PHYSICAL REVIEW C **92**, 021303(R) (2015)

Systematics of α-decay transitions to excited states

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(Received 21 April 2015; revised manuscript received 19 June 2015; published 4 August 2015)

We systematize the available experimental material concerning α-decay transitions to low-lying excited states in even-even and odd-mass emitters. We generalize our previous theoretical prediction concerning the linear dependence between hindrance factors and the excitation energy for transitions in even-even α emitters. Thus, we show that α intensities for transitions to excited states depend linearly upon the excitation energy for all known even-even and odd-mass α emitters. It turns out that the well-known Viola-Seaborg law for α-decay transitions between ground states can be generalized for transitions to excited states. This rule can be used to predict any α-decay half-life to a low-lying excited state.

DOI: 10.1103/PhysRevC.92.021303 PACS number(s): 21.60.Gx, 23.60.+e, 24.10.Eq

The phenomenon of α decay can be treated within the Gamow picture, where an α particle that is located on the nuclear surface penetrates through the Coulomb barrier [1]. A microscopic approach requires the computation of the α-particle formation probability, thus defining the reduced width within the R-matrix theory [2–6]. In the case of transitions to excited states, only a few single-particle states around the Fermi surface are involved and therefore the decay widths to excited states are very sensitive to the structure of the wave function in the daughter nucleus.

Most calculations of α-decay transitions to excited states were performed for even-even α emitters, due to the many high precision measurements that are available; see, e.g., [7–9]. The first attempts to estimate the hindrance factor (HF), defined as the ratio between reduced widths for transitions to ground and excited states [10], were performed in vibrational nuclei within the quasiparticle random-phase approximation (QRPA) in Refs. [11,12], and later in Refs. [13–15]. The first calculations of the α-decay widths in rotational even-even nuclei within the coupled channels approach were performed in Ref. [16]. This formalism was applied in Refs. [17,18] by making use of the double folding potential [19–21] plus a repulsive core simulating the Pauli principle. Later on, several papers were devoted to the coupled channels analysis of the α-decay fine structure in even-even [22,23] as well as odd-mass emitters [24], using the same double folding potential together with the Wildermuth rule to simulate the Pauli principle [25]. A study of the influence of the $Z = 82$ shell closure on the α-particle formation probability for neutron deficient even-even nuclei in the region between Pb and Th can be seen in Ref. [26]. Interesting predictions concerning the competition between heavy particle or cluster radioactivity and α decay in superheavy nuclei can be found in Refs. [27,28].

The aim of this paper is to give a unified view on α-decay transitions to excited states in even-even and odd-mass nuclei in terms of some scaling laws for half-lives and transition intensities. A systematic study of α-decay transitions to excited states in odd-odd nuclei in the region $83 \leqslant Z \leqslant 101$ is given in Ref. [29]. A statistical overview of the transitions analyzed here can be seen in Table I. All the experimental data analyzed in this paper were taken from the ENSDF database maintained by BNL. Mass and separation energy tables together with related procedures can be investigated in Refs. [30,31]. In particular, we considered in our analysis only 338 g.s. to g.s. transitions which fulfill two conditions: (i) all experimental data that are required in the analysis are available on the ENSDF (total half-life, total α branch, Q value, initial and final angular momenta); and (ii) a spectroscopic factor for the g.s. to g.s. transition could be calculated in our approach using a fixed set of parameters for all data.

We consider the following α-decay process

$$P(J_P) \to D(J) + \alpha(L), \tag{1}$$

where J_P denotes the spin-parity of the parent nucleus, J the spin-parity of the daughter nucleus, and L the angular momentum of the emitted α particle. We suppose that the wave function has a clustered α-daughter ansatz [32] with the total spin of the initial state

$$\Psi_{J_P M_P}(\xi,\mathbf{R}) = \sum_{c=(J,L)} \frac{f_c(R)}{R} \mathcal{Y}^{(c)}_{J_P M_P}(\xi,\hat{R}), \tag{2}$$

where the core-angular harmonic is given by

$$\mathcal{Y}^{(c)}_{J_P M_P}(\xi,\hat{R}) = [\Phi_J(\xi) \otimes Y_L(\hat{R})]_{J_P M_P}. \tag{3}$$

Here, $\Phi_{JM_J}(\xi)$ denotes the daughter internal wave function with ξ the daughter degrees of freedom, while $Y_{LM_L}(\hat{R})$ is the standard spherical harmonic describing the angular motion of the α-daughter system. The radial function $f_c(R)$ describes the α-daughter radial motion in the channel $c \equiv (J,L)$ and at large distances has an outgoing asymptotic expression

$$f_c(R) \to N_c H^{(+)}_L(\kappa_c R,\chi_c), \tag{4}$$

TABLE I. Number of studied α-decay transitions between ground states (g.s.) and from ground to excited states (ex.s.). All experimental data regarding these transitions are taken from http://www.nndc.bnl.gov/ensdf/.

g.s. → g.s.	Transitions
Even-even	149
Even-odd	72
Odd-even	67
Odd-odd	50
Total	338
g.s. → ex.s.	Transitions
Even-even	238
Odd-mass favored	130
Odd-mass unfavored	333
Total	701

in terms of the Coulomb-Hankel spherical wave depending on the reduced radius $\kappa_c R$ and Coulomb parameter

$$\chi_c = \frac{2Z_D Z_\alpha}{\hbar v_c} \sim \frac{2Z_D Z_\alpha}{\sqrt{Q_\alpha - E_c}}, \tag{5}$$

where Q_α is the Q value of the decay process. The coefficient N_c is called scattering amplitude. From the continuity equation one obtains the total decay width as a sum of partial widths [32]

$$\Gamma = \sum_c \Gamma_c = \sum_c \hbar v_c \lim_{R\to\infty} |f_c(R)|^2$$
$$= \sum_c \hbar v_c |N_c|^2, \tag{6}$$

where $v_c = \hbar\kappa_c/\mu$ is the center-of-mass velocity at infinity in the α-daughter channel c. Let us mention that each partial width can be rewritten in a factorized form at some radius R

$$\Gamma_c = 2\gamma_c^2(R)P_c(R), \tag{7}$$

in terms of the squared reduced width and penetrability

$$\gamma_c^2(R) = \frac{\hbar^2}{2\mu R}|f_c(r)|^2$$
$$P_c(R) = \frac{\kappa_c R}{|H_L^{(+)}(\kappa_c R, \chi_c)|^2}. \tag{8}$$

Of course this representation does not depend upon the radius, but it allows us to compute the decay width by using the wave function on the nuclear surface. The simplest case is the emission between low-lying states of even-even nuclei. Most experimental data are concerned with transitions from the ground state of the parent nucleus with $J_P = 0$. In this case the angular momentum of the α-particle coincides with the daughter spin $J = L$, and the core-angular harmonic is given by $\mathcal{Y}_0^{(c)}(\xi_D, \hat{R})$, with $c = J$. In particular in Ref. [17] we described the ground band by a rigid rotator, while in Ref. [9] we generalized the core dynamics by using a projected coherent state depending on the deformation parameter. For small values of this parameter one obtains a vibrational spectrum, while for large values a rotational one.

For transitions from odd-mass nuclei, if the state of the unpaired nucleon remains unchanged during the decay process, then the transition is known as favored, otherwise it is called unfavored. In this case the wave function of the daughter nucleus is given by a particle-core ansatz

$$\Phi_{JM_J}(\xi, \mathbf{r}) = \sum_J X_J^{(J_D)} [\varphi_{J_D}(\xi) \otimes \psi_{j_D}(\mathbf{r})]_{JM_J}, \tag{9}$$

where $\varphi_{J_D}(\xi)$ is the wave function of the even-even core and $\psi_{j_D m_D}(\mathbf{r})$ is the single-particle orbital. The mixing coefficients are found by diagonalizing a quadrupole-quadrupole interaction between the even-even core and the odd particle. A more general ansatz assumes a quasiparticle-core coupling. Such a model describes bands built on top of single-particle states. In odd-odd nuclei the single-particle orbital is replaced by a proton-neutron wave function.

Let us mention that for some odd-mass nuclei around ^{208}Pb two quasiparticles can be broken, being coupled to some angular momentum $J_{\rm pair} = 2,4,6,\ldots$. Thus, one can use the three quasiparticle-core model [33]. In our analysis we considered only α-decay transitions in odd-mass nuclei between nuclei described by Eq. (9).

The α-daughter potential is usually estimated by using the double folding integration method of the nucleon-nucleon interaction with density distributions of the emitted fragments. A potential determined from α-scattering experiments is used for the nucleon-nucleon interaction, thus assuming that the α-particle exists with certainty. In reality, the particle forms with a probability given by the spectroscopic factor

$$S = \frac{\Gamma_{\rm expt}}{\Gamma_{\rm theor}} = \frac{T_{\rm theor}}{T_{\rm expt}}, \tag{10}$$

which should be less than unity. Equation (7) can be interpreted as a product between the α-formation probability γ^2 and the probability P of the penetration through the Coulomb barrier. Therefore the spectroscopic factor S and α-formation probability should be proportional.

This feature is nicely evidenced in Fig. 1, where we plotted the spectroscopic factor as a function of the squared

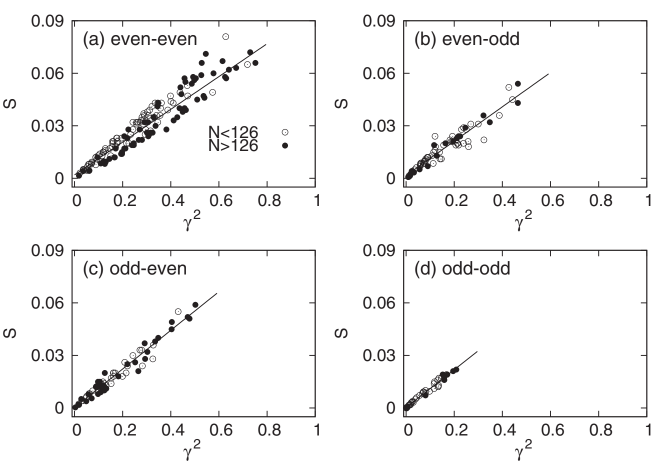

FIG. 1. Spectroscopic factor versus the α-formation probability for even-even, even-odd, odd-even, and odd-odd emitters.

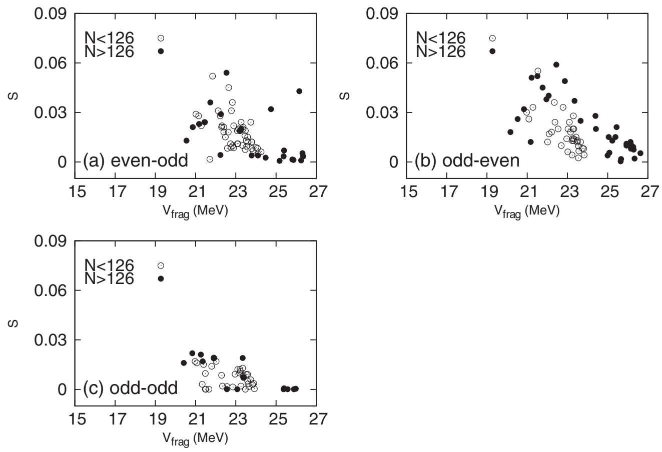

FIG. 2. Spectroscopic factor versus the fragmentation potential for even-odd, odd-even, and odd-odd emitters.

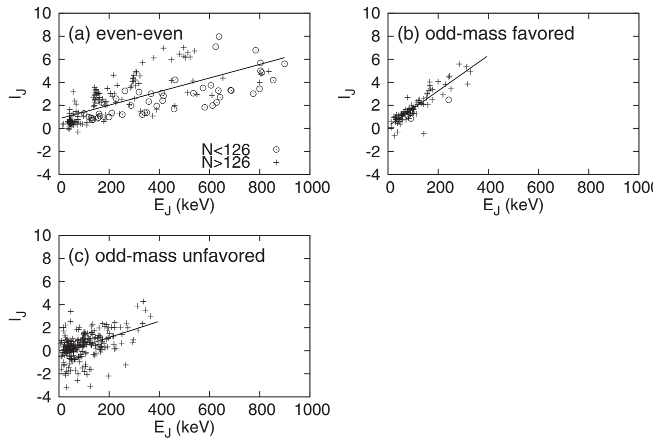

FIG. 3. Intensity I_J versus the excitation energy in the daughter nucleus E_J, $1 \leqslant J \leqslant 6$, in even-even nuclei and for odd-mass favored and unfavored transitions.

reduced width computed at the touching configuration $R = 1.2(A_D^{1/3} + 4^{1/3})$, for even-even, even-odd, odd-even, and odd-odd emitters, by using a double folding potential with the M3Y nucleon-nucleon interaction [19].

In Ref. [34] the following linear dependence has been derived:

$$\log_{10} \gamma_J^2(R_B) = -\frac{2\log_{10}\mathbf{e}}{\hbar\omega}[V_C(R_B) - (Q_\alpha - E_J)] + \log_{10}\frac{\hbar^2 A_J^2}{2\mathbf{e}\mu R_B}, \quad (11)$$

where $\mathbf{e}$ is the basis of natural logarithms, $\hbar\omega$ is the energy parameter of the pocket-like α-daughter potential, $V_C(R_B)$ the Coulomb barrier, and A_J the amplitude of the α cluster. For transitions between ground states with $E_J = 0$, the linear correlation with a negative slope between the spectroscopic factor and fragmentation potential $V_{\rm frag} = V_C - Q_\alpha$ was already evidenced in Ref. [34] for even-even emitters. It turns out that a similar picture holds for even-odd nuclei as well as for odd-even and odd-odd emitters, as can be seen in Fig. 2.

Equation (11) has as a direct consequence the linear dependence between the intensity of the ground to excited state transition and the excitation energy [9]

$$I_J \equiv \log_{10}\frac{\Gamma_0}{\Gamma_J} = gE_J + h, \quad (12)$$

where $g = \frac{2\log_{10}\mathbf{e}}{\hbar\omega} > 0$. This correlation with a positive slope is clearly evidenced for even-even emitters in Fig. 3(a). A similar correlation remains valid for odd-mass emitters. This feature is evidenced in Fig. 3(b) for favored transitions to excited states of rotational bands and in Fig. 3(c) for unfavored transitions to excited states of rotational bands. The fitting parameters are given in Table II, where in the last column we specify the standard deviations. Notice that this correlation is significantly improved if one analyzes each spin separately. These linear dependencies can be used to predict the order of magnitude of α-decay branching ratios.

The most popular systematics for α-decay transitions between ground states is given by the Viola-Seaborg rule [35], where the logarithm of the total half-life depends on the Coulomb parameter and charge number of the daughter nucleus. It was used to describe transitions between ground states in the case of α decay [36], as well as proton [37] and heavy-cluster emission [38]. Other simple formulas for α-emission have been provided in Refs. [39,40]. The Viola-Seaborg rule is a direct consequence of two facts, namely the exponential dependence of the penetrability upon the Coulomb parameter and the dependence of the squared reduced width upon the charge number, given by the fragmentation potential in Eq. (11). Due to the fact that the channel Coulomb parameter (5) depends on the excitation energy of the daughter nucleus this rule can be generalized for partial half-lives of transitions to excited states

$$\log_{10} T_J = \frac{aZ_D + b}{\sqrt{Q_\alpha - E_J}} + cZ_D + d \equiv V_J. \quad (13)$$

This generalized Viola-Seaborg law is very well satisfied by all available experimental data concerning transitions to excited states with $1 \leqslant J \leqslant 6$, as can be seen in Fig. 4 for even-even emitters as well as for odd-mass emitters in favored and unfavored transitions. Let us mention that this correlation is fulfilled much better than in the case of intensities. The fitting parameters are given in Table III, together with their standard deviations. We notice that one obtains similar values of the parameters both for even-even and odd-mass emitters, in the case of favored as well as unfavored transitions. This rule is useful for predicting the half-lives of α-decay transitions to excited states.

TABLE II. Fitting parameters of the linear dependence in Eq. (12) and standard deviations σ.

Transition	g	h	σ
Even-even	0.00590	0.84091	1.25829
Favored	0.01538	0.18944	0.54416
Unfavored	0.00705	− 0.26040	0.99511

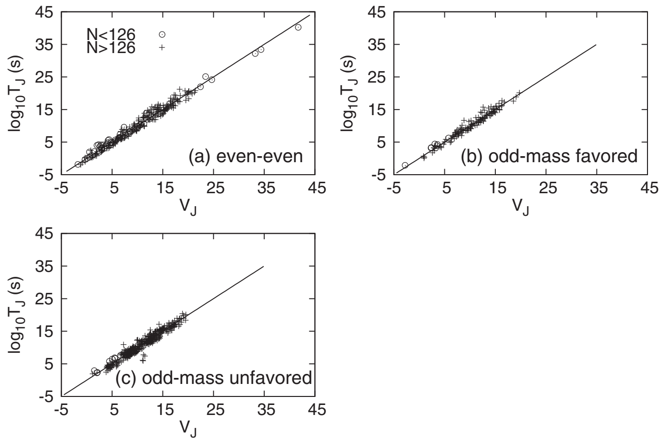

FIG. 4. Logarithm of the partial half-life versus the generalized Viola-Seaborg parameter (13) for even-even, odd-mass favored, and odd-mass unfavored emitters. Here we considered data with $1 \leqslant J \leqslant 6$ in daughter nuclei.

In conclusion, we analyzed the available experimental α-decay widths to excited states for even-even and odd-mass emitters. We have shown that α intensities to excited states depend linearly upon the excitation energy of the daughter nucleus in all known α-emission processes. We generalized the well-known Viola-Seaborg law for α-decay transitions between ground states to the case of transitions to excited states, thereby allowing the reliable prediction of the half-lives of α-decay transitions to any excited state.

TABLE III. Fitting parameters of the linear dependence in Eq. (13) and standard deviations σ.

Transition	a	b	c	d	σ
Even-even	1.575	7.884	−0.160	−38.879	1.057
Favored	1.536	6.962	−0.164	−36.777	0.861
Unfavored	1.531	7.063	−0.087	−42.514	1.039

This work was supported by the grants of the Romanian National Authority for Scientific Research, CNCS – UEFISCDI, PN-II-ID-PCE-2011-3-0092, and PN-09370102 and by the strategic grant POSDRU/159/1.5/S/137750.

[1] G. Gamow, Z. Phys. **51**, 204 (1928).
[2] A. M. Lane and R. G. Thomas, Rev. Mod. Phys. **30**, 257 (1958).
[3] H. J. Mang, Phys. Rev. **119**, 1069 (1960).
[4] A. Sandulescu, Nucl. Phys. A **37**, 332 (1962).
[5] J. K. Poggenburg, H. J. Mang, and J. O. Rasmussen, Phys. Rev. **181**, 1697 (1969).
[6] D. S. Delion, A. Săndulescu, and W. Greiner, Phys. Rev. C **69**, 044318 (2004).
[7] Y. A. Akovali, Nucl. Data Sheets **84**, 1 (1998).
[8] V. Yu. Denisov and A. A. Khudenko, At. Data Nucl. Data Tables **95**, 815 (2009).
[9] D. S. Delion and A. Dumitrescu, At. Data Nucl. Data Tables **101**, 1 (2015).
[10] J. O. Rasmussen, Phys. Rev. **113**, 1593 (1959).
[11] A. Săndulescu and O. Dumitrescu, Phys. Lett. **19**, 404 (1965); Phys. Lett. B **24**, 212 (1967).
[12] M. I. Cristu, O. Dumitrescu, N. I. Pyatov, and A. Săndulescu, Nucl. Phys. A **130**, 31 (1969).
[13] D. S. Delion and R. J. Liotta, Phys. Rev. C **56**, 1782 (1997).
[14] D. S. Delion and J. Suhonen, Phys. Rev. C **64**, 064302 (2001).
[15] S. Peltonen, D. S. Delion, and J. Suhonen, Phys. Rev. C **71**, 044315 (2005).
[16] J. O. Rasmussen and B. Segal, Phys. Rev. **103**, 1298 (1956).
[17] D. S. Delion, S. Peltonen, and J. Suhonen, Phys. Rev. C **73**, 014315 (2006).
[18] S. Peltonen, D. S. Delion, and J. Suhonen, Phys. Rev. C **78**, 034608 (2008).
[19] G. Bertsch, J. Borysowicz, H. McManus, and W. G. Love, Nucl. Phys. A **284**, 399 (1977).
[20] G. R. Satchler and W. G. Love, Phys. Rep. **55**, 183 (1979).
[21] F. Cârstoiu and R. J. Lombard, Ann. Phys. (NY) **217**, 279 (1992).
[22] D. D. Ni and Z. Z. Ren, Phys. Rev. C **80**, 051303(R) (2009); **81**, 064318 (2010); **83**, 067302 (2011).
[23] X. Zhang, C. Xu, and Z. Z. Ren, Phys. Rev. C **84**, 044312 (2011).
[24] D. D. Ni and Z. Z. Ren, Phys. Rev. C **86**, 054608 (2012).
[25] C. Xu and Z. Z. Ren, Phys. Rev. C **73**, 041301(R) (2006); Nucl. Phys. A **778**, 1 (2006).
[26] A. N. Andreyev *et al.*, Phys. Rev. Lett. **110**, 242502 (2013).
[27] D. N. Poenaru, R. A. Gherghescu, and W. Greiner, Phys. Rev. Lett. **107**, 062503 (2011).
[28] D. N. Poenaru, R. A. Gherghescu, and W. Greiner, Phys. Rev. C **85**, 034615 (2012).
[29] K. P. Santhosh and J. G. Joseph, Phys. Rev. C **86**, 024613 (2012).
[30] Peter Möller *et al.*, Phys. Rev. Lett. **108**, 052501 (2012).
[31] M. Wang *et al.*, Chin. Phys. C **36**, 1603 (2012).
[32] D. S. Delion, *Theory of Particle and Cluster Emission* (Springer-Verlag, Berlin, 2010).
[33] A. A. Raduta, *Nuclear Structure with Coherent States* (Springer, Switzerland, 2015).
[34] D. S. Delion, Phys. Rev. C **80**, 024310 (2009).
[35] V. E. Viola and G. T. Seaborg, J. Inorg. Nucl. Chem. **28**, 741 (1966).
[36] Y. J. Ren and Z. Z. Ren, Phys. Rev. C **85**, 044608 (2012).
[37] D. S. Delion, R. J. Liotta, and R. Wyss, Phys. Rev. Lett. **96**, 072501 (2006).
[38] C. Qi, F. R. Xu, R. J. Liotta, and R. Wyss, Phys. Rev. Lett. **103**, 072501 (2009).
[39] D. N. Poenaru, R. A. Gherghescu, and W. Greiner, J. Phys. G: Nucl. Part. Phys. **39**, 015105 (2012).
[40] A. Parkhomenko and A. Sobiczewski, Acta Phys. Polon. B **36**, 3095 (2005).