frac{3n_0}{8}\frac{\partial^3\epsilon_N}{\partial n\partial x^2}\bigg|_{n=n_0,x=1/2}. \end{aligned} \tag{8}$$

The analysis of nuclear masses predicts $B = 16 \pm 0.1$ MeV and $n_0 = 0.16 \pm 0.01$ fm^{-3} [31] and the study of giant resonances imply $K = 235 \pm 25$ MeV for the nuclear incompressibility. Finally, a wide range of experimental data from nuclear masses, dipole polarizabilities, and giant resonances implies $S = 32 \pm 2$ MeV for the symmetry energy and $L = 50 \pm 15$ MeV (see [32,33] and references therein). Given values of B, n_0, and K, one can determine α, η, and γ, and then S and L can be used to obtain α_L and η_L.

After a set of parameters is chosen, the β-equilibrated state is found by minimizing $\epsilon(n,x,y)$ in relation to x and y for any given value of n. At the highest density considered and for all parameters used $x < 25\%$, confirming that the extrapolation is relatively small. In any viable equation of state y is very rarely larger than 10% substantiating the assumed linear dependence on y on Eq. (4).

It follows the discussion above that all but two the parameters in our equation of state [Eq. (4)], namely, $\alpha,\alpha_L,\eta,\eta_L,\gamma,E_\Lambda,S_\Lambda$, are constrained, to a larger or lesser degree, by empirical information. We can now use the existence of neutron stars with $M \approx 2\text{M}_\odot$ to constrain the remaining two, Δ and δ. The parameter Δ sets the density, in units of the saturation density, beyond which the Λ is repelled, as opposed to attracted, to dense matter. If Δ is too large, there is a large range of densities where the presence of Λ particles is energetically favored, making the equation of state to soft too support $M \approx 2\text{M}_\odot$ stars. From microphysics, the only thing we know about Δ is that $\Delta > 1$. There is no analytical expression for the density at which Λ particles appear in our model, because it depends on the equation of state of nonstrange matter. This behavior is typical of other more microscopic approaches to hyperonic matter. Our model does not allow any Λ particles in matter at the saturation density. The parameter δ fixes the shape of the in-medium Λ energy as a function of the density. As we will see, neutron star masses puts very loose bounds on it and δ plays very little role in our discussion.

Taking $B = 16$ MeV, $n_0 = 0.16$ fm^{-3}, we perform a Monte Carlo simulation (based on the work in Refs. [1,34,35]) over the estimated values of the parameters K, S, L as in Ref. [36] and also over the new parameters E_Λ, S_Λ, Δ, and δ. We use Gaussian distributions for all parameters except for Δ and δ. For Δ, we use a uniform distribution between 1 and 10 and for δ we use a log-normal distribution centered at 1. The width is fixed at 1/2 (taking logs with base 10) and the one-σ range is approximately 1/3 to 3. We compute the equation of state using 4 and solve the Tolman-Oppenheimer-Volkov equations, rejecting all points which do not have $M > 2\text{M}_\odot$. The results are given in Fig. 7. As Δ becomes large, the pressure of matter decreases with increasing energy density, so values of $\Delta > 4$ are not explored by the Monte Carlo. This puts a strict, if not unsurprising, bound on how fast the Λ binding has to change with density. The upper left corner of the plot is ruled out (small values of δ and large values of Δ) by the maximum mass constraint. This result is consistent with many other previous works which found that

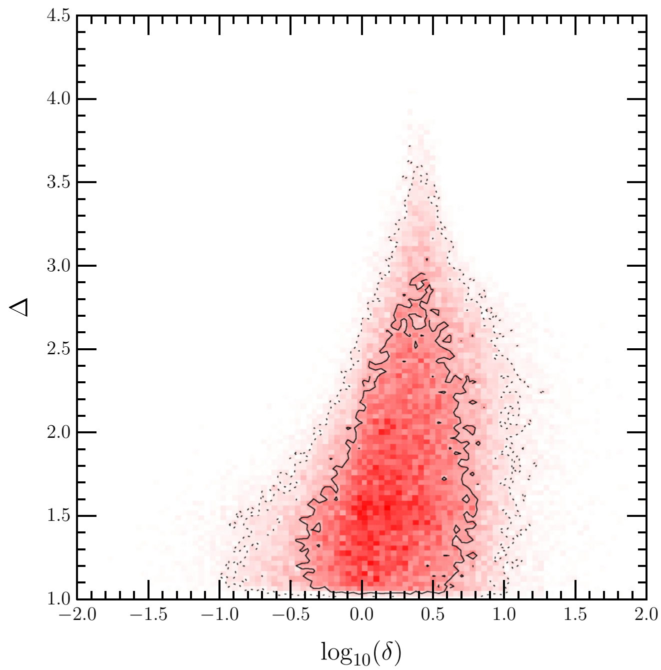

FIG. 7. (Color online) Monte Carlo probability distribution of neutron star models with $M > 2\mathrm{M}_{\odot}$ in the δ-Δ plane.

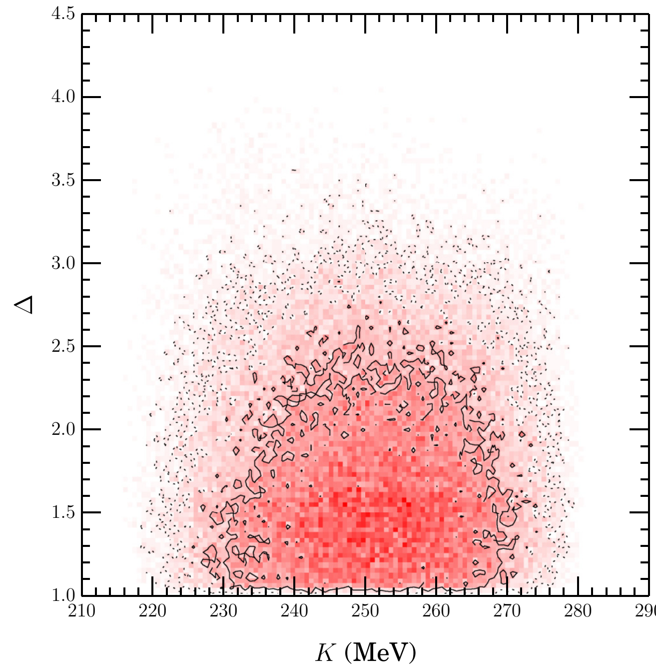

FIG. 8. (Color online) Monte Carlo probability distribution of incompressibility (x axis) and the parameter Δ (y axis) for star models with $M > 2\mathrm{M}_{\odot}$

the "hyperon problem" could be solved with an appropriate variation of model parameters. We find this is also the case in our phenomenological model for small Δ or large δ.

The results above were obtained under some assumptions that we will now discuss. The first was the nonstrange equation of state Eq. (7); while it parametrizes well the fairly well-known low-density ($n \lesssim 2n_0$) equation of state, it is unknown how well it does at higher density. The high density behavior of Eq. (7) is dominated by the value of the exponent γ: higher γ corresponding to stiffer equations of state. The exponent γ itself is largely determined by the value of the nuclear matter compressibility K and is little affected by the remaining parameters in Eq. (8). Thus, the nonstrange equations of state with larger values of K are the ones stiffer at high densities. It turns out, however, that increasing the value of K—even outside the empirically acceptable range—does not increase the range of value of Δ consistent with $M > M_{\odot}$ stars. Figure 8 demonstrates this, where it is clear that K and Δ are essentially uncorrelated. The reason is that, as mentioned above, increase the energetic cost of nucleons only triggers the formation of more Λs without an increase of pressure on energy density. It should be pointed out, however, that with the assumed form of the Λ energy density [Eq. (4)], very stiff equations of state lead to an instability where a large number of Λ's appear. In a more complete model including Λ-Λ interactions this instability would be cutoff by the repulsion between Λ's. Thus, the possibility remains that a stiff nonstrange equation of state coupled to a nonlinear dependence of the energy density with the Λ fraction (y) would support $M > 2.4\mathrm{M}_{\odot}$ stars, regardless of the value of Δ. The few doubly strange hypernuclei presently known might be used to extract some information about Λ-Λ interaction in nuclear matter but the extrapolation of that to neutron matter at higher densities seem too unconstrained to be pursued at the moment. Another possibility is the onset of quark matter at relatively low densities [5,8,9], maybe even at lower densities than the threshold for hyperon formation. That possibility, of course, obviates the hyperon problem while, at the same time, leading to problems of their own that could be solved by postulating a stiffer quark matter equation of state [37].

We find that our parametrization allows Λ hyperons to appear at moderate densities and disappear at higher densities, especially for larger values of Δ. This appears to be a unique feature of our parametrization and provides a possible solution of the hyperon problem. In relativistic mean-field models, for example, the density dependence of the Λ chemical potential closely follows the nucleon chemical potential because they are both controlled by the isoscalar-vector meson which is coupled only to the baryon density. However, this simple behavior of the chemical potentials is highly nongeneric, and our model explores the possibility that μ_n and μ_Λ behave differently at high-densities.

IV. CONCLUSION

We argued that the empirical knowledge of the Λ separation energy in Λ nuclei provides information not only about the mass shift of the Λ particle in nuclear matter but also some bounds on the mass shift in *neutron* matter. We use this bound to construct a phenomenological model for the energy density of neutron/proton/Λ/electron dense matter. This model contains, besides the parameters constrained by microphysics, two new parameters describing the change of the Λ mass shift with density. One of them, Δ, determines the density at which the Λ-neutron matter interaction changes from attractive to repulsive. We found that Δ is constrained to lie in the range

$1 < \Delta \lesssim 3$ in order to ensure hydrodynamic stability and to ensure that stars with $M > 2\mathrm{M}_\odot$ are supported. We do not find strong correlations between our model parameters and the radius of low-mass neutron stars. This implies that other works which have found that neutron star radius measurements are strong probes of Λ-nucleon interactions are at least somewhat model-dependent. The possible presence of quarks will only strengthen this conclusion.

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