PHYSICAL REVIEW C **92**, 024622 (2015)

Dynamics of neutron-induced fission of ^{235}U using four-dimensional Langevin equations

M. R. Pahlavani* and S. M. Mirfathi

Department of Nuclear Physics, Faculty of Basic Science, University of Mazandaran, P.O. Box 47415-416, Babolsar, Iran

(Received 27 June 2015; revised manuscript received 27 July 2015; published 28 August 2015)

Background: Langevin equations have been suggested as a key approach to the dynamical analysis of energy dissipation in excited nuclei, formed during heavy-ion fusion-fission reactions. Recently, a few researchers theoretically reported investigations of fission for light nuclei in a low excitation energy using the Langevin approach, without considering the contribution of pre- and post-scission particles and γ-ray emission.

Purpose: We study the dynamical evolution of mass distribution of fission fragments, and neutron and γ-ray multiplicity for ^{236}U as compound nuclei that are constructed after fusion of a neutron and ^{235}U.

Method: Energy dissipation of the compound nucleus through fission is calculated using the Langevin dynamical approach combined with a Monte Carlo method. Also the shape of the fissioning nucleus is restricted to "funny hills" parametrization.

Results: Fission fragment mass distribution, neutron and γ-ray multiplicity, and the average kinetic energy of emitted neutrons and γ rays at a low excitation energy are calculated using a dynamical model, based on the four-dimensional Langevin equations.

Conclusions: The theoretical results show reasonable agreement with experimental data and the proposed dynamical model can well explain the energy dissipation in low energy induced fission.

DOI: 10.1103/PhysRevC.92.024622 PACS number(s): 25.70.Jj, 25.85.Ge, 24.10.Lx, 24.10.Pa

I. INTRODUCTION

The characteristics of fission fragments such as mass distribution and the multiplicity of γ rays and emitted particles during fission processes have been studied extensively both theoretically and experimentally [1–7]. Statistical descriptions of the fission process are often used to explain experimental fission characteristics. Weisskopf-type or Hauser-Feshbach approaches have been developed to simulate the particle emission based on sequential emission from the excited fission fragments without considering time and the dynamical evolution of the fission process [5,6]. A large amount of experimental data indicated that the pre-scission particle multiplicities from excited nuclei exceeded the values expected from the statistical model. The emerged concept of dissipation that has been used for many years in order to evaluate the fission process was first presented by Kramers. Also the origin and mechanism of mass-asymmetric fission have not been investigated using statistical descriptions. To give a possibly unified picture of the fission process, it is necessary to introduce a dynamical model, which includes a dynamical treatment of a fissioning nucleus based on the classical Langevin equations [8]. This dynamical approach goes beyond the limitations of the statistical model, taking into account the dissipation effects along with the time evolution of the fissioning system. A few dynamical models have been applied to investigate the time evolution of the nuclear shape during fission at low excitation energies along with the shell structure of certain nuclei [1,9–11]. In the present work, we attempt to clarify the relation between the origin of the asymmetry in the mass distribution of fission fragments and the dissipation effects based on the "funny hills" shape parametrization, by taking account of the mean neutron and γ-ray multiplicity.

The paper is organized as follows. The theoretical framework is briefly described in Sec. II. In Sec. III results of the present model calculations are gathered and compared with available experimental data. A brief summary of the present study with concluding remarks is given in Sec. IV.

II. DESCRIPTION OF THE MODEL

The process of neutron-induced fission is described in terms of collective motion through the Langevin equations coupled with a Monte Carlo simulation allowing discrete emission of light particles and γ rays.

In the present study, the shape of the fissioning nucleus is restricted to the funny hills (FH) parametrization. The elongation (c), neck thickness (h), and asymmetry parameter (α) of a compound system define the shape of the compound nucleus in cylindrical coordinates as FH parameters [12]:

$$\rho_s^2(z) = \begin{cases} (c^2 - z^2)(A_s\, c^2 + B_{sh}\, z^2/c^2 + \alpha\, z/c) & \text{if } B_{sh} \geqslant 0 \\ (c^2 - z^2)(A_s\, c^2 + \alpha\, z/c)\exp(B_{sh}\, c\, z^2) & \text{otherwise,} \end{cases} \tag{1}$$

where ρ_s is the radial coordinate of the compound nucleus, whose rotation about the symmetry axis determines the nuclear surface. z is the coordinate along the symmetry axis. Parameters A_s and B_{sh} are determined using the conservation of nuclear volume. The evolution of the above mentioned shape coordinates is calculated by the coupled four-dimensional set of Langevin equations of motion [13],

$$\frac{dq_i}{dt} = \frac{p_j}{m_{ij}}, \tag{2}$$

*m.pahlavani@umz.ac.ir

$$\frac{dp_i}{dt} = -\frac{p_j\, p_k}{2}\frac{\partial}{\partial q_i}\left(\frac{1}{m_{jk}}\right) - \frac{\partial V}{\partial q_i} - T^2\frac{\partial a}{\partial q_i} - \eta_i\frac{dq_i}{dt} + R(t), \quad (3)$$

with q_i as deformation coordinates (c,h,α) and p_i as their conjugate momenta. m_{ij} stands for the shape-dependent collective inertia and η_i represents the friction tensor. Depending on the inertia of the system, the topology of the Langevin trajectory is observed to substantially differ within different approaches. Among numerous methods, the Werner-Wheeler model is used to calculate the mass tensor for the fusion channel of heavy ion reactions, as fission. Here regarding overlapping nuclei, the inertia tensor m_{ij} is evaluated using the Werner-Wheeler formula [14,15]

$$m_{ij} = \pi\, \rho_m \int_{z_{\min}}^{z_{\max}} \rho_s^2(z)(A_i\, A_j + \rho_s^2(z)\, A_i'\, A_j'/8)\, dz, \quad (4)$$

where ρ_m is the mass density of the compound nucleus and $\bar{v}$ is the average speed of nucleons inside the nucleus; $z_{\min}$ and $z_{\max}$ are respectively the left and right boundaries of the compound nuclear surface. According to the Werner-Wheeler formula, A_i can be expressed as

$$A_i = -\frac{1}{\rho_s^2(z)}\frac{\partial}{\partial q_i}\int_{-c}^{z}\rho_s^2(z')\, dz'. \quad (5)$$

The quantity A' is the first derivative of A with respect to z. T is the temperature of the compound nucleus which is related to its intrinsic energy through $T = \sqrt{(\frac{E_{\rm int}}{a})}$, where $E_{\rm int}$ and a are respectively the intrinsic energy of the system and the level density parameter, which are defined in advance.

The friction tensor can be obtained using one- and two-body dissipation models, considering the chaotic motion of nucleons within the nucleus. The dissipation tensor is then calculated by the following relation [14,16]:

$$\eta_i = \eta_i^{\rm TB} + \eta_i^{\rm OB}, \quad (6)$$

where $\eta_i^{\rm OB}$ and $\eta_i^{\rm TB}$ respectively refer to one- and two-body dissipation and are obtained using the relations

$$\eta_i^{\rm TB} = \pi\mu_0 R_{\rm CN} f_i \int_{z_{\min}}^{z_{\max}} \rho_s^2(z)\big[3A_i'^2 + A_i''^2\rho_s^2(z)/8\big]\, dz, \quad (7)$$

$$\eta_i^{\rm OB} = 2\pi\, \rho_m\, \overline{v}\, R_{\rm CN}^2\, f_i \int_{z_{\min}}^{z_{\max}} \rho_s(z)[A_i\, \rho_s^{'}(z) + A_i'\, \rho_s(z)/2]^2 \times \big[1 + \rho'^2_s(z)\big]^{-1/2} dz. \quad (8)$$

In the above relations μ_0 is the viscosity coefficient. Moreover A' and A'' are respectively the first and second derivatives of A with respect to z. Also $\rho_s^{'}(z)$ is the first derivative of $\rho_s(z)$ with respect to z, $R_{\rm CN}$ is the radius of the compound nucleus, and f is defined by the following relation [17]:

$$f_i = \left(\frac{d\, q_i}{d\, x}\right)^2 + 2\frac{d\, q_i}{d\, x}. \quad (9)$$

Here $x = r_{cm}/R_{\rm CN}$. The coordinate r_{cm} is the center-to-center distance between two parts of nascent fragments. The evolution of the orientation degree of freedom (K coordinate) [18] is obtained from the solution of Langevin equations

$$dK = -\frac{\gamma_K^2 I^2}{2}\frac{\partial V}{\partial K}dt + \gamma_K I \xi(t)\sqrt{\frac{T dt}{2}}, \quad (10)$$

where γ_K is the friction parameter which controls the coupling between the orientation degree of freedom, K, and the heat bath [18],

$$\gamma_K = \frac{1}{R_N\, R_{\rm c.m.}\sqrt{2\pi^3 n_0}}\sqrt{\frac{J_{||} J_{\rm eff} J_R}{J_\perp^3}} \quad (11)$$

where R_N, $R_{\rm c.m.}$, and n_0 are the neck radius, the distance between the centers of mass of nascent fragments, and the bulk flux in standard nuclear matter, respectively. $J_{||}$ and $J_\perp$ are the rigid body moments of inertia, about and perpendicular to the symmetry axis and $J_R = MR_{\rm c.m.}^2/4$ where M is the compound nucleus mass. Also $\xi(t)$ is the normalized random variable which is assumed to be white noise. The potential energy of the system is defined as a sum of the liquid-drop parts, namely rotational and microscopic parts [8]:

$$V(q,l,T) = V_{\rm LD}(q) + \frac{\hbar^2 l(l+1)}{2I(q)} + V_{\rm SH}(q,T). \quad (12)$$

Here $V_{\rm LD}(q)$ stands for the potential energy, which is calculated based on the liquid-drop model, given as a sum of the surface energy (E_S), the Coulomb energy (E_C), and the temperature-dependent shell correction energy as a microscopic part of the potential energy which is denoted by $V_{\rm SH}$ as the following relation:

$$V_{\rm SH}(q,T) = [\Delta E_{\rm pair}(q) + \Delta E_{\rm shell}(q)]\Phi(T), \quad (13)$$

where $\Delta E_{\rm pair}(q)$ is the pairing correlation energy which is evaluated in the BCS approximation. Also $\Delta E_{\rm shell}(q)$ is the shell correction energy based on the Strutinsky method which would also be defined as the difference between the sum of the single particle energies of occupied states and the average over density of single particle states [19,20],

$$\Delta E_{\rm shell}(q) = \Sigma\epsilon_k - \int_{\infty}^{\mu} eg(e)de, \quad (14)$$

where ϵ_k, μ, and $g(e)$ are the energy, the chemical potential, and the density of states parameter for single particle representation, respectively. Also $\Phi(T)$ is the temperature-dependent shell correction which is evaluated through the following relation:

$$\Phi(T) = \exp\left(-\frac{aT^2}{E_d}\right), \quad (15)$$

where E_d is the shell damping energy that is chosen to be 25 MeV and a is the level density parameter [21]

$$a = \left\{1 + \frac{V_{\rm SH}(T=0)}{E_{\rm int}}\left[1 - \exp\left(-\frac{E_{\rm int}}{E_d}\right)\right]\right\}\widetilde{a}(q), \quad (16)$$

where

$$\widetilde{a}(q) = a_1 A_{\rm CN} + a_2 A^{2/3} B_s(q). \quad (17)$$

The intrinsic energy of the system (E_{int}) is calculated at each step of a Langevin trajectory by the following relation:

$$E_{\mathrm{int}} = E_{\mathrm{c.m.}} - Q - \frac{p_i\, p_j}{2\, m_{ij}} - V(q,l,T=0), \tag{18}$$

where $E_{\mathrm{c.m.}}$ and Q are the energy of the system in the center-of-mass framework and the Q value of the reaction, respectively. In the present dynamical model, each fission event is defined as the instance that a Langevin trajectory overcomes the scission point on the potential energy surface in which the configuration of the neck radius becomes zero. Shape evolution of the compound nucleus proceeds in competition with pre-scission particle emissions and fission. Emission of neutrons is simulated through Weisskopf's conventional evaporation theory under the following outline. The neutron decay width is calculated using the following relation [22]:

$$\Gamma_n = \frac{2m_n}{[\pi\hbar]^2 \rho_c(E_{\mathrm{int}})} \int_0^{E_{\mathrm{int}}-B_n} \rho_d(E_{\mathrm{int}} - \varepsilon_n)\varepsilon_n \sigma_{\mathrm{inv}} d\varepsilon_n, \tag{19}$$

where m_n is the reduced mass of the neutron with respect to the residual nucleus and B_n shows the binding energy of the compound nucleus. Also, ρ_c is the level density of the compound nucleus and $\sigma_{\mathrm{inv}}(\varepsilon_n)$ is the inverse cross section for the reaction $(A-1)+n \rightarrow A$ and ε_n is the mean kinetic energy of the emitted neutrons. A Monte Carlo algorithm is used to calculate the competition between neutron emission and fission. The kinetic energy of the emitted neutrons has been sampled from the Watt spectrum. After emission of a neutron, the intrinsic excitation energy of the compound nucleus is recalculated and the process is continued. After each fission event, the mass numbers of conjugate fragments are calculated as well.

III. RESULTS

The time evaluation of the calculated mass distribution of fission fragments for ^{236}U at $E^* = 20$ MeV together with corresponding experimental data is shown in Fig. 1. It is clear from this figure that the number of fission events increases with time since $t = 2.5 \times 10^{-20}$ s, and they become saturated until $t = 1.0 \times 10^{-18}$ s, which indicates the occurrence of almost all of the fission events. At this time evaluation is in agreement with available experimental data, which is shown by solid squares in Fig. 1 [23]. Figure 2 shows the average angular spin carried by a fission fragment as a function of the fragment's mass. It is evident from this figure that the calculated average spin of fragments is higher than those obtained from isomeric yield ratio experiments of Durell [24], Aumann *et al.* [25], and Naik *et al.* [26], which are denoted by solid squares, triangles, and circles in the figure, respectively, for heavier fragments.

Decelerating the fission process and increasing the time from saddle to scission point in ^{236}U as a viscous system

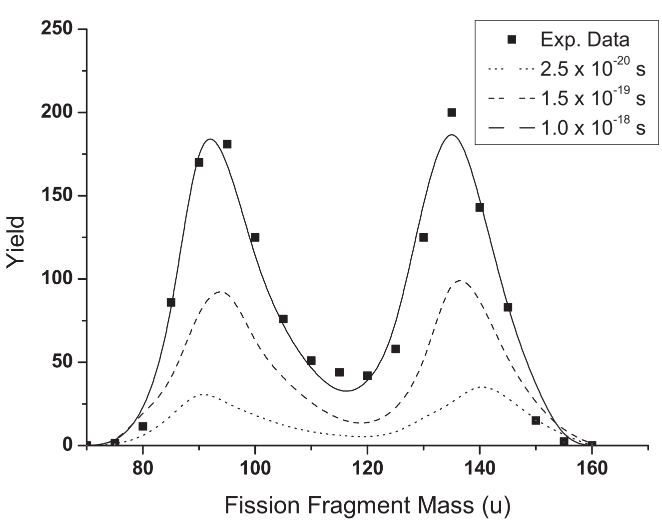

FIG. 1. Time evolution of mass distribution of fission fragments of ^{236}U at $E^* = 20$ MeV. Experimental data [23] are indicated by solid squares.

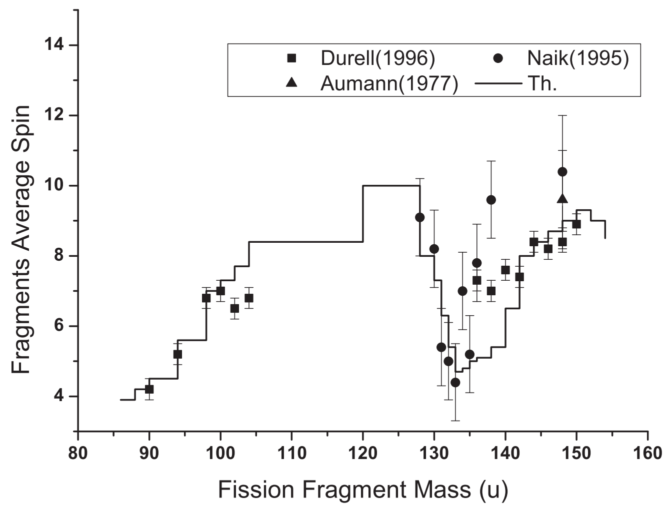

FIG. 2. Average initial fission fragment spin ($\hbar$) of ^{236}U at $E^* = 20$ MeV as a function of the fragment mass. Experimental data of Durell [24], Aumann *et al.* [25], and Naik *et al.* [26], are denoted by solid squares, triangles, and circles, respectively.

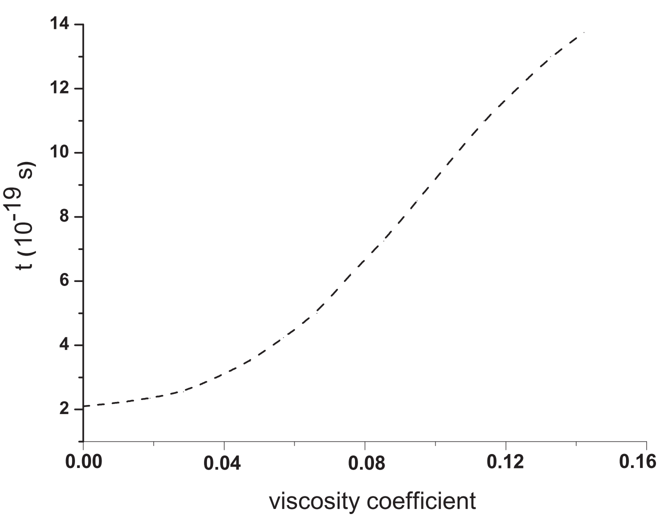

FIG. 3. Time scale of fission of ^{236}U nucleus as a function of the viscosity coefficient ($\hbar/\mathrm{fm}^3$).

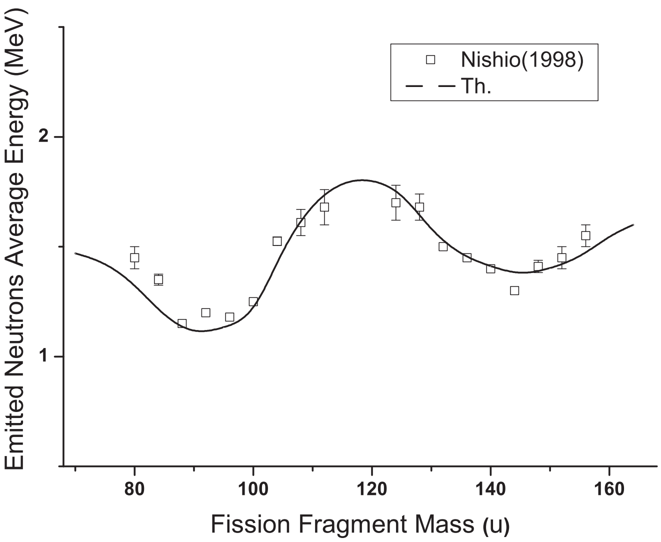

FIG. 4. The energy carried away by emitted neutrons in the fission of ^{236}U as a function of fission fragment mass. The theoretical result of the present approach and experimental data of Nishio *et al.* [28] are denoted by the solid line and open squares, respectively.

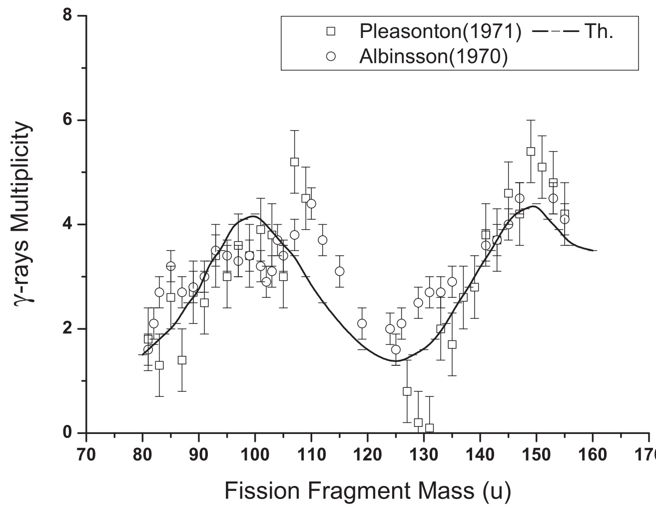

FIG. 6. Average prompt fission γ-ray multiplicity for ^{236}U at $E^* = 20$ MeV as a function of the fragment mass. Experimental data of Pleasonton *et al.* [27] and Albinsson *et al.* [29] are denoted by open squares and circles, respectively.

presumably was due to decreased potential energy that is converted into internal excitation energy; however, the evaluation time of neutron-induced fission of ^{236}U is shorter as compared with heavy-ion fusion-fission reactions. It is well established that for small viscosity coefficient, by increasing the kinetic energy in the fission mode, the decrease in the time of fission is expected [15]. As it is shown in Fig. 3 the time scale of fission smoothly increases as a function of viscosity coefficient. The magnitude of this coefficient for each Langevin trajectory was chosen by fitting and comparing with the time evaluation of fission fragments' mass distribution. In the present study the magnitude of the viscosity coefficient is limited to a range of $0.0095 < \mu_0 < 0.1422$ $(\hbar/\text{fm}^3)$.

The average kinetic energy of emitted neutrons and γ rays from a given initial fission fragment as a function of the fission fragment's mass is shown in Figs. 4 and 5, which correspond to the available experimental data [27,28]. In these figures the open squares show the experimental data and the solid line represents the calculated result in our macroscopic dynamical approach. It should be noted that in our dynamical treatment the average kinetic energy carried away by γ rays is considered when no further neutron emission occurs.

The average γ-ray multiplicity as a function of fragment mass is also calculated and is shown in Fig. 6. This figure

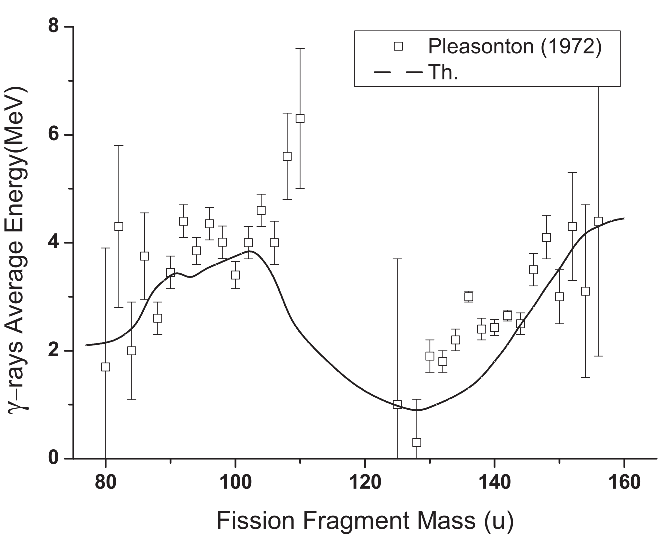

FIG. 5. The energy carried away by emitted γ rays in the fission of ^{236}U as a function of fission fragment mass distribution. The theoretical result of the present approach and experimental data of Pleasonton *et al.* [27] are denoted by the solid line and open squares, respectively.

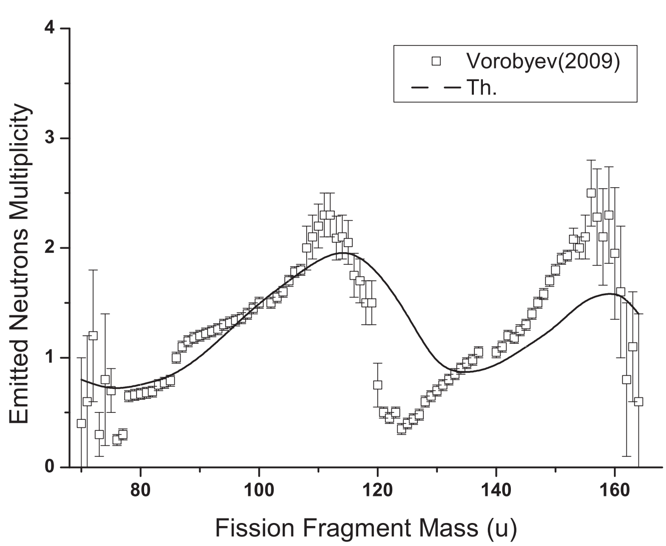

FIG. 7. Average prompt fission neutron multiplicity for ^{236}U at $E^* = 20$ MeV as a function of the fragment mass. Experimental data of Vorobyev *et al.* [30] are denoted by open squares.

shows a good agreement of the theoretical result based on the dynamical approach with data from the experiments of Pleasonton *et al.* [27] and Albinsson *et al.* [29], as shown by open squares and circles, respectively. As is clear from this figure, the calculated emission of the light fragment was to be slightly higher than measured data. Figure 7 shows the average neutron multiplicity as a function of the fission fragment mass. A very good agreement with the experimental data is achieved. These data were taken from the experiment of Vorobyev *et al.* [30] and are denoted by open squares in Fig. 7. The only exception was for mass regions 115–135 u and 145–155 u, which belong to heavier fragments. The theoretical results in these regions are greater than experimental data.

IV. CONCLUSION

In this study, the Langevin equations have been used to investigate the dynamical evaluation of the fission process of ^{236}U induced fission at $E^* = 20$ MeV. The mass distribution, initial spin, and γ-ray and neutron multiplicity of fission fragments have been obtained as a function of the fragment mass. By comparing the mass distribution in this model with the experimental data, it is shown that the time scale of fission is of order $t = 1.0 \times 10^{-18}$ s. Since the initial spin distribution strongly influences the neutron and γ-ray emission competition, we compared the initial spin of the nascent fragments with the available data. The calculated initial fission fragment spin, based on our dynamical approach, has been found to have a good agreement with the experimental data.

Based on our macroscopic description of fission, the magnitude of viscosity coefficient in the two-body friction tensor is fitted by comparing its time evaluation with experimental mass distribution of fission fragments in each Langevin trajectory. This method leads to obtaining the average value of $\mu_0 = 0.12$ ($\hbar/\text{fm}^3$), which has a highly acceptable accordance with the results of the literature [15]. In order to obtain more information about the energy dissipation in the fission process of ^{236}U, the energy carried away by the neutrons and the γ rays is calculated and compared with experimental data [28]. Reasonable agreement between the theoretical results and the experimental data indicates the validity of our approach, based on one- and two-body friction tensors.

Eventually, the calculations of prompt γ-ray and neutron emission from nascent fission fragments were presented and compared with the available experimental data. Ultimately, the theoretical findings and the experimental data were greatly accordant. Here, the remarkable point is that the mass distribution of fission fragments has been calculated by considering the dissipation effects and the γ-ray and neutron emission, based on the four-dimensional Langevin equations. As a subject under study by our group, much more detailed and thorough investigations such as the particle emission, and the influence of the level density parameter and dissipation coefficients on neutron-induced fissions, could improve the model for various systems.

[1] H. Goutte, J. F. Berger, P. Casoli, and D. Gogny, Phys. Rev. C **71**, 024316 (2005).
[2] S. Lemaire, P. Talou, T. Kawano, M. B. Chadwick, and D. G. Madland, Phys. Rev. C **73**, 014602 (2006).
[3] J. Randrup and R. Vogt, Phys. Rev. C **80**, 024601 (2009).
[4] P. Talou, B. Becker, T. Kawano, M. B. Chadwick, and Y. Danon, Phys. Rev. C **83**, 064612 (2011).
[5] R. Vogt, J. Randrup, D. A. Brown, M. A. Descalle, and W. E. Ormand, Phys. Rev. C **85**, 024608 (2012).
[6] B. Becker, P. Talou, T. Kawano, Y. Danon, and I. Stetcu, Phys. Rev. C **87**, 014617 (2013).
[7] Y. Aritomo, S. Chiba, and F. Ivanyuk, Phys. Rev. C **90**, 054609 (2014).
[8] Y. Aritomo and S. Chiba, Phys. Rev. C **88**, 044614 (2013).
[9] C. Schmitt, J. Bartel, K. Pomorski, and A. Surowiec, Acta Phys. Pol. B **34**, 1651 (2003).
[10] T. Asano, T. Wada, M. Ohta, T. Ichikawa, S. Yamaji, and H. Nakahara, J. Nucl. Radiochem. Sci. **5**, 1 (2004).
[11] J. Randrup and P. Moller, Phys. Rev. Lett. **106**, 132503 (2011).
[12] G. D. Adeev and P. A. Cherdantsev, Phys. Lett. B **39**, 485 (1972).
[13] G. Chaudhuri and S. Pal, Phys. Rev. C **65**, 054612 (2002).
[14] H. Feldmeier, Rep. Prog. Phys. **50**, 915 (1987).
[15] K. T. R. Davies, A. J. Sierk, and J. R. Nix, Phys. Rev. C **13**, 2385 (1976).
[16] J. R. Nix, Nucl. Phys. A **130**, 241 (1969).
[17] G. D. Adeev, A. V. Karpov, P. N. Nadtochii, and D. V. Vanin, Phys. Part. Nucl. **36**, 378 (2005).
[18] P. N. Nadtochy, E. G. Ryabov, A. E. Gegechkori, Yu. A. Anischenko, and G. D. Adeev, Phys. Rev. C **85**, 064619 (2012).
[19] V. M. Strutinsky, Nucl. Phys. A **95**, 420 (1967); **122**, 1 (1968).
[20] M. Brack, J. Damgaard, A. S. Jensen, H. C. Pauli, V. M. Strutinsky, and C. Y. Wong, Rev. Mod. Phys. **44**, 320 (1972).
[21] J. Toke and W. J. Swiatecki, Nucl. Phys. A **372**, 141 (1981).
[22] M. Blann, Phys. Rev. C **21**, 1770 (1980).
[23] K. Katakura, JENDL FP Decay Data File 2011 and Fission Yields Data File 2011 (Japan Atomic Energy Agency, 2012).
[24] J. L. Durell, in *Proceedings of the Third International Conference on Dynamical Aspects of Nuclear Fission, Casta Papiernicka, Slovakia, Aug. 30–Sep. 4, 1996*, edited by J. Kliman and B. L. Pustylnik (Joint Institute for Nuclear Research, Dubna, 1996), p. 270.
[25] D. C. Aumann, W. Guckel, E. Nirschl, and H. Zeising, Phys. Rev. C **16**, 254 (1977).
[26] H. Naik, S. P. Dange, R. J. Singh, and T. Datta, Nucl. Phys. A **587**, 273 (1995).
[27] F. Pleasonton, R. L. Ferguson, and H. W. Schmitt, Phys. Rev. C **6**, 1023 (1972).
[28] K. Nishio, Y. Nakagome, H. Yamamoto, and I. Kimura, Nucl. Phys. A **632**, 540 (1998).
[29] H. Albinsson and L. Lindow, Aktiebolaget Atomenergi Studsvik, Report No. AE-398, Nykoping, Sweden, 1970.
[30] A. S. Vorobyev, O. A. Shcherbakov, Yu. S. Pleva, A. M. Gagarski, G. V. Valski, G. A. Petrov, V. I. Petrova, and T. A. Zavarukhina, in *Proceedings of ISINN-17, Dubna, Russia, May 27–30* (Joint Institute for Nuclear Research, Dubna, 2009).