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Vorticity and hydrodynamic helicity in heavy-ion collisions in the hadron-string dynamics model

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The hydrodynamic helicity separation effect in noncentral heavy-ion collisions is investigated using the hadron-string dynamics (HSD) model. Computer simulations are done to calculate velocity and hydrodynamic helicity on a mesh in a small volume around the center of the reaction. The time dependence of hydrodynamic helicity is observed for various impact parameters and different calculation methods. Comparison with a similar earlier work is carried out. A new quantity related to jet handedness is used to probe for p-odd effects in the final state.

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I. INTRODUCTION

$C(P)$ odd effects in heavy-ion collisions are under intensive theoretical investigation nowadays [1]. It was noted that $(C)P$-violating processes in quark gluon plasma (QGP) can result in induced electromagnetic currents and separation of charge under the influence of a magnetic field—the chiral magnetic effect (CME) [2,3]. This effect has been studied theoretically and experimental techniques have been proposed in order to register it [4–6]. Another possible result of $(C)P$-violating processes in QGP is the chiral vortical effect (CVE). Just like CME, CVE leads to induced currents and charge separation (not necessarily electromagnetic) [7]. CVE is caused by coupling to medium vorticity, in a way similar to CME being caused by coupling to magnetic field, and leads to contribution to the electromagnetic current:

$$J_e^\gamma = \frac{N_c}{4\pi^2 N_f}\varepsilon^{\gamma\beta\alpha\rho}\partial_\alpha u_\rho \partial_\beta\left(\theta\sum_j e_j\mu_j\right), \tag{1}$$

where N_c and N_f are the number of colors and flavors respectively, e_j and μ_j are electric charge and chemical potential of particle of flavor j respectively, u_ρ is the medium four-velocity, and θ is the topological QCD field. Therefore, one of the key ingredients for the development of CVE is medium vorticity. The presence of vorticity developing in noncentral heavy-ion collisions may lead [7] to neutron asymmetries.

Classic vorticity as well as its relativistic generalization were calculated in a 3+1-dimensional hydrodynamic model. Velocity circulation has also been analyzed in Ref. [8].

Another quantity that can be studied is hydrodynamic helicity:

$$H = \int (\vec{v}, rot\vec{v})dV, \tag{2}$$

where $\vec{v}$ is the medium velocity Euclidean vector and integration is carried out over three dimensional space. Theoretical [7,9] and numerical [10,11] studies show that hydrodynamic helicity can cause polarization of particles in the final state. Triangle anomaly requires a new vorticity-dependent term ω^μ in the expression for current in hydrodynamics [12]:

$$\omega^\mu = \frac{1}{2}e^{\mu\nu\lambda\rho}u_\nu\partial_\lambda u_\rho,$$

and ω^0 is proportional to the integrand in Eq. (2). This makes hydrodynamic helicity an important parameter in heavy-ion collisions. It can be divided into two parts depending on the v_y component of velocity:

$$H_\uparrow = \int (\vec{v}, rot\vec{v})dV, v_y > 0 \tag{3}$$

and

$$H_\downarrow = \int (\vec{v}, rot\vec{v})dV, v_y < 0. \tag{4}$$

If there is nonzero medium vorticity with a dominating direction, these two quantities will have different signs. Time dependence of these quantities can give additional information on medium vorticity. Hydrodynamic helicity has been studied in Ref. [10] with special emphasis on its time dependence. It was shown that hydrodynamic helicity separation can be observed in the QGSM model. The time dependence of other integral quantities regarding hydrodynamic helicity and vorticity were also calculated in that work. Vorticity and hydrodynamic helicity in heavy-ion collisions were investigated in Ref. [8,10,13].

Vorticity and hydrodynamic helicity are the sources of chiral effects just like electric and magnetic fields are responsible for anomalous pion decay. Thus it is important to find out if vorticity really exists in different models and calculate related quantities such as hydrodynamic helicity to observe their time evolution.

The aim of current paper is to investigate vorticity and hydrodynamic helicity in heavy-ion collisions with the help of

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the HSD model [14]. The HSD modeling program provides the numerical solution of a set of relativistic transport equations for particles with in-medium self-energies. Comparison to the similar results obtained in a QGSM model [10] is also made.

We also study the directly observable quantity: handedness. It is a modification of the variables proposed in Refs. [15,16] to study particle polarization.

II. MODELLING VELOCITY FIELD

The heavy-ion collision modeling was done with the help of slightly modified HSD program [14]. Au+Au collisions with different impact parameters and with bombarding energy of 12.38 GeV per nucleon were simulated, which corresponds to $\sqrt{s} = 5$ GeV in the center-of-mass frame. Before collision nuclei travel along the Z axis. The distance between centers of the nuclei is b along the X axis and 7 fm in the Z axis direction. Plane $y = 0$ is called the reaction plane.

The velocity field was calculated using energies and momenta of all particles in the final state. All final-state quantities are given in the center-of-mass frame, and calculations are carried out in the same frame of reference. The space was represented with a three-dimensional mesh. Each cell is a rectangular cuboid with the following parameters: $\Delta x = \Delta y = \gamma \Delta z = 0.6$ fm, where γ is the γ factor of the center-of-mass frame. Medium velocity in each cell (m,n,k) was computed as follows:

$$\vec{v}(m,n,k) = \frac{\sum_{i,j} \vec{P}_{ij}(m,n,k)}{\sum_{i,j} E_{ij}(m,n,k)}, \tag{5}$$

where $\vec{P}_{ij}(m,n,k)$ is the momentum of particle j in cell (m,n,k) and event i, and $E_{ij}(m,n,k)$ is defined in a similar manner. For each cell (m,n,k) sums are carried out over all events and all particles in that cell. Cells with at least two particles were taken into account.

Velocities of fluid cells obtained this way were used to calculate hydrodynamic helicity (H) and vorticity ($rot\vec{v}$). All results presented were calculated using a basic two-point difference formula for derivatives. As we will see later, a more sophisticated formula for derivatives (7), (8), and (9) does not give better results. For vorticity distribution weighted $(rot\vec{v})_y$ was used, as suggested in Ref. [8]. This enables easier comparison of results to those obtained in a different approach (purely hydrodynamic) in Ref. [8]. Vorticity in each cell (m,n,k) was weighted by the factor of $w(m,n,k)$:

$$w(m,n,k) = \frac{E(m,n,k) N_{\text{cells}}}{2 E_{\text{total}}}, \tag{6}$$

where $E(m,n,k)$ is the sum of energies of all particles in the cell (m,n,k), N_{cells} is the total number of cells, and E_{total} is the total energy in the whole volume. This weighted vorticity was averaged over all x-z layers to get a single x-z layer at different times. In order to observe hydrodynamic helicity separation cells are divided into two groups depending on sign of velocity component v_y normal to the reaction plane. H was calculated for both kinds of cells separately.

III. RESULTS AND COMPARISON

A. Weighted vorticity

In this subsection we present the weighted y-component of vorticity averaged over all x-z layers at different times. To observe time evolution of weighted vorticity it was plotted at three different time moments: 7.5, 10.5, and 14 fm/c, impact parameter $b = 8$ fm (Figs. 1, 2, and 3). A similar plot for $t = 10.5$ fm/c and impact parameter $b = 0$ fm is included (Fig. 4). In the latter case, the weighted y component of the vorticity is less in magnitude and there are regions with positive as well as negative y components of the vorticity. The overall average is decreasing in time. As for the $b = 0$ fm case (Fig. 4), we notice that the average value of weighted y component of vorticity is negligible with relation to the same time moment $t = 10.5$ fm/c with impact parameter $b = 8$ fm (Fig. 2).

B. Hydrodynamic helicity

The main results obtained in the HSD model for hydrodynamic helicity separation are presented in Fig. 5. Simulations in the HSD model manifest hydrodynamic helicity separation similar to the separation in QGSM model. Along with this there is a notable shift in time (about 6-7 fm/c). Hydrodynamic helicity separation begins later than in Ref. [10]. This may be explained by the difference in initial state of the nuclei. Since they start off at a significant distance along the Z axis as mentioned above it takes some time for them to collide.[1]

Magnitudes of $H_\uparrow$ and $H_\downarrow$ are also different from the same quantities in Ref. [10]. $H_\uparrow$ subdivided by components in scalar product is shown in Fig. 5. In both models the x component does not give a significant contribution in H. For the QGSM model there is a difference between y- and z-component contributions. However, there is no such tendency for the HSD model: Both components give contributions of similar magnitude.

The same quantity was calculated using another formula for velocity derivatives in hydrodynamic helicity for comparison.

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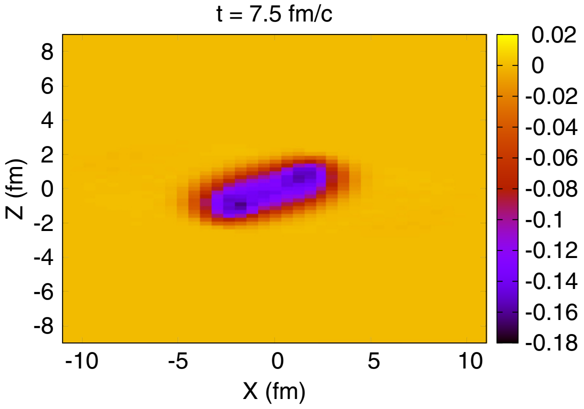

FIG. 1. (Color online) weighted y component of the vorticity (c/fm) averaged over all x-z layers at 7.5 fm/c, impact parameter $b = 8$ fm. Average value is -4.4395×10^{-2} c/fm.

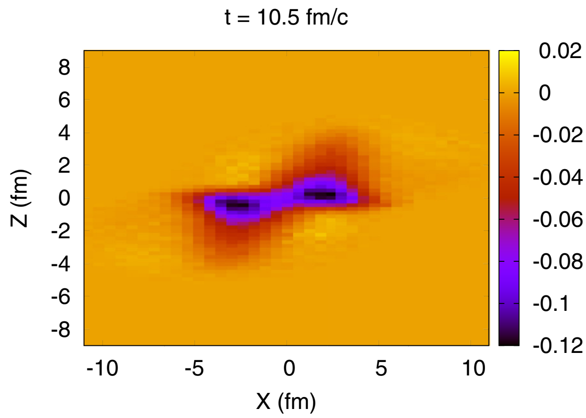

FIG. 2. (Color online) weighted y component of the vorticity (c/fm) averaged over all x-z layers at 10.5 fm/c, impact parameter $b = 8$ fm. Average value is -2.28×10^{-2} c/fm.

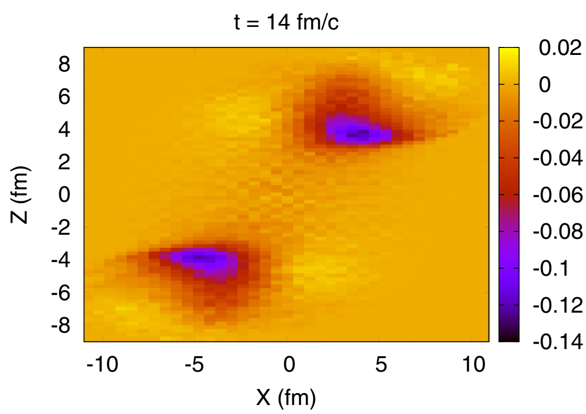

FIG. 3. (Color online) Weighted y component of the vorticity (c/fm) averaged over all x-z layers at 14 fm/c, impact parameter $b = 8$ fm. Average value is -1.2452×10^{-2} c/fm.

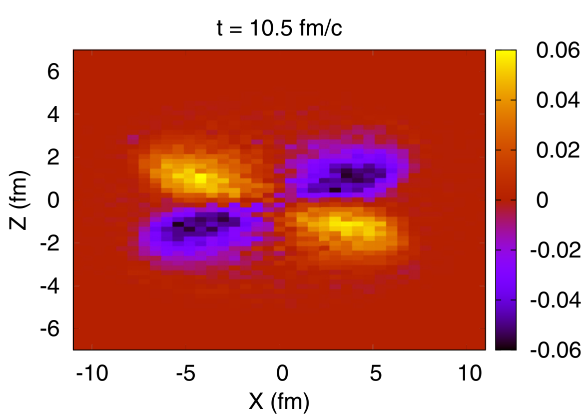

FIG. 4. (Color online) Weighted y component of the vorticity (c/fm) averaged over all x-z layers at 10.5 fm/c, impact parameter $b = 0$ fm. Average value is -1.2×10^{-5} c/fm.

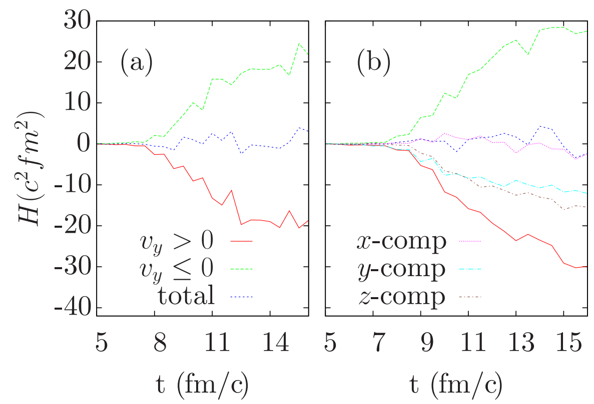

FIG. 5. (Color online) Hydrodynamic helicity: $H_\uparrow$ and $H_\downarrow$, impact parameter $b = 4$ fm/c (a) and $b = 8$ fm/c (b).

It can be interesting to see if a more accurate formula can improve the result. Let us calculate derivatives as follows:

$$\partial_x v_\alpha(m,n,k) = \frac{1}{8h_x} \sum_{\substack{i=-1,1\\ j=-1,1}} \{v_\alpha(m+1,n+i,k+j) - v_\alpha(m-1,n+i,k+j)\}, \tag{7}$$

$$\partial_y v_\alpha(m,n,k) = \frac{1}{8h_y} \sum_{\substack{i=-1,1\\ j=-1,1}} \{v_\alpha(m+i,n+1,k+j) - v_\alpha(m+i,n-1,k+j)\}, \tag{8}$$

$$\partial_z v_\alpha(m,n,k) = \frac{1}{8h_z} \sum_{\substack{i=-1,1\\ j=-1,1}} \{v_\alpha(m+i,n+j,k+1) - v_\alpha(m+i,n+j,k-1)\}, \tag{9}$$

where h_x,h_y,h_z are the cell sizes along the x,y, and z axis respectively; m,n,k are discrete coordinates on the mesh. This method of calculation uses a higher order discrete derivative and averages over four derivatives calculated at different points. The new formula for derivatives, however, does not give any improved results (Fig. 6).

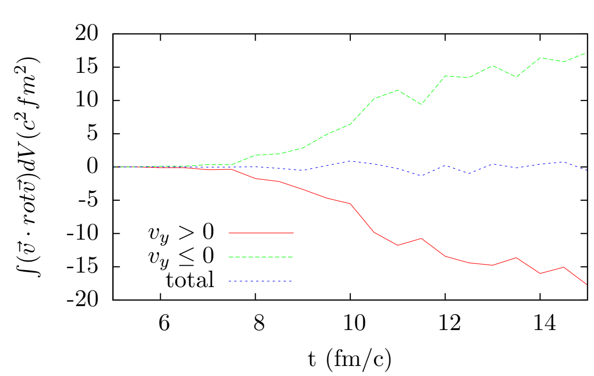

FIG. 6. (Color online) Results for new derivative formulas parameter $b = 4$ fm/c.

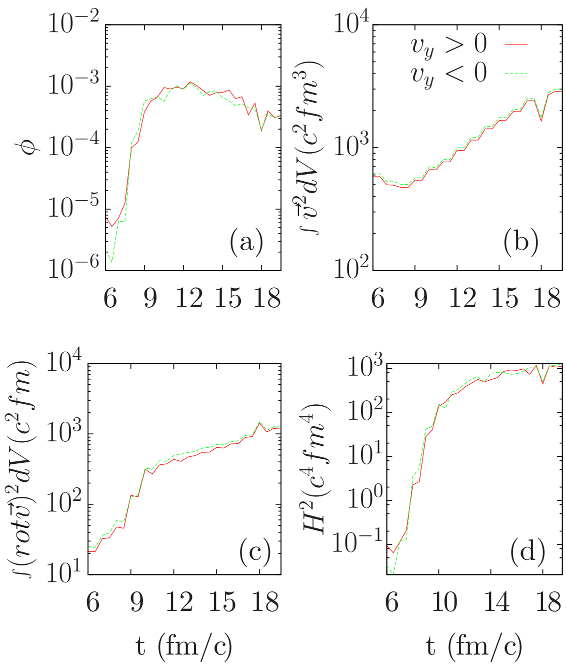

FIG. 7. (Color online) Integral values calculated for impact parameter $b = 8$ fm.

To determine the relative direction velocity and vorticity, the quantity

$$\phi = \frac{(\int(\vec{v}, rot\vec{v})dV)^2}{\int v^2 dV \int(rot\vec{v})^2 dV} \tag{10}$$

has been plotted over time. The value of 1 corresponds to the velocity being parallel to vorticity. Plots for this value, along with related integrals, are presented in Fig. 7. For both models this value appears to be very small (10^{-2}–10^{-3}), but non-negligible.

In this section we have studied hydrodynamic vorticity, hydrodynamic helicity, and some integral values in heavy-ion collisions. The results were compared to those obtained with the help of the QGSM model. They are mostly similar except for an explainable shift in time. Although the integral values are small in magnitude, their time dependence resembles the results obtained in the QGSM model.

The possible observable result of nonzero medium hydrodynamic helicity is polarization of Λ hyperons with different signs of the y component of momentum. A quantitative estimate of such polarization is given in Ref. [10]. At hydrodynamic helicity values calculated here and in Ref. [10], the Λ-hyperon polarization is possible to observe. The possibility of Λ-hyperon polarization effect is also discussed in Ref. [7]. Λ-hyperon polarization is considered in a hydrodynamic model in Ref. [11].

IV. HANDEDNESS

Since nuclei have nonzero angular momentum in noncentral collisions we can expect to find some p-odd effects in the final state. In this part of the article we try to find a relation between observable properties of particles in the final state with explicit parity breaking by angular momentum in the initial state.

TABLE I. Octant enumeration.

Octant	Momentum
0	$p_x > 0, p_y > 0, p_z > 0$
1	$p_x > 0, p_y > 0, p_z \leqslant 0$
2	$p_x > 0, p_y \leqslant 0, p_z > 0$
3	$p_x > 0, p_y \leqslant 0, p_z \leqslant 0$
4	$p_x \leqslant 0, p_y > 0, p_z > 0$
5	$p_x \leqslant 0, p_y > 0, p_z \leqslant 0$
6	$p_x \leqslant 0, p_y \leqslant 0, p_z > 0$
7	$p_x \leqslant 0, p_y \leqslant 0, p_z \leqslant 0$

Several methods were proposed [15,16] to obtain information about spin polarization of particles in the initial state by exploring the properties of particles in the final state. These methods are based on computation of vector or triple product of 3-momenta of particles in the final state. They are suitable for processing experimental data and may be used in heavy-ion collisions. In this case, the similar correlation should be related to orbital rather than spin angular momentum in the initial state so that the spin-orbital interaction and related imaginary phases are not necessary.

In the first article [16] a pseudoscalar T was introduced:

$$T = \frac{1}{|\vec{p}|}([\vec{p}_1, \vec{p}_2], \vec{p}_3),$$

with $|\vec{p}_1| > |\vec{p}_2| > |\vec{p}_3|$, where $\vec{p}_1$, $\vec{p}_2$, and $\vec{p}_3$ are 3-momenta of particles in the final state, and $\vec{p}$ is momentum of the particle in the initial state. Using T and quantities derived from it, some reactions including electron-positron annihilation to hadrons and nucleon collisions were considered.

Independently, in Ref. [15] a quantity called handedness was defined. It was proposed to investigate polarization of the initial quark or gluon. Longitudinal handedness is defined as follows:

$$H_{\parallel} = \frac{N_l - N_r}{N_l + N_r},$$

where N_l and N_r are the numbers of left- and right-handed combinations $\vec{k}$, $\vec{k}_1$, $\vec{k}_2$:

$$e_{ijk} k^i k_1^j k_2^k > 0, \quad \text{for } N_l,$$

$$e_{ijk} k^i k_1^j k_2^k < 0, \quad \text{for } N_r.$$

Here, $\vec{k}$ is momentum of the initial particle, and $\vec{k}_1$,$\vec{k}_2$ are momenta of particles (pions) in the final state. It was proposed to sort particles $\vec{k}_1$ and $\vec{k}_2$ according to their charge or magnitudes of momenta.

A. Methods and results

Based on these articles we can introduce the following quantity:

$$\eta = \frac{\sum(\vec{p}_3, \vec{p}_2, \vec{p}_1)}{\sum|(\vec{p}_3, \vec{p}_2, \vec{p}_1)|}, \tag{11}$$

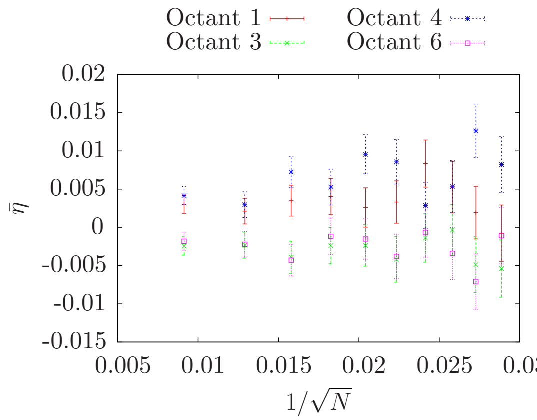

FIG. 8. (Color online) Dependence of $\bar{\eta}$ on $1/\sqrt{N}$, for impact parameter $b = 7$ fm. Octants 1, 3, 4, and 6.

where $(\vec{p}_3,\vec{p}_2,\vec{p}_1)$ is triple product $(\vec{p}_3,[\vec{p}_2,\vec{p}_1])$ with all vectors in a triplet in the same octant in the momentum space, and $\vec{p}_1,\vec{p}_2,\vec{p}_3$ are momenta of pions in the final state. Momenta in each triple product were sorted:

$$|p_3|^2 < |p_2|^2 < |p_1|^2.$$

Hence eight values $\eta_i, i = 0\ldots7$, one for each octant, were calculated. Octants were enumerated in the way described in Table I. It is difficult to say at the moment whether η possesses any special physical meaning; this pseudoscalar variable is expected to reveal the p oddness (existence of orbital angular momentum pseudovector) of the noncentral heavy-ion collisions. Au+Au collisions were considered with projectile energy of 5 GeV per nucleon in the laboratory frame with impact parameter $b = 7$ fm/c. Heavy-ion collisions were modeled, as before, in the hadron-string dynamics model [14].

Since collisions are noncentral, nonzero values of η are expected. To take into account statistical errors, η was averaged over a number of events and an estimate of standard deviation for every average value was taken to be the statistical error. Average $\bar{\eta}$ is plotted with the estimate of standard deviation

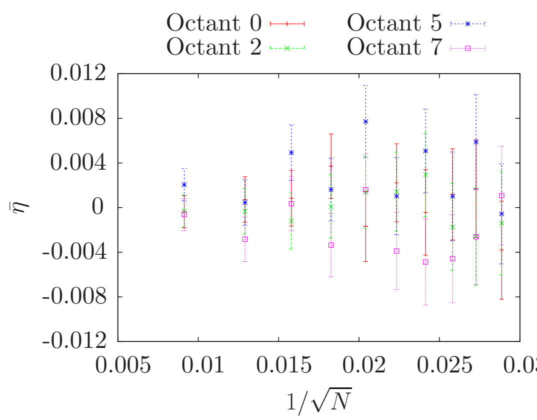

FIG. 9. (Color online) Dependence of $\bar{\eta}$ on $1/\sqrt{N}$, for impact parameter $b = 7$ fm. Octants 0, 2, 5, and 7.

for every octant over $1/\sqrt{N}$, where N is the number of events used to calculate the average value (Figs. 8 and 9).

Although the statistical error is high at low N, we can see that it decreases at higher N. As the number of events N increases, $\bar{\eta}$ in octants 1, 3, 4, and 6 do not completely vanish. Moreover $|\bar{\eta}|$ is higher than one standard deviation. This points to the possibility of nonzero values of $\bar{\eta}$ in noncentral collisions.

V. CONCLUSION

We have studied vorticity and hydrodynamic helicity in heavy-ion collisions in the HSD model for Au+Au reactions at small energy $\sqrt{s} = 5$ GeV and for different impact parameters.

We have calculated the velocity field of the final-state particles. Using this velocity field we calculated the averaged weighted voritcity and studied its time evolution. We noticed that the average weighted y component of vorticity decreases over time in noncentral heavy-ion collisions and disappears for the central collisions. The spacial distribution averaged over all x-z planes was also considered. The difference of the emerging picture with that in the hydrodynamic approach [8] is due to the viscosity effects.

Hydrodynamic helicity separation was observed in the HSD model. The results in this model are similar to those that were obtained in the QGSM model [10], with some differences in time dependence and magnitude. The most significant discrepancy in the time dependence can be explained by details of heavy-ion collision simulation. At the initial moment of time $t = 0$ there is a significant distance between the nuclei, so the reaction happens later. The difference in magnitude is not significant. Generally the integral values have similar time dependence but smaller magnitude in the HSD model. Being purely classical quantities, hydrodynamic helicity and vorticity can be a source for quantum anomalies. Nonzero hydrodynamic helicity in such reactions can result in Λ-hyperon polarization which can be observed.

We have also proposed a pseudoscalar quantity η for investigation of parity-odd effects in heavy-ion collisions based on previous suggestions [15,16]. The advantage of this approach is suitability for experimental observations without additional calculations. Using computer simulations in the HSD model we have obtained preliminary results for $\bar{\eta}(1/\sqrt{N})$ dependence, indicating that it could be used to probe for p-odd effects in noncentral collisions. Note the "handedness separation" to the different sides of reaction plane similar to hydrodynamic helicity separation discussed above.

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