Reassessing nuclear matter incompressibility and its density dependence

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Experimental giant monopole resonance energies are now known to constrain nuclear incompressibility of symmetric nuclear matter K and its density slope M at a particular value of subsaturation density, the crossing density ρ_c. Consistent with these constraints, we propose a reasonable way to construct a plausible equation of state of symmetric nuclear matter in a broad density region around the saturation density ρ_0. Help of two additional empirical inputs, the value of ρ_0 and that of the energy per nucleon $e(\rho_0)$ are needed. The value of $K(\rho_0)$ comes out to be 211.9 ± 24.5 MeV.

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I. INTRODUCTION

The nuclear incompressibility parameter K_0 defined for symmetric nuclear matter (SNM) at saturation density ρ_0 stands out as an irreducible element of physical reality. It has an umbilical association with the isoscalar giant monopole resonances (ISGMR) in microscopic nuclei; it also underlies in a proper understanding of supernova explosion in the cosmic domain [1]. From careful microscopic analysis of ISGMR energies with suitably constructed energy density functional (EDF) $\mathcal{E}(\rho)$ in a nonrelativistic framework as applicable to finite and infinite nuclear systems, its value had initially been fixed at $K_0 \simeq 210 \pm 30$ MeV [2,3]. In microscopic relativistic approaches on the other hand, a higher value of $K_0 \sim 260$ MeV was obtained [4]. After several revisions from different corners, however, its value settled to $K_0 \simeq 230 \pm 20$ MeV [5–7]. It gives good agreement with the experimentally determined centroids of ISGMR, in particular, for ^{208}Pb, ^{90}Zr, and ^{144}Sm nuclei, calculated both with nonrelativistic [8,9] and relativistic [7] energy density functionals. The near-settled problem was, however, left open with the apparent incompatibility of the said value of K_0 with the recent ISGMR data for Sn and Cd-isotopes [6,10–17]. These nuclei showed remarkable softness toward compression, the ISGMR data appeared explained best with $K_0 \sim 200$ MeV [6].

A plausible explanation was recently put forward by Khan *et al.* [18] for the apparent discrepancy. It is argued that there may not be a unique relation between the value of K_0 associated with an effective force and the monopole energy of a nucleus predicted by the force [19]. The region between the center and the surface of the nucleus is the most sensitive toward displaying the compression as manifested in the ISGMR. The ISGMR centroid E_G is related to the integral of incompressibility $[\int K(\rho)d\rho]$ over the whole density range [20]. As a result, a larger value of $K(\rho_0)$ for a given EDF can be compensated by lower values of $K(\rho)$ at subsaturation densities so as to predict a similar value of ISGMR energy in nuclei. It is seen that the incompressibility $K(\rho)$ calculated with a multitude of energy density functionals when plotted against density cross close to a single density point [18], this universality possibly arising from the constraints encoded in the EDF from empirical nuclear observables. This crossing density $\rho_c[= (0.71 \pm 0.005)\rho_0]$ [21] seems more relevant as an indicator for the ISGMR centroid. Because of the incompressibility integral, the centroid seems more intimately correlated to the derivative of the compression modulus [defined as $M = 3\rho K'(\rho)$] at the crossing density rather than to K_0. The value of $K_c[= K(\rho_c)]$ is seen to be $\sim 35 \pm 4$ MeV [21]. From various functionals, the calculated values of $M_c[= M(\rho_c)]$ are found to be linearly correlated with the correspondingly calculated values of ISGMR centroids for ^{208}Pb and also for ^{120}Sn. From the known experimental ISGMR data for these nuclei, a value of $M_c \simeq 1050 \pm 100$ MeV [21] is then obtained, revised from an earlier estimate of 1100 ± 70 MeV [18]. Using a further assumption of a linear correlation between K_0 and E_G calculated from different EDF, a value for $K_0 \simeq 230$ MeV with an uncertainty of $\simeq 40$ MeV is reported, the uncertainty being inferred from the spread of K_0 values obtained with the different functionals used.

The universality of the crossing point ρ_c and the values of K_c and M_c can be readily acknowledged; M_c is seen to be well correlated to E_G. The Pearson correlation coefficient r [22] of M_c with E_G for ^{120}Sn is 0.80 and is 0.94 for ^{208}Pb. However, assumption of a linear correlation between K_0 and E_G may not be justified, they seem to be very weakly correlated ($r = 0.67$ for ^{120}Sn and 0.79 for ^{208}Pb) [21]. The inferred value of incompressibility around saturation may then be called into question. One can see that a linear Taylor expansion $K_0(\rho_0) = K(\rho_c) + (\rho_0 - \rho_c)K'(\rho_c)$ yields for $K_0 \simeq 185 \pm 14.3$ MeV, noting that $K'(\rho_c) = M_c/(3\rho_c)$.

The absence of a strong linear correlation between K_0 and E_G calculated from different effective forces prompts one to think that K_c and M_c alone are not sufficient to yield the correct value of K_0. Further empirical information is possibly needed to arrive at that. In this paper, we show that with given values of only K_c and M_c along with some time-tested values of empirical nuclear constants, it is possible to address to a proper assessment of the value of incompressibility K and its density dependence. The empirical constants are the saturation density ρ_0, taken as 0.155 ± 0.008 fm^{-3} for SNM and the energy per nucleon at that density $e(\rho_0)$, taken as -16.0 ± 0.1 MeV [23,24]. An acceptable value of the effective nucleon mass m^*/m, which lies in the range $m^*/m \sim 0.8 \pm 0.2$ [25] at saturation density is also used.

This paper is structured as follows. In Sec. II, we introduce the theoretical elements to calculate the nuclear equation of state from K_c and ρ_c with the aid of empirical inputs mentioned. Results and discussions are presented in Sec. III. Section IV contains the concluding remarks.

II. THEORETICAL EDIFICE

We keep the discussions pertinent for SNM at any density ρ at zero temperature ($T = 0$). The chemical potential of a nucleon is given by the single-particle energy at the Fermi surface,

$$\mu = \varepsilon_F = \frac{p_F^2}{2m} + U, \tag{1}$$

where $p_F(\rho)$ is the Fermi momentum and $U(\rho)$ the single-particle potential. Assuming the nucleonic interaction to be momentum and density dependent, the single-particle potential separates into three parts [26]:

$$U = V_0 + p_F^2 V_1 + V_2. \tag{2}$$

The last term V_2 is the rearrangement potential that arises only for density-dependent interactions, and the second is the momentum-dependent term that defines the effective mass m^*,

$$\frac{p_F^2}{2m^*} = \frac{p_F^2}{2m} + p_F^2 V_1, \tag{3}$$

so that

$$\frac{1}{m^*} = \frac{1}{m} + 2V_1. \tag{4}$$

The energy per nucleon at density ρ is given by

$$e = \left\langle \frac{p^2}{2m} \right\rangle + \frac{1}{2}\langle p^2\rangle V_1 + \frac{1}{2}V_0$$
$$= \frac{1}{2}\left(1 + \frac{m^*}{m}\right)\left\langle \frac{p^2}{2m^*} \right\rangle + \frac{1}{2}V_0. \tag{5}$$

From Gibbs-Duhem relation,

$$\mu = e + \frac{P}{\rho}, \tag{6}$$

where P is the pressure. Keeping this in mind, from Eqs. (1), (5), and (6), we get

$$e(\rho) = \frac{p_F^2}{10m}\left[3 - \frac{2m}{m^*}\right] - V_2 + \frac{P}{\rho}, \tag{7}$$

where we have put $\langle p^2 \rangle = \frac{3}{5}p_F^2$.

The density dependence of the effective mass [27] can be cast as $\frac{m}{m^*} = 1 + k\rho$; the rearrangement potential can be written in the form $V_2 = a\rho^\alpha$. This is the form that emerges for finite range density-dependent forces [26] in a nonrelativistic framework or for Skyrme interactions. The quantities a, α, and k are numbers. If $\frac{m^*}{m}(\rho_0)$ is chosen, k is known.

At $\rho = \rho_0, P = 0$, then from Eq. (7), writing for $\frac{p_F^2}{2m} = b\rho^{2/3}$ with $b = \frac{(\frac{3}{2}\pi^2)^{2/3}\hbar^2}{2m}$,

$$e_0 = e(\rho_0) = \frac{b}{5}\rho_0^{2/3}[1 - 2k\rho_0] - a\rho_0^\alpha. \tag{8}$$

Since $P = \rho^2\frac{\partial e}{\partial \rho}$, from Eq. (7) again we get

$$P = \frac{b}{15}\rho^{5/3} - \frac{1}{3}bk\rho^{8/3} - \frac{1}{2}\alpha a\rho^{\alpha+1} + \frac{1}{2}\rho\frac{\partial P}{\partial \rho}. \tag{9}$$

At ρ_0, this yields (since $K_0 = 9\frac{\partial P}{\partial \rho}|_{\rho_0}$)

$$\frac{1}{2}\alpha a\rho_0^\alpha + \frac{1}{3}bk\rho_0^{5/3} - \left(\frac{K_0}{18} + \frac{b}{15}\rho_0^{2/3}\right) = 0. \tag{10}$$

Furthermore, Eq. (9) gives

$$K(\rho) = 9\frac{\partial P}{\partial \rho} = 2b\rho^{2/3} - 16bk\rho^{5/3}$$
$$- 9\alpha(\alpha + 1)a\rho^\alpha + 9\rho\frac{\partial^2 P}{\partial \rho^2}. \tag{11}$$

Defining $M = 3\rho\frac{dK}{d\rho} = 27\rho\frac{\partial^2 P}{\partial \rho^2}$, this leads, at $\rho = \rho_c$, to

$$9\alpha(\alpha + 1)a\rho_c^\alpha + 16bk\rho_c^{5/3} - \left(2b\rho_c^{2/3} + \frac{M_c}{3} - K_c\right) = 0. \tag{12}$$

Since k is a given entity and ρ_c and $(M_c/3 - K_c)$ are known, Eqs. (8) and (12) can be solved for a and α, Eq. (10) then gives the value of the nuclear incompressibility K_0. Once K_0 is obtained, $M_0[=M(\rho_0)]$ is evaluated from Eq. (12) by choosing ρ_0 for ρ_c. Then $Q_0 = 27\rho_0^3\frac{\partial^3 e}{\partial \rho^3}|_{\rho_0}$ is also known from $M_0 = 12K_0 + Q_0$.

The structure of Eq. (9) shows that the pressure and its first derivative are interrelated. One can then get higher density derivatives of P or of energy e recursively from Eq. (9) as is evident from Eq. (11). For the present, we show that

$$9\rho\frac{\partial^3 P}{\partial \rho^3} = 9\alpha^2(\alpha + 1)a\rho^{\alpha-1} + \frac{80}{3}bk\rho^{2/3} - \frac{4}{3}b\rho^{-1/3}. \tag{13}$$

Since

$$\frac{\partial^3 P}{\partial \rho^3} = 6\frac{\partial^2 e}{\partial \rho^2} + 6\rho\frac{\partial^3 e}{\partial \rho^3} + \rho^2\frac{\partial^4 e}{\partial \rho^4}, \tag{14}$$

we find

$$9\rho_0^2\frac{\partial^3 P}{\partial \rho^3}\bigg|_{\rho_0} = 6K_0 + 2Q_0 + \frac{1}{9}N_0. \tag{15}$$

where we have defined $N_0 = 81\rho_0^4\frac{\partial^4 e}{\partial \rho^4}|_{\rho_0}$. From Eqs. (13) and (15), knowing K_0 and Q_0, N_0 can be calculated. Similarly, one can calculate the fifth density derivative of energy ($R_0 = 243\rho_0^5\frac{\partial^5 e}{\partial \rho^5}|_{\rho_0}$) by exploiting Eqs. (13) and (14) from

$$9\rho_0^3\frac{\partial^4 P}{\partial \rho^4}\bigg|_{\rho_0} = 4Q_0 + \frac{8}{9}N_0 + \frac{1}{27}R_0. \tag{16}$$

These help to find the density variation of the energy and also of the incompressibility, as is seen,

$$e(\rho) = e(\rho_0) + \tfrac{1}{2}K_0\epsilon^2 + \tfrac{1}{6}Q_0\epsilon^3 + \tfrac{1}{24}N_0\epsilon^4 + \tfrac{1}{120}R_0\epsilon^5 + \cdots, \tag{17}$$

where $\epsilon = (\frac{\rho-\rho_0}{3\rho_0})$ (counting terms only up to ϵ^5 is seen to be a very good approximation in the density range of $\sim \rho_0/4 < \rho < 2.0\rho_0$, we retain terms up to them). Equations (7) and (17) give

$$\frac{P(\rho)}{\rho} = e(\rho_0) + \frac{1}{2}K_0\epsilon^2 + \frac{1}{6}Q_0\epsilon^3 + \frac{1}{24}N_0\epsilon^4 + \frac{1}{120}R_0\epsilon^5 - \frac{b}{5}\rho^{2/3}[1-2k\rho] + a\rho^\alpha, \quad (18)$$

and Eq. (9) gives

$$K(\rho) = 9\frac{dP}{d\rho} = 18\left[\frac{P}{\rho} - \frac{b}{15}\rho^{2/3} + \frac{1}{3}bk\rho^{5/3} + \frac{1}{2}\alpha a\rho^\alpha\right]. \quad (19)$$

We have thus the equation of state (EOS) of symmetric nuclear matter in a reasonably spread-out density domain around the saturation density.

The incompressibility K at any density ρ can be calculated directly from Eq. (19) or it may be calculated in terms of $K(\rho_c)$ and its higher density derivatives as

$$K(\rho) = K(\rho_c) + (\rho-\rho_c)K'(\rho_c) + \frac{(\rho-\rho_c)^2}{2}K''(\rho_c) + \frac{(\rho-\rho_c)^3}{6}K'''(\rho_c) + \cdots. \quad (20)$$

The different derivatives can be calculated from Eq. (19). With given values of $\rho_0, e_0, \frac{m^*}{m}(\rho_0)$, and ρ_c, one notes that the solutions for a and α do not depend separately on K_c and M_c but on $(M_c/3 - K_c)$.

III. RESULTS AND DISCUSSIONS

The values of the empirical constants ρ_0, e_0, and $\frac{m^*}{m}$ needed for our calculation have already been mentioned. As for the crossing density, we choose $\rho_c = 0.110 \pm 0.0008$ fm^{-3}. With given inputs of M_c and K_c, it should be noted that the output values for M_c and K_c may come out to be different, but $(M_c/3 - K_c)$ remains invariant. With inputs $M_c = 1050$ MeV and $K_c = 35$ MeV, the output M_c and K_c are found to be 1051.8 and 35.46 MeV, respectively. Since they are very close to the input values, they were not tinkered with for exact matching of the output and input values. The value of incompressibility at ρ_0 turns out to be $K_0 = 211.9 \pm 24.5$ MeV either from Eq. (19) or (20). We note that in Eq. (20), at saturation, the value of the second term on the right-hand side is 143.3 MeV, the third term is 35.9 MeV, the fourth term is -3.2 MeV, the fifth term [not shown in Eq. (20)] is 0.55 MeV and so on, which adds up to $\sim$211.9 MeV.

The uncertainty in an observable X (like K, M, etc.) is calculated from $\Delta X^2 = \sum_i(\frac{\partial X}{\partial y_i}\Delta y_i)^2$, where Δy_i are the uncertainties in the empirically known entities y_i. The sensitivity of K_0 on these entities that influence the incompressibility most is displayed in Fig. 1. The abscissa is scaled such that 0 refers to the central value of these entities, M_c, K_c, ρ_c, and ρ_0; ± 1 refer to the extrema of their domain (± 100 MeV, ± 5 MeV, $\pm 0.005\rho_0$, and ± 0.008 fm^{-3} from the central values of the entities, respectively). The value of K_0 is seen to be very sensitive with changes in either M_c or ρ_0 when all other input entities are kept fixed. Its sensitivity to K_c or ρ_c is weak; on $\frac{m^*}{m}$ or to the energy per nucleon e_0, it is rather insensitive. The near-insensitivity of incompressibility to the effective mass is observed for Skyrme density functionals also. From the data base for these functionals as tabulated by Dutra *et al.* [25], the correlation coefficient between K_0 and m^* is calculated to be only ~ -0.2.

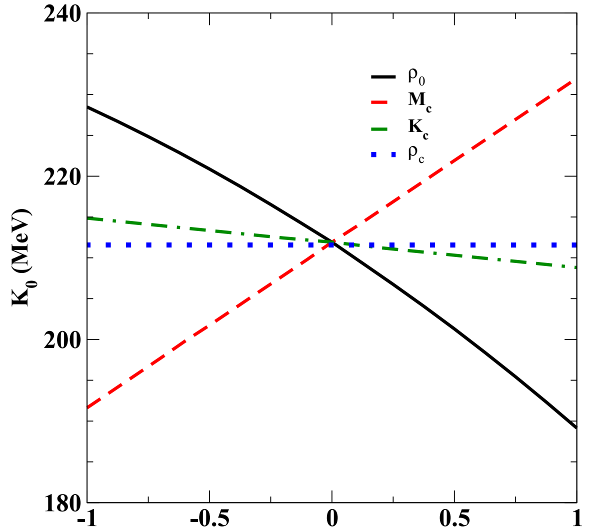

FIG. 1. (Color online) The sensitivity of the incompressibility K_0 at the saturation density (ρ_0) on the values of the incompressibility K_c (green dash-dotted line), its density slope M_c (red dashed line), the crossing density ρ_c (blue dotted line), and the value of ρ_0 (black full line). The abscissa extends from -1 to $+1$. These endpoints refer to the scaled lower and upper limits of K_c, M_c, ρ_c, and ρ_0, respectively (see text).

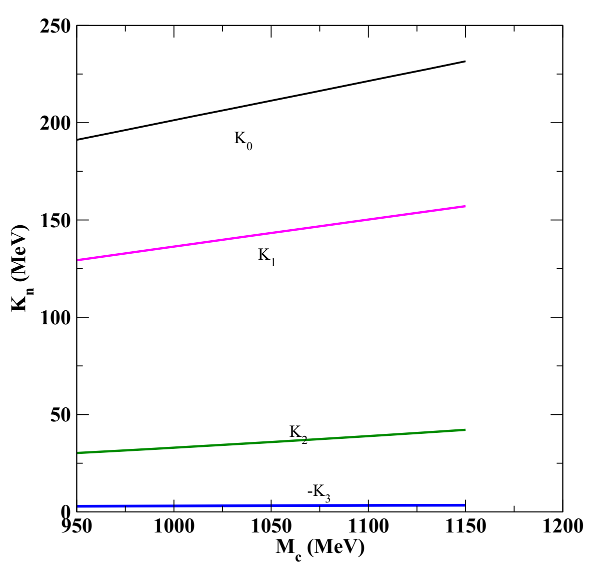

FIG. 2. (Color online) The incompressibility and its different density derivative as defined in the text plotted as a function of M_c.

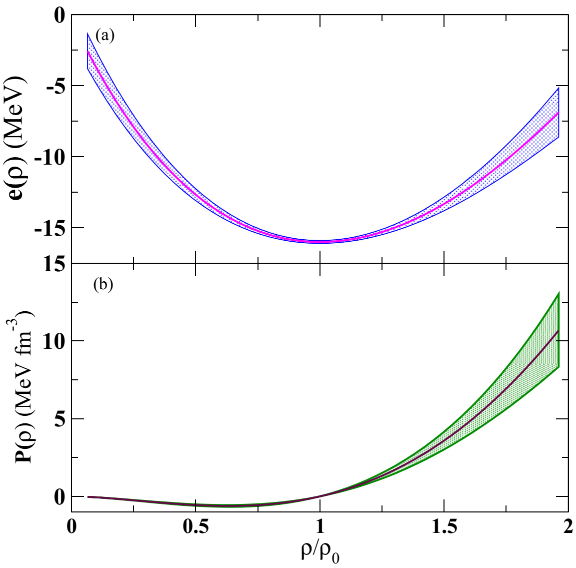

FIG. 3. (Color online) The nuclear EOS as a function of density. Panels (a) and (b) show the energy per nucleon and pressure, respectively, in a selected range around the saturation density.

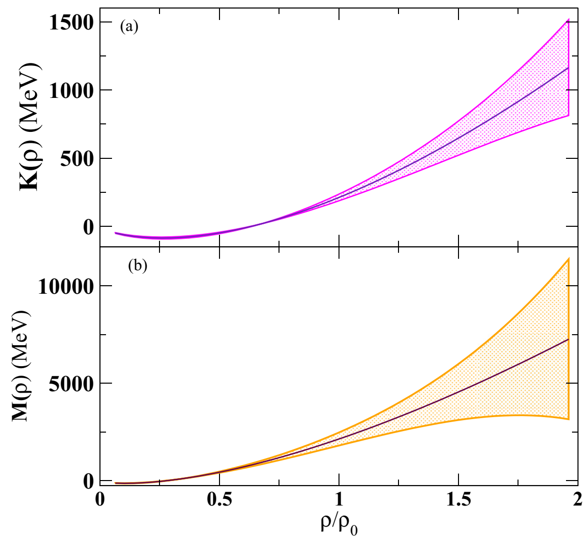

FIG. 4. (Color online) The nuclear EOS as a function of density. Panels (a) and (b) show the incompressibility and its density derivative M, respectively, in a selected range around the saturation density.

The near-perfect linear correlation of K_0 with M_c as seen in Fig. 1 is very startling. From Eq. (20), one may expect that the second- and higher-order derivatives of $K(\rho_c)$ would destroy this correlation. However, we find that both K'' and K''' are also linearly correlated with M_c and thus $K(\rho_0)$ retains its linear correlation with M_c. This is displayed in Fig. 2, where we define $K_1 = (\rho_0 - \rho_c)K'(\rho_c)$, $K_2 = \frac{(\rho_0-\rho_c)^2}{2}K''(\rho_c)$, and $K_3 = \frac{(\rho_0-\rho_c)^3}{6}K'''(\rho_c)$. The weak correlation between K_0 and M_c that can be inferred from the calculated correlation structure of $(M_c - E_G)$ and $(K_0 - E_G)$ in Refs. [18,21] possibly results from the use of different EDFs in getting the various relevant observables.

Figures 3 and 4 display the functional dependence of the nuclear EOS on density. Figures 3(a) and 3(b) show the energy per nucleon and the pressure, respectively; Figs. 4(a) and 4(b) show the incompressibility and its density derivative M, respectively. As one sees, the uncertainty in energy and pressure grows as one moves away from the saturation density; similarly, the uncertainty in incompressibility or its density derivative increases with distance from the crossing density.

IV. CONCLUSIONS

To sum up, we have made a modest attempt to reassess the value of $K(\rho_0)$ consistent with the new-found constraint on the incompressibility $K(\rho_c)$ and its density slope $M(\rho_c)$ at a particular value of density at subsaturation, the crossing density ρ_c. We have relied on some empirically well-known values of nuclear constants. We have further made the assumption of linear-density dependence of the effective mass and the power-law dependence of the rearrangement potential, which happens to be generally true for nonrelativistic momentum and density-dependent interactions. In relativistic models, the density dependence of the effective mass may not be linear [28]. The rearrangement potential appears explicitly there only in the case of density-dependent meson exchange models [29].

The value of incompressibility $K(\rho_0)$ turns out to be 211.9 ± 24.5 MeV. This is somewhat lower than the current value in vogue, $K_0 \sim 230 \pm 20$ MeV. From recursive relations, our method allows also estimates of higher-density derivatives of energy or of pressure and thus helps in constructing the nuclear EoS $e(\rho)$ at and around the saturation density.

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[1] G. E. Brown, Phys. Rep. **163**, 167 (1988).
[2] J. P. Blaizot, Phys. Rep. **64**, 171 (1980).
[3] M. Farine, J. M. Pearson, and F. Tondeur, Nucl. Phys. A **615**, 135 (1997).
[4] D. Vretenar, T. Niksic, and P. Ring, Phys. Rev. C **68**, 024310 (2003).
[5] B. G. Todd-Rutel and J. Piekarewicz, Phys. Rev. Lett. **95**, 122501 (2005).
[6] P. Avogadro and C. A. Bertulani, Phys. Rev. C **88**, 044319 (2013).
[7] T. Niksic, D. Vretenar, and P. Ring, Phys. Rev. C **78**, 034318 (2008).

[8] B. K. Agrawal, S. Shlomo, and V. K. Au, Phys. Rev. C **72**, 014310 (2005).
[9] J. Kvasil, D. Bozhik, A. Repko, P.-G. Reinhard, V. O. Nesterenko, and W. Kleinig, arXiv:1412.1997 [nucl-th].
[10] Y. W. Lui, D. H. Youngblood, Y. Tokimoto, H. L. Clark, and B. John, Phys. Rev. C **70**, 014307 (2004).
[11] D. H. Youngblood, Y.-W. Lui, B. John, Y. Tokimoto, H. L. Clark, and X. Chen, Phys. Rev. C **69**, 054312 (2004).
[12] T. Li *et al.*, Phys. Rev. Lett. **99**, 162503 (2007).
[13] J. Piekarewicz, Phys. Rev. C **76**, 031301(R) (2007).
[14] U. Garg *et al.*, Nucl. Phys. A **788**, 36 (2007).
[15] V. Tselyaev, J. Speth, S. Krewald, E. Litvinova, S. Kamerdzhiev, N. Lyutorovich, A. Avdeenkov, and F. Gruümmer, Phys. Rev. C **79**, 034309 (2009).
[16] T. Li *et al.*, Phys. Rev. C **81**, 034309 (2010).
[17] D. Patel *et al.*, Phys. Lett. B **718**, 447 (2012).
[18] E. Khan, J. Margueron, and I. Vidaña, Phys. Rev. Lett. **109**, 092501 (2012).
[19] G. Colò, N. V. Giai, J. Meyer, K. Bennaceur, and P. Bonche, Phys. Rev. C **70**, 024307 (2004).
[20] E. Khan, J. Margueron, G. Colò, K. Hagino, and H. Sagawa, Phys. Rev. C **82**, 024322 (2010).
[21] E. Khan and J. Margueron, Phys. Rev. C **88**, 034319 (2013).
[22] S. Brandt, *Statistical and Computational Methods in Data Analysis*, 3rd ed. (Springer, New York, 1997).
[23] B. K. Agrawal, J. N. De, and S. K. Samaddar, Phys. Rev. Lett. **109**, 262501 (2012).
[24] N. Alam, B. K. Agrawal, J. N. De, S. K. Samaddar, and G. Colo, Phys. Rev. C **90**, 054317 (2014).
[25] M. Dutra, O. Lourenco, J. S. Sá Martins, A. Delfino, J. R. Stone, and P. D. Stevenson, Phys. Rev. C **85**, 035201 (2012).
[26] D. Bandyopadhyay, C. Samanta, S. K. Samaddar, and J. N. De, Nucl. Phys. A **511**, 1 (1990).
[27] A. Bohr and B. R. Mottelson, *Nuclear Structure*, Vol. I (Benjamin, New York, 1969).
[28] B. D. Serot and J. D. Walecka, in *Advances Nuclear Physics*, edited by J. W. Negele and E. Vogt (Plenum Press, New York, 1986), Vol. 16, p. 1.
[29] S. Typel and H. H. Wolter, Nucl. Phys. A **656**, 331 (1999).