# Airy structure in ${ }^{16} \mathrm{O}+{ }^{14} \mathrm{C}$ nuclear rainbow scattering 

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#### Abstract

The Airy structure in ${ }^{16} \mathrm{O}+{ }^{14} \mathrm{C}$ rainbow scattering is studied with an extended double-folding (EDF) model that describes all the diagonal and off-diagonal coupling potentials derived from the microscopic realistic wave functions for ${ }^{16} \mathrm{O}$ by using a density-dependent nucleon-nucleon force. The experimental angular distributions at $E_{L}=132,281$, and 382.2 MeV are well reproduced by the calculations. By studying the energy evolution of the Airy structure, the Airy minimum around $\theta=76^{\circ}$ in the angular distribution at $E_{L}=132 \mathrm{MeV}$ is assigned as the second-order Airy minimum $A 2$ in contrast to the recent literature which assigns it as the third order A3. The Airy minima in the $90^{\circ}$ excitation function is investigated in comparison with well-known ${ }^{16} \mathrm{O}+{ }^{16} \mathrm{O}$ and ${ }^{12} \mathrm{C}+{ }^{12} \mathrm{C}$ systems. Evolution of the Airy structure into the molecular resonances with the ${ }^{16} \mathrm{O}+{ }^{14} \mathrm{C}$ cluster structure in the low-energy region around $E_{\text {c.m. }}=30 \mathrm{MeV}$ is discussed. It is predicted theoretically for the first time for a non- $4 N^{16} \mathrm{O}+{ }^{14} \mathrm{C}$ system that Airy elephants in the $90^{\circ}$ excitation function are present.


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## I. INTRODUCTION

Nuclear rainbow scattering, which is observed under incomplete absorption, can uniquely determine the interactionpotential family up to the internal region without ambiguity [1]. The interaction potential for the ${ }^{16} \mathrm{O}+{ }^{16} \mathrm{O}$ system has been most thoroughly investigated both experimentally and theoretically. Although a shallow potential had been used in heavy-ion scattering and reactions for many years [2], the observation of a nuclear rainbow in ${ }^{16} \mathrm{O}+{ }^{16} \mathrm{O}$ scattering at $E_{L}=350 \mathrm{MeV}$ finally showed that a global interaction potential for this system is deep [1]. It was shown in Ref. [3] that the global deep potential determined in nuclear rainbow scattering can describe in a unified way not only the prerainbows and the Airy structure in the $90^{\circ}$ excitation function, but also the molecular resonances in the low-energy region and the band structure with the ${ }^{16} \mathrm{O}+{ }^{16} \mathrm{O}$ cluster structure. It was also found [3] that the highest-order Airy structure [4] evolves into molecular resonances with the ${ }^{16} \mathrm{O}+{ }^{16} \mathrm{O}$ cluster structure in ${ }^{32} \mathrm{~S}$ as the incident energy decreases. The gross structures in the $90^{\circ}$ excitation function in rainbow scattering separated by the Airy minima have been visually interpreted as pachydermous Airy elephants in Ref. [5].

Rainbow scattering and interaction potentials for the asymmetric ${ }^{16} \mathrm{O}+{ }^{12} \mathrm{C}$ system have been studied systematically at $E_{L}=63-260 \mathrm{MeV}[6-9]$ and $E_{L}=608-1503 \mathrm{MeV}$ [10] and a global potential was determined. The global potential could explain not only the rainbows and prerainbows [1,6-10] but also the molecular resonances in the low-energy region and the superdeformation with the ${ }^{16} \mathrm{O}+{ }^{12} \mathrm{C}$ cluster structure in a unified way [11]. However, in order to explain the Airy minimum observed at much larger angles at around $E_{L}=300 \mathrm{MeV}$ [12], which was impossible to reproduce in the optical model calculations with the global potential, a deeper family potential was needed. In Ref. [12] the order of the Airy minimum was reassigned systematically to be one higher than that reported in previous literature [1,6-10]. For example, the Airy minimum at $\theta=82^{\circ}$ at $E_{L}=132 \mathrm{MeV}$ was assigned $A 3$ instead of $A 2$. Very recently this dilemma was rescued [13]
by noticing that the Airy minimum at the large angle is a new kind of Airy minimum caused dynamically by the coupling to an excited state of ${ }^{12} \mathrm{C}$ and it was found that the experimental angular distributions are reproduced by the coupled channel calculations with a global extended folding potential derived from the microscopic wave functions for ${ }^{12} \mathrm{C}$ and ${ }^{16} \mathrm{O}$.

The ${ }^{16} \mathrm{O}+{ }^{14} \mathrm{C}$ system is situated between ${ }^{16} \mathrm{O}+{ }^{16} \mathrm{O}$ and ${ }^{16} \mathrm{O}+{ }^{12} \mathrm{C}$. Ogloblin et al. [12] measured rainbow scattering for the ${ }^{16} \mathrm{O}+{ }^{14} \mathrm{C}$ system at 132,281 , and 382.2 MeV . Glukhov et al. [14] investigated the Airy structure and concluded that the order of the Airy minimum at $\theta=76^{\circ}$ in the angular distribution at $E_{L}=132 \mathrm{MeV}$ is $A 3$, which is similar to the Airy minimum $A 3$ at $\theta=82^{\circ}$ in the angular distribution of ${ }^{16} \mathrm{O}+{ }^{12} \mathrm{C}$ at $E_{L}=132 \mathrm{MeV}$ claimed with a deeper family potential in Ref. [12].

The purpose of this paper is to study the Airy structure of rainbow scattering for the ${ }^{16} \mathrm{O}+{ }^{14} \mathrm{C}$ system with the extended double-folding model used successfully in Ref. [13] for the ${ }^{16} \mathrm{O}+{ }^{12} \mathrm{C}$ system and to determine the order of the Airy minimum from the energy evolution of the Airy minimum over a wide range of incident energies. It is shown that the Airy minimum at $\theta=76^{\circ}$ in the angular distribution at $E_{L}=132 \mathrm{MeV}$ is $A 2$. This is different from the previous assignment in Ref. [14]. The evolution of the Airy structure into the molecular resonances and the cluster structure in the low-energy region is discussed in comparison with typical systems, such as ${ }^{16} \mathrm{O}+{ }^{16} \mathrm{O}$.

## II. EXTENDED DOUBLE-FOLDING MODEL

We study rainbow scattering for ${ }^{16} \mathrm{O}+{ }^{14} \mathrm{C}$ with an extended double-folding (EDF) model that describes all the diagonal and off-diagonal coupling potentials derived from the microscopic wave functions for ${ }^{16} \mathrm{O}$ by using a densitydependent nucleon-nucleon force. The diagonal and coupling potentials for the ${ }^{16} \mathrm{O}+{ }^{14} \mathrm{C}$ system are calculated by using the

EDF model and are given as follows:

$$
\begin{align*}
V_{i j}(\mathbf{R})= & \int \rho_{i j}^{\left({ }^{16} \mathrm{O}\right)}\left(\mathbf{r}_{1}\right) \rho_{00}^{\left({ }^{(44} \mathrm{C}\right)}\left(\mathbf{r}_{2}\right) \\
& \times v_{N N}\left(E, \rho, \mathbf{r}_{1}+\mathbf{R}-\mathbf{r}_{2}\right) d \mathbf{r}_{1} d \mathbf{r}_{2} \tag{1}
\end{align*}
$$

where $\rho_{00}^{\left({ }^{14} \mathrm{C}\right)}(\mathbf{r})$ represents the diagonal nucleon density of the ground state of ${ }^{14} \mathrm{C}$, which is obtained by the convolution of the proton size from the charge density distribution taken from Ref. [15]. $\rho_{i j}^{\left({ }^{16} \mathrm{O}\right)}(\mathbf{r})$ is the diagonal $(i=j)$ or transition $(i \neq j)$ nucleon density of ${ }^{16} \mathrm{O}$ taken from the microscopic $\alpha+{ }^{12} \mathrm{C}$ cluster-model wave functions calculated in the orthogonalitycondition model (OCM) in Ref. [16], which uses a realistic size parameter both for the $\alpha$ particle and ${ }^{12} \mathrm{C}$. This is an extended version of Ref. [17], which well reproduces almost all the energy levels up to $E_{x} \approx 13 \mathrm{MeV}$ and the electric transition probabilities in ${ }^{16} \mathrm{O}$. The wave functions have been successfully used for the systematic analysis of elastic and inelastic scattering over a wide range of incident energies [13,18-20]. We take into account the important transition densities available in Ref. [16], i.e., g.s. $\leftrightarrow 3^{-}$ (6.13 MeV) and $2^{+}(6.92 \mathrm{MeV})$ in addition to all the diagonal potentials. For the effective interaction $v_{\text {NN }}$ we use the DDM3Y-FR interaction [21], which takes into account the finite-range nucleon-exchange effect. In the calculations we introduce the normalization factor $N_{R}$ for the real part of the double-folding potential [22,23]. An imaginary potential with a Woods-Saxon volume-type form factor (nondeformed) is introduced phenomenologically to take into account the effect of absorption due to other channels.

## III. AIRY STRUCTURE IN ELASTIC ${ }^{16} \mathrm{O}+{ }^{14} \mathrm{C}$ SCATTERING

In Fig. 1 the angular distributions in elastic ${ }^{16} \mathrm{O}+{ }^{14} \mathrm{C}$ scattering calculated by using the single-channel double-folding (DF) model potential are compared with the experimental data [12] at $E_{L}=132,281$, and 382.2 MeV . The normalization factor and volume integral per nucleon pair, $J_{V}$, for the real folding potential, and the imaginary-potential parameters used are given in Table I. The experimental angular distributions are well reproduced by the single-channel calculations. The calculated cross sections are decomposed into the farside (dashed line) and nearside (dash-dotted line) components. The nearside component decreases rapidly beyond the diffraction region and the farside component dominates toward the intermediate-angle region. Thus the broad structure of the angular distribution is the Airy structure of the nuclear rainbow caused by refractive scattering.

The order of the Airy minimum is determined by calculating the angular distribution by switching off the imaginary potential at the highest energy $E_{L}=382.2 \mathrm{MeV}$ in Fig. 1(c). The falloff of the cross sections in the angular distribution, i.e., the darkside of the rainbow, starts beyond $\theta=40^{\circ}$, which means that the minimum at $30^{\circ}$ is the first-order Airy minimum A1. At $E_{L}=281 \mathrm{MeV}$ in Fig. 1(b) the second-order Airy minimum $A 2$ is seen at $30^{\circ}$ in addition to $A 1$ at $45^{\circ}$.

In order to determine the order of the Airy minimum at $\theta=76^{\circ}$ at the lowest energy $E_{L}=132 \mathrm{MeV}$ in Fig. 1(a)


FIG. 1. (Color online) Comparison of the single-channel DF potential model calculations (blue solid line) with the experimental angular distributions of cross sections (ratio to Rutherford scattering) (points) [12] in elastic ${ }^{16} \mathrm{O}+{ }^{14} \mathrm{C}$ scattering at (a) $E_{L}=132 \mathrm{MeV}$, (b) 281 MeV , and (c) 382.2 MeV . The dashed (green) and dash-dotted (gray) lines display the calculated farside and nearside components, respectively. The angular distributions calculated by switching off the imaginary potential ( $W=0$ ) are displayed by the dotted (pink) lines.
without ambiguity, the energy evolution of the Airy structure of the angular distribution between $E_{L}=281 \mathrm{MeV}$ and 116 MeV is calculated by using the single-channel double-folding potential by switching off the imaginary potential. This is

TABLE I. The volume integral per nucleon pair, $J_{V}$, of the the ground-state diagonal potential (in units of $\mathrm{MeV} \mathrm{fm}^{3}$ ) and the imaginary-potential parameters used in the single-channel doublefolding calculations in Fig. 1 and in the coupled channel calculations with EDF in Fig. 2. $N_{R}=1$ is used except for $N_{R}=0.95$ at 132 MeV (single channel) and 281 MeV (coupled channel).

| $E_{L}$ | $J_{V}$ | $W$ | $R$ | $a$ |  | $J_{V}$ |  | $W$ | $R$ | $a$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  |  |



FIG. 2. (Color online) The energy evolution of the Airy structure in the angular distributions of cross sections (ratio to Rutherford scattering) in ${ }^{16} \mathrm{O}+{ }^{14} \mathrm{C}$ scattering calculated by using the singlechannel DF potential by switching off the imaginary potential is shown by the solid lines. The dashed lines at 116 and 132 MeV are the farside component of the calculated cross sections.
displayed in Fig. 2. The value of $N_{R}$ was interpolated or extrapolated from those at $E_{L}=281 \mathrm{MeV}\left(N_{R}=1.0\right)$ and $132 \mathrm{MeV}\left(N_{R}=0.95\right)$. The energy dependence of the DF potential comes mostly from the DDM3Y-FR effective twobody interaction. At $E_{L}=140 \mathrm{MeV}$ the $A 1$ Airy minimum is clearly seen at $100^{\circ}$. Thus the Airy minimum at $76^{\circ}$ at $E_{L}=132 \mathrm{MeV}$ is found to be $A 2$. This assignment of the $A 2$ Airy minimum at $76^{\circ}$ for the ${ }^{16} \mathrm{O}+{ }^{14} \mathrm{C}$ system at 132 MeV corresponds well to the $A 2$ assignment of the Airy minimum at $82^{\circ}$ for the ${ }^{16} \mathrm{O}+{ }^{12} \mathrm{C}$ system at the same $E_{L}=132 \mathrm{MeV}$ in Refs. [7-9]. The energy evolution of the Airy minimum seems to support the interpretation that the minimum (not visible in Fig. 1) around $120^{\circ}$ at $E_{L}=132 \mathrm{MeV}$ is a remnant of the Airy minimum $A 1$. In fact, the calculated angular distribution beyond this angle turns into diffraction-like high-frequency oscillations rising toward the extreme backward angle of $180^{\circ}$. At $E_{L}=132 \mathrm{MeV}$ in Fig. 1(a) $A 3$ is observed at $50^{\circ}$.

It is worth mentioning that coupled reaction channel calculations for the ${ }^{16} \mathrm{O}+{ }^{14} \mathrm{C}$ system at $E_{L}=132$ and


FIG. 3. (Color online) Comparison of the coupled-channel calculations (blue solid line) with experimental angular distributions of cross sections (ratio to Rutherford scattering) (points) [12] in ${ }^{16} \mathrm{O}+{ }^{14} \mathrm{C}$ scattering at (a) $E_{L}=132 \mathrm{MeV}$, (b) 281 MeV , and (c) 382.2 MeV . The dashed (green) and dash-dotted (gray) lines display the calculated farside and nearside components, respectively. The angular distributions calculated by switching off the imaginary potential ( $W=0$ ) are displayed with dotted (pink) lines. Note that the upper horizon scale is for panels (a) and (b) and the lower horizontal scale is for panel (c).

281 MeV in Ref. [24] show that the potential scattering dominates at angles less than $90^{\circ}$ and the contribution of the two-proton cluster-transfer reaction dominates at large angles. The minimum at around $\theta=120^{\circ}$ in ${ }^{16} \mathrm{O}+{ }^{14} \mathrm{C}$ scattering at $E_{L}=132 \mathrm{MeV}$ visible in the farside component in Fig. 2 could be seen only as a remnant of the Airy minimum $A 1$ in experiment. We note that the wrong $A 3$ assignment to the Airy minimum at $76^{\circ}$ at 132 MeV in Ref. [14], which should be $A 2$ due to the Luneberg lens [25] of the mean-field potential, was done simply based on the similarity of the shape of the angular distributions and the Airy minimum between ${ }^{16} \mathrm{O}+{ }^{14} \mathrm{C}$ and ${ }^{16} \mathrm{O}+{ }^{12} \mathrm{C}$ scatterings at the same energy.

In Fig. 3 the angular distributions calculated by using the coupled-channel method are compared with the experimental data. The potential parameters used are given in Table I. The experimental angular distributions are well reproduced by the coupled-channel calculations. There is little difference
between the coupled-channel and the single-channel calculations in Fig. 1 at the higher energies, 382.2 and 281 MeV , although the Airy minimum $A 1$ is slightly shifted forward at 132 MeV compared with the single-channel calculation. Essentially, the effect of channel coupling on the Airy structure is not important and the angular distributions are well described by the mean-field DF potential. A dynamical secondary rainbow due to the coupling to the excited state of ${ }^{12} \mathrm{C}$ observed in ${ }^{16} \mathrm{O}+{ }^{12} \mathrm{C}$ rainbow scattering is not seen in the calculated angular distributions. In this sense, ${ }^{16} \mathrm{O}+{ }^{14} \mathrm{C}$ rainbow scattering is similar to the ${ }^{16} \mathrm{O}+{ }^{16} \mathrm{O}$ system $[1,26]$ rather than to the ${ }^{16} \mathrm{O}+{ }^{12} \mathrm{C}$ system [13] in the way that $\alpha+{ }^{14} \mathrm{C}$ scattering [27] is similar to $\alpha+{ }^{16} \mathrm{O}$ scattering [28].

## IV. AIRY MINIMA AND AIRY ELEPHANTS IN $90^{\circ}$ EXCITATION FUNCTION

Airy elephants in the $90^{\circ}$ excitation function in heavyion rainbow scattering has been a continuing interest $[1,33]$ since their famous discovery in the ${ }^{12} \mathrm{C}+{ }^{12} \mathrm{C}$ excitation function [5]. The existence of the Airy elephants and their numbers can be determined by calculating the Airy minima that cross the $90^{\circ}$ excitation function. To determine the Airy minima theoretically, the global interaction potential that describes rainbow scattering over a wide range of incident energies has to be determined uniquely. The energy of the $A 1$ minimum in the $90^{\circ}$ excitation function can be determined by using the global potential. In the ${ }^{16} \mathrm{O}+{ }^{16} \mathrm{O}$ system, which has been most thoroughly investigated, its unique global potential has made it possible to understand Airy Structure [4], molecular resonances, and cluster structure with the ${ }^{16} \mathrm{O}+{ }^{16} \mathrm{O}$ configuration at low energy in a unified way [3]. As seen in Fig. 4, the A1 Airy minimum in the $90^{\circ}$ excitation function appears at around $E_{\text {c.m. }}=95 \mathrm{MeV}$ and other higher-order Airy minima $A 2, A 3, A 4, A 5$, and $A 6$ appear at around $E_{\text {c.m. }}=75$, $62,47,40$, and 32 MeV , respectively [ $3,4,26,30$ ]. The highest


FIG. 4. (Color online) The observed minima (filled circles) in the $90^{\circ}$ excitation functions for the ${ }^{16} \mathrm{O}+{ }^{16} \mathrm{O}[3,26,29,30],{ }^{12} \mathrm{C}+$ ${ }^{12} \mathrm{C}[5,31,32]$, and ${ }^{16} \mathrm{O}+{ }^{12} \mathrm{C}$ [9] systems are shown as a function of reduced mass. The predicted Airy minima for the ${ }^{16} \mathrm{O}+{ }^{14} \mathrm{C}$ system, $A 1$ and $A 2$, are indicated by open squares. The line is to guide the eye.
order of the Airy minimum is $A 6$ for the ${ }^{16} \mathrm{O}+{ }^{16} \mathrm{O}$ system. At the lower energies below $E_{\text {c.m. }}=32 \mathrm{MeV}$, the gross structure of the Airy structure evolves into the gross structure of the molecular resonances with the ${ }^{16} \mathrm{O}+{ }^{16} \mathrm{O}$ structure, as was shown in Ref. [3]. In Fig. 4, the observed $A 1$ Airy minimum and the highest-order Airy minimum for the ${ }^{12} \mathrm{C}+{ }^{12} \mathrm{C}$ system determined from Refs. [5,31,32] and those for the ${ }^{16} \mathrm{O}+{ }^{12} \mathrm{C}$ system from Ref. [9] are also displayed. For the ${ }^{16} \mathrm{O}+{ }^{12} \mathrm{C}$ system the energy of the $A 1$ Airy minimum was interpolated from the experimental result at $E_{L}=170 \mathrm{MeV}$ ( $E_{\text {c.m. }}=$ 72.9 MeV ) and $132 \mathrm{MeV}\left(E_{\text {c.m. }}=56.6 \mathrm{MeV}\right)$ in Ref. [9]. In Fig. 4 there exist four Airy minima for the ${ }^{12} \mathrm{C}+{ }^{12} \mathrm{C}$ system and three Airy minima for the ${ }^{16} \mathrm{O}+{ }^{12} \mathrm{C}$ system. For the ${ }^{16} \mathrm{O}+{ }^{14} \mathrm{C}$ system, although there is no experimental data, the energy evolution of the Airy minimum in Fig. 2 predicts that the $A 1$ minimum appears at $90^{\circ}$ at $E_{L}=158 \mathrm{MeV}$ and the $A 2$ minimum appears at $E_{L}=116 \mathrm{MeV}$. We see in Fig. 4 that the ${ }^{16} \mathrm{O}+{ }^{14} \mathrm{C}$ system is situated between the two identical systems, ${ }^{16} \mathrm{O}+{ }^{16} \mathrm{O}$ and ${ }^{12} \mathrm{C}+{ }^{12} \mathrm{C}$. The energy between the $A 1$ and $A 2$ minima of about 20 MeV is similar to that of the ${ }^{16} \mathrm{O}+{ }^{16} \mathrm{O}$ system rather than the ${ }^{16} \mathrm{O}+{ }^{12} \mathrm{C}$ system. From this similarity, Airy minima with orders higher than A3 are likely to exist below $E_{L}=115 \mathrm{MeV}$ before the transition into the molecular resonances with the ${ }^{16} \mathrm{O}+{ }^{14} \mathrm{C}$ structure that have been observed in the $E_{\text {c.m. }}=30 \mathrm{MeV}$ region $[34,35]$.

In Fig. 5 the energy evolution of the volume integral of the real potential for the ${ }^{16} \mathrm{O}+{ }^{14} \mathrm{C}$ system is compared with the systematic data for the ${ }^{16} \mathrm{O}+{ }^{16} \mathrm{O},{ }^{12} \mathrm{C}+{ }^{12} \mathrm{C}$, and ${ }^{16} \mathrm{O}+{ }^{12} \mathrm{C}$ systems. The volume integrals for the ${ }^{16} \mathrm{O}+{ }^{14} \mathrm{C}$ system are consistent with the behavior of the other systems in the energy region where experimental data are available. It seems that the number of the Airy minima for identical systems is larger than that for the asymmetric systems. It is highly desired to observe the Airy minima $A 2$ and $A 3$ for the ${ }^{16} \mathrm{O}+{ }^{14} \mathrm{C}$ system experimentally. The lowest energy (highest order) Airy structure, Airy elephant, will evolve into the molecular resonances with the ${ }^{16} \mathrm{O}+{ }^{14} \mathrm{C}$ cluster structure


FIG. 5. (Color online) The values of the volume integrals per nucleon pair of the real potential, $J_{V}$, for ${ }^{16} \mathrm{O}+{ }^{14} \mathrm{C}$ rainbow scattering (red filled circles) are shown in comparison with those for ${ }^{16} \mathrm{O}+{ }^{12} \mathrm{C}$ (pink open squares [9], pink filled diamonds [13]), ${ }^{16} \mathrm{O}+{ }^{16} \mathrm{O}$ (blue up triangles) [26] and ${ }^{12} \mathrm{C}+{ }^{12} \mathrm{C}$ (black down triangles) [10] rainbow scattering. The line is to guide the eye.
in the lower-energy region similar to the ${ }^{16} \mathrm{O}+{ }^{16} \mathrm{O}$ system [3]. The molecular resonance with the ${ }^{16} \mathrm{O}+{ }^{14} \mathrm{C}$ structure has been studied theoretically [36] and experimentally [34,35]. The existence of the molecular resonances with $18^{+}, 20^{+}$, and $22^{+}$(or $20^{+}, 22^{+}$, and $24^{+}$) at $E_{\text {c.m. }}=23.4,27.4$, and 31.05 MeV , respectively, have been reported by Freeman et al. [35]. Abbondanno et al. reported the existence of the molecular resonances with $L=11,13,17$, and (15) at $E_{\text {c.m. }}=18.2,19.1,22.9$, and 23.8 MeV , respectively [34]. Therefore, it is expected that the gross structure, Airy elephant, corresponding to the fourth or fifth Airy minimum in the $90^{\circ}$ excitation function evolves into molecular resonance around $E_{\text {c.m. }}=30 \mathrm{MeV}$. The experimental study of ${ }^{16} \mathrm{O}+{ }^{14} \mathrm{C}$ elastic scattering in the energy region below $E_{L}=132 \mathrm{MeV}$ and above 65 MeV is highly desired to connect the Airy structures (Airy elephants) and the molecular resonances with the ${ }^{16} \mathrm{O}+{ }^{14} \mathrm{C}$ configuration.

## V. SUMMARY

To summarize, we studied the Airy structure in ${ }^{16} \mathrm{O}+{ }^{14} \mathrm{C}$ rainbow scattering with an extended double-folding (EDF)
model that describes all the diagonal and off-diagonal coupling potentials derived from the microscopic wave functions for ${ }^{16} \mathrm{O}$ by using a density-dependent nucleon-nucleon force. The experimental angular distributions at $E_{L}=132,281$, and 382.2 MeV were analyzed and well reproduced by the theoretical calculations. The Airy minimum at $\theta=76^{\circ}$ in the angular distribution at $E_{L}=132 \mathrm{MeV}$ was found to be a second-order Airy minimum $A 2$. The number of the Airy minima in the $90^{\circ}$ excitation function was investigated in comparison with the typical identical ${ }^{16} \mathrm{O}+{ }^{16} \mathrm{O}$ and ${ }^{12} \mathrm{C}+{ }^{12} \mathrm{C}$ systems and at least two Airy minima, Airy elephants, are predicted to exist above $E_{L}=110 \mathrm{MeV}\left(E_{\mathrm{c} . \mathrm{m} .}=51 \mathrm{MeV}\right)$. The evolution of the Airy minima in the $90^{\circ}$ excitation function related to the Airy elephants into molecular resonances with the ${ }^{16} \mathrm{O}+{ }^{14} \mathrm{C}$ cluster structure in the low-energy region around $E_{\text {c.m. }}=30 \mathrm{MeV}$ is discussed.

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