Critical density and impact of Δ(1232) resonance formation in neutron stars

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The critical densities and impact of forming Δ(1232) resonances in neutron stars are investigated within an extended nonlinear relativistic mean-field (RMF) model. The critical densities for the formation of four different charge states of Δ(1232) are found to depend differently on the separate kinetic and potential parts of nuclear symmetry energy, the first example of a microphysical property of neutron stars to do so. Moreover, they are sensitive to the in-medium Δ mass m_Δ and the completely unknown Δ-ρ coupling strength $g_{\rho\Delta}$. In the universal baryon-meson coupling scheme where the respective Δ-meson and nucleon-meson coupling constants are assumed to be the same, the critical density for the first $\Delta^-(1232)$ to appear is found to be $\rho_{\Delta}^{\rm crit} = (2.08 \pm 0.02)\rho_0$ using RMF model parameters consistent with current constraints on all seven macroscopic parameters usually used to characterize the equation of state of isospin-asymmetric nuclear matter at saturation density ρ_0. Moreover, the composition and the mass-radius relation of neutron stars are found to depend significantly on the values of the $g_{\rho\Delta}$ and m_Δ.

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I. INTRODUCTION

Understanding properties of Δ(1232) resonances in connection with possible pion condensation [1–3] in neutron stars and density isomers in dense nuclear matter [4–8] is a longstanding challenge of nuclear many-body physics. In fact, the role of Δ(1232) resonances in neutron stars has long been regarded as an important, and unresolved, issue [9]. Significant works have been carried out to understand in-medium properties of Δ(1232) resonances as well as their effects on saturation properties of nuclear matter and the equation of state (EOS) of dense matter using various many-body theories and interactions; see, e.g., Refs. [10–15]. However, compared to the numerous investigations on the possible appearance and effects of other particles, such as hyperons and deconfined quarks, much less effort has been devoted to the study of Δ(1232) resonances in neutron stars in recent years. This is probably partially because of the rather high Δ(1232) formation density $\rho_{\Delta^-}^{\rm crit}$ in the core of neutron stars predicted in the seminal work by Glendenning *et al.* [16–18] using a mean-field model with parameters well constrained by the experimental data available at the time. Using default parameters of their model Lagrangian leading to a symmetry energy of $E_{\rm sym}(\rho_0) = 36.8$ MeV and its density slope $L(\rho_0) \equiv 3\rho_0 {\rm d}E_{\rm sym}(\rho)/d\rho|_{\rho=\rho_0} \gtrsim 90$ MeV at saturation density ρ_0 [16–18], and using the universal baryon-meson coupling scheme in which the nucleon-meson couplings are set equal to the Δ(1232)-meson couplings ($g_{\sigma\Delta}/g_{\sigma {\rm N}} = g_{\omega\Delta}/g_{\omega {\rm N}} = g_{\rho\Delta}/g_{\rho {\rm N}} = 1$), the critical density $\rho_{\Delta^-}^{\rm crit}$ above which the first $\Delta^-(1232)$ appears is above $9\rho_0$. This led to the conclusion that Δ(1232) resonances played little role in the structure and composition of neutron stars. In the same studies, the extreme importance of symmetry energy for the formation of both hyperons and Δ(1232) resonances was emphasized. In particular, by turning off the Δ-ρ coupling which contributes the potential part of the symmetry energy (retaining thus kinetic symmetry energy only), the $\rho_{\Delta^-}^{\rm crit}$ is about $3\rho_0$ [16].

Interest has been renewed with recent studies using different symmetry energies and/or assumptions about the baryon-meson coupling constants which have found that the $\rho_{\Delta^-}^{\rm crit}$ can be as low as ρ_0 and the inclusion of the Δ(1232) has significant effects on both the composition and structure of neutron stars [19–24]. These studies generally use some individual sets of model parameters leading to macroscopic properties of asymmetric nuclear matter (ANM) at saturation density consistent with most if not all of the existing experimental constraints.

During the past three decades, much progress has been made in constraining the EOS of dense neutron-rich nuclear matter. In particular, reasonably tight constraints on the density dependence of the nuclear symmetry energy $E_{\rm sym}(\rho)$, especially around the saturation density, have been obtained in recent years, see, e.g., Refs. [25–34] for comprehensive reviews. For example, the 2013 global averages of the magnitude and slope of the $E_{\rm sym}(\rho)$ at ρ_0 are, respectively, $E_{\rm sym}(\rho_0) = 31.6 \pm 2.7$ MeV and $L = 58.9 \pm 16.5$ MeV based on 28 analyses of various terrestrial laboratory experiments and astrophysical observations [32]. Moreover, Δ(1232) resonances play a very important role in heavy-ion collisions, see, e.g., Ref. [35] for reviews, especially for the production of particles such as pions, kaons, and various exotic heavy mesons. In particular, the masses of Δ(1232) resonances primarily created in nucleon-nucleon (*NN*) collisions through the $NN \to N\Delta$ process act as an energy reservoir for subthreshold particle production. The release of this energy in subsequent collisions involving Δ(1232) resonances may help create new particles that cannot be produced otherwise in the direct, first-chance *NN* collisions. Thus, particle production has been widely used in probing in-medium properties of Δ(1232) resonances. Since Δ(1232) resonances and nucleons have isospins 3/2 and 1/2,

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respectively, the total isospin is 1 or 2 for the $N\Delta$ while it is 1 or 0 for the NN state. Because of the isospin conservation, the $\Delta(1232)$ production can only happen in the total isospin 1 NN channel. Therefore, the abundances and properties of $\Delta(1232)$ resonances are sensitive to the isospin asymmetry of the system as neutron-neutron pairs always have isospin 1 while neutron-proton pairs can have isospin 1 or 0.

Naturally, both heavy-ion collisions and neutron stars are places where the isovector properties and interactions of $\Delta(1232)$ resonances are expected to play a significant role. Indeed, useful information about the symmetry energy of dense neutron-rich matter has been extracted from studying pion and kaon productions in heavy-ion collisions [28,29]. It is especially worth noting that the isovector (symmetry) potential of $\Delta(1232)$ resonances was recently found to affect appreciably the ratio of charged pions in transport model simulations of heavy-ion collisions at intermediate energies [36]. However, to the best of our knowledge, no quantitative information about the isovector interaction of $\Delta(1232)$ resonances has been extracted yet from any terrestrial experiments. On the other hand, there are strong indications from both theoretical calculations and phenomenological model analyses of electron-nucleus, photoabsorption, and pion-nucleus scattering that the $\Delta(1232)$ isoscalar potential V_Δ (real part of its isoscalar self-energy Σ_S) is in the range of $-30\,\text{MeV} + V_N \leqslant V_\Delta \leqslant V_N$ with respect to the nucleon isoscalar potential V_N [24]. The in-medium masses and widths of $\Delta(1232)$ resonances are also the focuses of many experimental and theoretical studies using various reactions and techniques. To the best of our knowledge, however, no clear consensus has been reached yet. For instance, from analyzing the (p,π) invariant masses in the final state of heavy-ion collisions at SIS/GSI energies, indications were found for an approximately -60 MeV mass shift for $\Delta(1232)$ resonances at the freeze-out of about 1/3 the saturation density [37]. However, photoabsorption data and some advanced model calculations found no evidence of significant in-medium $\Delta(1232)$ mass shift [38,39]. It is thus exciting that new proposals to experimentally study at FAIR/GSI the $\Delta(1232)$ resonance spectroscopy and interactions in neutron-rich matter are being considered by the NUSTAR Collaboration [40].

Similarly to the appearance of any other new hadron above its production threshold in neutron stars, the addition of $\Delta(1232)$ resonances will soften the EOS and influence the composition of neutron stars [16–24]. Because of charge neutrality, depending on the individual populations of the four different charge states of $\Delta(1232)$ resonances, the density dependence of the proton fraction in neutron stars may be modified. Then different cooling mechanisms sensitive to the proton fraction may come into play above certain densities. Moreover, the formation of $\Delta(1232)$ resonances may also push up critical densities for the appearance of various hyperons [23]. As noticed earlier in the literature and emphasized in Ref. [9], there are many interesting questions regarding properties of $\Delta(1232)$ resonances in dense matter and their impact on observables of neutron stars. Obviously, answers to all of these questions naturally rely on the critical density of $\Delta(1232)$ formation in dense neutron star matter.

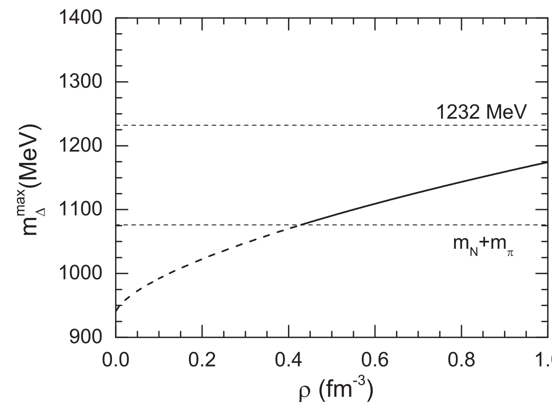

FIG. 1. The maximum mass of Δ resonances produced in the $NN \to N\Delta$ process in a free Fermi gas of nucleons at density ρ.

In this work, we first identify analytically key microphysics quantities determining the critical formation densities of the four charge states of $\Delta(1232)$ resonances. Then, within a nonlinear relativistic mean-field (RMF) model, we calculate consistently the $\rho_{\Delta^-}^{\text{crit}}$ as a function of the $\Delta(1232)$ mass m_Δ, the isovector Δ-ρ coupling strength $g_{\rho\Delta}$, and seven macroscopic variables characterizing the EOS of ANM at ρ_0 all within their latest constraints. Finally, effects of the $\Delta(1232)$ formation on the composition and mass-radius correlation of neutron stars are studied.

II. KEY MICROPHYSICS DETERMINING THE Δ FORMATION DENSITY IN NEUTRON STARS

To set a reference for our following studies we first estimate the $\rho_{\Delta^-}^{\text{crit}}$ in a free Fermi gas of nucleons. For the head-on collision of two nucleons both with Fermi momentum $|\mathbf{k}| = k_F = (3\pi^2\rho/2)^{1/3}$ in the $NN \to N\Delta$ process, the maximum mass of the produced $\Delta(1232)$ resonance is $m_\Delta^{\max} = 2(k_F^2 + m_N^2)^{1/2} - m_N$, where m_N is the average nucleon mass in free space. It is well known that $\Delta(1232)$ has a Breit-Wigner mass distribution around the centroid $m_\Delta^0 = 1232$ MeV with a width of about 120 MeV. The distribution starts at a minimum of $m_\Delta^{\min} \equiv m_N + m_\pi \simeq 1076$ MeV, where m_π is the pion mass. Shown in Fig. 1 is the $m_\Delta^{\max}$ reachable in the $NN \to N\Delta$ process as a function of density. From this simple estimate, where effects of the nuclear potentials are neglected, the critical density $\rho_\Delta^{\text{crit}}$ for producing the lightest $\Delta(1232)$ resonance is about $3\rho_0$. To reach the centroid $m_\Delta^0 = 1232$ MeV, the density has to be far above 1 fm^{-3}. This estimate also illustrates the importance of considering the mass dependence of the $\Delta(1232)$ formation density in more realistic calculations for matter in neutron stars.

In interacting nuclear systems, the masses of $\Delta(1232)$ resonances and their critical formation densities depend on the in-medium self-energies of all particles involved. Assuming neutron stars are made of neutrons, protons, $\Delta(1232)$ resonances, electrons, and muons, i.e., the $npe\mu\Delta$ matter in chemical and β equilibrium, the total baryon number density is

$$\rho = \rho_p + \rho_n + \rho_{\Delta^{++}} + \rho_{\Delta^+} + \rho_{\Delta^0} + \rho_{\Delta^-}, \tag{1}$$

where

$$\rho_{\rm n} = \frac{\left(k_{\rm F}^{\rm n}\right)^3}{3\pi^2}, \quad \rho_{\rm p} = \frac{\left(k_{\rm F}^{\rm p}\right)^3}{3\pi^2}, \tag{2}$$

$$\rho_{\Delta^{++}} = \frac{\left(k_{\rm F}^{\Delta^{++}}\right)^3}{\pi^2}, \quad \rho_{\Delta^-} = \frac{\left(k_{\rm F}^{\Delta^-}\right)^3}{\pi^2}, \tag{3}$$

$$\rho_{\Delta^{+}} = \frac{\left(k_{\rm F}^{\Delta^{+}}\right)^3}{3\pi^2}, \quad \rho_{\Delta^0} = \frac{\left(k_{\rm F}^{\Delta^0}\right)^3}{3\pi^2}, \tag{4}$$

with $k_{\rm F}^j$'s $(j = p,n,\Delta^{++},\Delta^+,\Delta^0,\Delta^-)$ being the corresponding Fermi momenta. The chemical equilibrium condition for reactions $n \to p + e + \overline{\nu}_e$ and $p + e \to n + \nu_e$ requires $\mu_{\rm e} = \mu_{\rm n} - \mu_{\rm p}$, where $\mu_{\rm e} = [m_{\rm e}^2 + (k_{\rm F}^{\rm e})^2]^{1/2} = [m_{\rm e}^2 + (3\pi^2\rho x_{\rm e})^{2/3}]^{1/2} \simeq (3\pi^2\rho x_{\rm e})^{1/3}$ with $x_{\rm e} \equiv \rho_{\rm e}/\rho$ the electron fraction. When the chemical potential of electron is larger than the static mass of a muon, reactions $e \to \mu + \nu_e + \overline{\nu}_\mu$, $p + \mu \to n + \nu_\mu$, and $n \to p + \mu + \overline{\nu}_\mu$ will also take place. The latter requires

$$\mu_{\rm n} - \mu_{\rm p} = \mu_\mu = \sqrt{m_\mu^2 + (3\pi^2\rho x_\mu)^{2/3}} \tag{5}$$

besides $\mu_{\rm n} - \mu_{\rm p} = \mu_{\rm e}$, where $m_\mu = 105.7$ MeV is the mass of a muon and $x_\mu \equiv \rho_\mu/\rho$ is the muon fraction. On the other hand, the following four types of inelastic reactions will take place between nucleons and the four charge states of Δ resonances:

$$\Delta^{++} + n \longleftrightarrow p + p, \tag{6}$$

$$\Delta^{+} + n \longleftrightarrow n + p, \tag{7}$$

$$\Delta^{0} + p \longleftrightarrow p + n, \tag{8}$$

$$\Delta^{-} + p \longleftrightarrow n + n. \tag{9}$$

Their chemical equilibrium then requires

$$\mu_{\Delta^{++}} = 2\mu_{\rm p} - \mu_{\rm n}, \tag{10}$$

$$\mu_{\Delta^{+}} = \mu_{\rm p}, \tag{11}$$

$$\mu_{\Delta^{0}} = \mu_{\rm n}, \tag{12}$$

$$\mu_{\Delta^{-}} = 2\mu_{\rm n} - \mu_{\rm p}. \tag{13}$$

In addition, the total charge neutrality in neutron stars requires that $x_{\rm p} + x_{\Delta^+} + 2x_{\Delta^{++}} = x_{\rm e} + x_\mu + x_{\Delta^-}$, where $x_{\Delta^-} \equiv \rho_{\Delta^-}/\rho$, $x_{\Delta^+} \equiv \rho_{\Delta^+}/\rho$, and $x_{\Delta^{++}} \equiv \rho_{\Delta^{++}}/\rho$, respectively.

Within the framework of a given nuclear many-body theory, Eqs. (10)–(13) can be used to calculate the critical formation densities for the four charge states of Δ(1232) resonances. Generally speaking, in relativistic mean-field models, a baryon of bare mass $m_{\rm baryon}$ obtains a Dirac effective mass $m^*_{\rm dirac}({\rm baryon}) = m_{\rm baryon} + \Sigma_{\rm S}$ and a chemical potential $\mu_{\rm baryon} = [k_{\rm F}^2 + m^{*2}_{\rm dirac}({\rm baryon})]^{1/2} + \Sigma_{\rm V}$, where $\Sigma_{\rm S}$ and $\Sigma_{\rm V}$ are the real parts of its scalar and vector self-energies, respectively. Consider the Δ^- formation, for example, noticing that $\mu_{\rm n} - \mu_{\rm p} \simeq 4E_{\rm sym}(\rho)\delta$ and using nonrelativistic kinematics, the Eq. (13) leads to the following condition for producing a Δ^- of bare mass m_{Δ^-} at rest:

$$\frac{\left(k_{\rm F}^{\rm n}\right)^2}{2m^*_{\rm dirac}} = \frac{[3\pi^2(1+\delta)\rho/2]^{2/3}}{2m^*_{\rm dirac}} = m_{\Delta^-} - m_{\rm N} - 4E_{\rm sym}(\rho)\delta + \Sigma_{\rm S}^{\Delta} - \Sigma_{\rm S}^{\rm N} + \Sigma_{\rm V}^{\Delta^-} - \Sigma_{\rm V}^{\rm n}, \tag{14}$$

where $m^*_{\rm dirac}$ is the nucleon Dirac effective mass and $\delta = (\rho_{\rm n} - \rho_{\rm p})/\rho$ is the isospin asymmetry of nucleons before Δ(1232) resonances are produced. Given the density dependencies of the symmetry energy and self-energies, this equation determines the $\rho^{\rm crit}_{\Delta^-}$ in neutron stars at β equilibrium. It also shows clearly what microphysics quantities determine the $\rho^{\rm crit}_{\Delta^-}$. In particular, the difference in Δ and nucleon masses $m_{\Delta^-} - m_{\rm N}$, the symmetry energy $E_{\rm sym}(\rho)$, and the difference in both scalar $\Sigma_{\rm S}^{\Delta} - \Sigma_{\rm S}^{\rm N}$ and vector $\Sigma_{\rm V}^{\Delta^-} - \Sigma_{\rm V}^{\rm n}$ self-energies are all characteristics of baryon interactions. It also indicates where the model dependence and uncertainties are. As we mentioned earlier, experimental data indicate that the difference in nucleon and Δ(1232) isoscalar self-energies can be up to about 30 MeV while there is simply no experimental indication so far about the difference in their isovector self-energies. We notice that in many studies in the literature the Δ(1232) resonances and nucleons are assumed to have the same scalar and vector self-energies. In this case, then, the $\rho^{\rm crit}_{\Delta^-}$ is completely determined by the Δ(1232) mass m_Δ and the nuclear symmetry energy as a function of density $E_{\rm sym}(\rho)$.

The nonlinear RMF model has been very successful in describing many nuclear properties and phenomena during the past few decades, see, e.g., Refs. [41–59]. The total Lagrangian density of the nonlinear RMF model of Ref. [54] augmented by the Yukawa couplings of the Δ fields to various isoscalar and isovector meson fields can be written as [5–8,16,19,22–24]

$$\begin{aligned}\mathcal{L} = {}& \overline{\psi}_{\rm N}[\gamma_\mu(i\partial^\mu - g_{\omega{\rm N}}\omega^\mu - g_{\rho{\rm N}}\vec{\tau}_{\rm N}\cdot\vec{\rho}^\mu) \\ & - (m_{\rm N} - g_{\sigma{\rm N}}\sigma)]\psi_{\rm N} \\ & + \overline{\psi}_{\Delta\nu}[\gamma_\mu(i\partial^\mu - g_{\omega\Delta}\omega^\mu - g_{\rho\Delta}\vec{\tau}_\Delta\cdot\vec{\rho}^\mu) \\ & - (m_\Delta - g_{\sigma\Delta}\sigma)]\psi_\Delta^\nu \\ & + \tfrac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \tfrac{1}{2}m_\sigma^2\sigma^2 - U_{\rm N}(\sigma) \\ & + \tfrac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \tfrac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \tfrac{1}{4}c_{\omega{\rm N}}\left(g_{\omega{\rm N}}^2\omega_\mu\omega^\mu\right)^2 \\ & + \tfrac{1}{2}m_\rho^2\vec{\rho}_\mu\cdot\vec{\rho}^\mu - \tfrac{1}{4}\vec{\rho}_{\mu\nu}\cdot\vec{\rho}^{\mu\nu} \\ & + \tfrac{1}{2}\left(g_{\rho{\rm N}}^2\vec{\rho}_\mu\cdot\vec{\rho}^\mu\right)\Lambda_{\rm V}g_{\omega{\rm N}}^2\omega_\mu\omega^\mu,\end{aligned} \tag{15}$$

where $\omega_{\mu\nu} \equiv \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$ and $\rho_{\mu\nu} \equiv \partial_\mu\vec{\rho}_\nu - \partial_\nu\vec{\rho}_\mu$ are strength tensors for the ω and ρ meson fields, respectively. $\psi_{\rm N}$, ψ_Δ^ν, σ, ω_μ, and $\vec{\rho}_\mu$ are the nucleon Dirac field, Schwinger-Rarita field for Δ resonances, isoscalar-scalar meson field, isoscalar-vector meson field, and isovector-vector meson field, respectively, and the arrows denote isovectors, $U_{\rm N}(\sigma) = b_{\sigma{\rm N}}m_{\rm N}(g_{\sigma{\rm N}}\sigma)^3/3 + c_{\sigma{\rm N}}(g_{\sigma{\rm N}}\sigma)^4/4$ is the self-interaction term of the σ field. The parameter $\Lambda_{\rm V}$ represents the coupling constant of mixed interaction between the isovector ρ and isoscalar ω mesons. It is known to be important for calculating the density dependence of the symmetry energy [54]. In terms of the expectation values of the meson fields, $\overline{\sigma}$, $\overline{\omega}_0$, and $\overline{\rho}_0^{(3)}$, where the subscript "0" denotes the zeroth component of the four-vector while the superscript "(3)" denotes the third component of isospin, the nucleon and Δ(1232) isoscalar self-energies are, respectively,

$$\Sigma_{\rm S}^{\rm N} = -g_{\sigma{\rm N}}\overline{\sigma}, \tag{16}$$

and

$$\Sigma_{\mathrm{S}}^{\Delta} = -g_{\sigma\Delta}\overline{\sigma}. \tag{17}$$

Their isovector self-energies are, respectively,

$$\Sigma_{\mathrm{V}}^{\mathrm{N}} = g_{\omega\mathrm{N}}\overline{\omega}_0 + \tau_{\mathrm{p/n}}^3 g_{\rho\mathrm{N}}\overline{\rho}_0^{(3)} \tag{18}$$

and

$$\Sigma_{\mathrm{V}}^{\Delta} = g_{\omega\Delta}\overline{\omega}_0 + \tau_i^3 g_{\rho\Delta}\overline{\rho}_0^{(3)} \tag{19}$$

with $\tau_{\mathrm{p}}^3 = +1$, $\tau_{\mathrm{n}}^3 = -1$, and $i = \Delta^{++}, \Delta^+, \Delta^0, \Delta^-$, $\tau_{\Delta^{++}}^3 = +3$, $\tau_{\Delta^+}^3 = +1$, $\tau_{\Delta^0}^3 = -1$, $\tau_{\Delta^-}^3 = -3$.

In terms of the ratios of Δ-meson over nucleon-meson coupling constants $x_\sigma \equiv g_{\sigma\Delta}/g_{\sigma\mathrm{N}}$, $x_\omega \equiv g_{\omega\Delta}/g_{\omega\mathrm{N}}$, and $x_\rho \equiv g_{\rho\Delta}/g_{\rho\mathrm{N}}$, the Eqs. (10)–(13) lead to the following conditions determining the critical densities for forming the four charge states of $\Delta(1232)$ resonances:

$$\rho_{\Delta^-}^{\mathrm{crit}}: \frac{(k_{\mathrm{F}}^{\mathrm{n}})^2}{2m_{\mathrm{dirac}}^*} \simeq \Phi_\Delta + g_{\sigma\mathrm{N}}(1-x_\sigma)\overline{\sigma} - g_{\omega\mathrm{N}}(1-x_\omega)\overline{\omega}_0 - 6(1-x_\rho)E_{\mathrm{sym}}^{\mathrm{pot}}(\rho)\delta - 4E_{\mathrm{sym}}^{\mathrm{kin}}(\rho)\delta, \tag{20}$$

$$\rho_{\Delta^0}^{\mathrm{crit}}: \frac{(k_{\mathrm{F}}^{\mathrm{n}})^2}{2m_{\mathrm{dirac}}^*} \simeq \Phi_\Delta + g_{\sigma\mathrm{N}}(1-x_\sigma)\overline{\sigma} - g_{\omega\mathrm{N}}(1-x_\omega)\overline{\omega}_0 - 2(1-x_\rho)E_{\mathrm{sym}}^{\mathrm{pot}}(\rho)\delta, \tag{21}$$

$$\rho_{\Delta^+}^{\mathrm{crit}}: \frac{(k_{\mathrm{F}}^{\mathrm{p}})^2}{2m_{\mathrm{dirac}}^*} \simeq \Phi_\Delta + g_{\sigma\mathrm{N}}(1-x_\sigma)\overline{\sigma} - g_{\omega\mathrm{N}}(1-x_\omega)\overline{\omega}_0 + 2(1-x_\rho)E_{\mathrm{sym}}^{\mathrm{pot}}(\rho)\delta, \tag{22}$$

$$\rho_{\Delta^{++}}^{\mathrm{crit}}: \frac{(k_{\mathrm{F}}^{\mathrm{p}})^2}{2m_{\mathrm{dirac}}^*} \simeq \Phi_\Delta + g_{\sigma\mathrm{N}}(1-x_\sigma)\overline{\sigma} - g_{\omega\mathrm{N}}(1-x_\omega)\overline{\omega}_0 + 6(1-x_\rho)E_{\mathrm{sym}}^{\mathrm{pot}}(\rho)\delta + 4E_{\mathrm{sym}}^{\mathrm{kin}}(\rho), \tag{23}$$

where $\Phi_\Delta \equiv m_\Delta - m_{\mathrm{N}}$ is the Δ-nucleon mass difference and $E_{\mathrm{sym}}^{\mathrm{pot}}(\rho) = 2^{-1}\rho g_{\rho\mathrm{N}}^2(m_\rho^2 + \Lambda_{\mathrm{V}} g_{\rho\mathrm{N}}^2 g_{\omega\mathrm{N}}^2 \overline{\omega}_0^2)^{-1}$ and $E_{\mathrm{sym}}^{\mathrm{kin}}(\rho) = 6^{-1}k_{\mathrm{F}}^2(k_{\mathrm{F}}^2 + m_{\mathrm{dirac}}^{*,2})^{-1/2}$ are, respectively, the potential and kinetic part of the symmetry energy in the nonlinear RMF model [29].

Several interesting conclusions can be made qualitatively from inspecting the above four conditions. Generally, the critical densities depend differently on the three coupling ratios x_σ, x_ω, and x_ρ as they have different natures. The isoscalar coupling ratios x_σ and x_ω affect the four $\Delta(1232)$ resonances the same way, i.e., the x_σ lowers while the x_ω raises their critical formation densities, while the isovector coupling ratio x_ρ acts differently on the four different charge states of $\Delta(1232)$ resonances. Moreover, the kinetic and potential parts of the symmetry energy have separate and different effects. In particular, in the universal baryon-meson coupling scheme, i.e., $x_\sigma = x_\omega = x_\rho = 1$, the critical densities for creating Δ^- and Δ^{++} depend only on the Φ_Δ and the kinetic symmetry energy $E_{\mathrm{sym}}^{\mathrm{kin}}(\rho)$ besides the m_{dirac}^*, while those for the Δ^0 and Δ^+ are determined only by the Φ_Δ and m_{dirac}^*. In this case, assuming the $E_{\mathrm{sym}}^{\mathrm{kin}}(\rho)$ is always positive as in the case of RMF, noticing that the Fermi surface of protons is lower than that of neutrons at any density in neutron-rich matter, i.e., $k_{\mathrm{F}}^{\mathrm{p}} < k_{\mathrm{F}}^{\mathrm{n}}$, one then sees immediately the following sequence of appearance $\rho_{\Delta^-}^{\mathrm{crit}} < \rho_{\Delta^0}^{\mathrm{crit}} < \rho_{\Delta^+}^{\mathrm{crit}} < \rho_{\Delta^{++}}^{\mathrm{crit}}$ [16,24]. However, we notice that the short-range nucleon-nucleon correlation (SRC) [60–62] may lead to negative kinetic symmetry energies even at normal density of nuclear matter, see, e.g., Refs. [63–69]. In this case, the order of appearance of Δ^- and Δ^{++}, thus the fraction of various particles and the structure of neutron stars may differ. We remark here that this is the first time that some physics quantities in neutron stars are found to depend separately on the kinetic and potential parts instead of the total symmetry energy.

Some earlier studies, see, e.g., Refs. [6,8,23,24], indicate that $x_\sigma \approx x_\omega \approx 1.0$. Effects of slight deviations from this value on properties of neutron stars have also been reported, see, e.g., Ref. [21]. To the best of our knowledge, however, little is known about the range of x_ρ and its effects in either heavy-ion collisions [36] or neutron stars. Moreover, most studies so far are limited to density isomers due to $\Delta(1232)$ formation in symmetric nuclear matter where it is sufficient to consider only effects of the x_σ and x_ω.

In the universal baryon-meson coupling scheme, the default set (SH-NJ) of RMF model parameters leads to the following values of macroscopic quantities characterizing the EOS of ANM at $\rho_0 = 0.149\ \mathrm{fm}^{-3}$: the binding energy $E_0(\rho_0) = -16.09$ MeV, the Dirac effective mass $m_{\mathrm{dirac}}^{*0}(\rho_0)/m_{\mathrm{N}} = 0.64$, the incompressibility $K_0(\rho_0) = 230$ MeV, the skewness coefficient $J_0(\rho_0) = -415$ MeV, the magnitude $E_{\mathrm{sym}}(\rho_0) = 31.17$ MeV, and slope $L(\rho_0) = 48.64$ MeV of symmetry energy. We remark that almost all of these bulk parameters are extracted from a fit to the properties of finite nuclei [59] with the exception of the skewness coefficient J_0 that has been obtained by fixing the maximum mass of a neutron star at $M_{\mathrm{max}} = 2.01 M_\odot$ [70]. Variations around this parameterization will be investigated in the following.

For Δ resonances of mass $m_\Delta = 1232$ MeV, we study in Fig. 2 the $\rho_{\Delta^-}^{\mathrm{crit}}$ dependence on the x_ρ while keeping all other quantities at their default values. It is seen that the $\rho_{\Delta^-}^{\mathrm{crit}}$ increases approximately linearly with x_ρ. At $x_\rho = 1$, the $\rho_{\Delta^-}^{\mathrm{crit}}$ is only about $2.1\rho_0$ consistent with that found in

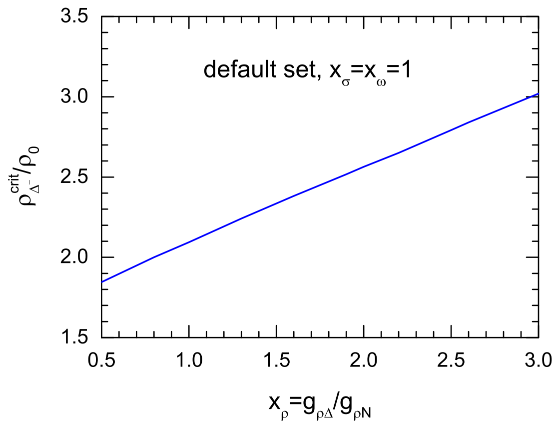

FIG. 2. (Color online) Dependence of the critical density $\rho^{\rm crit}_{\Delta^-}$ for Δ^- formation in neutron stars on the relative Δ-ρ coupling strength $x_\rho = g_{\rho\Delta}/g_{\rho\rm N}$.

Refs. [21,23,24] but much smaller than the one found by Glendenning *et al.* [16–18]. However, unless the value of x_ρ is somehow constrained, the $\rho^{\rm crit}_{\Delta^-}$ will remain underdetermined.

Delta resonances in free-space have the Breit-Wigner mass distribution

$$f(m_\Delta) = \frac{1}{4}\frac{\Gamma^2(m_\Delta)}{\left(m_\Delta - m^0_\Delta\right)^2 + \Gamma^2(m_\Delta)/4} \tag{24}$$

with the mass-dependent width given by [71,72]

$$\Gamma(m_\Delta) = 0.47q^3/\left(m^2_\pi + 0.6q^2\right)(\text{GeV}), \tag{25}$$

where $q = [([m^2_\Delta - m^2_{\rm N} + m^2_\pi]/2m_\Delta)^2 - m^2_\pi]^{1/2}$ is the pion momentum in the Δ rest frame in the $\Delta \to \pi + {\rm N}$ decay process. Shown in red in Fig. 3 is the free-space $\Delta(1232)$ mass distribution. It is known that the $\Delta(1232)$ mass distributions may be modified in a nuclear medium [40]. This effect is beyond the scope of the RMF model used here as it does not consider the imaginary part of the $\Delta(1232)$ self-energy self-consistently. However, we can examine how the $\rho^{\rm crit}_{\Delta^-}$ depends on the bare $\Delta(1232)$ mass m_Δ by varying its value in Eqs. (20)–(23). As one expects and as indicated by Eqs. (20)–(23), the $\rho^{\rm crit}_{\Delta^-}$ increases with m_Δ in the universal coupling scheme. Our numerical calculations shown with the blue line

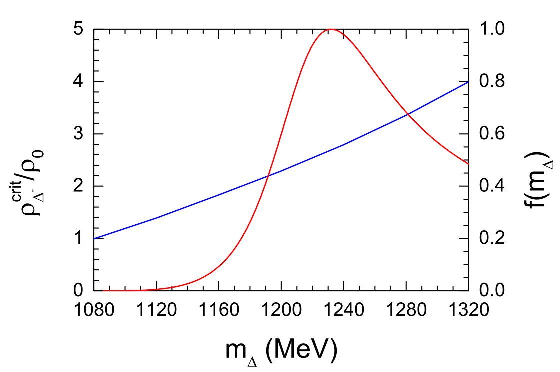

FIG. 3. (Color online) The Δ mass m_Δ dependence of the critical density $\rho^{\rm crit}_{\Delta^-}$ for Δ^- formation in neutron stars (blue) and the Breit-Wigner mass distribution of Δ resonances in free space (red).

indicate that the increase is almost linear. Considering the mass distribution, while the $\rho^{\rm crit}_{\Delta^-}$ for Δ resonances around m^0_Δ is about $2.1\rho_0$, it gradually decreases for lower Δ masses. Of course, these low-mass Δ resonances are less likely to be produced compared to the ones near m^0_Δ. On the other hand, the Δ mean lifetime $\tau_\Delta = \hbar/\Gamma(m_\Delta)$ is only about 1.7 fm/c at m^0_Δ but increases very quickly for lower masses. Thus, the main population of Δ resonances in neutron stars may not necessarily peak at $m_\Delta = 1232$ MeV. A detailed study of this issue will require a full account of the $\pi - N - \Delta$ dynamics in neutron stars that is also beyond the scope of the RMF model used here. Nevertheless, our results indicate that the appearance of Δ resonances, especially the ones with low masses around $2\rho_0$, may compete with other particles, such as hyperons, thus possibly modifying the widely accepted and long-time viewpoint that hyperons should appear earlier than $\Delta(1232)$ resonances in neutron stars [16,73]. To the best of our knowledge, however, no study to date has considered consistently the effects of the mass distribution and the associated mass-dependent lifetimes of Δ resonances in neutron stars.

III. EFFECTS OF NUCLEAR EQUATION OF STATE ON THE FORMATION OF Δ(1232) RESONANCES IN NEUTRON STARS

Equation (13) for determining the critical density $\rho^{\rm crit}_{\Delta^-}$ can be rewritten as

$$\mu^{\rm min}_{\Delta^-} = m_\Delta + \Sigma^{\Delta^-} = 2\mu_{\rm n} - \mu_{\rm p} \simeq \mu_{\rm n} + 4E_{\rm sym}(\rho)\delta. \tag{26}$$

The left-hand side is given in terms of the microscopic quantities, i.e., $\Delta(1232)$ mass and the three Δ-meson coupling constants in the total self-energy $\Sigma^{\Delta^-} = -g_{\sigma\Delta}\overline{\sigma} + g_{\omega\Delta}\overline{\omega}_0 - 3g_{\rho\Delta}\overline{\rho}^{(3)}_0$. Using the parabolic approximation for the EOS of ANM $E(\rho,\delta) \simeq E_0(\rho) + E_{\rm sym}(\rho)\delta^2 + \mathcal{O}(\delta^4)$ and assuming the density is not too far from ρ_0, the right-hand side of Eq. (26) can be expanded in terms of $\chi = (\rho - \rho_0)/3\rho_0$ and the isospin asymmetry δ as

$$\begin{aligned}\mu^{\rm min}_{\Delta^-} \simeq\ & m_{\rm N} + E_0(\rho_0) + \left(\frac{\chi}{3}\frac{\rho}{\rho_0} + \frac{\chi^2}{2}\right)K_0 \\ & + \frac{\chi^2}{6}\left(\frac{\rho}{\rho_0} + \chi\right)J_0 + (2\delta + 3\delta^2)E_{\rm sym}(\rho_0) \\ & + \left[\frac{\rho}{3\rho_0}\delta^2 + \chi(2\delta + 3\delta^2)\right]L(\rho_0),\end{aligned} \tag{27}$$

where higher-order terms in the expansion have been neglected (for full details see Ref. [58]). Here $K_0 = 9\rho_0^2 {\rm d}^2E_0(\rho)/d\rho^2|_{\rho=\rho_0}$ and $J_0 = 27\rho_0^3{\rm d}^3E_0(\rho)/d\rho^3|_{\rho=\rho_0}$ are the incompressibility and skewness coefficient of symmetric nuclear matter (SNM) at ρ_0, respectively. This expansion is very easy to understand considering the energy conservation in the Δ production process $NN \to N\Delta$, i.e., the minimum energy of the Δ is the energy of a nucleon (sum of nucleon rest mass and its mechanical energy). Since all seven macroscopic quantities used to characterize the EOS of ANM, i.e., (a) the saturation density ρ_0 of SNM where the pressure $P(\rho_0) = 0$, (b) the binding energy $E_0(\rho_0)$, (c) incompressibility K_0, (d)

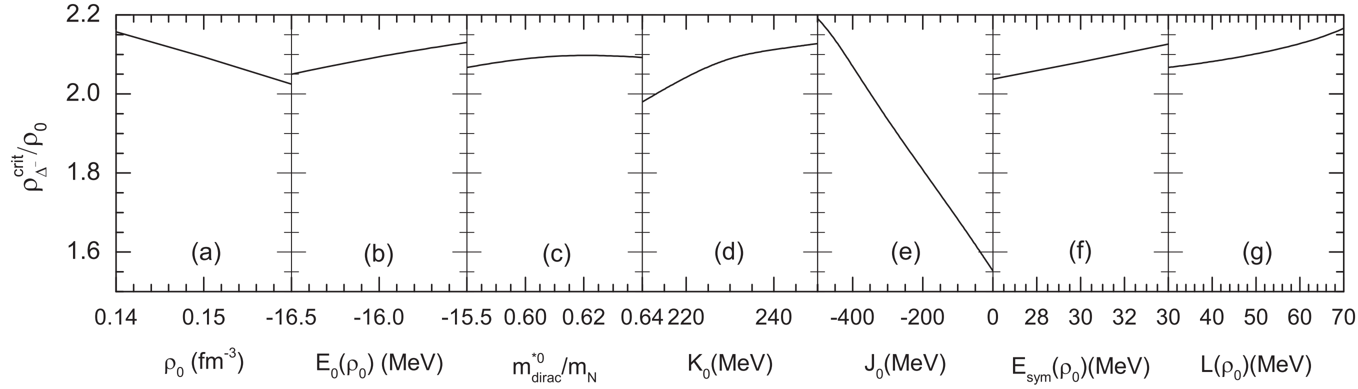

FIG. 4. Critical density for the formation of Δ^- resonance from the nonlinear RMF model by varying individually ρ_0 (a), $E_0(\rho_0)$ (b), $m^{*0}_{\rm dirac}$ (c), K_0 (d), J_0 (e), $E_{\rm sym}(\rho_0)$ (f), and $L(\rho_0)$ (g).

skewness coefficient J_0, (e) nucleon effective mass $m^{*0}_{\rm dirac}$, (f) magnitude $E_{\rm sym}(\rho_0)$, and (g) slope $L(\rho_0)$ of the symmetry energy, are all explicit functions (see, e.g., Refs. [58,74,75]) of the seven RMF microscopic model parameters, i.e., $g_{\sigma N}$, $g_{\omega N}$, $g_{\rho N}$, $b_{\sigma N}$, $c_{\sigma N}$, $c_{\omega N}$, and Λ_V, Eq. (27) allows us to explore the $\rho^{\rm crit}_{\Delta^-}$ dependence on the seven microscopic or macroscopic parameters. Since the macroscopic quantities are either empirical properties of nuclear matter or directly related to experimental observables, it is more useful for the purposes of this work to examine the $\rho^{\rm crit}_{\Delta^-}$ by varying individually the seven macroscopic quantities. We notice that within both the RMF and Skyrme-Hartree-Fock approaches similar correlation analyses [76] have been successfully applied to study the neutron skin [76,77], the giant monopole resonances (GMR) of finite nuclei [78], the higher-order bulk characteristic parameters of ANM [79], the electric dipole polarizability α_D in ^{208}Pb [80], the correlation between the maximum mass of neutron stars and the skewness coefficient of the SNM [58], as well as the relationship between the $E_{\rm sym}(\rho)$ and the symmetry energy coefficient in the mass formula of nuclei [81].

In Fig. 4 we show the $\rho^{\rm crit}_{\Delta^-}$ obtained in the universal coupling scheme as a function of the seven macroscopic parameters within their respective uncertain ranges. Among them, the $J_0 = -250 \pm 250$ MeV and $L = 50 \pm 20$ MeV currently have the largest uncertainties. Examining the results shown in Fig. 5, we make the following two observations: (i) The values of $\rho^{\rm crit}_{\Delta^-}$ from all seven correlations overlap around $\rho^{\rm crit}_{\Delta^-} = (2.08 \pm 0.02)\rho_0$. Notice that this result depends on the value of J_0 in the default set, which had been fixed to obtain the maximum stellar mass configuration matching the observed $2.01 M_\odot$ neutron star. With this assumption, this result is the most reliable prediction for the $\rho^{\rm crit}_{\Delta^-}$ consistent with all of the constraints within the RMF model considered. We note that increasing J_0 raises the maximum mass and lowers the critical density $\rho^{\rm crit}_{\Delta^-}$; hence the range $\rho^{\rm crit}_{\Delta^-} =(2.08 \pm 0.02)\rho_0$ constitutes an upper limit on $\rho^{\rm crit}_{\Delta^-}$. Since the inclusion of Δ resonances inevitably softens the EOS, the magnitude of J_0 should therefore be much larger than the one used in our default set to be consistent with the current observation of the two solar-mass neutron star. With the other six macroscopic quantities fixed within their current uncertainty ranges, the critical density $\rho^{\rm crit}_{\Delta^-}$ will take an even smaller value than the upper limit predicted above. And this is one of the main reasons why we have not considered hyperons in the present work, because they appear at much higher densities than $\rho^{\rm crit}_{\Delta^-}$. (ii) A reasonable and quantitative measure of the sensitivity of $\rho^{\rm crit}_{\Delta^-}$ to each individual variable is the response function $\mathcal{Q} \equiv |(dy/y)/(dx/x)|$, where dy/y is the relative change in $\rho^{\rm crit}_{\Delta^-}$ with respect to its mean value and dx/x is the relative change in the variable x with respect to its mean value. The value of $\mathcal{Q}$ is approximately $\mathcal{Q}[E_0(\rho_0)] = 0.53$, $\mathcal{Q}[\rho_0] = 0.40$, $\mathcal{Q}[K_0] = 0.29$, $\mathcal{Q}[m^{*0}_{\rm dirac}] = 0.18$, $\mathcal{Q}[J_0] = 0.17$, $\mathcal{Q}[E_{\rm sym}(\rho_0)] = 0.11$, and $\mathcal{Q}[L(\rho_0)] = 0.10$ following the same order of importance in the expansion of the minimum Δ chemical potential in Eq. (27).

The seemingly stronger correlation between the $\rho^{\rm crit}_{\Delta^-}$ and J_0 compared to the correlations of $\rho^{\rm crit}_{\Delta^-}$ with the other six variables is because of the relatively larger uncertainty in J_0. On the other hand, some of the weaker correlations shown in Fig. 4 may become much stronger if one goes beyond their current constraints. For example, shown in Fig. 5 are the correlations of the $\rho^{\rm crit}_{\Delta^-}$ with the reduced slope $L(\rho_r)/w$ of $E_{\rm sym}(\rho)$ at three reference densities of $\rho_r/\rho_0 = w = 0.7, 1.0$ and two while fixing the magnitudes of $E_{\rm sym}(\rho)$ and other variables at their default values. It is seen that the $\rho^{\rm crit}_{\Delta^-}$ increases much faster

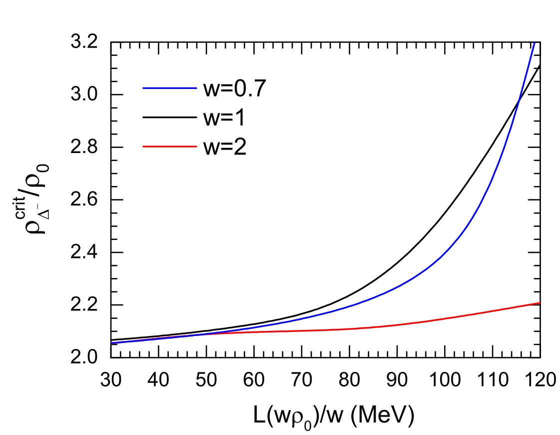

FIG. 5. (Color online) Critical density for the formation of Δ^- resonance as a function of the scaled density slope of symmetry energy $L(w\rho_0)/w$ at three reduced densities of $\rho_r/\rho_0 = w = 0.7, 1$ and 2, respectively.

for $L(\rho_r)/w \geqslant 70$ MeV. This feature is consistent with that found in Ref. [24]. Noting again that some earlier studies have used models predicting L values high than 90 MeV, they thus predicted correspondingly much large values for the $\rho_{\Delta^-}^{\rm crit}$.

IV. EFFECTS OF Δ RESONANCES ON THE COMPOSITION AND STRUCTURE OF NEUTRON STARS

Having shown that the $\rho_{\Delta^-}^{\rm crit}$ depends sensitively on the completely unknown Δ-ρ coupling strength $g_{\rho\Delta}$ and the Δ mass m_Δ, and it is about $\rho_{\Delta^-}^{\rm crit} = (2.08 \pm 0.02)\rho_0$ in the universal coupling scheme for $m_\Delta = 1232$ MeV using RMF model parameters consistent with all existing constraints on the nuclear EOS, we now turn to effects of Δ formation on properties of neutron stars. This study is carried out within the $npe\mu\Delta$ model omitting other particles such as hyperons and quarks at high densities. This model is sufficient for the purposes of this work. Moreover, we restrict ourselves to studying effects of the Δ formation on the composition and mass-radius relation of neutron stars by varying the Δ mass and its coupling strength with the ρ meson. In constructing the EOS of various layers in neutron stars for solving the Oppenheimer-Volkoff (OV) equation, we follow a rather standard scheme. For the core we use the EOS of β-stable and charge-neutral $npe\mu\Delta$ matter obtained from the nonlinear RMF model described earlier. The inner crust with densities ranging between $\rho_{\rm out} = 2.46 \times 10^{-4}{\rm fm}^{-3}$ corresponding to the neutron dripline and the core-crust transition density ρ_t determined self-consistently using the thermodynamical method [57,82] is the region where complex and exotic nuclear structure—collectively referred to as the "nuclear pasta"—may exist. Because of our poor knowledge about this region, we adopt the polytropic EOSs parameterized in terms of the pressure P as a function of total energy density ε according to $P = a + b\varepsilon^{4/3}$ [82,83]. The constants a and b are determined by the pressure and energy density at ρ_t and $\rho_{\rm out}$ [82]. For the outer crust [84], we use the Baym-Pethick-Sutherland EOS for the region with 6.93×10^{-13} fm^{-3} $< \rho < \rho_{\rm out}$ and the fundamental measure theory EOS for 4.73×10^{-15} fm^{-3} $< \rho < 6.93 \times 10^{-13}$ fm^{-3}, respectively.

Shown in Fig. 6 are fractions of different species, i.e., $x_i = \rho_i/\rho$, in neutron stars using two parameter sets with the Δ-ρ coupling strength corresponding to $x_\rho = 1$ and 2, respectively. These calculations are done with $m_\Delta = 1232$ MeV. It is seen that the appearance of Δ^- affects significantly the fractions of others particles depending on the value of the x_ρ as one expects. The modified fractions of the lighter particles e and μ will affect their weak decays and thus the possible kaon condensation. Moreover, the strong boost of the proton fraction above the Δ^- production threshold may have a significant impact on the cooling processes in neutron stars [85]. A detailed investigation of these consequences requires a self-consistent extension of the model considered here and is on the agenda of our proposed future work.

Finally, we study in Fig. 7 effects of both the Δ mass m_Δ and the x_ρ parameter on the mass-radius relation of neutron stars. Without including the Δ resonances (black line), as we mentioned earlier the default value of J_0 was chosen to predict a maximum neutron star mass of $2.01 M_\odot$, consistent with the

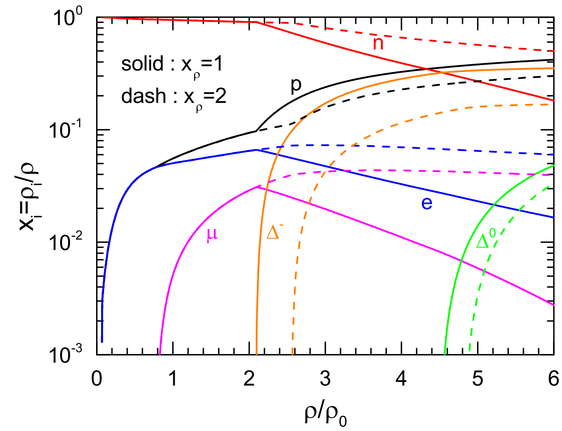

FIG. 6. (Color online) Fractions of different species in neutron stars composed of neutrons, protons, Δ resonances, electrons, and muons within the nonlinear RMF model using two sets of model parameters with $x_\rho = g_{\rho\Delta}/g_{\rho N} = 1$ and 2, respectively.

latest observations. As given in Eqs. (24) and (25), Δ(1232) has an intrinsic mass distribution and each mass has its own width (lifetime) in free space. Notably, it is possible to form Δ(1232) with masses much smaller than the centroid of 1232 MeV in low energy $NN \to N\Delta$ or $\pi + N \to \Delta$ reactions. As one expects, including Δ resonances reduces both the maximum mass and the corresponding radius. Moreover, the effect is stronger for Δ resonances with masses smaller than the centroid. These low-mass Δ resonances have lower production thresholds and are thus more abundant. They then soften the EOS more, starting at lower densities reached in the cores of lighter neutron stars. Of course, effects of the Δ resonance on properties of neutron stars depend on the value of x_ρ. As shown in Fig. 6 and discussed earlier, a larger value of x_ρ leads to a higher $\rho_{\Delta^-}^{\rm crit}$. Thus, the effect of varying m_Δ appears at a higher mass (higher central density) of neutron stars when the value of x_ρ is increased from 1 to 2.

Obviously, by including the Δ resonances we can no longer obtain a maximum neutron star mass of $2.01 M_\odot$ with the default parameter set. Moreover, we found that by varying

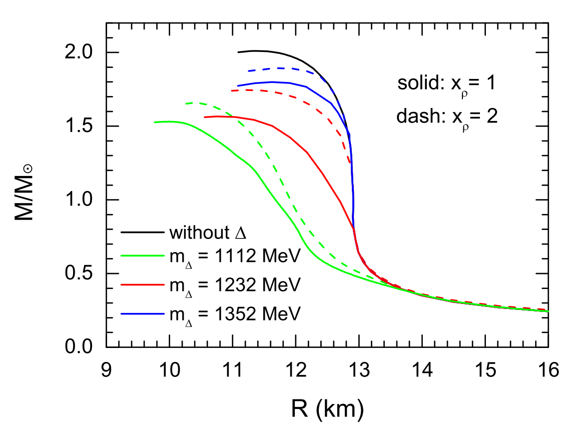

FIG. 7. (Color online) The mass-radius correlation of neutron stars without and with Δ resonances of different masses using $x_\rho = g_{\rho\Delta}/g_{\rho N} = 1$ and 2, respectively.

the value of J_0 in its uncertain range, we cannot recover a maximum mass of $2.01M_\odot$ with $x_\rho = 1$. We confirm that indeed there is a "Delta puzzle" consistent with the finding of Refs. [23,24]. Thus, if indeed Δ resonances can be formed around $2\rho_0$ as in the universal coupling scheme, our findings call for serious considerations of the various EOS stiffening mechanisms proposed in the literature. On the other hand, as we mentioned earlier, there is currently no constraint on the x_ρ neither theoretically nor experimentally. Since the critical density for Δ formation increases approximately linearly with x_ρ, it is therefore critical to first constrain independently the x_ρ, such as from heavy-ion collisions, before the "Delta puzzle" in understanding properties of neutron stars can be resolved.

V. SUMMARY AND DISCUSSIONS

The possible formation and roles of $\Delta(1232)$ resonances in neutron stars are outstanding issues in nuclear astrophysics. The first and most important piece of information necessary for resolving these issues is the critical density above which the $\Delta(1232)$ can be formed in neutron stars. Previous studies have indicated that the critical densities range from ρ_0 to very high values only reachable in the core of very massive neutron stars. In this work, within the extended nonlinear RMF model, we found that the critical formation densities for the four different charge states of $\Delta(1232)$ resonances depend differently on the separate kinetic and potential parts of the nuclear symmetry energy, the in-medium Δ mass m_Δ and the completely unknown Δ-ρ coupling strength $g_{\rho\Delta}$. *This is the first time a microphysical property of neutron star matter has been shown to depend differently on the potential and kinetic parts of the symmetry energy.* Assuming that the respective Δ-meson and nucleon-meson coupling constants are the same, the critical density for the first $\Delta^-(1232)$ to appear is found to be $\rho_{\Delta^-}^{\rm crit} = (2.08 \pm 0.02)\rho_0$ using RMF model parameters consistent with current constraints on all seven macroscopic parameters characterizing the EOS of ANM at ρ_0. We also found that the composition and the mass-radius relation of neutron stars are significantly affected by the formation of $\Delta(1232)$ resonances. In particular, the effects of the $\Delta(1232)$ formation depend sensitively on the values of the $g_{\rho\Delta}$ and the in-medium Δ mass m_Δ which are also being probed with terrestrial laboratory experiments.

To this end, it is interesting to note that, since the early work by Kubis and Kutschera [86], the isovector, Lorentz-scalar δ meson has been incorporated in several RMF models in predicting the EOS of dense neutron-rich matter and its applications in understanding properties of relativistic heavy-ion collisions and neutron stars [87–95]. Since the δ contributes negatively to the symmetry energy in contrast to the ρ meson, to fit the saturation properties of nuclear matter, reproduce the known symmetry energy of about 31 MeV at saturation density, and avoid the bounding of pure neutron matter, the ρ-nucleon coupling has to be readjusted significantly compared to its value in the RMF model without considering the δ meson since there is no known mechanism to determine the δ-nucleon coupling independently. The net effect of including the δ meson is thus to stiffen the symmetry energy at suprasaturation densities where the ρ contribution dominates. Indeed, applications of these models have found that the δ may play an appreciable role in determining the hadron-quark phase transition [91,92], the crust-core transition and the formation of a pasta phase [93], and the mass-radius relation of neutron stars [89,94], as well as the cooling mechanism of protoneutron stars [95]. We have no physical reason to preclude possible δ meson effects in determining the critical density for $\Delta(1232)$ formation in neutron stars. However, to consider effects of the δ meson one has to introduce all the necessary but unknown δ-baryon coupling constants. Given the fact that our main findings in this work already depend on the completely unknown Δ-ρ coupling, the inclusion of the δ into our current model is thus not beneficial for the main purposes of this work.

ACKNOWLEDGMENTS

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APPENDIX A: DERIVATION OF EQ. (20)

We start from the relation $\mu_{\Delta^-} = 2\mu_{\rm n} - \mu_{\rm p}$, where

$$\mu_{\rm n} = \sqrt{\left(k_{\rm F}^{\rm n}\right)^2 + m_{\rm dirac}^{*,2}} + g_{\omega{\rm N}}\overline{\omega}_0 - g_{\rho{\rm N}}\overline{\rho}_0^{(3)}, \tag{A1}$$

$$\mu_{\rm p} = \sqrt{\left(k_{\rm F}^{\rm p}\right)^2 + m_{\rm dirac}^{*,2}} + g_{\omega{\rm N}}\overline{\omega}_0 + g_{\rho{\rm N}}\overline{\rho}_0^{(3)}, \tag{A2}$$

$$\mu_{\Delta^-} = \sqrt{\left(k_{\rm F}^{\Delta^-}\right)^2 + (m_\Delta - g_{\sigma\Delta}\overline{\sigma})^2} + g_{\omega\Delta}\overline{\omega}_0 - 3g_{\rho\Delta}\overline{\rho}_0^{(3)}, \tag{A3}$$

are the chemical potential of neutron, proton, and Δ^-, and $m_{\rm dirac}^* = m_{\rm N} - g_{\sigma{\rm N}}\overline{\sigma}$. The condition to determine $\rho_{\Delta^-}^{\rm crit}$ suggests that we can set $\rho_{\Delta^-} = 0$, $k_{\rm F}^{\Delta^-} = 0$. Then, using the relation above, we find that

$$\begin{aligned} & m_\Delta - g_{\sigma\Delta}\overline{\sigma} + g_{\omega\Delta}\omega_0 - 3g_{\rho\Delta}\overline{\rho}_0^{(3)} \\ & \quad = \sqrt{\left(k_{\rm F}^{\rm n}\right)^2 + m_{\rm dirac}^{*,2}} + g_{\omega{\rm N}}\omega_0 - g_{\rho{\rm N}}\overline{\rho}_0^{(3)} + \mu_{\rm n} - \mu_{\rm p}, \end{aligned} \tag{A4}$$

where the last two terms on the right-hand side can be expressed as [57,82]

$$\mu_{\rm n} - \mu_{\rm p} \simeq 4E_{\rm sym}(\rho)\delta. \tag{A5}$$

The expression for the symmetry energy in the nonlinear RMF model can be written as [29]

$$\begin{aligned} E_{\rm sym}(\rho) = E_{\rm sym}^{\rm kin}(\rho) + E_{\rm sym}^{\rm pot}(\rho) = & \frac{k_{\rm F}}{6\sqrt{k_{\rm F}^2 + m_{\rm dirac}^{*,2}}} \\ & + \frac{1}{2}\frac{g_{\rho{\rm N}}^2\rho}{m_\rho^2 + \Lambda_{\rm V}g_{\rho{\rm N}}^2 g_{\omega{\rm N}}^2\overline{\omega}_0^2}. \end{aligned} \tag{A6}$$

The equation of motion for the $\overline{\rho}_0^{(3)}$ field is given as

$$m_\rho^2\overline{\rho}_0^{(3)} = g_{\rho\mathrm{N}}\big(-\rho\delta - \Lambda_{\mathrm{V}} g_{\rho\mathrm{N}}\overline{\rho}_0^{(3)} g_{\omega\mathrm{N}}^2\overline{\omega}_0^2\big). \quad \text{(A7)}$$

Rearranging this last equation, we find

$$\overline{\rho}_0^{(3)} = -\frac{g_{\rho\mathrm{N}}\rho\delta}{m_\rho^2 + \Lambda_{\mathrm{V}} g_{\rho\mathrm{N}} g_{\omega\mathrm{N}}^2\overline{\omega}_0^2}. \quad \text{(A8)}$$

This in turn can be rearranged to give

$$\begin{aligned} -2g_{\rho\mathrm{N}}\overline{\rho}_0^{(3)} &= \frac{2g_{\rho\mathrm{N}}^2\rho\delta}{m_\rho^2 + \Lambda_{\mathrm{V}} g_{\rho\mathrm{N}} g_{\omega\mathrm{N}}^2\overline{\omega}_0^2} \simeq 4E_{\mathrm{sym}}^{\mathrm{pot}}(\rho)\delta \\ &= 4\big[E_{\mathrm{sym}}(\rho) - E_{\mathrm{sym}}^{\mathrm{kin}}(\rho)\big]\delta. \end{aligned} \quad \text{(A9)}$$

The $\overline{\omega}_0$ in the expression of symmetry energy (A6) is evaluated at $\delta = 0$, whereas the one appearing in Eq. (A8) has the value of δ generally being nonzero, hence we used the "$\simeq$" symbol in the second line. Expanding now $[(k_{\mathrm{F}}^{\mathrm{n}})^2 + m_{\mathrm{dirac}}^{*,2}]^{1/2}$ as

$$\sqrt{\big(k_{\mathrm{F}}^{\mathrm{n}}\big)^2 + m_{\mathrm{dirac}}^{*,2}} \simeq \frac{k_{\mathrm{F}}^{\mathrm{n},2}}{2m_{\mathrm{dirac}}^*} + m_{\mathrm{dirac}}^*, \quad \text{(A10)}$$

and using the relation above we find

$$\begin{aligned} \frac{k_{\mathrm{F}}^{\mathrm{n},2}}{2m_{\mathrm{dirac}}^*} &= \Phi_\Delta + g_{\sigma\mathrm{N}}(1-x_\sigma)\overline{\sigma} - g_{\omega\mathrm{N}}(1-x_\omega)\overline{\omega}_0 + g_{\rho\mathrm{N}}(1-x_\rho)\overline{\rho}_0^{(3)} - 4x_\rho E_{\mathrm{sym}}^{\mathrm{kin}}(\rho)\delta - 4(1-x_\rho)E_{\mathrm{sym}}(\rho)\delta, && \text{(A11)} \\ &\simeq \Phi_\Delta + g_{\sigma\mathrm{N}}(1-x_\sigma)\overline{\sigma} - g_{\omega\mathrm{N}}(1-x_\omega)\overline{\omega}_0 + g_{\rho\mathrm{N}}\overline{\rho}_0^{(3)} + 6x_\rho E_{\mathrm{sym}}^{\mathrm{pot}}(\rho)\delta - 4E_{\mathrm{sym}}(\rho)\delta, && \text{(A12)} \\ &\simeq \Phi_\Delta + g_{\sigma\mathrm{N}}(1-x_\sigma)\overline{\sigma} - g_{\omega\mathrm{N}}(1-x_\omega)\overline{\omega}_0 + (6x_\rho - 2)E_{\mathrm{sym}}^{\mathrm{pot}}(\rho)\delta - 4E_{\mathrm{sym}}(\rho)\delta, && \text{(A13)} \\ &= \Phi_\Delta + g_{\sigma\mathrm{N}}(1-x_\sigma)\overline{\sigma} - g_{\omega\mathrm{N}}(1-x_\omega)\overline{\omega}_0 + (6x_\rho - 6)E_{\mathrm{sym}}(\rho)\delta - (6x_\rho - 2)E_{\mathrm{sym}}^{\mathrm{kin}}(\rho)\delta, && \text{(A14)} \\ &= \Phi_\Delta + g_{\sigma\mathrm{N}}(1-x_\sigma)\overline{\sigma} - g_{\omega\mathrm{N}}(1-x_\omega)\overline{\omega}_0 + (6x_\rho - 6)E_{\mathrm{sym}}^{\mathrm{pot}}(\rho)\delta + 4E_{\mathrm{sym}}^{\mathrm{kin}}(\rho)\delta, && \text{(A15)} \end{aligned}$$

where $x_\phi = g_{\phi\mathrm{N}}/g_{\phi\Delta}$ with $\phi = \sigma$, ω, and ρ, and $\Phi_\Delta = m_\Delta - m_{\mathrm{N}}$. This last equation is the Eq. (20) reported in the main part of this paper. The derivation of Eqs. (21)–(23) follows using similar arguments and steps.

APPENDIX B: DERIVATION OF EQ. (27)

Once again we start from the relation $\mu_{\Delta^-} = 2\mu_{\mathrm{n}} - \mu_{\mathrm{p}}$ and obtain

$$\mu_{\Delta^-} = 2\mu_{\mathrm{n}} - \mu_{\mathrm{p}} = \mu_{\mathrm{n}} + \mu_{\mathrm{n}} - \mu_{\mathrm{p}} \simeq \mu_{\mathrm{n}} + 4E_{\mathrm{sym}}(\rho)\delta + \mathcal{O}(\delta^3), \quad \text{(B1)}$$

where $E_{\mathrm{sym}}(\rho)$ is the nuclear symmetry energy, $\delta \equiv 1 - 2x_{\mathrm{p}}$ is the isospin asymmetry, and x_{p} is the proton fraction. The chemical potential for neutrons can be expressed as

$$\mu_{\mathrm{n}} = \frac{\partial\varepsilon_{\mathrm{N}}}{\partial\rho_{\mathrm{n}}} = \frac{\partial\varepsilon_{\mathrm{N}}}{\partial\rho} + \frac{2\tau_3^{\mathrm{n}}\rho_{\mathrm{p}}}{\rho^2}\frac{\partial\varepsilon_{\mathrm{N}}}{\partial\delta}, \quad \text{(B2)}$$

where ρ and δ are two independent variables. In obtaining the expression above we have used the relation

$$\frac{\partial}{\partial\rho_J} = \frac{\partial\rho}{\partial\rho_J}\frac{\partial}{\partial\rho} + \frac{\partial\delta}{\partial\rho_J}\frac{\partial}{\partial\delta} = \frac{\partial}{\partial\rho} + \frac{2\tau_3^J\rho_{\overline{J}}}{\rho^2}\frac{\partial}{\partial\delta}, \quad \text{(B3)}$$

where $J = n,p$, $\rho_{\overline{\mathrm{n}}} = \rho_{\mathrm{p}}$, and $\rho_{\overline{\mathrm{p}}} = \rho_{\mathrm{n}}$. When the density is smaller than $\rho_{\Delta^-}^{\mathrm{cric}}$, there are only neutrons, protons, electrons, and muons present in the system. Neglecting the contribution from electrons and muons, ε_{N} can be taken as the energy density of nuclear matter only. That is,

$$\varepsilon_{\mathrm{N}}(\rho,\delta) = [E(\rho,\delta) + m_{\mathrm{N}}]\rho, \quad \text{(B4)}$$

where $m_{\mathrm{N}} = 939$ MeV is the static mass of the nucleon and $E(\rho,\delta)$ is the equation of state of asymmetric nuclear matter, which in turn can be written as

$$E(\rho,\delta) \simeq E_0(\rho) + E_{\mathrm{sym}}(\rho)\delta^2 + \mathcal{O}(\delta^4). \quad \text{(B5)}$$

Then

$$\begin{aligned} \mu_{\Delta^-} &\simeq \mu_{\mathrm{n}} + 4E_{\mathrm{sym}}(\rho)\delta \\ &= E(\rho,\delta) + m_{\mathrm{N}} + \rho\frac{\partial E(\rho,\delta)}{\partial\rho} + \frac{2\tau_3^{\mathrm{n}}\rho_{\mathrm{p}}}{\rho^2}\frac{\partial}{\partial\delta}(\rho E_{\mathrm{sym}}(\rho)\delta^2) + 4E_{\mathrm{sym}}(\rho)\delta \\ &\simeq E_0(\rho) + E_{\mathrm{sym}}(\rho)\delta^2 + m_{\mathrm{N}} + \rho\frac{\partial E(\rho,\delta)}{\partial\rho} + \frac{2\tau_3^{\mathrm{n}}\rho_{\mathrm{p}}}{\rho}\frac{\partial}{\partial\delta}(E_{\mathrm{sym}}(\rho)\delta^2) + 4E_{\mathrm{sym}}(\rho)\delta \end{aligned}$$

$$
\begin{aligned}
&= E_0(\rho) + E_{\rm sym}(\rho)\delta^2 + m_{\rm N} + \rho\frac{\partial E(\rho,\delta)}{\partial\rho} + 4E_{\rm sym}(\rho)\delta(1-x_{\rm p})\\
&= E_0(\rho) + E_{\rm sym}(\rho)\delta^2 + m_{\rm N} + \rho\frac{\partial E(\rho,\delta)}{\partial\rho} + E_{\rm sym}(\rho)(2\delta+2\delta^2)\\
&= E_0(\rho) + m_{\rm N} + \rho\left[\frac{\partial E_0(\rho)}{\partial\rho} + \frac{\partial}{\partial\rho}(E_{\rm sym}(\rho)\delta^2)\right] + E_{\rm sym}(\rho)(2\delta+3\delta^2)\\
&\simeq E_0(\rho_0) + m_{\rm N} + \left(\frac{1}{2}\chi^2 + \frac{\rho}{3\rho_0}\chi\right)K_0 + \left(\frac{1}{6}\chi^3 + \frac{\rho}{6\rho_0}\chi^2\right)J_0\\
&\quad + (2\delta+3\delta^2)E_{\rm sym}(\rho_0) + \left((2\delta+3\delta^2)\chi + \frac{\rho}{3\rho_0}\delta^2\right)L(\rho_0), \qquad \text{(B6)}
\end{aligned}
$$

which is the desired Eq. (27). Note that $K_0 \equiv K_0(\rho_0)$ and $J_0 \equiv J_0(\rho_0)$, and, in the derivation above, we have made use of the following relation:

$$\frac{\partial\chi}{\partial\rho} = \frac{\partial}{\partial\rho}\frac{\rho-\rho_0}{3\rho_0} = \frac{1}{3\rho_0}. \qquad \text{(B7)}$$

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