Effective distributions of quasiparticles for thermal photons

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It has been found in recent heavy-ion experiments that the second and the third flow harmonics of direct photons are larger than most theoretical predictions. In this study, I construct effective parton phase-space distributions with in-medium interaction using quasiparticle models so that they are consistent with a lattice QCD equation of state. Then I investigate their effects on thermal photons using a hydrodynamic model. Numerical results indicate that elliptic flow and transverse momentum spectra are modified by the corrections to Fermi-Dirac and Bose-Einstein distributions.

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I. INTRODUCTION

The discovery of collective dynamics of the QCD media created in BNL Relativistic Heavy Ion Collider (RHIC) and CERN Large Hadron Collider (LHC) implies early equilibration of low-momentum gluons and light quarks and realization of a quark-gluon plasma (QGP) fluid [1,2]. Dynamical description of a QCD system in the vicinity of the quark-hadron crossover is generally a difficult issue. The fluidity of the system, however, is believed to allow an effective description of the bulk dynamics of heavy-ion collisions [3] where, aside from the initial conditions, the information of QCD comes into the formalism through the equation of state (EoS), the transport coefficients, and the chemical reaction rate. Particle spectra and flow harmonics [4,5] of hadrons are reasonably well explained in the framework of hydrodynamics with small viscosity. Direct photons, however, are found to be nontrivial, because elliptic flow v_2 and triangular flow v_3 of direct photons calculated by conventional theoretical models undershoot those measured in the experiments roughly by a factor of two [6–8]. Here direct photons are defined as the photons which does not originate from hadronic decay. Initial hard photons, named prompt photons, would have very small azimuthal anisotropy, but medium-induced soft photons, known as thermal photons, can inherit anisotropy from the medium. The apparent discrepancy poses theoretical challenges to the community [9–31].

Despite the experimental evidence that the system is likely to be strongly coupled, heavy-ion models often employ the phase-space distributions of ideal gases. In general the equilibrium phase-space distributions in an interacting system should be subject to nonideal corrections. They are known to play important roles in phenomenology in nonrelativistic systems; carbon dioxide, for example, has a solid phase owing to van der Waals force that arises from interaction corrections. It should be noted the deviations from the ideal gas picture constitute a different concept than viscous corrections because the system is in local equilibrium with no entropy production.

In heavy-ion collisions, hydrodynamic flow is converted into hadrons at freeze-out using the one-particle distribution so that energy-momentum tensor is conserved on the hypersurface using relativistic kinetic theory [32]. This is supported by the fact that the lattice QCD EoS agrees with that of hadron resonance gas well—but only approximately—below the crossover temperature. Also the second-order transport coefficients which appear in causal viscous hydrodynamic equations [33] are often analytically derived in kinetic theory and the energy density and pressure in the expressions are substituted by the hydrodynamic ones, which partially introduces the effects of interaction to the transport coefficients.

For photons, the parton distribution functions are used in thermal photon emission rates. Oftentimes calculations are performed simply assuming Fermi-Dirac and Bose-Einstein distributions for quarks and gluons, respectively, in the QGP phase, though lattice QCD results seem to deviate from the results implied from those assumptions even at relatively high temperatures [34]. Considering the recent status of the photon v_2 (and v_3) puzzle, the real gas effects should be worth investigating because the reduction of the effective degrees of freedom at high temperatures could suppress early-time photons with small anisotropy while hadronic photons with larger anisotropy would not be affected much owing to the aforementioned agreement of the kinetic theoretical estimations and the lattice QCD data. There have also been important studies to improve thermal photon models through the phase-space distribution. A few recent examples include viscous corrections to the distribution [35,36] and semi-QGP effects [30].

In this paper, I discuss the effects of the interaction in parton phase-space distributions on heavy-ion photons based on the quasiparticle parametrization. There are many variants of the quasiparticle models [37–43]. Here the model is phenomenologically constructed so that it reproduces thermodynamic variables of the lattice QCD EoS. Then I perform numerical estimations of thermal photon elliptic flow and particle spectra by consistently using the same EoS in a hydrodynamic model and show that modification of the emission rate of the QGP photons leads to visible corrections to the observables.

In Sec. II, the quasiparticle model is developed based on the lattice QCD results. The model for the estimations of thermal photons and background hydrodynamic flow is presented in Sec. III. Section IV is devoted to numerical results. Section V presents conclusions and discussion. The natural

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units $c = \hbar = k_B = 1$ and the Minkowski metric $g^{\mu\nu} = \mathrm{diag}(+, -, -, -)$ are used in the paper.

II. QUASIPARTICLE MODEL

I employ the quasiparticle picture to estimate the effects of in-medium interaction on thermal photon v_2, because the photon emission rate is usually given as a functional of distribution functions. Here the quasiparticle model is constructed so that it is compatible with lattice QCD results. The effects of interaction is represented by the (self-)interaction term $W^i_{\mathrm{eff}}(p,T)$, where p is the four-momentum and T is the temperature. The effective one-particle distribution is

$$f^i_{\mathrm{eff}} = \frac{1}{\exp(\omega_i/T) \pm 1}, \tag{1}$$

and the grand-canonical partition function Z_i in a logarithmic form is

$$\ln Z_i = \pm V \int \frac{g_i d^3p}{(2\pi)^3} \ln\left[1 \pm \exp\left(-\frac{\omega_i}{T}\right)\right] - \frac{V}{T}\Phi_i(T), \tag{2}$$

where V is the volume, g_i is the degeneracy, $\omega_i = \sqrt{p^2 + m_i^2} + W^i_{\mathrm{eff}}$ is the effective energy density, and i is the index for particle species, i.e., quarks and gluons. Here the numbers of quarks and antiquarks are assumed to be the same; i.e., the net baryon current is not considered. I consider the light quarks u, d, and s for the equilibrated quark components. The sign of quantum statistics is $+$ for fermions and $-$ for bosons. $\Phi_i(T)$ is the background field contribution, which is determined by the thermodynamic consistency condition [38]

$$\left.\frac{\partial \Phi_i}{\partial T}\right|_{\mu_B} = -\int \frac{g_i d^3p}{(2\pi)^3} \left.\frac{\partial \omega_i}{\partial T}\right|_{\mu_B} f^i_{\mathrm{eff}}, \tag{3}$$

where the baryon chemical potential $\mu_B = 0$ and the condition $P(T=0) = 0$. This corresponds to the bag constant in the bag model [44], but here it is temperature dependent. W^i_{eff} is assumed to include the information on interaction and self-energy correction to the Hamiltonian and can be regarded as an effective chemical potential as well as thermal correction to the particle mass. Energy density and hydrostatic pressure are written as

$$\begin{aligned} e &= -\frac{1}{V}\sum_i \left.\frac{\partial \ln Z_i}{\partial \beta}\right|_{\alpha_B} \\ &= \sum_i \int \frac{g_i d^3p}{(2\pi)^3}\left(\omega_i - T\left.\frac{\partial \omega^i}{\partial T}\right|_{\mu_B}\right) f^i_{\mathrm{eff}} + \Phi - T\left.\frac{\partial \Phi}{\partial T}\right|_{\mu_B} \\ &= \sum_i \int \frac{g_i d^3p}{(2\pi)^3}\omega_i f^i_{\mathrm{eff}} + \Phi, \end{aligned} \tag{4}$$

$$\begin{aligned} P &= \frac{1}{V}\sum_i T \ln Z_i \\ &= \pm T \sum_i \int \frac{g_i d^3p}{(2\pi)^3} \ln\left[1 \pm \exp\left(-\frac{\omega_i}{T}\right)\right] - \Phi \\ &= \frac{1}{3}\sum_i \int \frac{g_i d^3p}{(2\pi)^3} \mathbf{p}\frac{\partial \omega_i}{\partial \mathbf{p}} f^i_{\mathrm{eff}} - \Phi, \end{aligned} \tag{5}$$

where $\alpha_B = \mu_B/T = 0$, $\beta = 1/T$, and $\Phi = \sum_i \Phi_i$. The last line of Eq. (4) is obtained by using the consistency condition (3) and that of Eq. (5) by integration by parts. It should be noted that the trace anomaly $\Theta^\mu_\mu = e - 3P$ is no longer vanishing in the massless limit owing to the presence of the effective interaction. The entropy density is given through the thermodynamic relation $s = \partial P/\partial T = (e+P)/T$.

Determination of the effective interaction contribution W^i_{eff} is generally a nontrivial issue. Here it is constrained with the (2 + 1)-flavor lattice QCD EoS [45] assuming it depends only on the temperature. It is worth mentioning that the statement of temperature independence of W^i_{eff} in Ref. [38] does not apply to the model here because the parton numbers are not conserved in QCD systems. The condition (3) leads to

$$\left.\frac{\partial \Phi_i}{\partial T}\right|_{\mu_B} = -n_i \left.\frac{\partial \omega_i}{\partial T}\right|_{\mu_B}, \tag{6}$$

where n_i represents the quark and the gluon number densities and the equation cannot be simply integrated in terms of n_B following the prescription to derive the statement in Ref. [38]. For simplicity and the lack of additional constraints, I assume that W_{eff} is common for all partons in this paper to see its qualitative effects. It should be noted that a thermodynamically consistent formalism yields the energy density, the pressure, the entropy, and the trace anomaly simultaneously. One can use the entropy density to determine W_{eff} as the quantity is independent of Φ, and then determine the background contribution by the relation (3).

Figure 1 shows the temperature dependence of W_{eff} fit to the lattice data. It has a peak structure slightly below T_c because one has to mimic the crossover behavior which does not exist in the free parton gas picture. One can see that the effective fugacity would be non-negligible because it is comparable

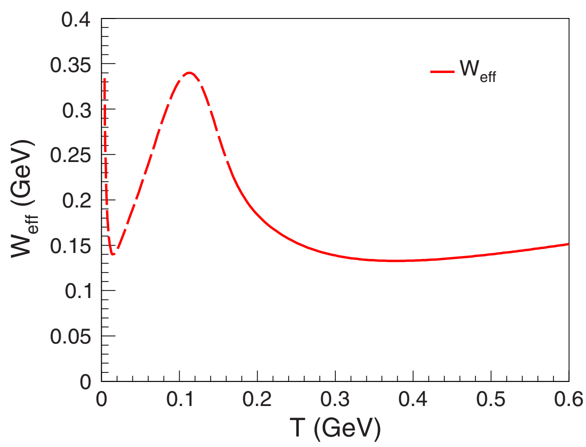

FIG. 1. (Color online) Lattice data fit of the effective interaction W_{eff} as a function of the temperature. The dashed line denotes the region below the crossover.

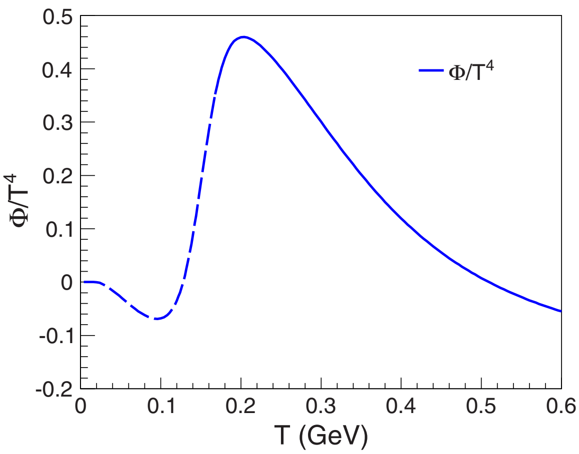

FIG. 2. (Color online) The background contribution in a dimensionless form Φ/T^4 as a function of the temperature. The dashed line denotes the region below the crossover.

to the temperature. The increasing behavior near $T = 0$ is coming from the fact that nearly massless parton gas requires large corrections to match the extrapolated lattice QCD results. However, it should be noted that the parton picture is at most valid above the crossover temperature. The corresponding background contribution Φ in a dimensionless form is shown in Fig. 2. Compared with the energy density and the pressure, the effective background contribution is small. Functional fits to W_{eff} and Φ are presented in the Appendix .

Although quasiparticle interpretations are commonly employed in hadron physics, I emphasize that the purpose of considering it is to improve the ideal gas distributions often assumed in thermal photon emission rates. The real QCD system is not necessarily a relativistic gas of quasiparticles especially near the crossover. One can also construct similar effective distributions for the hadronic phase, but it is not considered here because the difference between the EoS of hadronic resonance gas and that of lattice QCD is indicated to be relatively small as mentioned earlier.

III. THE MODEL

I develop the numerical model to estimate thermal photon elliptic flow and transverse momentum spectra with and without the effects of interaction corrections using the relativistic hydrodynamic model and the thermal photon emission rates in which the effective distributions obtained in Sec. II are embedded.

A. Thermal photons

The thermal photon emission rate is affected by the modification of the phase-space distribution. For the demonstrative purpose, the hard photon emissions in Compton scattering and pair annihilation are considered for the QGP phase. I employ the emission rate in Ref. [46] at the leading order in the fugacity expansion by substituting the parton distributions with the effective distributions. The emission rate reads

$$E\frac{dR}{d^3p} = \frac{5\alpha\alpha_s}{9\pi^2}T^2 e^{-E/T}\left\{\lambda_q\lambda_g\left[\ln\left(\frac{4ET}{k_c^2}\right)+\frac{1}{2}-\gamma\right] + \lambda_q^2\left[\ln\left(\frac{4ET}{k_c^2}\right)-1-\gamma\right]\right\}. \quad (7)$$

Here $\lambda_q = \lambda_g = e^{-W_{\text{eff}}/T}$ are the effective fugacities, γ is Euler's constant, and $k_c^2 = 2m_{\text{th}}^2 = g^2T^2/6$ is the infrared cutoff.

The hadronic photons are assumed to be unaffected. The emission rate in the hadronic phase is employed from Ref. [47]. It should be noted that the photon emission rate near the crossover can be, in principle, nontrivial [28]. Here they are simply interpolated with a hyperbolic function as

$$E\frac{dR}{d^3p} = c(T)E\frac{dR_{\text{lat}}}{d^3p} + [1-c(T)]E\frac{dR_{\text{had}}}{d^3p}, \quad (8)$$

where $c(T) = \{1 + \tanh[(T - T_c)/\Delta T]\}/2$, with the connecting temperature $T_c = 0.17$ GeV and the crossover width $\Delta T = 0.1T_c$. Here the value of ΔT is motivated by the observation regarding the pseudophase transition in the EoS ([48], Sec. 4.3).

B. Hydrodynamic flow

The flow and the temperature profiles are calculated using the (2 + 1)-dimensional ideal hydrodynamic model assuming boost invariance in the space-time rapidity direction [25]. The net baryon number is assumed to be vanishing. The hydrodynamic equation of motion is energy-momentum conservation $\partial_\mu T^{\mu\nu} = 0$, where

$$T^{\mu\nu} = (e + P)u^\mu u^\nu - Pg^{\mu\nu} \quad (9)$$

remains the same for ideal and effective distributions. u^μ is the flow. The medium property is determined by the EoS, which provides an additional equation for uniquely determining all the thermodynamic variables. Here it is employed from the aforementioned lattice QCD results [45], which, of course, is consistent with the quasiparticle description we use for the estimations of thermal photons. The initial conditions are generated with the Monte-Carlo Glauber model [49] and they are evolved through hydrodynamics after averaging over events. The nucleons are smeared with the Gaussian width parameter $\sigma^2 = 0.7$ fm for demonstration here. The numerical code is from Ref. [27]. More quantitative event-by-event analyses including the study on σ dependence will be performed in the future.

Thermal photon particle spectra can be calculated by integrating the photon emission rate over space-time,

$$\frac{dN^\gamma}{d\phi_p p_T dp_T dy} = \int dx^4 \frac{dR^\gamma}{d\phi_p p_T dp_T dy}, \quad (10)$$

and elliptic flow by taking its second-order Fourier harmonics in the azimuthal angle relative to the reference angle Ψ in momentum space as

$$v_2^\gamma(p_T,y) = \frac{\int_0^{2\pi} d\phi_p \cos(2\phi_p - \Psi)\frac{dN^\gamma}{d\phi_p p_T dp_T dy}}{\int_0^{2\pi} d\phi_p \frac{dN^\gamma}{d\phi_p p_T dp_T dy}}. \quad (11)$$

Here p_T is the transverse momentum, y is the rapidity, and ϕ_p is the angle in momentum space. For the smoothed initial conditions, Ψ is defined as

$$\Psi = \frac{1}{2}\arctan\frac{\langle r^2 \sin 2\phi\rangle}{\langle r^2 \cos 2\phi\rangle} + \frac{\pi}{2}, \tag{12}$$

where r and ϕ are the positions of participant nucleons [50]. The photon emission rate (7) is applied in the local rest frame of fluid.

IV. NUMERICAL RESULTS

For demonstrative purposes, Au-Au collisions at $\sqrt{s_{NN}} = 200$ GeV with the impact parameter $b = 6$ fm are considered. The energy density is normalized so that the maximum energy density is $e_{\max} = 50$ GeV in the most central collisions at the initial time of hydrodynamic evolution, which is chosen as $\tau_0 = 0.4$ fm/c. The contribution of the photon emission above the hadronic freeze-out temperature $T_f = 0.13$ GeV is taken into account. $\alpha = 1/137$ and $\alpha_s = 0.2$ are used for the photon emission rates in the present study.

A. Thermal photon v_2

The differential thermal photon elliptic flow with the ideal and the effective distributions at midrapidity are shown in Fig. 3 for the transverse momentum window $0.2 < p_T < 5$ GeV. One can see that the effective interaction corrections enhance $v_2^\gamma(p_T)$. This can be understood as follows. The emission of thermal photons are suppressed at early times owing to the fact that the effective degrees of freedom in the quasiparticle system are smaller than in the free-gas system when the system is in the QGP phase. The thermal photons emitted at late times have larger elliptic flow because azimuthal momentum anisotropy in the medium develops along with time evolution driven by the pressure gradients. Thus, when early-time contribution becomes effectively small, the overall elliptic flow can become large. It should be noted that, unlike hadrons, photons are emitted at each space-time point and they all contribute to the final spectra because their interaction with the bulk medium would be weak.

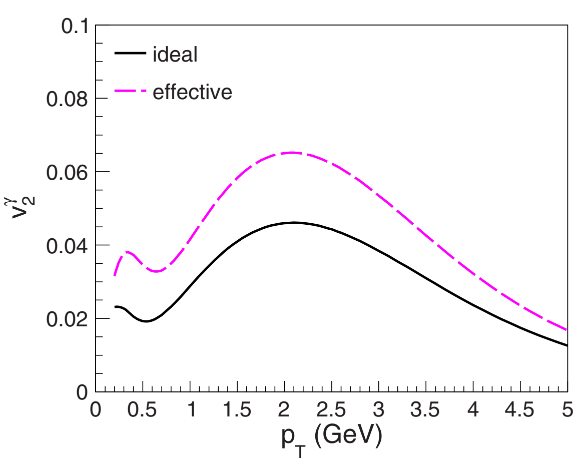

FIG. 3. (Color online) Thermal photon elliptic flow with effective distributions (dashed line) compared to that with ideal distributions (solid line).

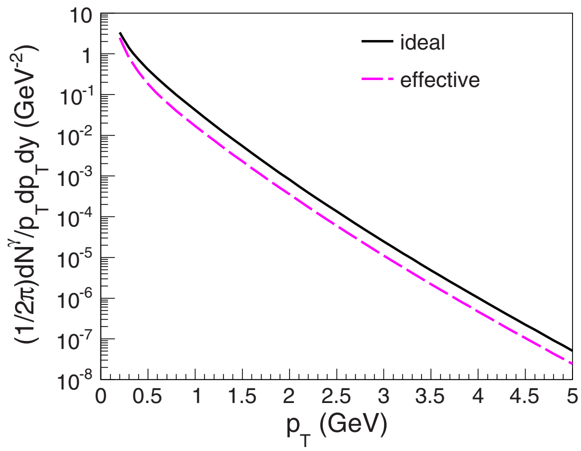

FIG. 4. (Color online) Thermal photon p_T spectrum with effective distributions (dashed line) compared to that with ideal distributions (solid line).

The effect of the nonideal gas corrections is consistent with the recently observed large photon $v_2^\gamma(p_T)$, though the numerical estimations suggest that the effect could be part of the cause for the excessive photon anisotropy but would not be large enough to be the sole reason with the current hydrodynamic parameter sets.

B. Thermal photon p_T spectra

Figure 4 shows the p_T spectra of thermal photons at midrapidity with ideal and effective distributions, respectively. The photon spectrum with the interaction effects is naturally suppressed by the suppression of initial QGP photon emission. This implies that an additional photon emission source might be required to explain the spectra when the interaction effects in the phase-space distributions are properly taken into account. Similar discussion can be found, for example, in the system with incomplete quark chemical equilibration [25]. The magnitude of the spectra suppression and v_2 enhancement effects is sensitive to the relative ratio of QGP photons to hadronic ones.

The proper time τ dependence of the photon emission from the medium is also investigated. Here it is defined as the emission rate integrated in the spatial and the transverse momentum directions,

$$I^\gamma(\tau,y) = \int \tau d\eta_s dx_T^2 \frac{dR^\gamma}{dy}(\tau,\eta_s,x_T,y), \tag{13}$$

where η_s is the space-time rapidity and x_T represents the transverse coordinates. The numerical results are shown in Fig. 5 for the systems with ideal and effective distributions, respectively, at $y = 0$. At the beginning, photon production is reduced along with time evolution because the temperature decreases rapidly owing to the longitudinal expansion. The in-medium corrections suppress the emission by a factor of $\sim e^{-2W_{\mathrm{eff}}/T}$, which becomes larger as it nears the pseudocritical temperature. However, hadronic photon contributions become

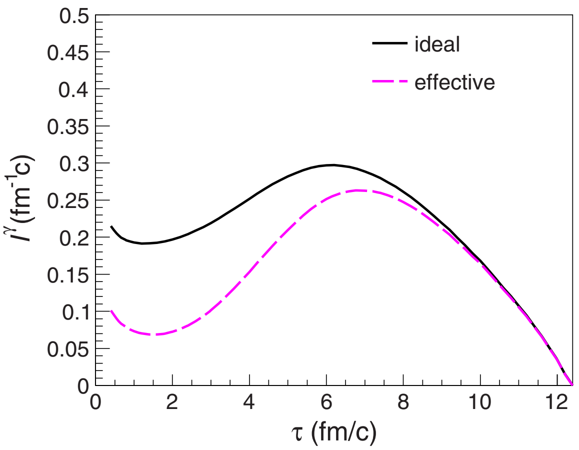

FIG. 5. (Color online) Spatially integrated photon emission as a function of proper time with effective distributions (dashed line) compared to that with ideal distributions.

larger as the system cools down and consequently the difference between the two systems disappears at later times.

V. DISCUSSION AND CONCLUSIONS

Effects of the interaction corrections in parton distributions on heavy-ion thermal photons are investigated. This allows one to preserve the consistency between the hydrodynamic evolution and the photon emission rate. The quasiparticle model is introduced and constrained so that it reproduces the thermodynamic variables from the lattice QCD estimations. Thermal photon emission is estimated in numerical simulations, and it is found that the suppression of the QGP photons in early stages leads to enhancement of thermal photon v_2. p_T spectrum of thermal photons, however, is suppressed by the corrections. The results imply that one has to take the modifications of phase-space distributions into account for the quantitative description of heavy-ion photon spectra and flow harmonics. Because the mechanism of early-time photon suppression can also lead to the enhancement of thermal photon v_3, it would be important to perform event-by-event analyses for calculating higher-order flow harmonics.

One should be careful before quantitative discussion that prompt photons, which will reduce the overall anisotropy in direct photons, are not included here and that only hard photons above k_c^2 are considered for the QGP photons. Also, though the corrections to ideal gas distributions would be a necessary ingredient in the heavy-ion modeling and the magnitude of v_2 enhancement is visible in the model calculations, it would not be enough to fully account for the large direct photon v_2 observed in the experiments, especially with prompt photon contributions. It will be interesting to include the effects of running coupling α_s for more quantitative analyses.

There would be several interesting applications of the quasiparticle model to heavy-ion phenomenology. First, the formalism can provide a model EoS for chemical nonequilibrated quark-gluon systems that is compatible with the lattice QCD EoS in the equilibrium limit, which is useful in studying the effects of quark chemical equilibration in hydrodynamic systems [25]. Second, it would be interesting to apply the effective distributions for the derivation of causal dissipative hydrodynamic equations because some formalism, such as Israel-Stewart theory [33], are explicitly dependent on the equilibrium distribution. The transport coefficients can then be fully consistent with hydrodynamic EoS. Third, this may also allow one to formulate an anisotropic hydrodynamic formalism which is compatible with the lattice EoS by using the effective distribution instead of the ideal gas distribution.

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APPENDIX: FUNCTIONAL FITS FOR THE EFFECTIVE DISTRIBUTIONS

In this appendix, the polynomial fits to the in-medium correction parameters in the parton phase-space distributions are presented. The contribution of gluons and up, down, and strange quarks are considered. The fit to the effective interaction energy W_{eff} for $0.15 < T < 0.6$ GeV is given as

$$W_{\text{eff}} = a_2 t^2 + a_1 t + a_0 + \frac{a_{-1}}{t} + \frac{a_{-2}}{t^2} + \frac{a_{-3}}{t^3} + \frac{a_{-4}}{t^4}, \quad \text{(A1)}$$

where $a_2 = -1.040$, $a_1 = 2.419$, $a_0 = -1.879$, $a_{-1} = 8.181\times10^{-1}$, $a_{-2} = -1.796\times10^{-1}$, $a_{-3} = 2.072\times10^{-2}$, and $a_{-4} = -9.111\times10^{-4}$. Here $t = T/1$ GeV is defined as the dimensionless temperature.

Likewise, the background contribution Φ is given by the functional fit

$$\Phi = b_3 t^3 + b_2 t^2 + b_1 t + b_0, \quad \text{(A2)}$$

where $b_3 = -3.303\times10^{-1}$, $b_2 = 2.424\times10^{-1}$, $b_1 = -4.152\times10^{-2}$, and $b_0 = 1.981\times10^{-3}$ for the aforementioned temperature range.

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