Null-test signal for T-invariance violation in pd scattering

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The integrated proton-deuteron cross section $\widetilde{\sigma}$ for the case of the incident proton vector polarization p_y^p and tensor polarization P_{xz} of the deuteron target provides a null-test signal for time-reversal invariance violating but P-parity conserving (TVPC) effects. We study the null-test observable $\widetilde{\sigma}$ within the Glauber theory of the double-polarized pd scattering. Full spin dependence of the ordinary strong pN scattering amplitudes and different types of the hypothetical TVPC pN amplitudes are taken into account. We show that the contribution from the exchange of the lowest-mass meson allowed in the TVPC interaction, i.e., the ρ meson, to the null-test observable $\widetilde{\sigma}$ is zero. The axial h_1 meson exchange makes a nonzero contribution. We find that inclusion of the Coulomb interaction does not lead to divergence of the cross section $\widetilde{\sigma}$ and we calculate its energy dependence at the proton beam energy 100–1000 MeV.

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I. INTRODUCTION

CP violation (or T-reversal invariance violation under CPT symmetry) is required to explain the baryon asymmetry of the universe [1]. In baryon systems violation of T invariance has not been observed yet. CP violation established in physics of kaons and B mesons leads to simultaneous CP and P-invariance violation. Under the assumption of CPT-invariance this implies existence of T-odd, P-odd interactions. These effects are parametrized in the Standard Model by the CP violating phase of the Cabibbo-Kobayashi-Maskawa matrix. Another source for T-odd P-odd effects is the QCD θ term, which can be related to electric dipole moments (EDM) of elementary particles and atoms in their ground states.

On the contrary, time-reversal-symmetry-violating (T-odd) P-parity-conserving (P-even) flavor-conserving (TVPC) interactions do not arise on the fundamental level within the standard model, although they can be generated from the T-odd P-odd interaction by weak radiative P-parity nonconserving corrections. However, in this case its intensity is too low [2,3] to be observed in experiments at present. Thus, observation of the TVPC effects would be considered as indication of physics beyond the standard model.

The existing experimental constraints on the TVPC effects in physics of nuclei are rather weak. So, the test of the detailed balance performed for the reactions $^{27}\text{Al}(p,\alpha)^{24}\text{Mg}$ and $^{24}\text{Mg}(\alpha,p)^{27}\text{Al}$ [4], and complemented by numerous statistical analyses of nuclear energy-level fluctuations, leads to the ratio of T-odd to T-even matrix elements as $\alpha_T < 2 \times 10^{-3}$ [5]. Another type of experiment, i.e., polarized neutron transmission through a polarized ^{165}Ho target, gives $\alpha_T \leqslant 7.1 \times 10^{-4}$ or $\bar{g}_\rho \leqslant 5.9 \times 10^{-2}$ [6]. Here $\bar{g}_\rho$ is the T-odd P-even coupling constant of the charged ρ-meson with the nucleon introduced in Ref. [7] to classify the TVPC interactions in terms of boson exchanges. Charge symmetry breaking determined as the difference in the scattering of polarized protons off unpolarized neutrons $\vec{p}n$ and polarized neutrons off unpolarized protons $\vec{n}p$ gives $\alpha_T \leqslant 8 \times 10^{-5}$ (or $\bar{g}_\rho < 6.7 \times 10^{-3}$) [7]. One should add that indirect model-dependent estimation based on the existing constraints on EDM gives $\alpha_T \leqslant 1.1 \times 10^{-5}$ ($\bar{g}_\rho \leqslant \times 10^{-3}$) [8]. However, a more recent analysis showed [9] that EDM may arise via another scenario which suggests no significant constraints on the TVPC forces.

The integrated cross section $\widetilde{\sigma}$ will be measured at Cooler Synhrotron in Forschungszentrum Jülich (Germany) (COSY) [10] in double polarized pd scattering with a transverse polarized proton beam (p_y^p) and a tensor polarized deuterium target (P_{xz}). This observable provides a real null test of the TVPC forces [11]. This signal is not affected by the initial- and final-state interaction and therefore its observation would directly indicate time-invariance violation, as in the case of the neutron EDM. The experiment [10] will be performed at a beam energy of 135 MeV. This energy choice was motivated by the theoretical analysis of the integrated pd cross section $\widetilde{\sigma}$ performed in Ref. [12]. The aim of this experiment is to diminish the upper bound on the TVPC effects previously obtained in the $\vec{n}^{167}$Ho scattering [6] by one order of magnitude.

The elastic channel and the deuteron breakup $dp \to pnp$ were considered in Ref. [12] in the impulse approximation (single scattering mechanism) for estimation of $\widetilde{\sigma}$. In the present work we study the null-test observable $\widetilde{\sigma}$ on the basis of the generalized optical theorem using the forward elastic pd scattering amplitude calculated within the Glauber theory. Both the single- and double-scattering mechanisms are considered.

The spin-dependent Glauber formalism recently developed in Ref. [13] was applied in our previous work [14] to calculate spin observables of the elastic pd scattering using the strong (time-invariance-conserving and P-parity conserving) pN scattering amplitudes as input at 135 MeV. The obtained differential cross section, vector and tensor analyzing powers, and spin-correlation parameters were found to be in reasonable agreement with the existing data [15,16]. Here we generalize this formalism to allow for TVPC pN scattering amplitudes of several types. This generalized formalism is applied below to derive formulas for the null-test observable $\widetilde{\sigma}$ and calculate its energy dependence. We show that within the single-scattering mechanism this observable is zero in the Glauber theory (for any type of the TVPC pN interactions considered in the general case in Ref. [17]) and, consequently, focus on the double-scattering mechanism. We investigate the contribution of several TVPC terms to the pN scattering amplitudes, in particular, the ρ-meson and axial h_1-meson exchanges. In addition, we investigate the influence of the Coulomb interaction on the $\widetilde{\sigma}$ cross section not considered in Ref. [12].

The paper is organized as follows. In Sec. II we consider the spin structure of the forward pd elastic scattering amplitude including the TVPC term and apply the generalized optical theorem to derive formulas for total spin-dependent cross sections in terms of the forward-scattering invariant amplitudes. In Sec. III we construct the Glauber scattering operator taking into account full spin dependence of the elementary pN-scattering amplitudes for strong and some other types of TVPC interactions and S and D components of the deuteron wave function. Analytical expressions for the TVPC forward-scattering amplitude $\widetilde{g}$ are derived for the double-scattering mechanism with different TVPC terms. The influence of the Coulomb effects on the $\widetilde{g}$ amplitude is discussed in Sec. IV. Numerical results are shown in Sec. IV.

II. FORWARD TRANSITION OPERATOR AND INTEGRATED CROSS SECTIONS

Time-reversal symmetry-conserving and P-parity conserving (TCPC or T-even P-even) interactions lead to the following transition amplitude of the elastic pd scattering at 0 deg [18]:

$$\begin{aligned} e'^{*}_{\beta} M(0)^{TCPC}_{\alpha\beta} e_\alpha = {} & g_1[\mathbf{e}\,\mathbf{e}'^{*} - (\mathbf{me})(\mathbf{me}'^{*})] + g_2(\mathbf{me})(\mathbf{me}'^{*}) \\ & + i g_3\{\boldsymbol{\sigma}[\mathbf{e}\times\mathbf{e}'^{*}] - (\boldsymbol{\sigma}\mathbf{m})(\mathbf{m}\cdot[\mathbf{e}\times\mathbf{e}'^{*}])\} \\ & + i g_4(\boldsymbol{\sigma}\mathbf{m})(\mathbf{m}\cdot[\mathbf{e}\times\mathbf{e}'^{*}]), \end{aligned} \tag{1}$$

where $\mathbf{e}$ ($\mathbf{e}'$) is the polarization vector of the initial (final) deuteron, $\mathbf{m}$ is the unit vector along the beam momentum, $\boldsymbol{\sigma}$ is the Pauli matrix, and g_i ($i = 1,\dots,4$) are complex amplitudes. To the right-hand side of Eq. (1) one can add the TVPC (T-odd P-even) term in a very general form:

$$\begin{aligned} e'^{*}_{\beta} M(0)^{TVPC}_{\alpha\beta} e_\alpha = {} & \widetilde{g}\{(\boldsymbol{\sigma}\cdot[\mathbf{m}\times\mathbf{e}])(\mathbf{m}\cdot\mathbf{e}'^{*}) \\ & + (\boldsymbol{\sigma}\cdot[\mathbf{m}\times\mathbf{e}'^{*}])(\mathbf{m}\cdot\mathbf{e})\}, \end{aligned} \tag{2}$$

where $\widetilde{g}$ is the TVPC transition amplitude. To find the total spin-dependent pd cross sections we use the generalized optical theorem [19],

$$\sigma_i^t = 4\sqrt{\pi}\,\mathrm{Im}\frac{\mathrm{Tr}(\rho_i M(0))}{\mathrm{Tr}\hat{\rho}_i}, \tag{3}$$

where $M(0) = M(0)^{TCPC} + M(0)^{TVPC}$ is the transition operator from Eqs. (1) and (2) for the pd elastic scattering at zero angle $\theta = 0$, ρ_i is the initial spin-density matrix, and σ_i^t is the total cross section corresponding to the density matrix ρ_i. The transition operator is normalized according to the following relation with the differential cross section [13]:

$$\frac{d\sigma}{dt} = \frac{1}{6}\mathrm{Tr}MM^{+}. \tag{4}$$

For the sum of Eqs. (1) and (2) one can write (see also Ref. [20])

$$\begin{aligned} M(0)_{\alpha\beta} = {} & g_1\delta_{\alpha\beta} + (g_2 - g_1)m_\alpha k_\beta + i g_3\sigma_i\epsilon_{\alpha\beta i} + i(g_4 - g_3) \\ & \times \sigma_i m_i m_j \epsilon_{\alpha\beta j} + \widetilde{g}\sigma_i(\epsilon_{z\alpha i} m_\beta m_z + \epsilon_{z\beta i} m_z m_\alpha), \end{aligned} \tag{5}$$

where σ_i ($i = x,y,z$) are the Pauli spin matrices, $\epsilon_{\alpha\beta\gamma}$ is the fully antisymmetric tensor, and m_α ($\alpha = x,y,z$) are the Cartesian components of the vector $\mathbf{m}$.

The initial-state spin density matrix $\rho_i = \rho^p\rho^d$ is the product of the spin density matrices for the proton

$$\rho^p = \tfrac{1}{2}(1 + \mathbf{p}^p\boldsymbol{\sigma}), \tag{6}$$

where $\mathbf{p}^p$ is the polarization vector of the proton, and for the deuteron

$$\rho^d = \tfrac{1}{3} + \tfrac{1}{2}S_j p_j^d + \tfrac{1}{9}S_{jk}P_{jk}; \tag{7}$$

here S_j is the spin-1 operator, p_j^d and P_{jk} ($j,k = x,y,z$) are the vector and tensor polarizations of the deuteron, and $S_{jk} = (S_jS_k + S_kS_j - \frac{4}{3}\delta_{jk})$ is the spin-tensor operator. Using Eqs. (3), (5), (6), and (7), one can find the total cross section of the pd scattering as

$$\begin{aligned} \sigma_{tot} = {} & \sigma_0^t + \sigma_1^t\mathbf{p}^p\cdot\mathbf{p}^d + \sigma_2^t(\mathbf{p}^p\cdot\mathbf{m})(\mathbf{p}^d\cdot\mathbf{m}) \\ & + \sigma_3^t P_{zz} + \widetilde{\sigma}\, p_y^p P_{xz}^d, \end{aligned} \tag{8}$$

where $\mathbf{p}^p$ ($\mathbf{p}^d$) is the vector polarization of the initial proton (deuteron) and P_{zz} and P_{xz} are the tensor polarizations of the deuteron. The OZ axis is directed along the $\mathbf{m}$, the OY axis is directed along the vector polarization of the proton beam $\mathbf{p}^p$, and the OX axis is chosen to form the right-hand reference frame. The following equations were found in Ref. [21] for the TVPC:

$$\begin{aligned} & \sigma_0^t = \tfrac{4}{3}\sqrt{\pi}\,\mathrm{Im}(2g_1 + g_2), \quad \sigma_1^t = -4\sqrt{\pi}\,\mathrm{Im}g_3, \\ & \sigma_2^t = -4\sqrt{\pi}\,\mathrm{Im}(g_4 - g_3), \quad \sigma_3^t = 4\sqrt{\pi}\,\mathrm{Im}(g_1 - g_2). \end{aligned} \tag{9}$$

We find that the TVPC term $\widetilde{g}$ in the forward pd elastic scattering amplitude (2) leads to the following integrated cross section:

$$\widetilde{\sigma} = -4\sqrt{\pi}\,\mathrm{Im}\tfrac{2}{3}\widetilde{g}. \tag{10}$$

In Eq. (8) the terms σ_i^t with $i = 0,1,2,3$ are nonzero only for the TCPC (T-even P-even) interactions and the last term $\widetilde{\sigma}$ is nonzero if the TVPC (T-odd P-even) interactions occur. Thus, the term $\widetilde{\sigma}$ constitutes the null-test signal for time-reversal invariance violating but P-parity-conserving effects. This term can be measured in the transmission experiment [10] as a difference of counting rates for the cases with $p_y^p P_{xz}^d > 0$ and $p_y^p P_{xz}^d < 0$.

We find the following matrix elements of the TVPC transition operator (2):

$$\left\langle \mu' = \tfrac{1}{2}, \lambda' = 0 \middle| M^{TVPC} \middle| \mu = -\tfrac{1}{2}, \lambda = 1 \right\rangle = i\sqrt{2}\widetilde{g}, \tag{11}$$

$$\langle\mu' = \tfrac{1}{2},\lambda' = -1|M^{TVPC}|\mu = -\tfrac{1}{2},\lambda = 0\rangle = -i\sqrt{2}\widetilde{g}, \tag{12}$$

where μ (μ') and λ (λ') are spin projections of the initial (final) proton and deuteron on the beam direction, respectively. The diagonal matrix elements of the M^{TVPC} operator are zeros. For the operator M^{TCPC} the corresponding matrix elements in Eqs. (11) and (12) are identical and equal to $\sqrt{2}g_3$.

III. SPIN-DEPENDENT GLAUBER FORMALISM WITH STRONG AND TVPC INTERACTION

A. Hadronic and Coulomb pN interaction

Hadronic amplitudes of pN scattering are taken in a form of [13]

$$\begin{aligned} &M_N(\mathbf{p},\mathbf{q};\boldsymbol{\sigma},\boldsymbol{\sigma}_N) \\ &\quad = A_N + C_N\boldsymbol{\sigma}\hat{\mathbf{n}} + C'_N\boldsymbol{\sigma}_N\hat{\mathbf{n}} + B_N(\boldsymbol{\sigma}\hat{\mathbf{k}})(\boldsymbol{\sigma}_N\hat{\mathbf{k}}) \\ &\qquad + (G_N + H_N)(\boldsymbol{\sigma}\hat{\mathbf{q}})(\boldsymbol{\sigma}_N\hat{\mathbf{q}}) + (G_N - H_N)(\boldsymbol{\sigma}\hat{\mathbf{n}})(\boldsymbol{\sigma}_N\hat{\mathbf{n}}), \end{aligned} \tag{13}$$

where $\hat{\mathbf{q}}$, $\hat{\mathbf{k}}$, and $\hat{\mathbf{n}}$ are defined as unit vectors along the vectors $\mathbf{q} = (\mathbf{p} - \mathbf{p}')$, $\mathbf{k} = (\mathbf{p} + \mathbf{p}')$ and $\mathbf{n} = [\mathbf{k} \times \mathbf{q}]$, respectively. Normalization of the amplitudes A_N, B_N, C_N, C'_N, G_N, and H_N is the same as in Ref. [13],

$$\frac{d\sigma}{dt} = \frac{1}{4}\mathrm{Tr}M_N M_N^{+}, \tag{14}$$

where $d\sigma/dt$ is the differential cross section of the elastic pN scattering. The Glauber formalism for the pd elastic scattering accounting for full spin dependence of the pN amplitudes (13) and S and D components of the deuteron wave function is given in Ref. [13]. The unpolarized differential cross section and analyzing powers of pd scattering calculated within this formalism are in reasonable agreement with existing data in the forward hemisphere at energies 250–1000 MeV [13,22]. Further development of this formalism was done in Ref. [14] to allow for calculation of spin correlation parameters and inclusion of the Coulomb interaction that is important at lower energies.

We include the Coulomb interaction by adding to the Glauber hadronic pd scattering amplitude the following pure Coulomb amplitude of the pd scattering:

$$M^{\mathrm{C}}_{pd}(\mathbf{q}) = \frac{\sqrt{\pi}}{k_{pp}} f^{\mathrm{C}}_{pp}(\mathbf{q}) S_d(\mathbf{q}/2), \tag{15}$$

where f^{C}_{pp} is the antisymmetric Coulomb amplitude of the pp scattering [23]:

$$f^{\mathrm{C}}_{pp}(\mathbf{q}) = f(\theta_{pp}) - \tfrac{1}{2}(1 + \boldsymbol{\sigma}\cdot\boldsymbol{\sigma}_p) f(\pi - \theta_{pp}) \tag{16}$$

with

$$f(\theta_{pp}) = -\frac{\alpha}{4vk_{pp}\sin^2\theta_{pp}/2}\exp\left\{-i\frac{\alpha}{v}\ln\sin\frac{\theta_{pp}}{2} + 2i\chi_0\right\}; \tag{17}$$

here α is the fine structure constant, v (k_{pp}) is the velocity (momentum) of the proton in the c.m. pp system, and χ_0 is the Coulomb phase. The momentum $\mathbf{q}$ transferred in the process $pd \to pd$ is related to the pp scattering angle θ_{pp} in the pp c.m. as $q = 2k_{pp}\sin\theta_{pp}/2$.

In Eq. (15) $S_d(\mathbf{q}/2)$ is the elastic form factor of the deuteron which can be presented in the form [13]

$$S_d(\mathbf{q}/2) = S_0(q/2) - \frac{1}{\sqrt{8}} S_2(q/2) S_{12}(\hat{\mathbf{q}};\boldsymbol{\sigma}_p,\boldsymbol{\sigma}_n). \tag{18}$$

Here

$$S_{12}(\hat{\mathbf{q}};\boldsymbol{\sigma}_p,\boldsymbol{\sigma}_n) = 3(\boldsymbol{\sigma}_p\cdot\hat{\mathbf{q}})(\boldsymbol{\sigma}_n\cdot\hat{\mathbf{q}}) - \boldsymbol{\sigma}_p\cdot\boldsymbol{\sigma}_n \tag{19}$$

is the tensor operator, $\boldsymbol{\sigma}_n(\boldsymbol{\sigma}_p)$ are the Pauli matrices acting on the spin states of the neutron and proton in the deuteron, and $\hat{\mathbf{q}}$ is the unit vector directed along the transferred momentum $\mathbf{q}$. The form factors S_0 and S_2 are related to the S and D components of the deuteron wave function [13]:

$$S_0(q) = S_0^{(0)}(q) + S_0^{(2)}(q), \quad S_2(q) = S_2^{(1)}(q) + S_2^{(2)}(q), \tag{20}$$

where

$$\begin{aligned} S_0^{(0)}(q) &= \int_0^\infty dr u^2(r) j_0(qr), \\ S_0^{(2)}(q) &= \int_0^\infty dr w^2(r) j_0(qr), \\ S_2^{(1)}(q) &= 2\int_0^\infty dr u(r)w(r) j_2(qr), \\ S_2^{(2)}(q) &= -\frac{1}{\sqrt{2}}\int_0^\infty dr w^2(r) j_2(qr). \end{aligned} \tag{21}$$

B. TVPC pN scattering amplitudes

In general, the TVPC NN interaction contains 18 different terms [17]. We consider here only the following terms of the t matrix of the elastic pN scattering investigated in Ref. [12]:

$$\begin{aligned} t_{pN} = {} & f_N(\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}_N)(\mathbf{q}\cdot\mathbf{k})/m_p^2 + h_N\big[(\boldsymbol{\sigma}\cdot\mathbf{k})(\boldsymbol{\sigma}_N\cdot\mathbf{q}) \\ & + (\boldsymbol{\sigma}_N\cdot\mathbf{k})(\boldsymbol{\sigma}\cdot\mathbf{q}) - \tfrac{2}{3}(\boldsymbol{\sigma}_N\cdot\boldsymbol{\sigma})(\mathbf{k}\cdot\mathbf{q})\big]/m_p^2 + \\ & + g_N[\boldsymbol{\sigma}\times\boldsymbol{\sigma}_N]\cdot[\mathbf{q}\times\mathbf{k}]/m_p^2 \\ & + g'_N(\boldsymbol{\sigma} - \boldsymbol{\sigma}_N)\cdot i\,[\mathbf{q}\times\mathbf{k}][\boldsymbol{\tau}\times\boldsymbol{\tau}_N]_z/m_p^2. \end{aligned} \tag{22}$$

Here $\boldsymbol{\sigma}$ ($\boldsymbol{\sigma}_N$) is the Pauli matrix acting on the spin state of the proton (nucleon $N = p,n$), and $\boldsymbol{\tau}$ ($\boldsymbol{\tau}_N$) is the isospin matrix acting on the isospin state of the proton (nucleon), $\mathbf{q} = \mathbf{p} - \mathbf{p}'$. In the framework of the phenomenological meson exchange interaction the term g'_N corresponds to the ρ-meson exchange, and the h_N term is caused by the axial meson exchange. As shown in Ref. [24], the contribution of the π- and σ-meson exchanges to TVPC NN interactions is excluded. The TVPC NN interaction potential corresponding to $h_1(1170)$, $I^G(J^{PC}) = 0^-(1^{+-})$ exchange in r space has the form [25]

$$V_h(\mathbf{r}) = -\frac{G_h\bar{G}_h m_h^2}{2\pi m_N^2} Y_1(x)[(\boldsymbol{\sigma}_1\bar{\mathbf{p}})(\boldsymbol{\sigma}_2\hat{\mathbf{r}}) + (\boldsymbol{\sigma}_2\bar{\mathbf{p}})(\boldsymbol{\sigma}_1\hat{\mathbf{r}})], \tag{23}$$

where G_h ($\bar{G}_h$) is the ordinary (TVPC) hNN coupling constant, m_h is the h_1-meson mass, $\bar{\mathbf{p}} = (\mathbf{p}_1 - \mathbf{p}_2)/2$, $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$, $Y_1(x) = (1 + x)e^{-x}/x^2$, $x = m_h r$; $\boldsymbol{\sigma}_i/2$ and $\mathbf{p}_i$ are the spin-operator and momentum of ith nucleon, respectively, and $\mathbf{r}_i$ is its r coordinate ($i = 1,2$). Making the Fourier transformation of Eq. (23) we obtain the interaction potential in p space and, therefore, find the factor h_N in Eq. (22) within the Born approximation for the t_{pN} amplitude as the following:

$$h_N = -i\phi_h \frac{2G_h^2}{m_h^2 + \mathbf{q}^2} F_{hNN}(\mathbf{q}^2), \tag{24}$$

where $\phi_h = \bar{G}_h/G_h$ is the strength of the T-invariance-violating potential of h_1-meson exchange relative to the T-conserving one, and $F_{hNN}(q^2) = (\Lambda^2 - m_h^2)/(\Lambda^2 + \mathbf{q}^2)$ is the phenomenological monopole form factor in the hNN vertex. Similarly, proceeding from the TVPC ρ-meson exchange NN potential in r space [25–27] we find for the g' term in Eq. (22)

$$g'_N = -\phi_\rho \frac{1}{2} \frac{g_\rho^2 \kappa}{m_\rho^2 + \mathbf{q}^2} F_{\rho NN}(\mathbf{q}^2), \tag{25}$$

where $\phi_\rho = \bar{g}_\rho/g_\rho$ is the ratio of the TVPC ρNN coupling constant $\bar{g}_\rho$ to the strong one g_ρ, m_ρ is the mass of the ρ meson, κ is the anomalous magnetic moment of the nucleon, and $F_{\rho NN}$ is the ρNN vertex form factor.

C. The Glauber operator

The Glauber operator of the elastic pd scattering in the general case can be written as

$$M(\mathbf{q},\mathbf{Q};\mathbf{S},\boldsymbol{\sigma}) = \iiint e^{i\mathbf{Qr}}\Psi_d^+(\mathbf{r})\mathrm{O}\Psi_d(\mathbf{r})d^3r, \tag{26}$$

where Ψ_d is the deuteron wave function, $\mathbf{q}$ is the transferred momentum, $\mathbf{S} = (\boldsymbol{\sigma}_\mathbf{p} + \boldsymbol{\sigma}_\mathbf{n})/\mathbf{2}$ is the spin operator of the deuteron nucleons, and $\boldsymbol{\sigma}/2$ is the spin operator of the incoming proton. We use the deuteron wave function generated by the NN interaction, which conserves time-reversal invariance and P parity and has the following standard form:

$$\Psi_d(\mathbf{r};\boldsymbol{\sigma}_n,\boldsymbol{\sigma}_p) = \frac{1}{\sqrt{4\pi}r}\left(u(r) + \frac{1}{\sqrt{8}}w(r)\cdot S_{12}(\hat{\mathbf{r}};\boldsymbol{\sigma}_n,\boldsymbol{\sigma}_p)\right), \tag{27}$$

where lower index n (p) refers to the neutron (proton) of the deuteron target; u and w denote the S and D wave of the deuteron, respectively; and the tensor operator $S_{12}(\hat{\mathbf{r}};\boldsymbol{\sigma}_n,\boldsymbol{\sigma}_p)$ is defined by Eq. (19).

The operator for the single- and double-scattering mechanisms of pd scattering in the general case (beyond the collinear kinematics) can be written as an expansion over the Pauli matrices $\boldsymbol{\sigma}_n$, $\boldsymbol{\sigma}_p$ and in notations of Ref. [13] takes the form

$$O(\boldsymbol{\sigma},\boldsymbol{\sigma}_n,\boldsymbol{\sigma}_p) = U(\boldsymbol{\sigma}) + \mathbf{V}_n(\boldsymbol{\sigma})\cdot\boldsymbol{\sigma}_n + \mathbf{V}_p(\boldsymbol{\sigma})\cdot\boldsymbol{\sigma}_p + W_{ij}(\boldsymbol{\sigma})\cdot(\sigma_{ni}\sigma_{pj} + \sigma_{nj}\sigma_{pi}), \tag{28}$$

where $i,j = q,n,k$ are indices of the projections onto directions of three orthogonal vectors $\hat{\mathbf{q}},\hat{\mathbf{n}},\hat{\mathbf{k}}$ introduced in Eq. (13). The operators U, V, and W act only on the spin state of the beam proton and do not depend on the spins and coordinates $\mathbf{r}$ of the target nucleons. When making the matrix element of the operator (28) over the deuteron wave functions (27) we obtain from Eq. (26)

$$\begin{aligned}
M(\mathbf{q},\mathbf{Q};\mathbf{S},\boldsymbol{\sigma}) &= \iiint e^{i\mathbf{Qr}}\Psi_d^+(\mathbf{r})\mathrm{O}\Psi_d(\mathbf{r})d^3r \\
&= US_0 + \mathbf{VS}S_0^{(0)} + [W_{ij}\{S_i,S_j\} - W_{ii}]S_0^{(0)} - \frac{1}{\sqrt{2}}US_{12}(\hat{\mathbf{Q}};\mathbf{S},\mathbf{S})S_2 \\
&\quad - \frac{1}{\sqrt{8}}S_{12}(\hat{\mathbf{Q}};\mathbf{V},\mathbf{S})S_2^{(1)} + \sqrt{8}\,W_{ii}S_{12}(\hat{\mathbf{Q}};\mathbf{S},\mathbf{S})S_2^{(1)} - \frac{1}{\sqrt{2}}S_{12}(\hat{\mathbf{Q}};\mathbf{V},\mathbf{S})S_2^{(2)} - \frac{1}{2}\mathbf{VS}S_0^{(2)} + \\
&\quad - W_{ii}S_{12}(\hat{\mathbf{Q}};\mathbf{S},\mathbf{S})S_2^{(2)} - 2\,W_{ii}S_0^{(2)} \\
&\quad - \sqrt{2}\,W_{ij}\,[\{S_i,S_j\}S_{12}(\hat{\mathbf{Q}};\mathbf{S},\mathbf{S}) + S_{12}(\hat{\mathbf{Q}};\mathbf{S},\mathbf{S})\{S_i,S_j\}]S_2^{(1)} \\
&\quad + \frac{1}{16\pi}W_{ij}\int d^3r\frac{1}{r^2}e^{i\mathbf{Qr}}w^2\,S_{12}(\hat{\mathbf{r}};\boldsymbol{\sigma}_n,\boldsymbol{\sigma}_p)\{S_i,S_j\}S_{12}(\hat{\mathbf{r}};\boldsymbol{\sigma}_n,\boldsymbol{\sigma}_p).
\end{aligned} \tag{29}$$

Here we use the notations $\mathbf{V} = \mathbf{V}_p + \mathbf{V}_n$ and $\{S_i,S_j\} = S_iS_j + S_jS_i$; the form factors $S_0(Q),S_2(Q)$, $S_0^{(0)}(Q),S_0^{(2)}(Q),S_2^{(1)}(Q)$, and $S_2^{(2)}(Q)$ are defined in Eqs. (20), (21); and the tensor operators $S_{12}(\hat{\mathbf{Q}};\mathbf{V},\mathbf{S})$ and $S_{12}(\hat{\mathbf{Q}};\mathbf{S},\mathbf{S})$ are defined in Eq. (19). In Eq. (29) summation is performed over repeating indexes i,j. To make the integration over directions of the vector $\mathbf{r}$ in Eq. (29), we used the following relation [28]:

$$\iint d\Omega_\mathbf{r}\exp(-i\mathbf{Qr})T_l(\hat{\mathbf{r}}) = 4\pi j_l(Qr)(-i)^l T_l(\hat{\mathbf{Q}}), \tag{30}$$

where $j_l(x)$ is the spherical Bessel function, $T_2(\hat{\mathbf{n}}) = (\boldsymbol{\sigma}_p\cdot\hat{\mathbf{n}})(\boldsymbol{\sigma}_n\cdot\hat{\mathbf{n}}) - \frac{1}{3}(\boldsymbol{\sigma}_p\cdot\boldsymbol{\sigma}_n)$, $T_0(\hat{\mathbf{n}}) = \boldsymbol{\sigma}_p\cdot\boldsymbol{\sigma}_n$; and $\hat{\mathbf{n}}$, $\hat{\mathbf{Q}}$, and $\hat{\mathbf{r}}$ are unit vectors along $\mathbf{n}$, $\mathbf{Q}$ and $\mathbf{r}$, respectively.

Equation (29) is a generalization of Eq. (18) from Ref. [13]. The difference from Ref. [13] consists in two following respects. First, the operators U, $\mathbf{V}$, and W_{ij} contain not only T-even P-even terms but T-odd P-even terms as well. Second, we present in Eq. (29) all terms allowed within the Glauber theory, whereas in Ref. [13] small spin-dependent terms (of the order higher than two in definitions of Ref. [13]) were neglected. These terms are small at high energies about 1 GeV but may be important at lower energies ∼100 MeV, corresponding the COSY experiment [10].

D. Differential spin observables

Vector analyzing powers A_y and spin correlation coefficients $C_{i,j}$, $C_{ij,k}$ of the elastic pd scattering are calculated as

$$\begin{aligned} A_y^p &= \mathrm{Tr} M\sigma_y M^+/\mathrm{Tr} MM^+, \\ A_y^d &= \mathrm{Tr} M S_y M^+/\mathrm{Tr} MM^+, \\ C_{xz,y} &= \mathrm{Tr} M S_{xz}\sigma_y M^+/\mathrm{Tr} MM^+, \\ C_{y,y} &= \mathrm{Tr} M S_y\sigma_y M^+ \mathrm{Tr} MM^+, \\ C_{x,z} &= \mathrm{Tr} M S_x\sigma_z M^+/\mathrm{Tr} MM^+, \\ C_{z,x} &= \mathrm{Tr} M S_z\sigma_x M^+/\mathrm{Tr} MM^+, \end{aligned} \tag{31}$$

where M is the transition operator given by Eq. (26). Details of these calculations in terms of invariant amplitudes are described in Ref. [14].

IV. NULL-TEST SIGNAL OF TVPC INTERACTIONS

Equation (29) gives the single-scattering pd amplitude if one put $\mathbf{Q} = \mathbf{q}/2$, where $\mathbf{q}$ is the momentum transferred in the pd scattering. Within the Glauber theory the amplitude of the single-scattering mechanism is proportional to the on-shell $t_{pN}(\mathbf{q})$ amplitude. At zero scattering angle the TVPC amplitude (22) vanishes, and therefore the corresponding pd-scattering amplitude and total cross section $\widetilde{\sigma}$ of pd scattering are equal to zero in the single-scattering Glauber approximation. Furthermore, the f term in Eq. (22) gives zero contribution within the Glauber theory both for the single and double scattering, because for the on-shell pN scattering involved in multistep scattering (28), one has $(\mathbf{q}\cdot\mathbf{k}) = 0$. For the same reason, the component of the h term proportional to $\boldsymbol{\sigma}_N\boldsymbol{\sigma}$ vanishes in Eq. (22) too. The rest g', g, and h terms contribute to the double-scattering forward pd-elastic amplitude.

The double-scattering amplitude is given by integration of Eq. (29) over $\mathbf{Q} \equiv \mathbf{q}'$ [13]

$$M^{(d)} = \frac{i}{2\pi^{3/2}} \iint d^2q' M(\mathbf{q},\mathbf{q}';\mathbf{S},\boldsymbol{\sigma}). \tag{32}$$

According to Eq. (11), in order to get the TVPC $\widetilde{g}$ amplitude one has to calculate the matrix element of the operator given by Eq. (32) at $\mathbf{q} = \mathbf{0}$ over definite initial $|\mu,\lambda\rangle$ and final $|\mu',\lambda'\rangle$ spin states:

$$\widetilde{g} = \frac{1}{(2\pi)^{3/2}} \int d^2q' \left\langle \mu' = \frac{1}{2}, \lambda' = 0 \middle| M(\mathbf{q}=0,\mathbf{q}';\mathbf{S},\boldsymbol{\sigma}) \middle| \mu = -\frac{1}{2}, \lambda = 1 \right\rangle. \tag{33}$$

When considering the double-scattering mechanism, in addition to three vectors $\{\hat{\mathbf{k}},\hat{\mathbf{q}},\hat{\mathbf{n}}\}$ defined after Eq. (13) it is convenient to introduce two more sets of orthonormal unit vectors $\{\hat{\mathbf{k}}_j,\hat{\mathbf{q}}_j,\hat{\mathbf{n}}_j\}$ for the first ($j = 1$) and second ($j = 2$) collision as was done in Ref. [13]. At zero scattering angle we have $\mathbf{q}_1 = -\mathbf{q}_2 = -\mathbf{q}'$, where $\mathbf{q}_1$ ($\mathbf{q}_2$) is the transferred momentum in the first (second) collision; $\mathbf{n}_1 = -[\mathbf{k}\times\mathbf{q}']$, $\mathbf{n}_2 = [\mathbf{k}\times\mathbf{q}']$, $\mathbf{k}_1 = \mathbf{k}_2 = \mathbf{k} + \mathbf{q}'$. In the eikonal approximation vectors $\mathbf{q}$ and $\mathbf{q}'$ are orthogonal to $\mathbf{k}$ and we assume $\mathbf{k}_1 = \mathbf{k}_2 = \mathbf{k}$ [13]. The Cartesian projections for unit vectors can be written in terms of components of the two-dimensional vector $\mathbf{q}' = (q'_x,q'_y)$ (OZ axis is directed along $\mathbf{k}$) as $\hat{n}_{1x} = q'_y/q'$, $\hat{n}_{1y} = -q'_x/q'$, $\hat{n}_{1z} = 0$.

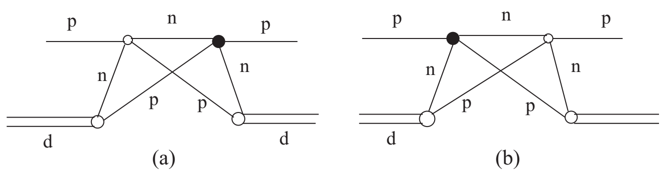

FIG. 1. Double-scattering mechanism with TVPC (black circle) and T-even P-even (open circle) charge-exchange pn interaction.

A. g' term

Nonzero matrix elements of the isospin operator connected with the g' term in Eq. (22) are

$$\langle n,p|[\boldsymbol{\tau}\times\boldsymbol{\tau}_N]_z|p,n\rangle = -i2, \quad \langle p,n|[\boldsymbol{\tau}\times\boldsymbol{\tau}_N]_z|n,p\rangle = i2. \tag{34}$$

Therefore, the g' term contributes only to the charge exchange transitions. Two allowed double-scattering amplitudes with one TVPC and another T-even P-even pN interaction are depicted in Fig. 1. Within the operator formalism these two terms can be evaluated in the following way. For pure T-even P-even (TCPC) interactions the transition operator for the charge-exchange mechanism of the $pd \to pd$ process has the form [29]

$$O^c_{TCPC} = -\tfrac{1}{2}[M_c(\mathbf{q}_2)M_c(\mathbf{q}_1)], \tag{35}$$

where $M_c(\mathbf{q}) = M_n(\mathbf{q}) - M_p(\mathbf{q})$ is the transition operator for the strong charge-exchange $pn \to np$ amplitude that is equal to the $np \to pn$ amplitude. We note that according to Eq. (34), for the TVPC NN interaction with the g' terms the amplitude $pn \to np$ differs from the amplitude $np \to pn$ by the sign. In order to get the TVPC operator of the charge-exchange pd scattering, we make in Eq. (35) the replacement $M_c(\mathbf{q}) \to M_c(\mathbf{q}) + T_c(\mathbf{q})$, where the index c means either $pn \to np$ or $np \to pn$ and T_c is the TVPC charge-exchange NN-scattering operator, normalized as M_N in Eq. (13) and related to the t_{pN} operator given by Eq. (22) as

$$T_{pN} = \frac{m_N}{4\sqrt{\pi}k_{pN}} t_{pN}. \tag{36}$$

Furthermore, we neglect the terms of the second order in T_c as compared to the first order and omit the T-even term $\sim M_cM_c$. As a result, the TVPC charge-exchange operator takes the form

$$\begin{aligned} O^c_{TVPC} = -\tfrac{1}{2}[&M_{np\to pn}(\mathbf{q}_2)T_{pn\to np}(\mathbf{q}_1) \\ &+ T_{np\to pn}(\mathbf{q}_2)M_{pn\to np}(\mathbf{q}_1)]. \end{aligned} \tag{37}$$

For further evaluation it is convenient to use $M_{np\to pn} = M_{pn\to np} = M_n - M_p$. Under the sign of the integral over $\mathbf{q}'$ in Eq. (33), the operator (37) is not changed after the substitution $\mathbf{q}_1 \leftrightarrow \mathbf{q}_2$. In order to find the operators U, $\mathbf{V}_p$, $\mathbf{V}_n$, and W_{ij} introduced in Eq. (28), it is convenient to add to the right side of Eq. (37) the term $O^c_{TVPC}(1 \leftrightarrow 2)$ and divide the obtained sum by the factor of 2:

$$O^c_{TVPC} = f_I + f_{II}, \tag{38}$$

where

$$\begin{aligned} f_I &= -\tfrac{1}{4}[(M_n(\mathbf{q}_2)T_{pn\to np}(\mathbf{q}_1) \\ &\quad + T_{np\to pn}(\mathbf{q}_2)M_n(\mathbf{q}_1)) + (\mathbf{q}_1 \leftrightarrow \mathbf{q}_2)], \\ f_{II} &= \tfrac{1}{4}[M_p(\mathbf{q}_2)T_{pn\to np}(\mathbf{q}_1) \\ &\quad + T_{np\to pn}(\mathbf{q}_2)M_p(\mathbf{q}_1) + (\mathbf{q}_1 \leftrightarrow \mathbf{q}_2)]. \end{aligned} \tag{39}$$

Using Eqs. (13) and (34) and symmetry in respect to the replacement $(\mathbf{q}_1 \leftrightarrow \mathbf{q}_2)$, we find that A_N, B_N, G_N, and H_N terms cancel in operators f_I and f_{II}:

$$\begin{aligned} f_I &= \frac{g'}{\Pi}[C_n(\boldsymbol{\sigma}\cdot\hat{\mathbf{n}}_1)(\boldsymbol{\sigma}_n - \boldsymbol{\sigma}_p)\cdot\mathbf{n}_1 \\ &\quad - C'_n(\boldsymbol{\sigma}_n\cdot\hat{\mathbf{n}}_1)(\boldsymbol{\sigma}_p\cdot\mathbf{n}_1) + C'_n\mathbf{n}_1\hat{\mathbf{n}}_1], \\ f_{II} &= \frac{g'}{\Pi}[C_p(\boldsymbol{\sigma}\cdot\hat{\mathbf{n}}_1)(\boldsymbol{\sigma}_p - \boldsymbol{\sigma}_n)\cdot\mathbf{n}_1 \\ &\quad - C'_p(\boldsymbol{\sigma}_p\cdot\hat{\mathbf{n}}_1)(\boldsymbol{\sigma}_n\cdot\mathbf{n}_1) + C'_p\mathbf{n}_1\hat{\mathbf{n}}_1], \end{aligned} \tag{40}$$

where

$$\Pi = 4\sqrt{\pi}m_N k_{pN}. \tag{41}$$

Making the sum $f_I + f_{II}$ we find the operators U, V, and W_{ij} in Eq. (28) for the g' term as the following:

$$\begin{aligned} U &= \frac{g'}{\Pi}(C'_n + C'_p)\mathbf{n}_1\hat{\mathbf{n}}_1, \\ \mathbf{V}_p &= (C_p - C_n)(\boldsymbol{\sigma}\cdot\mathbf{n}_1)\hat{\mathbf{n}}_1, \quad \mathbf{V}_n = (C_n - C_p)(\boldsymbol{\sigma}\cdot\mathbf{n}_1)\hat{\mathbf{n}}_1, \\ W_{ij} &= -\frac{g'}{\Pi}(C'_n + C'_p)n_{1i}\hat{n}_{1j}, \quad W_{ii} = -\frac{g'}{\Pi}(C'_n + C'_p)n_1\hat{n}_1. \end{aligned} \tag{42}$$

One can see from Eqs. (42) that $\mathbf{V} = \mathbf{V}_p + \mathbf{V}_n \equiv 0$. Furthermore, taking into account the relation $\mathbf{V}_p\boldsymbol{\sigma}_p + \mathbf{V}_n\boldsymbol{\sigma}_n = \mathbf{V}\mathbf{S}$, we find that for the g' term the operator Eq. (28) does not depend on the spin of the proton beam $\boldsymbol{\sigma}$. As a result the transition operator given by Eq. (29) is diagonal with respect to spins of the proton beam. According to Eq. (11), it means that the contribution of g' term to the TVPC amplitude $\widetilde{g}$ is equal to zero. We emphasize that this result is true for the S and D components of the deuteron wave function and for all spin terms in the transition amplitude (29) allowed in the Glauber formalism.

It is easy to find that this result is valid for the nd scattering too.

B. *h* and *g* terms

The TVPC interaction corresponding to the h and g terms in Eq. (22) occurs both in the pp and pn elastic scattering. Following Refs. [29] [see Eq. (2.7) in it] and [13] we consider the symmetric $O^{(d)}_+$ and antisymmetric $O^{(d)}_-$ parts of the operator $Q^{(d)}$:

$$\begin{aligned} Q^{(d)}_+ &= \tfrac{1}{2}[(T_{pp}(\mathbf{q}_1) + M_p(\mathbf{q}_1)(T_{pn}(\mathbf{q}_2) + M_n(\mathbf{q}_2)) \\ &\quad + (T_{pn}(\mathbf{q}_2) + M_n(\mathbf{q}_2))(T_{pp}(\mathbf{q}_1) + M_p(\mathbf{q}_1)], \\ Q^{(d)}_- &= \tfrac{1}{2}[(T_{pp}(\mathbf{q}_1) + M_p(\mathbf{q}_1)(T_{pn}(\mathbf{q}_2) + M_n(\mathbf{q}_2)) \\ &\quad - (T_{pn}(\mathbf{q}_2) + M_n(\mathbf{q}_2))(T_{pp}(\mathbf{q}_1) + M_p(\mathbf{q}_1)]. \end{aligned} \tag{43}$$

According to Refs. [29] and [13] the matrix elements of the antisymmetric operator $Q^{(d)}_-$ over the spin $S = 1$ states of the deuteron are negligible and therefore we drop this term. In the operator $Q^{(d)}_+$ one can neglect terms of the second order in the TVPC interaction ($\sim T_{pp}T_{pn}$) as compared to the first order T_{pN} and should omit the pure T-even terms $\sim M_pM_n$. Thus, the double-scattering TVPC operator consists of the following four terms:

$$\begin{aligned} Q^{(d)}_+ &= \tfrac{1}{2}[T_{pp}(\mathbf{q}_1)M_n(\mathbf{q}_2) + M_n(\mathbf{q}_2)T_{pp}(\mathbf{q}_1) \\ &\quad + T_{pn}(\mathbf{q}_2)M_p(\mathbf{q}_1) + M_p(\mathbf{q}_1)T_{pn}(\mathbf{q}_2)]. \end{aligned} \tag{44}$$

As in the case of g' term, it is convenient to add to the right side of Eq. (44) the term $O^{(d)}_+(1 \leftrightarrow 2)$ and divide the obtained sum by the factor of 2.

For the h term we find the operators U, V, and W_{ij} in Eq. (28) for the forward double scattering as

$$U = 0, \quad \mathbf{V}_p = 0, \quad \mathbf{V}_n = 0, \quad W_{ii} = 0, \tag{45}$$

$$\begin{aligned} W_{ij} &= \frac{1}{\Pi}\{C'_n(q')h_p(q')[\boldsymbol{\sigma}\mathbf{k}_1\hat{n}_{2i}q_{1j} + \boldsymbol{\sigma}\mathbf{q}_1k_{1j}\hat{n}_{2i}] \\ &\quad + C'_p(q')h_n(q')[\boldsymbol{\sigma}\mathbf{k}_1\hat{n}_{2j}q_{1i} + \boldsymbol{\sigma}\mathbf{q}_1k_{1i}\hat{n}_{2j}]\}, \\ W_{ij}\{S_i,S_j\} &= \frac{C'_nh_p + C'_ph_n}{\Pi}\{(\boldsymbol{\sigma}\mathbf{k}_1)[(\hat{\mathbf{n}}_2\mathbf{S})(\mathbf{q}_1\mathbf{S}) + (\mathbf{q}_1\mathbf{S})(\hat{\mathbf{n}}_2\mathbf{S})] \\ &\quad + (\boldsymbol{\sigma}\mathbf{q}_1)[(\mathbf{k}_1\mathbf{S})(\hat{\mathbf{n}}_2\mathbf{S}) + (\hat{\mathbf{n}}_2\mathbf{S})(\mathbf{k}_1\mathbf{S})]\}. \end{aligned} \tag{46}$$

In Eq. (28) in this case only the operator W_{ij} depends on the beam proton spin and, therefore, gives nonzero contribution to $\widetilde{g}$. The matrix elements over the proton spin states are

$$\begin{aligned} &\left\langle \mu' = +\frac{1}{2}\middle| W_{ij}\{S_i,S_j\}\middle|\mu = -\frac{1}{2}\right\rangle \\ &= (C'_nh_p + C'_ph_n)(-q'_x + iq'_y)[S_zS_xq'_y - S_zS_yq'_x \\ &\quad - q'_yS_xS_z + q'_xS_yS_z]\frac{k^2}{\Pi|\mathbf{n}_1|}. \end{aligned} \tag{47}$$

For the deuteron spin matrix elements of the operator M one has $\langle\lambda' = 0|M|\lambda = 1\rangle = -(M_{zx} + iM_{zy})/\sqrt{2}$. Finally the spin matrix element in the S-wave approximation is the following:

$$\begin{aligned} &\left\langle \mu' = \frac{1}{2}, \lambda' = 0\middle| M(\mathbf{q} = 0, \mathbf{q}'; \mathbf{S}, \boldsymbol{\sigma})\middle|\mu = -\frac{1}{2}, \lambda = 1\right\rangle \\ &= -\frac{ik}{\sqrt{2}}\frac{(C'_nh_p + C'_ph_n)q'}{\Pi}S^{(0)}_0(q'). \end{aligned} \tag{48}$$

For the g term we have got

$$\begin{aligned} U &= 0, \quad \mathbf{V}_p = 0, \quad \mathbf{V}_n = 0, \quad W_{ii} = 0, \\ W_{ij}\{S_i,S_j\} &= \frac{C'_ng_p + C'_pg_n}{\Pi}\{(\hat{\mathbf{n}}_2\cdot\mathbf{S})([\mathbf{n}_1\times\boldsymbol{\sigma}]\cdot\mathbf{S}) \\ &\quad + ([\mathbf{n}_1\times\boldsymbol{\sigma}]\cdot\mathbf{S})(\hat{\mathbf{n}}_2\cdot\mathbf{S})\}. \end{aligned} \tag{49}$$

Furthermore, using the proton spin matrix element

$$\left\langle \mu' = \tfrac{1}{2} \middle| ([\mathbf{n}_1 \times \boldsymbol{\sigma}] \cdot \mathbf{S} \middle| \mu = -\tfrac{1}{2} \right\rangle = (-iq'_y + q'_x)S_z k, \quad (50)$$

we find in the S-wave approximation

$$\left\langle \mu' = \frac{1}{2}, \lambda' = 0 \middle| M(\mathbf{q} = 0, \mathbf{Q}; \mathbf{S}, \boldsymbol{\sigma}) \middle| \mu = -\frac{1}{2}, \lambda = 1 \right\rangle = \frac{ik}{\sqrt{2}} \frac{(C'_n g_p + C'_p g_n) q'}{\Pi} S_0^{(0)}(q'). \quad (51)$$

The operator (28) does not depend on the deuteron state. Therefore, the contribution of the D component does not change the factor $C'_n h_p + C'_p h_n$ in Eq. (48) and the factor $C'_n g_p + C'_p g_n$ in Eq. (51). The D component can contribute due to the last two terms in Eq. (29) with W_{ij} caused by interference with the S wave and by the w^2 term. According to analysis performed in Ref. [13], at energies ∼100 MeV the D-wave contribution as well as the spin-dependent pN-scattering amplitudes are less important than the S-wave contribution and spin-independent pN amplitudes. Therefore, we postpone investigation of the D-wave contribution to the next paper.

C. $\widetilde{g}$ amplitude

Thus, the double-scattering mechanism with TVPC interaction from Eq. (22) leads to the following result for the $\widetilde{g}$ amplitude in the S-wave approximation:

$$\widetilde{g} = \frac{i}{4m_p\pi} \int_0^\infty dq q^2 S_0^{(0)}(q)[C'_n(q)(g_p - h_p) + C'_p(q)(g_n - h_n)]. \quad (52)$$

V. NUMERICAL RESULTS AND DISCUSSION

The main aim of this study is to analyze the null-test signal $\widetilde{\sigma}$ within the Glauber theory. In order to demonstrate capability of the Glauber model at energies of the planned COSY experiment [10] we calculated several spin observables of the pd scattering at 135 MeV in comparison with the existing data. The results of our calculations for the unpolarized differential cross section, vector A_y and tensor A_{ij} analyzing powers, and spin correlations parameters $C_{i,j}$, $C_{ij,k}$ given in Eqs. (31) are in reasonable agreement with the available experimental data and/or Faddeev calculations [15,16] at 135 and 250 MeV in the forward hemisphere ($\theta_{\rm cm} < 30^\circ$). Some of these calculations are shown in Fig. 2. We also found that Coulomb effects taken into account as explained above improve agreement with the data on the nonpolarized differential cross section and vector analyzing powers A_y^p and A_y^d at these energies at small angles $\theta_{\rm cm} \leqslant$ 20–30°. The obtained results lead us to the conclusion that the Glauber theory is quite suitable for studying the null-test signal for TVPC effects in the pd scattering because the corresponding signal is not affected by the strong background of T-even P-even interactions.

The previous study of the null-test signal in pd scattering was performed in Ref. [12]. The integrated cross section $\widetilde{\sigma}$ was calculated within an approach accounting separately for the elastic channel and the deuteron breakup $pd \to pnp$ estimated within the single-scattering approximation. The double-scattering mechanism was not considered. Extension of the calculation in Ref. [12] to higher energies above the pion threshold is questionable because the meson production is not taken into account in Ref. [12]. In contrast, our approach based on the optical theorem is more general and allows one to overcome these drawbacks. In particular, we find that the $\widetilde{\sigma}$ observable is determined by the double-scattering mechanism.

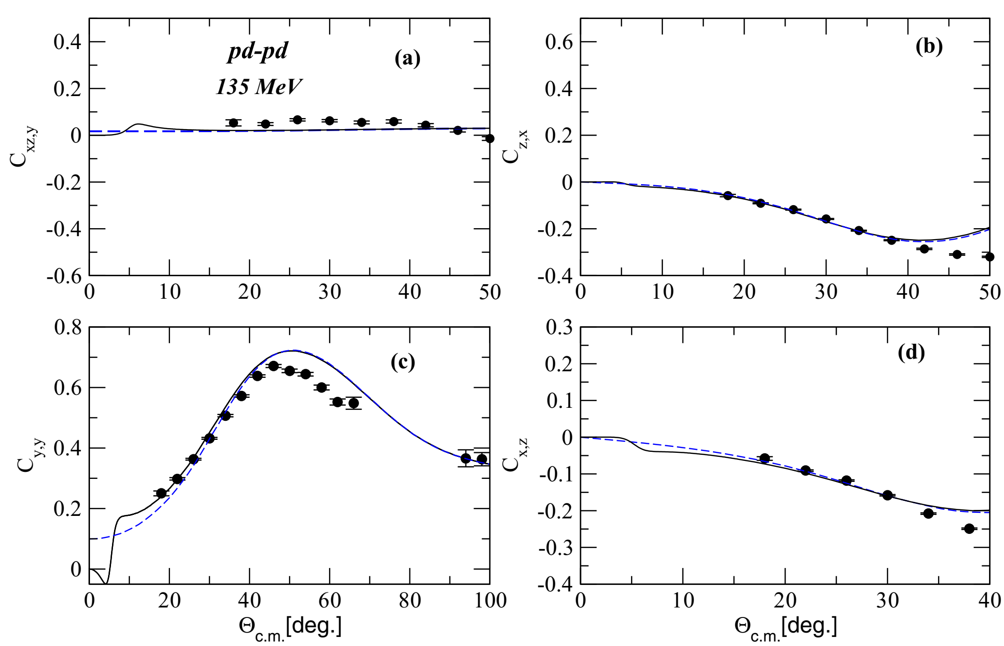

FIG. 2. (Color online) Results of our calculation [14] of the spin observables $C_{xz,y}$ (a), $C_{z,x}$ (b), $C_{y,y}$ (c), and $C_{x,z}$ (d) for the pd elastic scattering in comparison with the data [16] at 135 MeV: without (dashed line) and with the Coulomb interaction included (full).

Our main result obtained within the Glauber theory is formulated by Eq. (52) for the null-test signal. It is worth noting that in our approach only the amplitude C'_N appears in Eq. (52). Some other T-even P-even pN amplitudes, which were found in Ref. [12] to contribute to the TVPC null-test signal, are absent in Eq. (52). There are two other points worth mentioning in relation to Eq. (52). First, the g' term makes a zero contribution to $\widetilde{g}$ and this result is true in the general case when both the S and D components of the deuteron wave function are taken into account. Therefore, an exchange by the lightest meson, that is the ρ meson, is allowed in a general case in the TVPC NN interaction [7] and, as expected, makes the most important contribution to the TVPC NN interaction but does not contribute to the null-test signal $\widetilde{\sigma}$. Contribution of other heavier mesons is usually expected to be less important due to the NN repulsive core at short distances between nucleons. A microscopic T-violating optical potential for the nucleon-nucleus interaction was derived in Ref. [27] starting from the T-violating ρ-meson interaction between nucleons. This potential and the corresponding coupling constant of the ρ meson to the nucleon $\bar{g}_\rho$ is widely used [6,7] as a measure of intensity of the TVPC effects. However, as we have shown, for the nucleon-deuteron scattering this parameter cannot be applied strightforwardly as a scale of the TVPC interactions.

Strong suppression of the contribution of the ρ meson as compared to the axial h_1 meson was found numerically in the Faddeev calculations [25] of the null-test signal for the nd scattering at 100 keV, but no explanation of this result was offered. We suppose that the cause for this suppression is the same spin-isospin structure of the scattering amplitude, which leads to the vanishing ρ-meson contribution in the Glauber approach. A qualitative explanation of the vanishing contribution of the TVPC charge-exchange amplitude for the TCPC terms C'_N is the following. For the strong (TCPC) interaction, the charge-exchange amplitude $M_{pn\to np}(\mathbf{q})$ appearing in Fig. 1(a) coincides with the charge-exchange amplitude $M_{np\to pn}(\mathbf{q})$ in Fig. 1(b). In contrast, for the TVPC interaction caused by the g' term, the corresponding charge-exchange amplitudes have the opposite signs due to Eqs. (34). The corresponding deuteron vertices are the same in Figs. 1(a) and 1(b). Taking into account the symmetry with respect to the substitution $\mathbf{q}_1 \leftrightarrow \mathbf{q}_2$ under the sign of the integral over $\mathbf{q}'$ and keeping in mind that spin dependence of the strong NN scattering amplitudes with the C'_N term is identical to that for the TVPC g' term, we find that the double-scattering amplitude in Fig. 1(a) differs from that in Fig. 1(b) only by the sign. Therefore, the sum of these diagrams for the C'_N and g' terms is zero.[1]

The second point is connected with the role of the Coulomb interaction in the cross section $\widetilde{\sigma}$. Being a T-even P-even interaction, the Coulomb pp scattering cannot generate the TVPC amplitude $\widetilde{g}$ within the single-scattering mechanism; therefore its contribution to $\widetilde{g}$ is zero in this approximation. In order to include the Coulomb interaction within the double-scattering mechanism of the Glauber theory, one should replace the pure hadronic T-even P-even pp amplitude M_p given in Eq. (13) by the sum $M_p + \tilde{f}^C_{pp}$, where $\tilde{f}^C_{pp}$ is the properly normalized Coulomb pp-scattering amplitude (16). It is evident from the spin structure of this amplitudes that the Coulomb term is added to the spin-independent term A_p, $A_p \to A_p + \tilde{f}^C_{pp}$ and double-spin terms B_p, G_p, H_p but does not enter into the single-spin terms C_p and C'_p. However, all amplitudes A_p, B_p, G_p, H_p are excluded from the above derived formula for the TVPC amplitude (52) due to the specific spin structure of Eq. (2). As was noted in Sec. IV B the latter statement is also true if the D wave of the deuteron is taken into account. The only factor in Eq. (52) which contains the Coulomb effects is $C'_p(q)$. This is because the Coulomb scattering pp amplitude enters into the spin-independent term A_p, which, in turn, enters into the amplitude C'_p with the relativistic correction factor $q/2m_p$ [30] (m_p is the nucleon mass) $C'_p = C_p + iA_p\frac{q}{2m_p}$. When substituting this amplitude C'_p into Eq. (52) and making integration over the transferred momentum q, one can see that due to the presence of the factor $\sim q^3$ in the integrand the singularity of the Coulomb amplitude (17) at $\theta_{pp} = 0$ does not lead to divergence of the $\widetilde{g}$ amplitude.

Energy dependence of $\widetilde{\sigma}$ is calculated here for the h term in units of the unknown hNN coupling constant $\phi_h = \bar{G}_h/G_h$ with $G_h^2/4\pi = 1.56$ and $m_h = 1.17$ GeV [25]. For the monopole form factor F_{hNN} we used $\Lambda = 2$ GeV [12]. The pN scattering amplitudes C_N and A_N were calculated using the scattering analysis interactive dial-in system (SAID) database [31]. The results obtained with and without allowance for Coulomb effects are shown in Fig. 3. One can see that the magnitude of $\widetilde{\sigma}$ smoothly decreases with increasing beam energy. This is caused be the energy dependence of the strong T-even P-even pN-scattering amplitude. The role of the Coulomb interaction in the null-test signal $\widetilde{\sigma}$ is rather

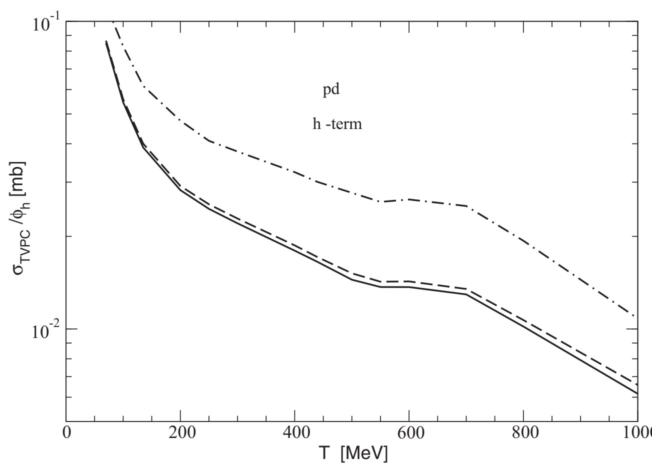

FIG. 3. The calculated energy dependence of the TVPC cross section $|\widetilde{\sigma}|$ for the h term in units of the constant $\phi_h = \bar{G}_h/G_h$ for $m_h = 1.17$ GeV, $\Lambda = 2$ GeV in h_N by Eq. (24) with the Coulomb interaction included (dashed line) and excluded (full). The dot-dashed curve is obtained at $\Lambda \to \infty$, $m_h^2 + \mathbf{q}^2 \to m_h^2$ without accounting for the Coulomb effects.

[1]The TVPC nucleon-nucleon interaction with the g' term can give a nonzero contribution to the pd and nd scattering if this interaction is included into the deuteron wave function. This is evident from Fig. 1 if these TVPC effects are included only into the first (second) deuteron vertex in both Figs. 1(a) and 1(b), but not included into the upper vertices. This dynamics will be investigated in a special paper.

unimportant in the considered interval of energies (Fig. 3). At a beam energy 135 MeV we obtain $\widetilde{\sigma} = 0.039\phi_h$ mb and $\sigma_0 = 78$ mb. Assuming the accuracy of the experiment allows us to measure the ratio $\widetilde{\sigma}/\sigma_0$ at the level $\sim 10^{-6}$, the bound on ϕ_h can be achieved as $\phi_h \leqslant 0.002$. If we neglect some details of the phenomenological h exchange and put $F_{hNN} = 1$ and $(m_h^2 + \mathbf{q}^2)^{-1} \to m_h^2$ in Eq. (24), the energy dependence of $\widetilde{\sigma}$ corresponds to the dot-dashed curve in Fig. 3. In this case we obtain at 135 MeV $\widetilde{\sigma} = 0.063\phi_h$ mb and, therefore, at the same accuracy of the experiment the bound is $\phi_h \leqslant 0.0012$. The g term leads to a very similar energy dependence. Figure 3 shows that the TVPC cross section $\widetilde{\sigma}$ increases with decreasing energy. Perhaps, at energies below$\sim$50 MeV the sensitivity of the experiment [10] to the TVPC effects is higher. However, at these low energies the applicability of the Glauber theory to the spin observables of the pd scattering is not validated, and therefore rigorous calculations like the Faddeev ones are needed.

Let us consider possible false effects in the planned experiment [10]. One source of these effects is connected with the non-zro vector polarization of the deuteron $p_y^d \neq 0$ directed along the vector polarization of the proton beam p_y^p. In this case the term $\sigma_1 p_y^p\, p_y^d$ in Eq. (8) contributes to the asymmetry corresponding to the cases of $p_y^p P_{xz}^d > 0$ and $p_y^p P_{xz}^d < 0$, which is planned to be measured in the TRIC experiment [10]. According to our calculation, at a beam energy of 135 MeV the total cross sections are $\sigma_0^t = 78.5$ mb, $\sigma_1^t = 3.7$ mb, $\sigma_2^t = 17.4$ mb, and $\sigma_3^t = -1.1$ mb. Therefore, the ratio $r = \sigma_1^t/\sigma_0^t$ is ≈ 0.05. If the TRIC project is going to measure the ratio $R_T = \widetilde{\sigma}/\sigma_0$ with an uncertainty about $\leqslant 10^{-6}$ (upper limit for R_T), one can find from the obtained ratio r that the vector polarization of the deuteron p_y^d has to be less than $\approx 2 \times 10^{-6}$. When making this estimation, we assume that the background-to-signal ratio is $p_y^d\, \sigma_1^t/\widetilde{\sigma} \sim 10^{-1}$. The total hadronic polarized cross sections σ_i $(i = 0,1,2,3)$ are calculated here using the optical theorem. The Coulomb effects for these observables can be taken into account along the line of Ref. [32] using the beam acceptance angle.

VI. CONCLUSION

Using the representation of Ref. [18] for the forward elastic pd scattering amplitude and including the phenomenological TVPC term in the most general form, we show on the basis of the optical theorem that this term generates an extra spin-dependent total cross section of the pd scattering.

This additional term is zero if the TVPC interaction is absent and nonzero only in the presence of the TVPC interaction. Earlier this conclusion was found in different representations [11,12]. Obviously, this null-test signal for T-invariance violation is not affected by the initial- or/and final-state interactions, because it is derived from a genuine pd-scattering amplitude considered beyond the perturbation theory. Furthermore, using the Glauber theory we show that (i) the TVPC interaction caused by the ρ-meson exchange does not contribute to the null-test observable $\widetilde{\sigma}$ and (ii) the Coulomb interaction does not lead to divergence of this observable. Numerical calculation of the energy dependence of $\widetilde{\sigma}$ shows that the choice of $\sim$100 MeV made in Ref. [10] is more preferable than $\sim$1 GeV. If the cross section $\widetilde{\sigma}$ will be measured with the planned accuracy [10], the bounds on T-odd coupling constants of the NN interaction can be achieved as $\phi_h \leqslant (1 \div 2) \times 10^{-3}$.

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