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Deduction of compound nucleus formation probability from the fragment angular distributions in heavy-ion reactions

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The presence of various fissionlike reactions in heavy-ion induced reactions is a major hurdle in the path to laboratory synthesis of heavy and super-heavy nuclei. It is known that the cross section of forming a heavy evaporation residue in fusion reactions depends on the three factors—the capture cross section, probability of compound nucleus formation P_{CN}, and the survival probability of the compound nucleus against fission. As the probability of compound nucleus formation, P_{CN} is difficult to theoretically estimate because of its complex dependence on several parameters; attempts have been made in the past to deduce it from the fission fragment anisotropy data. In the present work, the fragment anisotropy data for a number of heavy-ion reactions are analyzed and it is found that deduction of P_{CN} from the anisotropy data also requires the knowledge of the ratio of relaxation time of the K degree of freedom to pre-equilibrium fission time.

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I. INTRODUCTION

In recent years, considerable efforts have been devoted worldwide to the synthesis of super-heavy nuclei from heavy-ion fusion reactions. The cross section (σ_{ER}) for heavy element formation via fusion evaporation can be written as

$$\sigma_{ER} = \sigma_{cap} P_{CN} P_{surv}, \tag{1}$$

where σ_{cap} is the capture cross section, P_{CN} is the probability of compound nucleus formation, and P_{surv} is the probability that the compound nucleus survives equilibrium fission decay predominantly through neutron evaporation leading to evaporation residue (ER) of the fused heavy nucleus. While the first and third factors are simple to calculate, the second factor P_{CN} is difficult to estimate because of its complex dependence on various parameters. The capture of the projectile and target nuclei in the potential pocket of the entrance channel can lead to: (i) fusion, in which the di-nuclear configuration evolves towards the mononuclear shape and forms an equilibrated compound nucleus, or (ii) noncompound nucleus fission (NCNF) of the intermediate composite system. The fast fission [1,2] observed when the fission barrier height becomes vanishingly small for compound nuclei with large atomic number Z and at very high angular momenta and the quasifission [3] which occurs when the charge product of colliding nuclei exceeds a certain limit (∼1600), are well-known contributors to the NCNF. In addition, there is another mechanism which we call pre-equilibrium fission (PEF) [4–8] which can contribute to NCNF. According to PEF, even in cases where the contact configuration is more compact than the fission barrier shape, the system can fission before reaching K equilibration by diffusing over the fission barrier height as seen by the system relaxing in the K degree of freedom, where K is the projection of total angular momentum onto the fissioning axis. PEF is expected only for the systems with $\alpha < \alpha_{BG}$ where α is the mass asymmetry of the target-projectile system defined as $\frac{A_T - A_P}{A_T + A_P}$ and α_{BG} is the Businaro-Gallone critical mass asymmetry [5,9].

For light projectile induced fission, the angular distributions of the fission fragments are successfully explained by the transition state model (TSM) of Halpern and Strutinsky [10]. However, in several cases of heavy-ion induced reactions, the observed fission-fragments angular distribution were more anisotropic than predicted by the TSM. Considerable investigations have been carried out in this regard and it was pointed out that this deviation from standard theory arises from the presence of NCNF processes such as fast-fission, quasifission, and pre-equilibrium fission. The presence of these noncompound nucleus fission processes is a major hurdle in the synthesis of heavy and super-heavy nuclei by heavy-ion fusion reactions [11]. Many factors such as entrance channel mass asymmetry (α), charge product and deformation of colliding nuclei, and fissility of the fused system are known to affect the dynamics of systems on the potential energy surface and hence the outcome after capture. Thus it is important to determine and to understand the dependence of compound nucleus formation probability P_{CN} on the various entrance channel parameters.

II. P_{CN} DEDUCTION AND DISCUSSION

Recently, the extraction of P_{CN} from various fragment angular distribution data was reported [12,13] on the assumption that compound nucleus fission occurs for $J < J_{CN}$ and NCNF occurs when $J > J_{CN}$, where J is the angular momentum and J_{CN} is a parameter obtained by fitting the fragment angular distribution data. However, for lighter systems with much smaller $Z_1 Z_2$ (∼1300 and below) where quasifission and fast fission are expected to be absent, this assumption does not appear to be justified. Also, the value of P_{CN} was deduced in Ref. [12] with the assumption that $\frac{\Im_0}{\Im_{eff}} = 1.5$ for NCNF, where $\Im_0$ is the moment of inertia of a spherical nucleus and $\Im_{eff}$ is the effective moment of inertia of the transition state shape corresponding to NCNF. It is evident that the values of P_{CN} thus deduced depend on the assumed $\frac{\Im_0}{\Im_{eff}}$ for NCNF and thus another value of $\frac{\Im_0}{\Im_{eff}}$ would have yielded a different set of P_{CN} values.

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TABLE I. Various entrance channel parameters such as Z_1Z_2, mass asymmetry α, compound nucleus fissility (χ_{CN}), and effective fissility (χ_{eff}) for the various reactions analyzed in the present work. The deduced values of $P_{NCN}(t_K/t_f)$ are also presented in the table, along with P_{CN} for a specific case of σ_θ^2, and those deduced in Ref. [12].

Reaction	CN	$(E_{c.m.}/V_b)$	Z_1Z_2	α	χ_{CN}	χ_{eff}	$P_{NCN}(t_K/t_f)$	$P_{CN}(\sigma_\theta^2 = 0.06)$	P_{CN} [12]	Ref.
$^{12}C + ^{232}Th$	^{244}Cm	1.09	540	0.902	0.808	0.39	0.4362	0.962	–	[14]
$^{11}B + ^{204}Pb$	^{215}Fr	1.09	410	0.898	0.740	0.325	0	1	1	[15]
$^{16}O + ^{238}U$	^{254}Fm	1.083	736	0.874	0.842	0.4623	0.5152	0.9	–	[16]
$^{16}O + ^{232}Th$	^{248}Cf	1.07	720	0.871	0.826	0.458	0.7547	0.89	–	[17]
$^{16}O + ^{208}Pb$	^{224}Th	1.072	656	0.857	0.763	0.439	0	1	–	[17]
$^{18}O + ^{197}Au$	^{215}Fr	1.13	632	0.833	0.7398	0.413	0.014	0.99	1	[15]
$^{18}O + ^{197}Au$	^{215}Fr	1.24	632	0.833	0.7398	0.413	0.0077	0.987	0.66	[12]
$^{19}F + ^{208}Pb$	^{227}Pa	1.174	738	0.833	0.771	0.46	0	1	0.78	[17]
$^{24}Mg + ^{208}Pb$	^{232}Pu	1.124	984	0.793	0.7998	0.549	0.249	0.87	0.64	[17]
$^{26}Mg + ^{197}Au$	^{223}Pa	1.205	948	0.767	0.7766	0.524	0.1712	0.61	1	[12]
$^{28}Si + ^{208}Pb$	^{236}Cm	1.1	1148	0.763	0.8182	0.5967	0.5462	0.752	0.37	[17]
$^{30}Si + ^{197}Au$	^{227}Np	1.2	1106	0.736	0.795	0.572	0.3421	0.213	0.06	[12]
$^{32}S + ^{208}Pb$	^{240}Cf	1.082	1312	0.733	0.8366	0.641	0.8963	0.671	0.45	[17]
$^{32}S + ^{197}Au$	^{229}Am	1.13	1264	0.721	0.817	0.631	0.503	0.643	–	[17]
$^{32}S + ^{182}W$	^{214}Th	1.054	1184	0.701	0.779	0.613	0.7052	0.472	0.14	[18]
$^{36}S + ^{197}Au$	^{233}Am	1.192	1264	0.691	0.8104	0.604	0.9192	–	0.13	[12]

In the present work, we have carried out analysis of the fragment anisotropy data for various systems with Z_1Z_2 much smaller than 1600 where quasifission and fast fission are not expected to be present and anomalous anisotropy can arise from PEF. Table I gives the details of the reactions which are analyzed in the present work. Let P_{NCN} be the probability of noncompound nucleus fission (without K equilibration) and P_{CN} ($=1 - P_{NCN}$) be the probability that after the capture, the intermediate composite system fully equilibrates and forms a compound nucleus. The observed angular anisotropy of fission fragments in heavy-ion induced reactions can then be approximately written as

$$A_{exp} = P_{CN}A_{CN} + P_{NCN}A_{NCN}, \tag{2}$$

where A_{CN} is the anisotropy corresponding to the compound nucleus fission component P_{CN} and A_{NCN} is the ansiotropy from the nonequilibrium fission component P_{NCN}.

The anisotropy A_{CN} for the compound nucleus fission component can be calculated using standard statistical theory [10] and is approximately given by

$$A_{CN} = 1 + \frac{\langle J^2\rangle}{4K_0^2}, \tag{3}$$

where $K_0^2 = (T\Im_{eff}/\hbar^2)$ is the variance of the K distribution, $\Im_{eff}$ is the effective moment of inertia of the transition state saddle shapes determined by the moment of inertia for rotation around axes parallel and perpendicular to the nuclear symmetry axis, and T is the nuclear temperature at the saddle point given by

$$T = \sqrt{\frac{E_{c.m.} + Q - B_f - E_{rot} - E_{pre}}{a}}, \tag{4}$$

where Q is the "Q value" of the reaction, B_f is fission barrier, E_{rot} is the rotational energy of the fissioning nucleus, E_{pre} is the energy taken away by any prefission neutrons (ν_{pre}), and a is the level density parameter. In determining the saddle point temperature T, the quantity B_f, E_{rot}, and $\Im_{eff}$ were calculated from the Sierk model [19], ν_{pre} were taken from [20], and a was taken as $A/8.5$. The J values were calculated using the code CCFUS [21].

The anisotropy A_{NCN} for the pre-equilibrium fission component, where the K degree of freedom is not equilibrated can be calculated from the variance σ_K^2 of the K distribution ($\sigma_K^2 < K_0^2$) at the moment of pre-equilibrium fission and it is approximately given by

$$A_{NCN} = 1 + \frac{\langle J^2\rangle}{4\sigma_K^2}. \tag{5}$$

In an earlier work [8], the effective variance σ_K^2 of the K distribution for the nonequilibrated fission component was taken as the product of the initial K distribution $[\exp(-K^2/2\sigma_{K_I}^2)]$ and the saddle point K distribution $[\exp(-K^2/2K_0^2)]$, and is given as follows:

$$\sigma_K^2 = \sigma_{K_I}^2\left(\frac{K_0^2}{K_0^2 + \sigma_{K_I}^2}\right) = \sigma_\theta^2 J^2, \tag{6}$$

where $\sigma_{\theta_I}^2$ is the angular variance representing the misalignment of the symmetry axis of the fused system with respect to the $K = 0$ plane as defined by $\sigma_{K_I}^2 = J^2\sigma_{\theta_I}^2$ and σ_θ^2 is a parameter. For $\sigma_{K_I} << K_0$, $\sigma_K \sim \sigma_{K_I}$ while for large values of σ_{K_I} such as for a flat distribution when K is equilibrated, $\sigma_K \sim K_0$. Substituting the expressions for A_{CN} and A_{NCN} with σ_K^2 in Eq. (2) we get

$$A_{expt} = P_{CN}\left(1 + \frac{\langle J^2\rangle}{4K_0^2}\right) + P_{NCN}\left(1 + \frac{\langle J^2\rangle}{4\sigma_K^2}\right). \tag{7}$$

It may be noted that the initial variance σ_K^2 of the entrance channel K distribution starts from a delta function broadening gradually in time to K_0^2 with the time evolution of the di-nuclear system, and above σ_K^2 is the variance achieved at the instant of pre-equilibrium fission. Thus from Eqs. (6) and (7), the values of $P_{\rm NCN}$ can be deduced if the value of σ_θ^2 is known.

Time dependence of the variance σ_K^2 which can be written by the following expression as given in Ref. [6],

$$\sigma_K^2(t) = K_0^2[1 - \exp(-t/t_K)], \tag{8}$$

where t_K is characteristic K equilibration time. Based on this approach, we derive the weighted average value of σ_K^2 at the instant of fission as follows;

$$\sigma_K^2 = \frac{\int_0^\infty \sigma_K^2(t)\frac{dN}{dt}dt}{\int_0^\infty \frac{dN}{dt}dt}. \tag{9}$$

Here $\frac{dN}{dt}$ is the fission decay rate, where $N = N_0\exp(-t/t_f)$ and t_f is the pre-equilibrium fission time averaged over all values of J.

Solving Eq. (9), the weighted average value of σ_K^2 is thus obtained as

$$\sigma_K^2 = \frac{K_0^2}{\left[1 + \frac{t_K}{t_f}\right]}. \tag{10}$$

When the di-nuclear system is on its way to compound nucleus formation with equilibration in the K degree of freedom, the variance of the K distribution reaches K_0^2 starting from a delta function. The parameter σ_θ^2 of Eq. (6) is related to the ratio of the time of equilibration t_K of the K degree of freedom and the pre-equilibrium fission time t_f. Taking the angular width of the equilibrated K distribution as $\sigma_{\theta_m}^2$, it then follows that $\sigma_{\theta_m}^2 = K_0^2/J^2$ and $\sigma_\theta^2 = \sigma_{\theta_m}^2/(1 + t_K/t_f)$.

From Eq. (2), $P_{\rm NCN}$ can be written as

$$P_{\rm NCN} = \frac{(A_{\rm exp} - A_{\rm CN})}{(A_{\rm NCN} - A_{\rm CN})}. \tag{11}$$

Substituting the expressions for $A_{\rm NCN}$ and $A_{\rm CN}$ in the denominator of the above equation and after simplification, the above equation leads to

$$P_{\rm NCN} = \frac{(A_{\rm exp} - A_{\rm CN})}{(A_{\rm CN} - 1)\left(\frac{t_K}{t_f}\right)} \tag{12}$$

or

$$P_{\rm NCN}(t_K/t_f) = \frac{(A_{\rm exp} - A_{\rm CN})}{(A_{\rm CN} - 1)}. \tag{13}$$

From Eq. (13), it can be seen that the deviation from the fragment anisotropy corresponding to the compound nucleus fission (with K equilibration) depends on the probability of compound nucleus formation $P_{\rm CN}$, as well as the ratio (t_K/t_f). The value of $P_{\rm NCN}$ (or $P_{\rm CN}$) can be obtained by an analysis of the fragment anisotropy data, only if the absolute value of (t_K/t_f) is known. Thus extraction of $P_{\rm CN}$ from the fragment anisotropy data requires knowledge of the ratio of K relaxation time to fission time (t_K/t_f). The average fission time (t_f) deduced using the full fission barrier height may not be valid as the PEF takes place from an intermediate potential energy surface and not from the ground state.

Figure 1 shows the deduced values of $P_{\rm NCN}(t_K/t_f)$ as a function of the effective fissility $\chi_{\rm eff}$ for the various reactions. While most of the data points in Fig. 1 fall on a smooth curve,

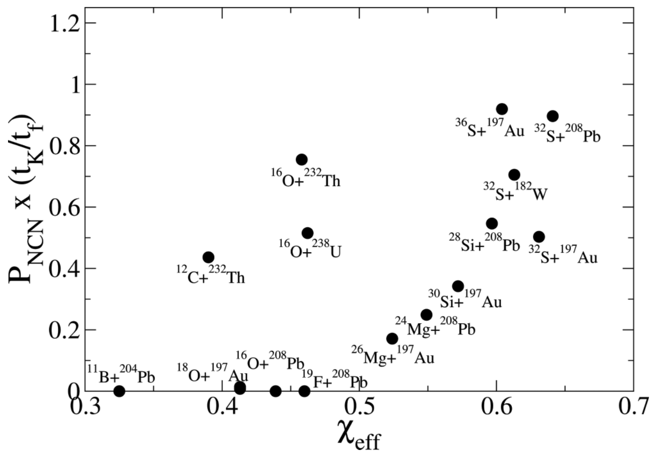

FIG. 1. Plot showing $P_{\rm NCN}(t_K/t_f)$ versus $\chi_{\rm eff}$ for the various reactions.

the three data points corresponding to ^{12}C + ^{232}Th, ^{16}O + ^{238}U, and ^{16}O + ^{232}Th reactions show a much larger value of $P_{\rm NCN}(t_K/t_f)$. This may be because of the fact that these systems have highly fissile statically deformed targets and some contribution from fast fission from the larger compound nucleus fissility, $\chi_{\rm CN}$. In Fig. 2, the values of $\chi_{\rm eff}$ are plotted as a function of the entrance channel mass asymmetry α for various reactions, showing that the above two parameters are correlated. Thus, while Fig. 1 shows smooth dependence of $P_{\rm NCN}(t_K/t_f)$ on $\chi_{\rm eff}$, it is not clear if this arises from intrinsic dependence on $\chi_{\rm eff}$ or on α. This result may be an indication that $P_{\rm NCN}(t_K/t_f)$ has larger values for larger $\chi_{\rm eff}$ because of the fact that these corresponds to smaller α and for small α the compound nucleus formation probability decreases as the difference between entrance channel mass asymmetry and Businaro-Gallone critical mass asymmetry increases. This is because the nature of collective dynamics leading to fusion exhibits abrupt changes across the critical Businaro-Gallone mass asymmetry point. Figure 3 shows $P_{\rm NCN}(t_K/t_f)$ as a function of mass asymmetry (α) for various

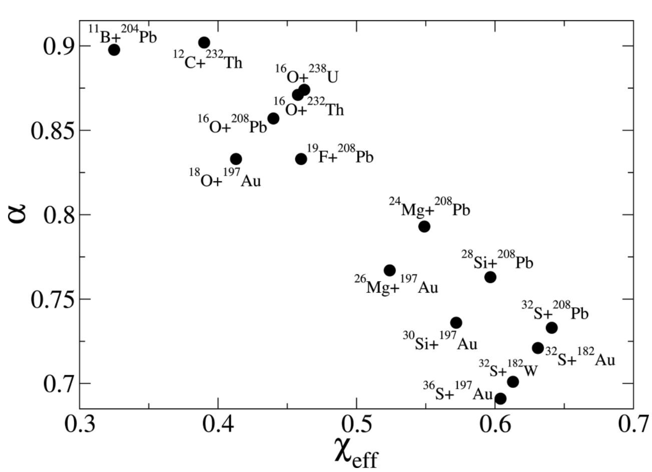

FIG. 2. Plot showing $\chi_{\rm eff}$ versus α for various reactions as tabulated in Table I.

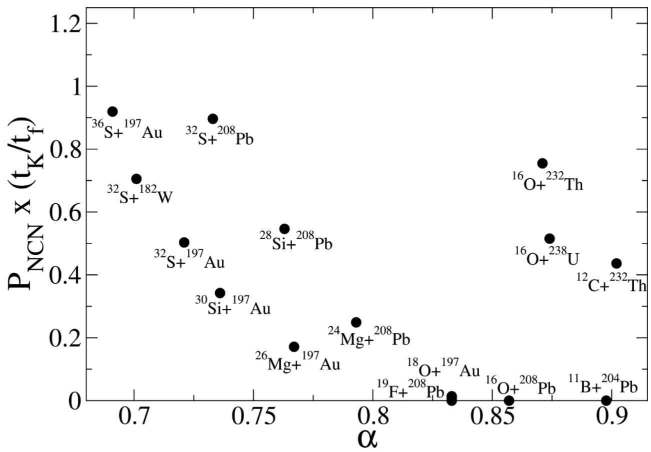

FIG. 3. Plot showing $P_{\rm NCN}(t_K/t_f)$ versus α for the various reactions.

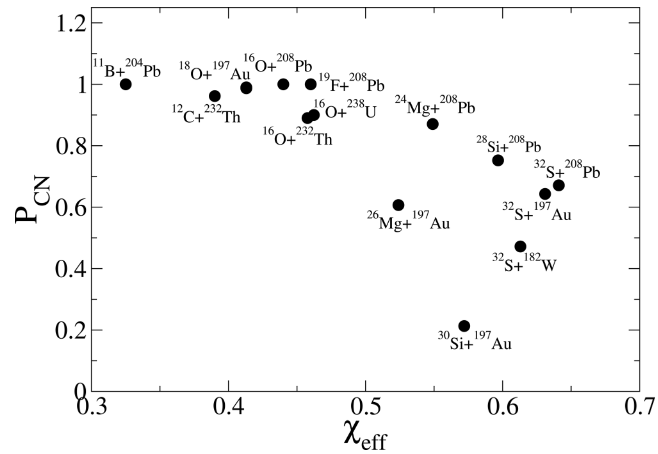

FIG. 4. $P_{\rm CN}$ deduced for the value of $\sigma_\theta^2 = 0.06$ from Eq. (7) versus $\chi_{\rm eff}$ for the various reactions analyzed in the present work.

reactions analyzed in the present work. In an earlier work [4], fission-fragment angular distribution data were fitted taking into account the PEF component for values of angular variance $\sigma_\theta^2 = 0.06$ corresponding to a K-relaxation time 8×10^{-21} s and pre-equilibrium fission time for the reduced fission barrier by a factor of 0.5. For the value of $\sigma_\theta^2 = 0.06$, the values of $P_{\rm CN}$ deduced from Eq. (7) for the various reactions analyzed in this work are plotted with respect to the effective fissility parameter $\chi_{\rm eff}$ in Fig. 4. The results of Fig. 4 do indicate that $P_{\rm CN}$ decreases for larger values of $\chi_{\rm eff}$ or smaller values of α, but as pointed out earlier, an unambiguous deduction of $P_{\rm CN}$ would also require an independent knowledge of t_K/t_f or σ_θ^2 on the basis of Fig. 1.

III. SUMMARY AND CONCLUSION

The probability of compound nucleus formation $P_{\rm CN}$ is a very important parameter required to correctly estimate the heavy element formation cross section. In the present work, we discussed our approach to deduce this parameter for various systems where quasifission and fast fission are expected to be absent and only pre-equilibrium fission can contribute to the observed noncompound nucleus fission. While pre-equilibrium fission can occur for all values of J, the anisotropy from the pre-equilibrium fission component depends both on $P_{\rm CN}$ as well as the ratio of K-equilibration time to the pre-equilibrium fission time. Thus, the analysis of $P_{\rm CN}$ for various fragment anisotropy data using pre-equilibrium fission formalism shows that unambiguous extraction of $P_{\rm CN}$ also requires the knowledge of the ratio of K-relaxation time to pre-equilibrium fission time.

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