Strange and nonstrange baryon spectra in the relativistic interacting quark-diquark model with a Gürsey and Radicati-inspired exchange interaction

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The relativistic interacting quark-diquark model, constructed in the framework of point form dynamics, is extended to strange baryons. The strange and nonstrange baryon spectra are calculated and compared with the experimental data.

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I. INTRODUCTION

The three-quark constituent quark models (QMs) [1–6] are quite successful in describing many baryon observables, like the magnetic moments, the open-flavor decays, and the electromagnetic form factors of the nucleon. These models show some differences, for example, concerning the particular form of Hamiltonian they are based on, but share the main features: (1) they are built upon the effective degrees of freedom of three constituent quarks and (2) their mass operator contains a confining potential which, in general, is linear or quadratic in the quark relative coordinate. In the 1980s, the predictive power of QMs was extended with the unquenched quark model (UQM) formalism [7–16], which introduces the higher Fock $qqq - q\bar{q}$ components in baryon wave functions, arising from the coupling to the meson-baryon continuum. This formalism makes it possible to access a number of problems which cannot be treated in naïve three-quark QMs, such as the calculation of the flavor asymmetry of the nucleon sea [11] or the strange content of the nucleon electromagnetic form factors [12].

One of the main difficulties of three-quark QMs is that they predict a number of states much larger than that of the experimentally observed baryons [17]. This is the well-known problem of the missing resonances. One may try to look for these resonances in channels such as $N\eta$, $N\eta'$, $N\omega$, and $K^+\Lambda$, where the final-state meson is different from the pion [18,19]. Indeed, it is well known that the majority of baryon resonances is seen in reactions in which the pion is present either in the incoming (e.g., $N\pi \to N\pi$) or outgoing (e.g., $N\gamma \to N\pi$) channel. Thus, it would not be surprising if some baryon resonances, very weakly coupled to the single pion, were missing from the experimental results. The other possibility is that the problem of missing resonances has to do with the choice of the effective degrees of freedom. Thus, in quark-diquark models, the effective degree of freedom of the diquark is introduced to describe baryons as bound states of a constituent quark and diquark [20,21]. Since the degrees of freedom of two quarks are frozen in the diquark, the state space will be greatly reduced.

The diquark concept dates back to 1964, when its possibility was hypothesized by Gell-Mann [22] in his original paper on quarks. Many articles have been written on this subject since its introduction (for a review, see Ref. [23]) and more recently the diquark concept has been used in several studies, ranging from one-gluon exchange to lattice QCD calculations [24–34]. Important phenomenological indications for diquark-like correlations [28–30,35,36] and for diquark confinement [37] have also been provided. This makes it plausible enough that diquarks are a part of baryon wave functions.

In this paper, we provide a mass formula for the strange and nonstrange baryon resonances within the interacting quark-diquark model, and then compute the strange and nonstrange baryon spectra. The relativistic interacting quark-diquark model [38–42] is constructed with the point form formalism [43], which was already used to develop point-form three-quark QMs for baryons such as those of Refs. [44,45]. In our model [38–42], baryon excitations are described as two-body quark-diquark bound states. The relative motion between the two constituents and the Hamiltonian of the model are functions of the relative coordinate $\vec{r}$ and its conjugate momentum $\vec{q}$. The Hamiltonian contains a direct (Coulomb + linear confining) interaction and an exchange one, depending on the spins and isospins of the quark and the diquark. The extension of the model [38–42] to strange and heavy (e.g., charmed and bottomed [46]) baryons needs only some small changes in the spin-flavor exchange potential of the model. Specifically, it requires the substitution of the previous spin- and isospin-dependent exchange interaction [39] with a more general one inspired by Gürsey-Radicati [47].

In the end, we compare our theoretical results to the experimental data [17]. Results for the strange baryon spectrum can also be found in Refs. [2,48–52].

II. NONRELATIVISTIC QUARK-DIQUARK STATES

In a quark-diquark model, baryons are assumed to be composed of a constituent quark, q, and a constituent diquark, Q^2. In the energy range we are interested in, i.e., up to 2 GeV, the diquark can be described as two correlated quarks with

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TABLE I. Mass difference (in GeV) between scalar and axial-vector diquarks according to some previous studies.

$M_{[n,n]}$	$M_{\{n,n\}} - M_{[n,n]}$	$M_{[n,s]} - M_{[n,n]}$	$M_{\{n,s\}} - M_{[n,s]}$	$M_{\{n,s\}} - M_{\{n,n\}}$	$M_{\{s,s\}} - M_{\{n,s\}}$	Source
	0.29		0.11			[28]
	0.210		0.150			[29]
0.50	0.30					[38]
0.60	0.35					[39]
0.74	0.21	0.14	0.17	0.10	0.08	[53,54]
0.78	0.28					[55]
0.420	0.520					[56]
0.692	0.330					[57]
0.595	0.205	0.240	0.140	0.175		[58]
0.688	0.202	0.272				[59]
	0.360					[60]
	0.183	0.218	0.176	0.211		[61]
	0.135	0.201	0.138	0.204	0.101	[62]
0.852	0.224	0.288	0.148	0.212	0.084	[63]
0.607	0.356	0.249	0.360	0.253	0.136	This work ("fit 2")

no internal spatial excitations, thus in S wave [38,39]. Then, its color-spin-flavor wave functions must be antisymmetric. Rainbow-ladder DSE calculations confirmed that the first spatially excited diquark, the vector diquark, has a mass much larger than those of the scalar and axial-vector diquark, i.e., the ground-state diquarks [53–55]. Moreover, as we take only light baryons into account, composed of u, d, s quarks, the internal group is restricted to $\mathrm{SU_{sf}}(6)$. Using the conventional notation of denoting spin by its value and flavor and color by the dimension of the representation, the quark has spin $s_2 = \frac{1}{2}$, flavor $F_2 = \mathbf{3}$, and color $C_2 = \mathbf{3}$. Since the hadron must be colorless, the diquark must transform as $\overline{\mathbf{3}}$ under $\mathrm{SU_c}(3)$ and therefore one can have only the symmetric $\mathrm{SU_{sf}}(6)$ representation $\mathbf{21}_{\mathrm{sf}}(\mathrm{S})$, containing $s_1 = 0$, $F_1 = \overline{\mathbf{3}}$, and $s_1 = 1$, $F_1 = \mathbf{6}$, i.e., the scalar and axial-vector diquarks, respectively.

In the following, we will indicate the possible diquark states by their constituent quarks (denoted by s if strange or n otherwise) in square (scalar diquarks) or curly brackets (axial-vector diquarks). The possible scalar diquark configurations are thus $[n,n]$ and $[n,s]$, while the possible axial-vector diquark configurations are $\{n,n\}$, $\{n,s\}$, and $\{s,s\}$ [29]. For quark-diquark states, we use the notation

$$|[q,q]q;\ (\boldsymbol{F}_1,\boldsymbol{F}_2)\boldsymbol{F};\ (t_1,t_2)T;\ (s_1,s_2)S\rangle \tag{1a}$$

or

$$|\{q,q\}q;\ (\boldsymbol{F}_1,\boldsymbol{F}_2)\boldsymbol{F};\ (t_1,t_2)T;\ (s_1,s_2)S\rangle, \tag{1b}$$

where the $\mathrm{SU_f}(3)$ representations of the diquark, $\boldsymbol{F}_1 = \overline{\mathbf{3}}$ or $\mathbf{6}$, and the quark, $\boldsymbol{F}_2 = \mathbf{3}$, are coupled to the $\mathrm{SU_f}(3)$ representation of the baryon, $\boldsymbol{F}$. Similarly, the spins (isospins) of the diquark, s_1 (t_1), and of the quark, s_2 (t_2), are coupled to the total spin (isospin) of the baryon, S (T).

Finally, the quark-diquark basis states for N-, Δ-, Λ-, Σ-, Ξ-, and Ω-type baryons, written in the notation of Eq. (1), are given in the Appendix. See also Table I, where we report some estimations of the masses of axial-vector and scalar diquarks according to some previous studies [28,29,38,39,53–63].

III. THE MASS OPERATOR

We consider a quark-diquark system, where $\vec{r}$ and $\vec{q}$ are the relative coordinate between the two constituents and its conjugate momentum, respectively. The baryon rest-frame mass operator we consider is

$$M = E_0 + \sqrt{\vec{q}^{\,2} + m_1^2} + \sqrt{\vec{q}^{\,2} + m_2^2} + M_{\mathrm{dir}}(r) + M_{\mathrm{ex}}(r), \tag{2}$$

where E_0 is a constant, $M_{\mathrm{dir}}(r)$ and $M_{\mathrm{ex}}(r)$ respectively are the direct and the exchange diquark-quark interactions, m_1 and m_2 stands for diquark and quark masses, where m_1 is either $m_{[q,q]}$ or $m_{\{q,q\}}$ depending if the mass operator acts on a scalar or axial-vector diquark [28,29,53,61–66], with $[q,q] = [n,n]$ or $[n,s]$ and $\{q,q\} = \{n,n\}$, $\{n,s\}$ or $\{s,s\}$.

The direct term we consider,

$$M_{\mathrm{dir}}(r) = -\frac{\tau}{r}(1 - e^{-\mu r}) + \beta r\ , \tag{3}$$

is the sum of a Coulomb-like interaction with a cutoff and a linear confinement term.

We also need an exchange interaction, since this is the crucial ingredient of a quark-diquark description of

TABLE II. Resulting values of the model parameters. The values denoted as "fit 1" are obtained by fitting the mass formula to nonstrange and strange baryons, those denoted as "fit 2" are fitted to the strange sector only.

Parameter	Value (fit 1)	Value (fit 2)	Parameter	Value (fit 1)	Value (fit 2)
m_n	200 MeV	159 MeV	m_s	550 MeV	213 Mev
$m_{[n,n]}$	600 MeV	607 MeV	$m_{[n,s]}$	900 MeV	856 MeV
$m_{\{n,n\}}$	950 MeV	963 MeV	$m_{\{n,s\}}$	1200 MeV	1216 MeV
$m_{\{s,s\}}$	1580 MeV	1352 MeV	τ	1.20	1.02
μ	75.0 fm^{-1}	28.4 fm^{-1}	β	2.15 fm^{-2}	2.36 fm^{-2}
A_S	350 MeV	–436 MeV	A_F	100 MeV	193 MeV
A_I	250 MeV	791 MeV	σ	2.30 fm^{-1}	2.25 fm^{-1}
E_0	141 MeV	150 MeV	ϵ	0.37	
D	6.13 fm^2		η	11.0 fm^{-1}	

baryons [38,67]. Thus, we consider the following interaction, inspired by Gürsey-Radicati [47]:

$$M_{\text{ex}}(r) = (-1)^{L+1} e^{-\sigma r}\left[A_S \vec{s}_1 \cdot \vec{s}_2 + A_F \vec{\lambda}_1^f \cdot \vec{\lambda}_2^f + A_I \vec{t}_1 \cdot \vec{t}_2\right], \tag{4}$$

where $\vec{s}$ and $\vec{t}$ are the spin and isospin operators and $\vec{\lambda}^f$ are the $SU_f(3)$ Gell-Mann matrices. In the nonstrange sector, we also have a contact interaction

$$M_{\text{cont}} = \left(\frac{m_1 m_2}{E_1 E_2}\right)^{1/2+\epsilon} \frac{\eta^3 D}{\pi^{3/2}} e^{-\eta^2 r^2} \delta_{L,0}\delta_{s_1,1} \left(\frac{m_1 m_2}{E_1 E_2}\right)^{1/2+\epsilon}, \tag{5}$$

which was introduced in the mass operator of Ref. [39] to reproduce the $\Delta - N$ mass splitting. It is worthwhile to compare the exchange interactions of Eq. (4) and that of Ref. [39],

$$M_{\text{ex}}(r) = (-1)^{L+1} e^{-\sigma r}[A_S \vec{s}_1 \cdot \vec{s}_2 + A_I \vec{t}_1 \cdot \vec{t}_2 + A_{SI}(\vec{s}_1 \cdot \vec{s}_2)(\vec{t}_1 \cdot \vec{t}_2)]; \tag{6}$$

one can notice that the spin-isospin $(\vec{s}_1 \cdot \vec{s}_2)(\vec{t}_1 \cdot \vec{t}_2)$ term of Eq. (6) has here been substituted with a flavor-dependent one. The isospin dependence is still necessary in Eq. (4), because there are resonances which have the same quantum numbers except the isospins. These baryons, belonging to the same $SU_f(3)$ representation, have different isospins that result from different combinations of the isospins of the quark and the diquark, like $\Lambda(1600)$ and $\Sigma(1193)$ (see Tables V and VII). Thus, without the introduction of an isospin dependence into the exchange interaction, the previous states, $\Lambda(1600)$ and $\Sigma(1193)$, would become degenerate and lie at the same energy.

TABLE III. Comparison between the experimental [17] values of non strange baryon resonances masses (up to 2 GeV) and the numerical ones, from "Fit 1". J^P and L^P are respectively the total angular momentum and the orbital angular momentum of the baryon, including the parity P; S is the total spin, obtained coupling the spin of the diquark, s_1, and that of the quark; finally n_r is the number of nodes in the radial wave function. Since in the nonstrange sector we can only have two type of diquarks, the scalar, $[n,n]$, and axial-vector diquark, $\{n,n\}$, with spin $s_1 = 0$ and 1, respectively, for simplicity here we use the notation of Refs. [39,42].

Resonance	Status	$M^{\text{exp.}}$ (MeV)	J^P	L^P	S	s_1	n_r	$M^{\text{calc.}}$ (fit 1) (MeV)
$N(939)\ P_{11}$	****	939	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	0	0	939
$N(1440)\ P_{11}$	****	1420–1470	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	0	1	1511
$N(1520)\ D_{13}$	****	1515–1525	$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	0	0	1537
$N(1535)\ S_{11}$	****	1525–1545	$\frac{1}{2}^-$	1^-	$\frac{1}{2}$	0	0	1537
$N(1650)\ S_{11}$	****	1645–1670	$\frac{1}{2}^-$	1^-	$\frac{1}{2}$	1	0	1625
$N(1675)\ D_{15}$	****	1670–1680	$\frac{5}{2}^-$	1^-	$\frac{3}{2}$	1	0	1746
$N(1680)\ F_{15}$	****	1680–1690	$\frac{5}{2}^+$	2^+	$\frac{1}{2}$	0	0	1799
$N(1700)\ D_{13}$	***	1650–1750	$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	1	0	1625
$N(1710)\ P_{11}$	***	1680–1740	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	1	0	1776
$N(1720)\ P_{13}$	****	1700–1750	$\frac{3}{2}^+$	0^+	$\frac{3}{2}$	1	0	1648
Missing			$\frac{1}{2}^-$	1^-	$\frac{3}{2}$	1	0	1746
Missing			$\frac{3}{2}^-$	1^-	$\frac{3}{2}$	1	0	1746
Missing			$\frac{3}{2}^+$	2^+	$\frac{1}{2}$	0	0	1799
$N(1875)\ D_{13}$	***	1820–1920	$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	0	1	1888
$N(1880)\ P_{11}$	**	1835–1905	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	0	2	1890
$N(1895)\ S_{11}$	**	1880–1910	$\frac{1}{2}^-$	1^-	$\frac{1}{2}$	0	1	1888
$N(1900)\ P_{13}$	***	1875–1935	$\frac{3}{2}^+$	0^+	$\frac{3}{2}$	1	1	1947
$\Delta(1232)\ P_{33}$	****	1230–1234	$\frac{3}{2}^+$	0^+	$\frac{3}{2}$	1	0	1247
$\Delta(1600)\ P_{33}$	***	1500–1700	$\frac{3}{2}^+$	0^+	$\frac{3}{2}$	1	1	1689
$\Delta(1620)\ S_{31}$	****	1600–1660	$\frac{1}{2}^-$	1^-	$\frac{1}{2}$	1	0	1830
$\Delta(1700)\ D_{33}$	****	1670–1750	$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	1	0	1830
$\Delta(1750)\ P_{31}$	*	1708–1780	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	1	0	1489
$\Delta(1900)\ S_{31}$	**	1840–1920	$\frac{1}{2}^-$	1^-	$\frac{3}{2}$	1	0	1910
$\Delta(1905)\ F_{35}$	****	1855–1910	$\frac{5}{2}^+$	2^+	$\frac{3}{2}$	1	0	2042
$\Delta(1910)\ P_{31}$	****	1860–1920	$\frac{1}{2}^+$	2^+	$\frac{3}{2}$	1	0	1827
$\Delta(1920)\ P_{33}$	***	1900–1970	$\frac{3}{2}^+$	2^+	$\frac{3}{2}$	1	0	2042
$\Delta(1930)\ D_{35}$	***	1900–2000	$\frac{5}{2}^-$	1^-	$\frac{3}{2}$	1	0	1910
$\Delta(1940)\ D_{33}$	**	1940–2060	$\frac{3}{2}^-$	1^-	$\frac{3}{2}$	1	0	1910
$\Delta(1950)\ F_{37}$	****	1915–1950	$\frac{7}{2}^+$	2^+	$\frac{3}{2}$	1	0	2042

Finally, it has to be noted that in the present work all the calculations are performed without any perturbative approximation.

The eigenfunctions of the mass operator of Eq. (2) can be seen as eigenstates of the mass operator with interaction in a Bakamjian-Thomas construction [44,68]. The interaction is introduced by adding an interaction term to the free mass operator $M_0 = \sqrt{\vec{q}^{\,2} + m_1^2} + \sqrt{\vec{q}^{\,2} + m_2^2}$, in such a way that the interaction commutes with the noninteracting Lorenz generators and with the noninteracting four-velocity [69].

The dynamics is given by a point-form Bakamjian-Thomas construction. *Point form* means that the Lorentz group is kinematic. Furthermore, since we are doing a point-form Bakamjian-Thomas construction, $P = M V_0$, where V_0 is the noninteracting four-velocity (with eigenvalue v).

The general quark-diquark state, defined on the product space $H_1 \otimes H_2$ of the one-particle spin s_1 (0 or 1) and spin s_2 (1/2) positive energy representations $H_1 = L^2(R^3) \otimes S_1^0$ or $H_1 = L^2(R^3) \otimes S_1^1$ and $H_2 = L^2(R^3) \otimes S_2^{1/2}$ of the Poincaré

TABLE IV. Comparison between the experimental values [17] of Σ, Ξ and Λ-type resonance masses (up to 2 GeV) and the numerical ones (all values are expressed in MeV), from "Fit 1". J^P and L^P are respectively the total angular momentum and the orbital angular momentum of the baryon, including the parity P; S is the total spin, obtained by coupling the spin of the diquark s_1 and that of the quark; Q^2q stands for the diquark-quark structure of the state; $\mathbf{F}$ and $\mathbf{F_1}$ are the dimensions of the $\mathrm{SU_f(3)}$ representations for the baryon and the diquark, respectively; I and t_1 are the isospins of the baryon and the diquark, respectively; finally n_r is the number of nodes in the radial wave function.

Resonance	Status	$M^{\text{exp.}}$ (MeV)	J^P	L^P	S	s_1	Q^2q	$\mathbf{F}$	$\mathbf{F_1}$	I	t_1	n_r	$M^{\text{calc.}}$ (fit 1) (MeV)
$\Sigma(1193)$ P_{11}	****	1189–1197	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	0	$[n,s]n$	**8**	$\bar{\mathbf{3}}$	1	$\frac{1}{2}$	0	1134
$\Sigma(1660)$ P_{11}	***	1630–1690	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	1	$\{n,s\}n$	**8**	**6**	1	$\frac{1}{2}$	0	1734
$\Sigma(1670)$ D_{13}	****	1665–1685	$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	0	$[n,s]n$	**8**	$\bar{\mathbf{3}}$	1	$\frac{1}{2}$	0	1800
$\Sigma(1750)$ S_{11}	***	1730–1800	$\frac{1}{2}^-$	1^-	$\frac{1}{2}$	0	$[n,s]n$	**8**	$\bar{\mathbf{3}}$	1	$\frac{1}{2}$	0	1800
$\Sigma(1770)$ P_{11}	*	≈1770	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	0	$[n,s]n$	**8**	$\bar{\mathbf{3}}$	1	$\frac{1}{2}$	1	1739
$\Sigma(1775)$ D_{15}	****	1770–1780	$\frac{5}{2}^-$	1^-	$\frac{3}{2}$	1	$\{n,s\}n$	**8**	**6**	1	$\frac{1}{2}$	0	2030
Missing			$\frac{1}{2}^-$	1^-	$\frac{1}{2}$	1	$\{n,n\}s$	**8**	**6**	1	1	0	1872
Missing			$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	1	$\{n,n\}s$	**8**	**6**	1	1	0	1872
$\Sigma(1880)$ P_{11}	**	≈1880	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	1	$\{n,n\}s$	**8**	**6**	1	1	0	1751
$\Sigma(1915)$ F_{15}	****	1900–1935	$\frac{5}{2}^+$	2^+	$\frac{1}{2}$	0	$[n,s]n$	**8**	$\bar{\mathbf{3}}$	1	$\frac{1}{2}$	0	2041
$\Sigma(1940)$ D_{13}	***	1900–1950	$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	1	$\{n,s\}n$	**8**	**6**	1	$\frac{1}{2}$	0	1916
$\Sigma(2000)$ S_{11}	*	≈2000	$\frac{1}{2}^-$	1^-	$\frac{1}{2}$	1	$\{n,s\}n$	**8**	**6**	1	$\frac{1}{2}$	0	1916
$\Xi(1318)$ P_{11}	****	1315–1322	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	0	$[n,s]s$	**8**	$\bar{\mathbf{3}}$	$\frac{1}{2}$	$\frac{1}{2}$	0	1343
$\Xi(1820)$ D_{13}	***	1818–1828	$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	0	$[n,s]s$	**8**	$\bar{\mathbf{3}}$	$\frac{1}{2}$	$\frac{1}{2}$	0	2002
Missing			$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	1	$\{n,s\}s$	**8**	**6**	$\frac{1}{2}$	$\frac{1}{2}$	0	1965
Missing			$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	0	$[n,s]s$	**8**	$\bar{\mathbf{3}}$	$\frac{1}{2}$	$\frac{1}{2}$	1	1978
$\Lambda(1116)$ P_{01}	****	1116	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	0	$[n,n]s$	**8**	$\bar{\mathbf{3}}$	0	0	0	1128
$\Lambda(1600)$ P_{01}	***	1560–1700	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	0	$[n,s]n$	**8**	$\bar{\mathbf{3}}$	0	$\frac{1}{2}$	0	1256
Missing			$\frac{3}{2}^+$	0^+	$\frac{3}{2}$	1	$\{n,s\}n$	**8**	**6**	0	$\frac{1}{2}$	0	1613
$\Lambda(1670)$ S_{01}	****	1660–1680	$\frac{1}{2}^-$	1^-	$\frac{1}{2}$	0	$[n,n]s$	**8**	$\bar{\mathbf{3}}$	0	0	0	1756
$\Lambda(1690)$ D_{03}	****	1685–1695	$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	0	$[n,n]s$	**8**	$\bar{\mathbf{3}}$	0	0	0	1756
Missing			$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	0	$[n,n]s$	**8**	$\bar{\mathbf{3}}$	0	0	1	1738
Missing			$\frac{1}{2}^-$	1^-	$\frac{1}{2}$	0	$[n,s]n$	**8**	$\bar{\mathbf{3}}$	0	$\frac{1}{2}$	0	1758
Missing			$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	0	$[n,s]n$	**8**	$\bar{\mathbf{3}}$	0	$\frac{1}{2}$	0	1758
$\Lambda(1800)$ S_{01}	***	1720–1850	$\frac{1}{2}^-$	1^-	$\frac{3}{2}$	1	$\{n,s\}n$	**8**	**6**	0	$\frac{1}{2}$	0	1853
$\Lambda(1810)$ P_{01}	***	1750–1850	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	0	$[n,s]n$	**8**	$\bar{\mathbf{3}}$	0	$\frac{1}{2}$	1	1794
$\Lambda(1820)$ F_{05}	****	1815–1825	$\frac{5}{2}^+$	2^+	$\frac{1}{2}$	0	$[n,n]s$	**8**	$\bar{\mathbf{3}}$	0	0	0	2006
$\Lambda(1830)$ D_{05}	****	1810–1830	$\frac{5}{2}^-$	1^-	$\frac{3}{2}$	1	$\{n,s\}n$	**8**	**6**	0	$\frac{1}{2}$	0	1979
Missing			$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	1	$\{n,s\}n$	**8**	**6**	0	$\frac{1}{2}$	0	1832
Missing			$\frac{3}{2}^-$	1^-	$\frac{3}{2}$	1	$\{n,s\}n$	**8**	**6**	0	$\frac{1}{2}$	0	1853
$\Lambda(1890)$ P_{03}	****	1850–1910	$\frac{3}{2}^+$	2^+	$\frac{1}{2}$	0	$[n,n]s$	**8**	$\bar{\mathbf{3}}$	0	0	0	2006
Missing			$\frac{1}{2}^-$	1^-	$\frac{3}{2}$	1	$\{n,s\}n$	**8**	**6**	0	$\frac{1}{2}$	0	1979
Missing			$\frac{3}{2}^-$	1^-	$\frac{3}{2}$	1	$\{n,s\}n$	**8**	**6**	0	$\frac{1}{2}$	0	1979

Group, is given by [39]

$$|p_1,p_2,\lambda_1,\lambda_2\rangle, \tag{7}$$

where p_1 and p_2 are the four-momenta of the diquark and the quark, respectively, while λ_1 and λ_2 are, respectively, the z projections of their spins.

The velocity states are introduced as [39,43,44]

$$|v,\vec{k}_1,\lambda_1,\vec{k}_2,\lambda_2\rangle = U_{B(v)}|k_1,s_1,\lambda_1,k_2,s_2,\lambda_2\rangle_0, \tag{8}$$

where the suffix 0 means that the diquark and the quark three-momenta $\vec{k}_1$ and $\vec{k}_2$ satisfy the condition

$$\vec{k}_1+\vec{k}_2=0\,. \tag{9}$$

Following the standard rules of the point-form approach, the boost operator $U_{B(v)}$ is taken as a canonical one, showing that the transformed four-momenta are given by $p_{1,2}=B(v)k_{1,2}$ and satisfy

$$p_1^\mu+p_2^\mu=\frac{P_N^\mu}{M_N}\left(\sqrt{\vec{q}^{\;2}+m_1^2}+\sqrt{\vec{q}^{\;2}+m_2^2}\right), \tag{10}$$

where P_N^μ is the observed nucleon four-momentum and M_N is its mass. The important point is that Eq. (8) redefines the single-particle spins. Since canonical boosts are applied, the conditions for a point-form approach [43,70] are satisfied. Thus, the spins on the left-hand state of Eq. (8) perform the same Wigner rotations as $\vec{k}_1$ and $\vec{k}_2$, allowing us to couple the spin and the orbital angular momentum as in the nonrelativistic case [43], while the spins in the ket on the right-hand side of Eq. (8) undergo the single-particle Wigner rotations.

In point-form dynamics, Eq. (2) corresponds to a good mass operator as it commutes with the Lorentz generators and with the four-velocity. We diagonalize (2) in the Hilbert space spanned by the velocity states. Instead of the internal momenta $\vec{k}_1$ and $\vec{k}_2$, one can also use the relative momentum $\vec{q}$, conjugate to the relative coordinate $\vec{r}=\vec{r}_1-\vec{r}_2$, thus considering the following velocity basis states:

$$|v,\vec{q},\lambda_1,\lambda_2\rangle = U_{B(v)}|k_1,s_1,\lambda_1,k_2,s_2,\lambda_2\rangle_0\,. \tag{11}$$

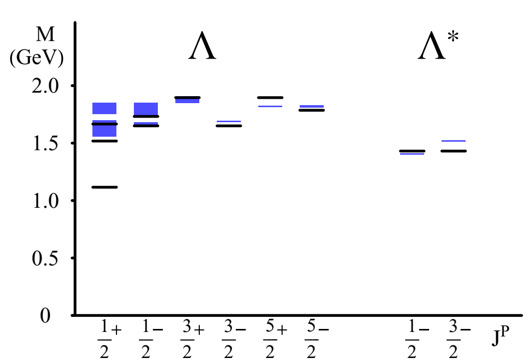

FIG. 1. (Color online) Comparison between the calculated masses (black lines) of the 3* and 4* Λ and Λ* resonances (up to 2 GeV; from fit 2) and the experimental masses from PDG [17] (blue [gray] boxes).

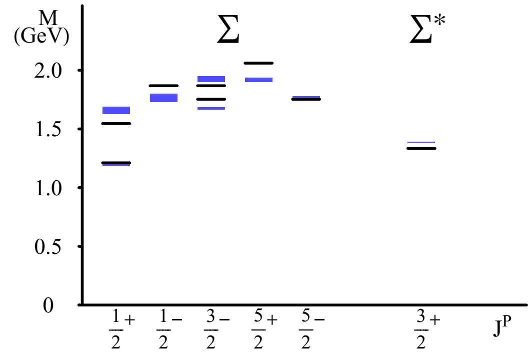

FIG. 2. (Color online) Comparison between the calculated masses (black lines) of the 3* and 4* Σ and Σ* resonances (up to 2 GeV; from fit 2) and the experimental masses from PDG [17] (blue [gray] boxes).

IV. RESULTS AND DISCUSSION

In this section, we show our results for the strange and nonstrange baryon spectra. Because this paper is mainly focused on the extension of the interacting quark-diquark model to strange baryons, here we present the results of two fits to the experimental data [17]. In the first, "fit 1," we fit the model mass formula to the strange and nonstrange baryon spectra, while in the second, "fit 2," we focus our attention on the strange sector only. Obviously, in this second case we expect to get a better reproduction of the experimental data in the strange baryon sector and, perhaps, to increase the predictive power of our model for still unobserved strange baryon resonances. Using the set of parameters of Table II (fit 1), Tables III and IV show the comparison between the experimental data and the results of our quark-diquark model calculation. In this case, the rms deviation is 146 MeV. This value corresponds to the rms deviation corrected for the number of free parameters of the model (fit 1). Figures 1–3 and Tables V–VII show our quark-diquark model results, obtained with the set of parameters of Table II (fit 2). In this second case, the rms deviation is 89 MeV. This value corresponds to

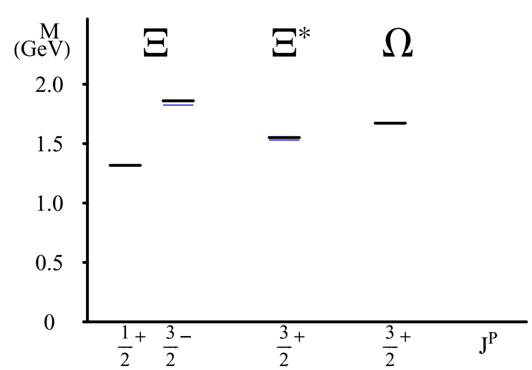

FIG. 3. (Color online) Comparison between the calculated masses (black lines) of the 3* and 4* Ξ, Ξ*, and Ω resonances (up to 2 GeV; from fit 2) and the experimental masses from PDG [17] (blue [gray] boxes).

TABLE V. Comparison between the experimental values [17] of Σ- and Σ*-type resonance masses (up to 2 GeV) and the numerical ones (all values are expressed in MeV), from fit 2. J^P and L^P are respectively the total angular momentum and the orbital angular momentum of the baryon, including the parity P; S is the total spin, obtained by coupling the spin of the diquark s_1 and that of the quark; Q^2q stands for the diquark-quark structure of the state; **F** and $\mathbf{F_1}$ are the dimensions of the $SU_f(3)$ representations for the baryon and the diquark, respectively; I and t_1 are the isospins of the baryon and the diquark, respectively; finally n_r is the number of nodes in the radial wave function.

Resonance	Status	$M^{exp.}$ (MeV)	J^P	L^P	S	s_1	Q^2q	**F**	$\mathbf{F_1}$	I	t_1	n_r	$M^{calc.}$ (fit 2) (MeV)
Σ(1193) P_{11}	****	1189—1197	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	0	$[n,s]n$	**8**	$\bar{\mathbf{3}}$	1	$\frac{1}{2}$	0	1211
Σ(1620) S_{11}	**	≈1620	$\frac{1}{2}^-$	1^-	$\frac{3}{2}$	1	$\{n,n\}s$	**8**	**6**	1	1	0	1753
Σ(1660) P_{11}	***	1630–1690	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	1	$\{n,n\}s$	**8**	**6**	1	1	0	1546
Σ(1670) D_{13}	****	1665–1685	$\frac{3}{2}^-$	1^-	$\frac{3}{2}$	1	$\{n,n\}s$	**8**	**6**	1	1	0	1753
Σ(1750) S_{11}	***	1730–1800	$\frac{1}{2}^-$	1^-	$\frac{1}{2}$	0	$[n,s]n$	**8**	$\bar{\mathbf{3}}$	1	$\frac{1}{2}$	0	1868
Σ(1770) P_{11}	*	≈1770	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	1	$\{n,s\}n$	**8**	**6**	1	$\frac{1}{2}$	0	1668
Σ(1775) D_{15}	****	1770–1780	$\frac{5}{2}^-$	1^-	$\frac{3}{2}$	1	$\{n,n\}s$	**8**	**6**	1	1	0	1753
Σ(1880) P_{11}	**	≈1880	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	0	$[n,s]n$	**8**	$\bar{\mathbf{3}}$	1	$\frac{1}{2}$	1	1801
Σ(1915) F_{15}	****	1900–1935	$\frac{5}{2}^+$	2^+	$\frac{1}{2}$	0	$[n,s]n$	**8**	$\bar{\mathbf{3}}$	1	$\frac{1}{2}$	0	2061
Σ(1940) D_{13}	***	1900–1950	$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	0	$[n,s]n$	**8**	$\bar{\mathbf{3}}$	1	$\frac{1}{2}$	0	1868
Missing			$\frac{3}{2}^-$	1^-	$\frac{3}{2}$	1	$\{n,n\}s$	**8**	**6**	1	1	0	1895
Σ(2000) S_{11}	*	≈2000	$\frac{1}{2}^-$	1^-	$\frac{3}{2}$	1	$\{n,n\}s$	**8**	**6**	1	1	0	1895
Σ*(1385) P_{13}	****	1382–1388	$\frac{3}{2}^+$	0^+	$\frac{3}{2}$	1	$\{n,n\}s$	**10**	**6**	1	1	0	1334
Σ*(1840) P_{13}	*	≈1840	$\frac{3}{2}^+$	0^+	$\frac{3}{2}$	1	$\{n,s\}n$	**10**	**6**	1	$\frac{1}{2}$	0	1439
Σ*(2080) P_{13}	**	≈2080	$\frac{3}{2}^+$	0^+	$\frac{3}{2}$	1	$\{n,n\}s$	**10**	**6**	1	1	1	1924

the rms deviation corrected for the number of free parameters of the model (fit 2).

There is a certain difference between the values of the model parameters used in the two fits. This is especially evident in the case of the quark masses and the exchange potential parameters. The values of the parameters strongly depend from one another. Thus, if we modify those for the exchange potential, this will also have an effect on the constituent quark masses. Moreover, and most important, some parameters are present in the first fit and not in second, because they were introduced to reproduce the $\Delta - N$ mass splitting, and thus they are inessential in the strange sector. In fact, we can say that the nonstrange sector is a special case. This is because spin forces are stronger in this sector than in the others. This can be seen not only in baryons, but also in meson spectroscopy, where light meson masses result from very large hyperfine contributions, while, for example, in the strange or charmed sectors spin forces are much weaker. This is the reason why we expect to get better results for heavy baryons [46], where spin forces are weaker and can be treated more easily.

It is also interesting to note that in our model Λ(1116) and Λ*(1520) are described as bound states of a scalar diquark $[n,n]$ and a quark s, where the quark-diquark system is in S or P wave, respectively. This is in accordance with the observations of Refs. [29,30] on Λ's fragmentation functions, that the two resonances can be described as $[n,n] - s$ systems. See Table VII.

TABLE VI. As Table V, but for Ξ-, Ξ*-, and Ω-type resonances.

Resonance	Status	$M^{exp.}$ (MeV)	J^P	L^P	S	s_1	Q^2q	**F**	$\mathbf{F_1}$	I	t_1	n_r	$M^{calc.}$ (fit 2) (MeV)
Ξ(1318) P_{11}	****	1315–1322	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	0	$[n,s]s$	**8**	$\bar{\mathbf{3}}$	$\frac{1}{2}$	$\frac{1}{2}$	0	1317
Missing			$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	1	$\{n,s\}s$	**8**	**6**	$\frac{1}{2}$	$\frac{1}{2}$	0	1772
Ξ(1820) D_{13}	***	1818–1828	$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	0	$[n,s]s$	**8**	$\bar{\mathbf{3}}$	$\frac{1}{2}$	$\frac{1}{2}$	0	1861
Missing			$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	0	$[n,s]s$	**8**	$\bar{\mathbf{3}}$	$\frac{1}{2}$	$\frac{1}{2}$	1	1868
Missing			$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	1	$\{s,s\}n$	**8**	**6**	$\frac{1}{2}$	0	0	1874
Missing			$\frac{3}{2}^-$	1^-	$\frac{3}{2}$	1	$\{n,s\}s$	**8**	**6**	$\frac{1}{2}$	$\frac{1}{2}$	0	1971
Ξ*(1530) P_{13}	****	1531–1532	$\frac{3}{2}^+$	0^+	$\frac{3}{2}$	1	$\{n,s\}s$	**10**	**6**	$\frac{1}{2}$	$\frac{1}{2}$	0	1552
Missing			$\frac{3}{2}^+$	0^+	$\frac{3}{2}$	1	$\{s,s\}n$	**10**	**6**	$\frac{1}{2}$	0	0	1653
Ω(1672) P_{03}	****	1672–1673	$\frac{3}{2}^+$	0^+	$\frac{3}{2}$	1	$\{s,s\}s$	**10**	**6**	0	0	0	1672

TABLE VII. As Table V, but for Λ- and Λ^*-type resonances.

Resonance	Status	$M^{\text{exp.}}$ (MeV)	J^P	L^P	S	s_1	Q^2q	$\mathbf{F}$	$\mathbf{F_1}$	I	t_1	n_r	$M^{\text{calc.}}$ (fit 2) (MeV)
$\Lambda(1116)\ P_{01}$	****	1116	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	0	$[n,n]s$	$\mathbf{8}$	$\bar{\mathbf{3}}$	0	0	0	1116
$\Lambda(1600)\ P_{01}$	***	1560–1700	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	0	$[n,s]n$	$\mathbf{8}$	$\bar{\mathbf{3}}$	0	$\frac{1}{2}$	0	1518
$\Lambda(1670)\ S_{01}$	****	1660–1680	$\frac{1}{2}^-$	1^-	$\frac{1}{2}$	0	$[n,n]s$	$\mathbf{8}$	$\bar{\mathbf{3}}$	0	0	0	1650
$\Lambda(1690)\ D_{03}$	****	1685–1695	$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	0	$[n,n]s$	$\mathbf{8}$	$\bar{\mathbf{3}}$	0	0	0	1650
Missing			$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	0	$[n,s]n$	$\mathbf{8}$	$\bar{\mathbf{3}}$	0	$\frac{1}{2}$	0	1732
Missing			$\frac{1}{2}^-$	1^-	$\frac{3}{2}$	1	$\{n,s\}n$	$\mathbf{8}$	$\mathbf{6}$	0	$\frac{1}{2}$	0	1785
Missing			$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	0	$[n,n]s$	$\mathbf{8}$	$\bar{\mathbf{3}}$	0	0	1	1785
$\Lambda(1800)\ S_{01}$	***	1720–1850	$\frac{1}{2}^-$	1^-	$\frac{1}{2}$	0	$[n,s]n$	$\mathbf{8}$	$\bar{\mathbf{3}}$	0	$\frac{1}{2}$	0	1732
$\Lambda(1810)\ P_{01}$	***	1750–1850	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	0	$[n,n]s$	$\mathbf{8}$	$\bar{\mathbf{3}}$	0	0	1	1666
$\Lambda(1820)\ F_{05}$	****	1815–1825	$\frac{5}{2}^+$	2^+	$\frac{1}{2}$	0	$[n,n]s$	$\mathbf{8}$	$\bar{\mathbf{3}}$	0	0	0	1896
$\Lambda(1830)\ D_{05}$	****	1810–1830	$\frac{5}{2}^-$	1^-	$\frac{3}{2}$	1	$\{n,s\}n$	$\mathbf{8}$	$\mathbf{6}$	0	$\frac{1}{2}$	0	1785
$\Lambda(1890)\ P_{03}$	****	1850–1910	$\frac{3}{2}^+$	0^+	$\frac{3}{2}$	1	$\{n,s\}n$	$\mathbf{8}$	$\mathbf{6}$	0	$\frac{1}{2}$	0	1896
Missing			$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	1	$\{n,s\}n$	$\mathbf{8}$	$\mathbf{6}$	0	$\frac{1}{2}$	0	1955
Missing			$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	0	$[n,s]n$	$\mathbf{8}$	$\bar{\mathbf{3}}$	0	$\frac{1}{2}$	1	1960
Missing			$\frac{1}{2}^-$	1^-	$\frac{1}{2}$	1	$\{n,s\}n$	$\mathbf{8}$	$\mathbf{6}$	0	$\frac{1}{2}$	0	1969
Missing			$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	1	$\{n,s\}n$	$\mathbf{8}$	$\mathbf{6}$	0	$\frac{1}{2}$	0	1969
$\Lambda^*(1405)\ S_{01}$	****	1402–1410	$\frac{1}{2}^-$	1^-	$\frac{1}{2}$	0	$[n,n]s$	$\mathbf{1}$	$\bar{\mathbf{3}}$	0	0	0	1431
$\Lambda^*(1520)\ D_{03}$	****	1519–1521	$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	0	$[n,n]s$	$\mathbf{1}$	$\bar{\mathbf{3}}$	0	0	0	1431
Missing			$\frac{1}{2}^-$	1^-	$\frac{1}{2}$	0	$[n,s]n$	$\mathbf{1}$	$\bar{\mathbf{3}}$	0	$\frac{1}{2}$	0	1443
Missing			$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	0	$[n,s]n$	$\mathbf{1}$	$\bar{\mathbf{3}}$	0	$\frac{1}{2}$	0	1443
Missing			$\frac{1}{2}^-$	1^-	$\frac{1}{2}$	0	$[n,n]s$	$\mathbf{1}$	$\bar{\mathbf{3}}$	0	0	1	1854
Missing			$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	0	$[n,n]s$	$\mathbf{1}$	$\bar{\mathbf{3}}$	0	0	1	1854
Missing			$\frac{1}{2}^-$	1^-	$\frac{1}{2}$	0	$[n,s]n$	$\mathbf{1}$	$\bar{\mathbf{3}}$	0	$\frac{1}{2}$	1	1928
Missing			$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	0	$[n,s]n$	$\mathbf{1}$	$\bar{\mathbf{3}}$	0	$\frac{1}{2}$	1	1928

The presence of more diquark types, with respect to the nonstrange case of Ref. [39], makes the reproduction of the experimental data below the energy of 2 GeV more difficult than before. In particular, one can notice that in the present case (see results from fit 2, Tables V–VII) there are 19 missing resonances below the energy of 2 GeV, while in the nonstrange sector [39] there were no missing states under 2 GeV. Indeed, in the strange sector one has two scalar diquarks, $[n,n]$ and $[n,s]$, and three axial-vector diquarks, $\{n,n\}$, $\{n,s\}$, and $\{s,s\}$, while in the nonstrange sector one only has a scalar diquark, $[n,n]$, and an axial-vector diquark, $\{n,n\}$. Nevertheless, we think that the number of missing resonances of our model may decrease when experimental data from more powerful experiments and more precise data analyses are extracted. The search for these resonances should be one of the main goals of the baryon research programs at JLab, BES, ELSA, Crystal Barrel, and TAPS. See also the latest multichannel Bonn-Gatchina partial wave analysis results, including data from Crystal Barrel and TAPS at ELSA and other laboratories [71].

Baryon resonance problems have already been treated with an algebraic U(4) quark-diquark models [63], unquenched quark models [7–16], and hypercentral models [4,72], but in the end baryon resonances still remain an open problem [73]. In three-quark QMs for baryons, light baryons are ordered according to the approximate $SU_f(3)$ symmetry. Nevertheless, on one hand many unseen excited resonances are predicted by every three-quark model; on the other hand, states with certain quantum numbers appear in the spectrum at excitation energies much lower than predicted [17]. For example, in the nonstrange sector up to an excitation energy of 2.41 GeV, on average about 45 N states are predicted, but only 12 have been established (four- or three-star) and 7 are tentative (two- or one-star) [17]. A possible solution to the puzzle of missing resonances is the introduction of a new effective degree of freedom: the diquark. This is what we tried to do in the present paper and in Ref. [39] in the nonstrange sector.

While the absolute values of the diquark masses are model dependent, their difference is not. Comparing our result for the mass difference between the axial-vector and scalar diquarks to those of Table I, it is interesting to note that our estimations are comparable with the other ones. The main deviation from the evaluations reported in the table arises in the difference $\{n,s\} - [n,s]$.

The whole mass operator of Eq. (2) has been diagonalized by means of a numerical variational procedure, based on harmonic oscillator trial wave functions. With a variational basis of 100 harmonic oscillator shells, the results converge very well.

The present work can be expanded to include charmed and/or bottomed baryons [46], which can be quite interesting in light of the recent experimental effort to study the properties of heavy hadrons [74–78]. The application of our model to the description of heavy baryons is straightforward and does not require a modification of the mass operator.

APPENDIX: QUARK-DIQUARK BASIS

For N-type states, we have the following states:

$$\left|[n,n]n;(\bar{\mathbf{3}}\otimes\mathbf{3})\mathbf{8};\left(0,\tfrac{1}{2}\right)\tfrac{1}{2};\left(0,\tfrac{1}{2}\right)\tfrac{1}{2}\right\rangle, \tag{A1a}$$

$$\left|\{n,n\}n;(\mathbf{6}\otimes\mathbf{3})\mathbf{8};\left(1,\tfrac{1}{2}\right)\tfrac{1}{2};\left(1,\tfrac{1}{2}\right)\tfrac{1}{2}\right\rangle, \tag{A1b}$$

$$\left|\{n,n\}n;(\mathbf{6}\otimes\mathbf{3})\mathbf{8};\left(1,\tfrac{1}{2}\right)\tfrac{1}{2};\left(1,\tfrac{1}{2}\right)\tfrac{3}{2}\right\rangle; \tag{A1c}$$

For Δ-type states, one has

$$\left|\{n,n\}n;(\mathbf{6}\otimes\mathbf{3})\mathbf{10};\left(1,\tfrac{1}{2}\right)\tfrac{3}{2};\left(1,\tfrac{1}{2}\right)\tfrac{1}{2}\right\rangle, \tag{A2a}$$

$$\left|\{n,n\}n;(\mathbf{6}\otimes\mathbf{3})\mathbf{10};\left(1,\tfrac{1}{2}\right)\tfrac{3}{2};\left(1,\tfrac{1}{2}\right)\tfrac{3}{2}\right\rangle; \tag{A2b}$$

For Λ-type states one has

$$\left|[n,n]s;(\bar{\mathbf{3}}\otimes\mathbf{3})\mathbf{8};(0,0)0;\left(0,\tfrac{1}{2}\right)\tfrac{1}{2}\right\rangle, \tag{A3a}$$

$$\left|[n,s]n;(\bar{\mathbf{3}}\otimes\mathbf{3})\mathbf{8};\left(\tfrac{1}{2},\tfrac{1}{2}\right)0;\left(0,\tfrac{1}{2}\right)\tfrac{1}{2}\right\rangle, \tag{A3b}$$

$$\left|\{n,s\}n;(\mathbf{6}\otimes\mathbf{3})\mathbf{8};\left(\tfrac{1}{2},\tfrac{1}{2}\right)0;\left(1,\tfrac{1}{2}\right)\tfrac{1}{2}\right\rangle, \tag{A3c}$$

$$\left|\{n,s\}n;(\mathbf{6}\otimes\mathbf{3})\mathbf{8};\left(\tfrac{1}{2},\tfrac{1}{2}\right)0;\left(1,\tfrac{1}{2}\right)\tfrac{3}{2}\right\rangle; \tag{A3d}$$

for Λ^*-type states one has

$$\left|[n,n]s;(\bar{\mathbf{3}}\otimes\mathbf{3})\mathbf{1};(0,0)0;\left(0,\tfrac{1}{2}\right)\tfrac{1}{2}\right\rangle, \tag{A4a}$$

$$\left|[n,s]n;(\bar{\mathbf{3}}\otimes\mathbf{3})\mathbf{1};\left(\tfrac{1}{2},\tfrac{1}{2}\right)0;\left(0,\tfrac{1}{2}\right)\tfrac{1}{2}\right\rangle; \tag{A4b}$$

for Σ-type states one has

$$\left|[n,s]n;(\bar{\mathbf{3}}\otimes\mathbf{3})\mathbf{8};\left(\tfrac{1}{2},\tfrac{1}{2}\right)1;\left(0,\tfrac{1}{2}\right)\tfrac{1}{2}\right\rangle, \tag{A5a}$$

$$\left|\{n,n\}s;(\mathbf{6}\otimes\mathbf{3})\mathbf{8};(1,0)1;\left(1,\tfrac{1}{2}\right)\tfrac{1}{2}\right\rangle, \tag{A5b}$$

$$\left|\{n,n\}s;(\mathbf{6}\otimes\mathbf{3})\mathbf{8};\ (1,0)1;\left(1,\tfrac{1}{2}\right)\tfrac{3}{2}\right\rangle, \tag{A5c}$$

$$\left|\{n,s\}n;(\mathbf{6}\otimes\mathbf{3})\mathbf{8};\left(\tfrac{1}{2},\tfrac{1}{2}\right)1;\left(1,\tfrac{1}{2}\right)\tfrac{1}{2}\right\rangle, \tag{A5d}$$

$$\left|\{n,s\}n;(\mathbf{6}\otimes\mathbf{3})\mathbf{8};\left(\tfrac{1}{2},\tfrac{1}{2}\right)1;\left(1,\tfrac{1}{2}\right)\tfrac{3}{2}\right\rangle; \tag{A5e}$$

for Σ^*-type states one has

$$\left|\{n,n\}s;(\mathbf{6}\otimes\mathbf{3})\mathbf{10};(1,0)1;\left(1,\tfrac{1}{2}\right)\tfrac{1}{2}\right\rangle, \tag{A6a}$$

$$\left|\{n,n\}s;(\mathbf{6}\otimes\mathbf{3})\mathbf{10};(1,0)1;\left(1,\tfrac{1}{2}\right)\tfrac{3}{2}\right\rangle, \tag{A6b}$$

$$\left|\{n,s\}n;(\mathbf{6}\otimes\mathbf{3})\mathbf{10};\left(\tfrac{1}{2},\tfrac{1}{2}\right)1;\left(1,\tfrac{1}{2}\right)\tfrac{1}{2}\right\rangle, \tag{A6c}$$

$$\left|\{n,s\}n;(\mathbf{6}\otimes\mathbf{3})\mathbf{10};\left(\tfrac{1}{2},\tfrac{1}{2}\right)1;\left(1,\tfrac{1}{2}\right)\tfrac{3}{2}\right\rangle; \tag{A6d}$$

for Ξ-type states one has

$$\left|[n,s]s;(\bar{\mathbf{3}}\otimes\mathbf{3})\mathbf{8};\left(\tfrac{1}{2},0\right)\tfrac{1}{2};\left(0,\tfrac{1}{2}\right)\tfrac{1}{2}\right\rangle, \tag{A7a}$$

$$\left|\{n,s\}s;(\mathbf{6}\otimes\mathbf{3})\mathbf{8};\left(\tfrac{1}{2},0\right)\tfrac{1}{2};\left(1,\tfrac{1}{2}\right)\tfrac{1}{2}\right\rangle, \tag{A7b}$$

$$\left|\{n,s\}s;(\mathbf{6}\otimes\mathbf{3})\mathbf{8};\left(\tfrac{1}{2},0\right)\tfrac{1}{2};\left(1,\tfrac{1}{2}\right)\tfrac{3}{2}\right\rangle, \tag{A7c}$$

$$\left|\{s,s\}n;(\mathbf{6}\otimes\mathbf{3})\mathbf{8};\left(0,\tfrac{1}{2}\right)\tfrac{1}{2};\left(1,\tfrac{1}{2}\right)\tfrac{1}{2}\right\rangle, \tag{A7d}$$

$$\left|\{s,s\}n;(\mathbf{6}\otimes\mathbf{3})\mathbf{8};\left(0,\tfrac{1}{2}\right)\tfrac{1}{2};\left(1,\tfrac{1}{2}\right)\tfrac{3}{2}\right\rangle; \tag{A7e}$$

for Ξ^*-type states one has

$$\left|\{n,s\}s;(\mathbf{6}\otimes\mathbf{3})\mathbf{10};\left(\tfrac{1}{2},0\right)\tfrac{1}{2};\left(1,\tfrac{1}{2}\right)\tfrac{1}{2}\right\rangle, \tag{A8a}$$

$$\left|\{n,s\}s;(\mathbf{6}\otimes\mathbf{3})\mathbf{10};\left(\tfrac{1}{2},0\right)\tfrac{1}{2};\left(1,\tfrac{1}{2}\right)\tfrac{3}{2}\right\rangle, \tag{A8b}$$

$$\left|\{s,s\}n;(\mathbf{6}\otimes\mathbf{3})\mathbf{10};\left(0,\tfrac{1}{2}\right)\tfrac{1}{2};\left(1,\tfrac{1}{2}\right)\tfrac{1}{2}\right\rangle, \tag{A8c}$$

$$\left|\{s,s\}n;(\mathbf{6}\otimes\mathbf{3})\mathbf{10};\left(0,\tfrac{1}{2}\right)\tfrac{1}{2};\left(1,\tfrac{1}{2}\right)\tfrac{3}{2}\right\rangle; \tag{A8d}$$

finally for Ω-type states the only possibility is

$$\left|\{s,s\}s;(\mathbf{6}\otimes\mathbf{3})\mathbf{10};(0,0)0;\left(1,\tfrac{1}{2}\right)\tfrac{3}{2}\right\rangle. \tag{A9}$$

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