PHYSICAL REVIEW C **92**, 024301 (2015)

Effects of electron screening on α-decay half-lives in different external environments

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(Received 28 April 2015; revised manuscript received 29 June 2015; published 3 August 2015)

In this paper, the electron screening effects on α-decay half-lives are investigated in different external environments, including α decays in neutral atoms, in metal, and in extremely strong magnetic-field environments. Systematic calculations of α-decay half-lives are performed for α emitters with proton number $Z = 52$–105. By taking into account the electron screening effects, the interaction potential between α particle and daughter nucleus, and the decay energy are both changed in external environments. From the numerical results, it is found that the α-decay half-lives in external environments are changed by a factor of from $5 \times 10^{-4}\%$ to 11.46% due to the electron screening effects. Moreover, we find that the electron screening effect is closely related to the decay energy of the bare nucleus and its proton number. To measure the electron screening effects in experiments, it is suggested to select the α-decay candidates with relatively low decay energies and proper decay half-lives.

DOI: 10.1103/PhysRevC.92.024301 PACS number(s): 23.60.+e, 21.10.Tg, 21.60.Gx

I. INTRODUCTION

In 1928, Gamow successfully explained α decay as a pure quantum tunneling effect by simply treating the α particle and daughter nucleus as point particles [1]. Since the pioneering work of Gamow, the α decays of unstable nuclei have attracted much interest and several theoretical models have been developed to calculate the α-decay width such as the shell model [2–4], the cluster model [5,6], the liquid-drop model [7], and the fission-like model [8]. Usually the α-decay process is considered to be an α particle penetrating a Coulomb barrier after its formation in the parent nucleus. The barrier penetration probability of the α particle is known to be the most important term in the α-decay width, which depends exponentially on the so-called Gamow factor [1]. The pre-exponential factor of the penetration probability was treated in a classic way for a long time, which can be called the frequency of collisions. In 1980s, Gurvitz *et al.* successfully derived the pre-exponential factor appearing in the α-decay width by applying the two-potential approach [9]. The quasiclassical limit of the width formula given by Gurvitz *et al.* leads to the famous Gamow formula [9]. In contrast to the penetration probability, the α-particle preformation factor is very difficult to handle in theory. Fortunately, its value does not vary significantly for most open-shell nuclei. There have been only a few microscopic calculations on the α-particle preformation factor. For instance, Varga *et al.* calculated the α-particle preformation factor in ^{212}Po by using a combined shell and cluster model [10]. Very recently, Röpke *et al.* presented the microscopic calculation for ^{212}Po and found that the α-particle preformation factor in ^{212}Po is around 0.3 [11], which is consistent with the previous result [10].

Several analyses have been performed to calculate the half-lives of α emitters throughout the nuclide chart. For instance, Buck *et al.* calculated the half-lives of many α emitters by applying the two-potential approach and a phenomenological potential [5]. Royer *et al.* performed systematic calculations on the half-lives of α emitters by using the generalized liquid-drop model [7]. Delion and coworkers analyzed α-decay transitions to both the ground and the excited states for many nuclei [12]. By combining the two-potential approach and a microscopic potential, we investigated the α-decay half-lives of both spherical and deformed nuclei by using the density-dependent cluster model (DDCM) [13–21]. Due to the efforts of the whole community, α decay is now known not only as a reliable way to identify the newly synthesized superheavy elements and nuclides, but also as an effective tool to extract detailed nuclear structure information such as the ground-state properties of α emitters, the cluster preformation factors in shell closure regions, and the shape-coexistence of ground and excited states [22–25].

Although there have been many theoretical studies of α decays, the effects of different external environments on α-decay half-lives have not been systematically studied. This topic is interesting because the electron screening effect is expected to play an important role in different external environments, and the α-decay half-lives will be changed correspondingly. There are only a few theoretical works discussing electron screening effects on α decays. For example, the electron screening effects in neutral atoms are studied within different approaches [26–33]. Electron screening effects in a metal environment are also studied in Refs.[34–36]. The α-decay half-lives of some nuclides like ^{221}Fr were measured in different metal environments [37]. Electron screening effects on nuclear decays and reactions are also discussed at astrophysical energies [38,39] and in dense astrophysical plasmas and superstrong magnetic fields [40–45].

Previous research has been devoted to the studies of screening effects on α decays in one specific environment. To our best knowledge, a systematic analysis of electron screening effects on α decay in different external environments has not been done. The main purpose of this paper is to present a

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systematic analysis of electron screening effects on α decays in different external environments, namely, α decays in neutral atoms, in metal, and in an extremely strong magnetic-field environment. Systematic calculations of the α-decay half-lives of unstable nuclei are performed using the DDCM. Electron screening effects on the interaction potential as well as the decay energy are discussed and the variations of α-decay half-lives in different external environments are presented.

The outline of this paper is as follows. In Sec. II, variations of both the interaction potential and the α-decay energy are discussed. In Sec. III, the corresponding results of the relative variations of the α-decay half-lives in external environments are compared with the α-decay half-lives of bare nuclei in details. A summary is given in Sec. IV.

II. VARIATIONS OF THE INTERACTION POTENTIAL AND DECAY ENERGY IN AN EXTERNAL ENVIRONMENT

A. General analysis of interaction potential and decay energy in an external environment

In the DDCM, the α-decay width Γ can be expressed as [5,6,17–21]

$$\Gamma = P_\alpha F \frac{\hbar^2}{4\mu} \exp\left[-2\int_{R_2}^{R_3} dR \sqrt{\frac{2\mu}{\hbar^2}|P(R)|}\right], \quad (1)$$

where P_α is the preformation factor of the α particle and $F\frac{\hbar^2}{4\mu}$ is the pre-exponential factor. Electron screening affects mainly the exponential term in Eq. (1). The function $P(R)$ is defined by $P(R) = V(R) - Q$, where $V(R)$ is the interaction potential and Q is the decay energy. R_2 and R_3 are the second and third classical turning points.

For a bare nucleus, $P(R)$ can be written as

$$P_B(R) = V_B(R) - Q_B. \quad (2)$$

In external environments, both the Coulomb potential and the decay energy will be changed due to the electron screening,

$$\begin{aligned} P(R) &= V(R) - Q \\ &= [V_B(R) + \Delta V(R)] - [Q_B + \delta Q] \\ &= V_{\text{equ}}(R) - Q_B, \end{aligned} \quad (3)$$

where $\Delta V(R)$ is the variation of the Coulomb potential and δQ is the variation of the decay energy. The term $\Delta V(R)$ can be divided into two terms [31–33],

$$\Delta V(R) = \delta V + \delta V(R), \quad (4)$$

where δV is a constant and $\delta V(R)$ depends on the relative distance R. Then the equivalent interaction potential function $V_{\text{equ}}(R)$ given in Eq. (3) can be rewritten as

$$\begin{aligned} V_{\text{equ}}(R) &= V_B(R) + \Delta V(R) - \delta Q \\ &= V_B(R) + \delta V(R) + [\delta V - \delta Q]. \end{aligned} \quad (5)$$

Here δV is equal to δQ according to the recent studies presented by Karpeshin and coworker [31,32]. By applying the fact that the inner atomic electrons are much swifter than the emitted α particle, the α-decay process is described in the adiabatic approximation [31,32]. The results of [31] and [32] were also made use of in a more recent work [33]. Therefore Eq. (3) can be rewritten as

$$P(R) = V_B(R) + \delta V(R) - Q_B. \quad (6)$$

Compared to Eq. (2), it is clear that the variation of the α-decay half-life between the bare nucleus and the external environment is mainly determined by the term $\delta V(R)$. A sketch of $V_{\text{equ}}(R) = V_B(R) + \delta V(R)$ and $V_B(R)$ is shown in Fig. 1. It can be seen that the screened potential is different from the bare one, which will change the corresponding α-decay widths and half-lives.

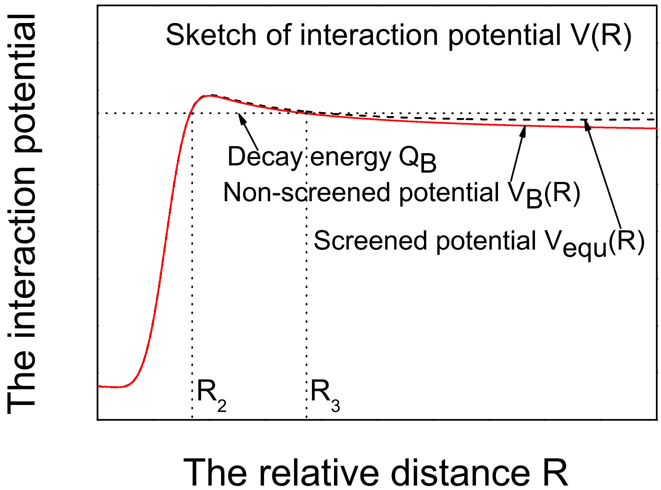

FIG. 1. (Color online) Sketch of the screened potential $V_{\text{equ}}(R)$ and the nonscreened potential $V_B(R)$. R is the relative distance and Q_B is the decay energy of the bare nucleus.

B. Electron screened α decay in neutral atoms

In neutral atoms, a fitting formula for the variation of the Coulomb potential $\delta V(R)$ is given in Ref. [34],

$$\delta V(R) = 0.10277\left(\frac{Z}{54}\right)^{6.45} R + 0.00626\left(\frac{Z}{54}\right)^{4.06} R^2, \quad (7)$$

where the units of $\delta V(R)$ and R are electron volts and femtometers, respectively. Z is the proton number of the mother nucleus. Moreover, the variation of the Coulomb potential $\delta V(R)$ is also analytically derived [33],

$$\delta V(R) = \frac{2(2Z)^{2\gamma+1}}{\Gamma(2\gamma+2)\gamma} \frac{e^2}{a_0} \left(\frac{R}{a_0}\right)^{2\gamma}, \quad (8)$$

where $\gamma = \sqrt{1-\beta^2 Z^2}$, with the fine-structure constant $\beta = e^2/\hbar c$, and a_0 is the Bohr radius.

The variation of the decay energy δQ can be easily obtained from the electron binding energies of mother and daughter atoms [34]:

$$\delta Q = B(Z,Z) - B(Z-2,Z-2) - B(2,2). \quad (9)$$

The quantity $B(Z,N_e)$ denotes the electron binding energy of N_e electrons in the field of a nucleus with Z protons [34] and the values of the quantity $B(Z,N_e)$ $(N_e \leqslant Z)$ are obtained from Ref. [46]. Since the decay energies of neutral atoms Q

can be obtained from the atomic mass evaluation [47], the decay energies of bare nuclei are $Q_B = Q - \delta Q$. Once the Q_B, $\delta V(R)$, and δQ are obtained, the α-decay half-lives for bare nuclei and neutral atoms can be calculated to study the electron screening effects. Both Eq. (7) and Eq. (8) have been used to calculate the electron screening effects and we find that their numerical results are nearly the same. In the following, we show only numerical results in neutral atoms by using the analytically derived formula, Eq. (8).

C. Electron screened α decay in a metal environment

In a metal environment, the screened Coulomb potential $V_C(R)$ can be analytically derived from the Thomas-Fermi approach [48], which is found to be a Yukawa-type potential [36,48],

$$V_C(R) = \frac{Z_1 Z_2 e^2}{R} \exp\left(-\frac{R}{R_{\mathrm{TF}}}\right), \tag{10}$$

where Z_1 and Z_2 are the proton numbers of the α particle and daughter nucleus, respectively. R_{TF} is the electron screening radius. As pointed out in a recent work [33], the corresponding electron charge distribution of the exponential screened potential [Eq. (10)] will diverge at small distances [33]. In Ref. [33], the variation of the Coulomb potential $\delta V(R)$ in a metal environment is divided into two parts,

$$\delta V(R) = \delta V_1(R) + \delta V_2(R), \tag{11}$$

where $\delta V_1(R)$ is the variation caused by electrons outside the mother nucleus and $\delta V_2(R)$ is the variation caused by electrons in the metal environment. $\delta V_1(R)$ is the same as in Eq. (8) and $\delta V_2(R)$ is given by [33]

$$\delta V_2(R) = \frac{8e^2}{\pi} \int_0^{q_F} dq \int_0^{qR} \frac{dy}{y^2} \int_0^{y} dx \times \left[F_0^2(x) - \frac{1}{3}\left(\frac{q_F}{q}\right)^2 x^2 \right], \tag{12}$$

where $F_0(x) = C_0(q) x y_0(x)$ is the radial function and $C_0(q)$ is the amplitude [33]. The function $y_0(x)$ satisfies the equation [33]

$$x y_0''(x) + 2y_0'(x) + [x - 2\eta g(x)] y_0(x) = 0, \tag{13}$$

with the initial condition $y_0(0) = 1$. $\eta = -\frac{Z}{qa_0}$ is the dimensionless Coulomb parameter and $g(x) = \exp(-x/x_0)$ is the screening factor with $x_0 = qr_a$, where $r_a = a_0 Z^{-1/3}$ [33]. The Fermi vector q_F is related to the average electron density n_0 by [33]

$$q_F = (3\pi^2 n_0)^{1/3}. \tag{14}$$

Here we calculate the electron screening effects on α decays in metal copper (Cu) and $n_0 = 8.48 \times 10^{22}$ cm^{-3}.

D. Electron screened α decay in an extremely strong magnetic-field environment

In an extremely strong magnetic-field environment, the screened Coulomb potential $V_C(R)$ is usually obtained by introducing the function $\phi(x)$: $V_C(R) = \frac{Z_1 Z_2 e^2}{R}\phi(x)$, in which $\phi(x)$ can be analytically derived [41–44]. It fulfills the equation $\frac{d^2\phi(x)}{dx^2} = (x\phi)^{1/2}$ [40–45] with the boundary condition $\phi(0) = 1$ and $\phi'(0) = -0.938\,966$ [42,44]. The parameter x is equal to R/R_s where the screening radius $R_s = 1.041\,863 Z^{1/5} b^{-2/5} a_0$ [42–45]. The parameter $b = B/B_0$ is a dimensionless magnetic-field strength [41], where B is the magnetic-field strength in the environment. B_0 is equal to $\frac{m_e^2 e^3 c}{\hbar^3} = 2.3505 \times 10^9$ G, which is the typical magnetic-field strength on the surfaces of neutron stars [41]. With Baker's small-x expansion [49], the first few terms are given by

$$\phi(x) = 1 + Sx + \frac{4}{15}x^{2.5} + \frac{2}{35}Sx^{3.5} - \frac{1}{126}S^2 x^{4.5}, \tag{15}$$

where $S = \phi'(0) = -0.938\,966$ [42,44]. The first term on the right-hand side represents the nonscreened Coulomb potential for a bare nucleus. The second term represents δV and the remaining terms represent $\delta V(R)$, namely,

$$\delta V = \frac{Z_1 Z_2 e^2}{R} Sx, \tag{16}$$

$$\delta V(R) = \frac{Z_1 Z_2 e^2}{R}\left[\frac{4}{15}x^{2.5} + \frac{2}{35}Sx^{3.5} - \frac{1}{126}S^2 x^{4.5}\right]. \tag{17}$$

Here the condition $\delta Q = \delta V$ is also used in the magnetic-field environment. The electron screening effects can be calculated once the $\delta V(R)$ values in different magnetic field environments are given by Eq. (17).

III. NUMERICAL RESULTS AND DISCUSSION

We apply the DDCM to perform systematic calculations of the α-decay half-lives of nuclei with proton number $Z = 52$–105 by taking into account the electron screening effects in several different external environments. Note that there still exist some uncertainties and more accurate theory is needed to study α decay in external environments. To avoid the uncertainties from the effects of nonzero angular momentum, only the favored α transitions are considered. The relative variation between the α-decay half-lives of the screened and those of the bare nucleus is defined by Δ_{sc},

$$\Delta_{\mathrm{sc}} = \frac{T_{\mathrm{sc}} - T_{\mathrm{bare}}}{T_{\mathrm{bare}}}, \tag{18}$$

where T_{sc} and T_{bare} are the α-decay half-lives of the screened and bare nucleus, respectively. Δ_{sc} includes Δ_{Atom}, Δ_{Metal}, and Δ_{Mag}, corresponding to α decay in neutral atoms, in metal, and in a magnetic-field environment.

In Fig. 2, the relative variations Δ_{sc} in different external environments are given. Figure 2(a) represents neutral atoms, Fig. 2(b) a metal environment, and Figs. 2(c)–2(f) are all for a magnetic environment but with different magnetic field strengths: $b = 10^3$ (c), $b = 10^4$ (d), $b = 10^5$ (e), and $b = 10^6$ (f). From these six plots, it can be seen that the values of Δ_{sc} are all positive, which means that the α-decay half-lives are all increased in external environments compared with bare nuclei. This is not surprising because the potential barrier of $V_{\mathrm{equ}}(R)$ is slightly higher in external environments than that of $V_B(R)$ (shown in Fig. 1). Thus, it becomes relatively more difficult for the α particle to penetrate the barrier, resulting in longer

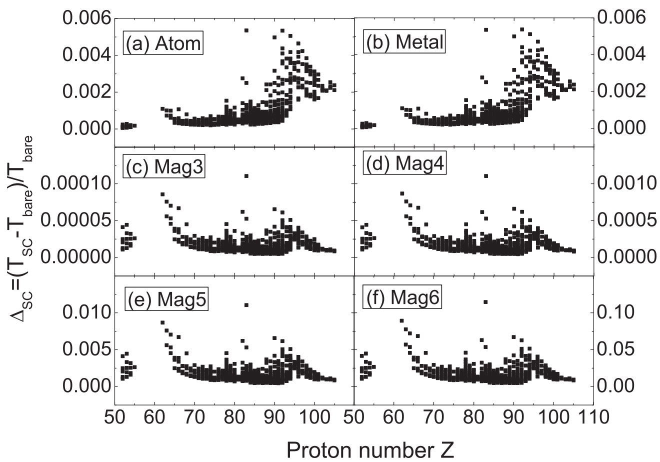

FIG. 2. The relative variation $\Delta_{sc} = (T_{sc} - T_{bare})/T_{bare}$ in neutral atoms (a), in metal (b), and in a magnetic-field environment with $b = 10^3$ (c), $b = 10^4$ (d), $b = 10^5$ (e), and $b = 10^6$ (f).

α-decay half-lives compared to bare nuclei. It is also shown in Fig. 2 that the majority of the values of Δ_{sc} are distributed in the range from 0.01% to 0.4% in Figs. 2(a) and 2(b). So the variations of α-decay half-lives in neutral atoms and in a metal environment are moderate (usually $<1\%$). But the variations of α-decay half-lives in a magnetic-field environment can be very large, and are closely dependent on the magnetic-field strength b. For example, Δ_{sc} can be as large as 11.46% in a strong magnetic-field environment with $b = 10^6$.

Besides, there are several values of Δ_{sc} significantly larger than the others along an isotopic chain in each chart of Fig. 2. We find that these large values of Δ_{sc} are closely related to the small decay energies of bare nucleus Q_B. To show the relationship between them more clearly, we plot the dependence of Δ_{SC} on the decay energy Q_B for a typical isotopic chain Re in Fig. 3. It is clearly seen that the variation of the decay half-life decreases with the decay energy for the Re isotopic chain. For instance, the decay energy of ^{168}Re is the smallest one, but the variation of the α-decay half-life of ^{168}Re

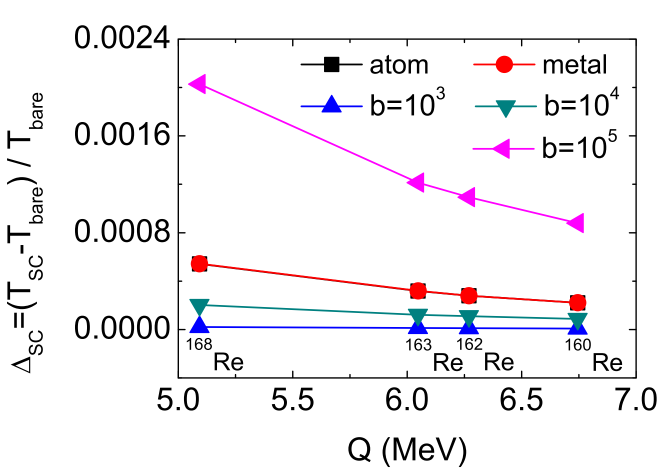

FIG. 3. (Color online) The dependence of Δ_{SC} on the decay energy of the bare nucleus Q_B for a typical isotopic chain of $Z = 75$.

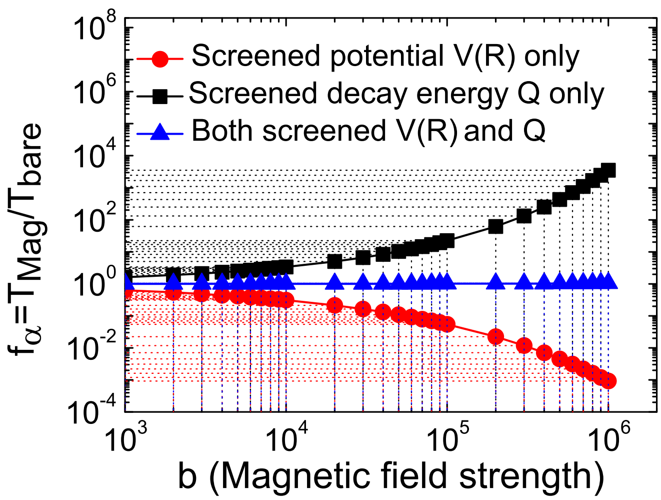

FIG. 4. (Color online) The ratio of magnetic screened to bare half-lives $f_\alpha = T_{Mag}/T_{bare}$ for ^{235}U with different magnetic field strengths $b = B/B_0$.

is bigger than the others. This is because the electron screening effects are approximately the same for all Re isotopes. Thus the value of Δ_{SC} mainly depends on the decay energy, namely, the lower the α-decay energy Q_B, the larger the effect of electron screening on the α decay for isotopic chains. The variation of the α-decay half-life Δ_{SC} is also related to the proton number of the mother nucleus. To measure the electron screening effects in experiments, it is suggested to select α-decay candidates with relatively low decay energies and proper decay half-lives (e.g., tens of seconds or several minutes).

In an extremely strong magnetic-field environment, electron screening effects on α-decay half-lives increase quickly with the magnetic-field strength as shown in Fig. 3. The relative variation Δ_{sc} can be as large as 11.46% in magnetar, where the magnetic-field strength can reach 10^{15} G, corresponding to $b = 10^6$. Thus the extremely strong magnetic-field environment could exert a significant influence on the α decays. To show the detailed electron screening effects in a magnetic-field environment, we plot the ratio of magnetic screened to bare half-lives $f_\alpha = T_{Mag}/T_{bare}$ for ^{235}U in Fig. 4. As shown in Fig. 4, if only the potential correction is considered, the decay half-lives decrease sharply with the magnetic-field strength ($f_\alpha \ll 1$). On the contrary, the decay half-lives increase sharply with the magnetic-field strength ($f_\alpha \gg 1$) if only the decay energy correction is considered. However, when both these corrections are included, the decay half-lives still increase with the magnetic-field strength, but the increases are much slower and f_α is in the range from 1.000 04 ($b = 10^3$) to 1.048 ($b = 10^6$). Thus the Coulomb potential correction and the decay energy correction compete with each other and they are both important factors for the electron screening effects.

IV. SUMMARY

Within the DDCM, a systematic analysis of electron screening effects on α decay has been performed for different external environments, including neutral atoms, metal, and extremely strong magnetic-field environments. Both the variations of the Coulomb potential and the decay energy are found to be important in determining the electron screening effects on α decay. It is further found that the variations of α-decay half-lives in neutral atoms and in a metal environment are mainly distributed in the range from 0.01% to 0.4%. But the variations of α-decay half-lives in magnetic-field environments depend closely on the magnetic-field strength ($\Delta_{sc} \sim 5 \times 10^{-4}$%–11.46%). More specifically, it is as large as 11.46% in magnetar, where the magnetic-field strength reaches 10^{15} G, corresponding to $b = 10^6$. Similarly to previous studies [31,32], it can be concluded that the variation of the α-decay half-life in the external environment is closely related to the α-decay energy of the bare nucleus and its proton number. Thus it is suggested to select α-decay candidates with relatively low decay energies and proper decay half-lives to measure electron screening effects in experiments.

ACKNOWLEDGMENTS

This work was supported by the National Natural Science Foundation of China (Grant Nos. 11175085, 11235001, 11375086, and 11120101005), by the 973 Program of China (Grant No. 2013CB834400), by the Project Funded by the Priority Academic Program Development of Jiangsu Higher Education Institutions (PAPD), and by the Open Project Program of the State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences (Grant No.Y5KF141CJ1).

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