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Proton pairing vibrational states around the doubly magic nucleus ^{208}Pb

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Proton pairing vibrational states in ^{208}Pb isotones with $76 \leqslant Z \leqslant 88$ protons are described like neutron pairing vibrational states in lead isotopes with $120 \leqslant N \leqslant 132$ neutrons within the model of pair addition and pair removal phonons. The anharmonicity calculated with three parameters reproduces the known energies within less than 5%. The interaction between the pairing phonons for protons and neutrons is similar in a striking manner.

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The doubly magic nucleus ^{208}Pb is an interesting object used to study nuclear forces. Both non-collective states with dominant one-particle one-hole configuration described by the shell model and collective states are manifest. Among a few hundred known levels [1] most bound states have a simple shell model structure. However, at rather low excitation energies collective states are found. The neutron pairing vibrational state with the excitation energy $E_x = 4868$ keV [2,3] has been known for a long time.

The proton pairing vibrational state was suggested to be at $E_x = 5.26$ MeV [4]. Yet the state at $E_x = 5241$ keV was later identified as the the double octupole state [5–7]. Thus the search for the proton pairing vibrational state was unsuccessful until recently. It was identified at $E_x = 5667$ keV using the Q3D magnetic spectrograph of the Maier-Leibnitz-Laboratorium at Garching (Germany) [8]. The identification is based on the clear excitation in the ^{208}Pb(p,p') reaction with $14.8 < E_p < 18.2$ MeV at scattering angles $20^\circ \leqslant \theta \leqslant 138^\circ$ and the excitation in the ^{208}Pb(d,d') reaction with $E_d = 22$ and 24 MeV.

The knowledge of all negative parity states predicted below $E_x = 6.3$ MeV [9] rules out negative parity. The excitation in the ^{208}Pb(α,α') reaction [10] indicates the spin to be even.

No one-particle one-hole state with the spin of 0^+ is expected at excitation energies $E_x < 10$ MeV. Two-particle two-hole states with the spin of 0^+, besides the pairing vibrational and the double octopole states, are predicted by the shell model above $E_x = 6.0$ MeV [7].

The knowledge of the Coulomb interaction between the protons among six particle-hole configurations completely detected [9] and the prediction of the proton pairing vibrational state to have an excitation energy in the range $5.4 \lesssim E_x \lesssim 5.9$ MeV [4] assigns the spin of 0^+ to the state at $E_x = 5667$ keV [8].

The idea of pairing vibrational excitations results from the concept associated with the collective field that creates or annihilates pairs of nucleons: the monopole mode. This is in contrast to the fields associated with the quadrupole and octupole modes creating particle-hole excitations which therefore conserve the number of particles.

The pairing field is associated with the particle-particle channel of the effective nucleon-nucleon interaction which binds pairs of nucleons into correlated states with zero angular momentum (0^+). The addition (or removal) of such pairs to the closed shells creates excitations which can be repeated; therefore they can be considered as quanta of the pairing vibrational mode or phonons.

Pairing vibrational spectra exist in two modes, namely the creation and destruction of either proton pairs or neutron pairs starting from closed shells. Thus, a pairing vibrational spectrum combines states belonging to different nuclei. In nuclei with many particles outside closed shells a condensate of the correlated nucleon pairs can be created and the phase transition from the normal fluid to the superfluid nuclei takes place [11,12].

The investigation of the pairing field in nuclei is realized in a close interplay of experimental and theoretical work. The decisive contribution to clarify the role of the pairing field in nuclei came from the studies of the two-nucleon transfer reactions. The pairing interaction of nucleons produces strong correlations which enhance the amplitudes of the two-nucleon transfer. Thus, doing two-nucleon transfer reactions is a suitable tool for the search of the collective pairing vibrational excitations.

Two-proton transfer reactions can be realized only with difficulty. In the lead region no two-proton transfer reaction can be envisaged at all. Since the (t,p) and (p,t) reactions are easily studied, mainly neutron pairing vibrations have been investigated in the past.

The theoretical approach to describe the collective pairing vibrational states has been developed by A. Bohr [11–18]. It has been successfully applied to describe the neutron pairing vibrational spectrum around ^{208}Pb. The difference between the experimental energy of the neutron pairing vibrational state in ^{208}Pb and the energy of the state calculated in the harmonic approximation is less than 3% of the excitation energy (115 from 4868 keV).

Naturally, the approach suggested by A. Bohr [13] can be also applied to describe the proton pairing vibrational spectrum around ^{208}Pb. In order to calculate the energies, the binding energies (E_B) of the isotones with $N = 126$ neutrons and $76 \leqslant Z \leqslant 88$ protons have to be known, namely ^{202}Os, ^{204}Pt, ^{206}Hg, ^{208}Pb, ^{210}Po, ^{212}Rn, and ^{214}Ra. Not all binding energies are known, but the recent tables [19] yield interpolated values with a systematic uncertainty of only 400 keV for ^{202}Os and 200 keV for ^{204}Pt.

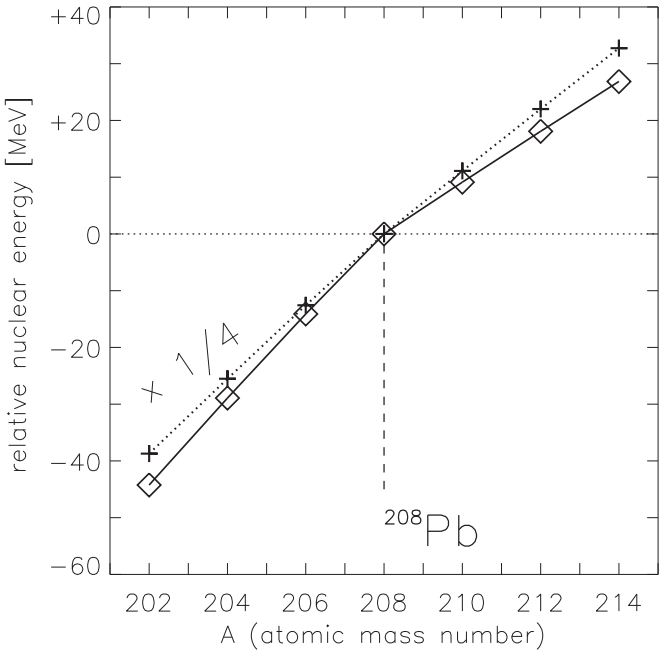

FIG. 1. Binding energies $E^{\rm rel}$ [Eq. (3)] of neutrons (solid lines with diamonds) and protons (dotted lines with crosses, reduced by a factor of 4) relative to ^{208}Pb.

The binding energies include the contribution from the Coulomb interaction between the protons. In order to obtain the proton pairing vibrational energies, it is necessary to separate the pure nuclear contribution from the binding energies E_B, i.e., to subtract the Coulomb energy $E^{\rm Coul}$ between the Z protons (Eq. (2-19) in [20]):

$$E^{\rm nucl}(Z,N,A) = E_B(Z,N,A) + E^{\rm Coul}(Z,N,A), \quad (1)$$

$$E^{\rm Coul}(Z,N,A) = 0.70Z^2(1-0.76Z^{-2/3})A^{-1/3}\ {\rm MeV}. \quad (2)$$

For convenience, energies are calculated relative to ^{208}Pb:

$$E^{\rm rel}(Z,N,A) = E^{\rm nucl}(Z,N,A) - E^{\rm nucl}(82,126,208); \quad (3)$$

they are presented in Fig. 1. The binding energies of the lead isotopes are shown for comparison, too. Here the Coulomb energy can be neglected; there is a small effect related to the change of the radius in the isotopes [Eq. (2)].

In order to obtain the proton pairing vibrational spectrum, it is convenient to add to $E^{\rm rel}$ [Eq. (3), Fig. 1] a term linear in $(Z-82)$,

$$\mathcal{E}(Z,N,A) = \lambda(Z-82) - E^{\rm rel}(Z,N,A), \quad (4)$$

where λ is determined such that

$$\mathcal{E}(^{206}{\rm Hg}) = \mathcal{E}(^{210}{\rm Po}). \quad (5)$$

We obtain the spectrum presented in Fig. 2(a). Here the solid lines with diamonds indicate the values of $\mathcal{E}$ corresponding to the nuclear ground states.

In analogy to the consideration for neutrons with the doubly magic nucleus ^{208}Pb as the basic state (Fig. 2(b), see also [11–14]), this spectrum can be interpreted in the following way. The states with fewer protons, $Z < 82$ (left from ^{208}Pb), are the multiphonon states constructed by the pair removal mode; the states with more protons, $Z > 82$ (right from ^{208}Pb), are the multiphonon states constructed by the pair addition mode.

The dotted lines in Fig. 2(a) indicate the energies of the proton pairing vibrational states determined in the harmonic approximation using the value of $\mathcal{E}(^{206}{\rm Hg})$ as the energy of the pair removal and $\mathcal{E}(^{210}{\rm Po})$ as the energy of pair addition phonons.

In order to obtain the energies of the excited proton vibrational states the combination of the pair removal and the pair addition phonons is considered. Every state with the pairing vibrational nature is characterized by two numbers: the number of the pair removal phonons n_- and the number of the pair addition phonons n_+ [14]; in Fig. 2 they are marked by $[n_-n_+]$, $n = 0,1,2,\ldots$.

In the harmonic approximation the proton pairing vibrational state of ^{208}Pb is $[1_-1_+]$. Its energy of 5873 keV is only 4% higher than the experimental energy.

Comparing the experimental energies [Eq. (4)] with the energies calculated in the harmonic approximation (Fig. 2), the interaction of the proton pairing phonons becomes visible. Namely, the repulsion between the pair removal phonons is stronger than between the pair addition phonons. For the pairing vibration state in ^{208}Pb, in contrast, there is a weak attraction between the pair removal and pair addition phonons.

The effect of anharmonicity is taken into account by the Hamiltonian [14]

$$H = \hbar\omega(b_-^\dagger b_- + b_+^\dagger b_+) + V_{+-}b_+^\dagger b_-^\dagger b_- b_+ + \frac{V_{++}}{2}\, b_+^\dagger b_+^\dagger b_+ b_+ + \frac{V_{--}}{2}\, b_-^\dagger b_-^\dagger b_- b_-, \quad (6)$$

$$\begin{aligned} \text{where}\quad \hbar\omega &= \mathcal{E}(^{206}{\rm Hg}) = \mathcal{E}(^{210}{\rm Po}) = 2992\ {\rm keV},\\ V_{+-} &= 5667 - 2\hbar\omega = -206\ {\rm keV},\\ V_{++} &= \mathcal{E}(^{212}{\rm Rn}) - 2\hbar\omega = +776\ {\rm keV},\\ \text{and}\quad V_{--} &= \mathcal{E}(^{204}{\rm Pt}) - 2\hbar\omega = +1495\ {\rm keV}. \end{aligned} \quad (7)$$

The parameters V_{+-}, V_{++}, and V_{--} are fixed by the experimental energy of the proton pairing vibrational state in ^{208}Pb [8] and the ground state energies of ^{204}Pt and ^{212}Rn [19].

We can compare the predictions of the Hamiltonian [Eq. (6)] with the known data for the ground states of ^{202}Os and ^{214}Ra [19]. The calculated energies are indicated by dashed lines in Fig. 2. The agreement between the calculated and the experimental energies is much improved by including the anharmonic terms in Eq. (6); for ^{214}Ra the difference is reduced from 25% to 1%, for ^{202}Os from 45% to the uncertainties of about 5%.

The anharmonicity shows a striking similarity between neutrons and protons (Fig. 2). The pairing for neutrons around ^{208}Pb is described similarly to the pairing for protons [Eq. (7)] by

$$\begin{aligned} \hbar\omega &= \mathcal{E}(^{206}{\rm Pb}) = \mathcal{E}(^{210}{\rm Pb}) = 2493\ {\rm keV},\\ V_{+-} &= 4868 - 2\hbar\omega = -115\ {\rm keV},\\ V_{++} &= \mathcal{E}(^{212}{\rm Pb}) - 2\hbar\omega = +161\ {\rm keV},\\ \text{and}\quad V_{--} &= \mathcal{E}(^{204}{\rm Pb}) - 2\hbar\omega = +713\ {\rm keV}. \end{aligned} \quad (8)$$

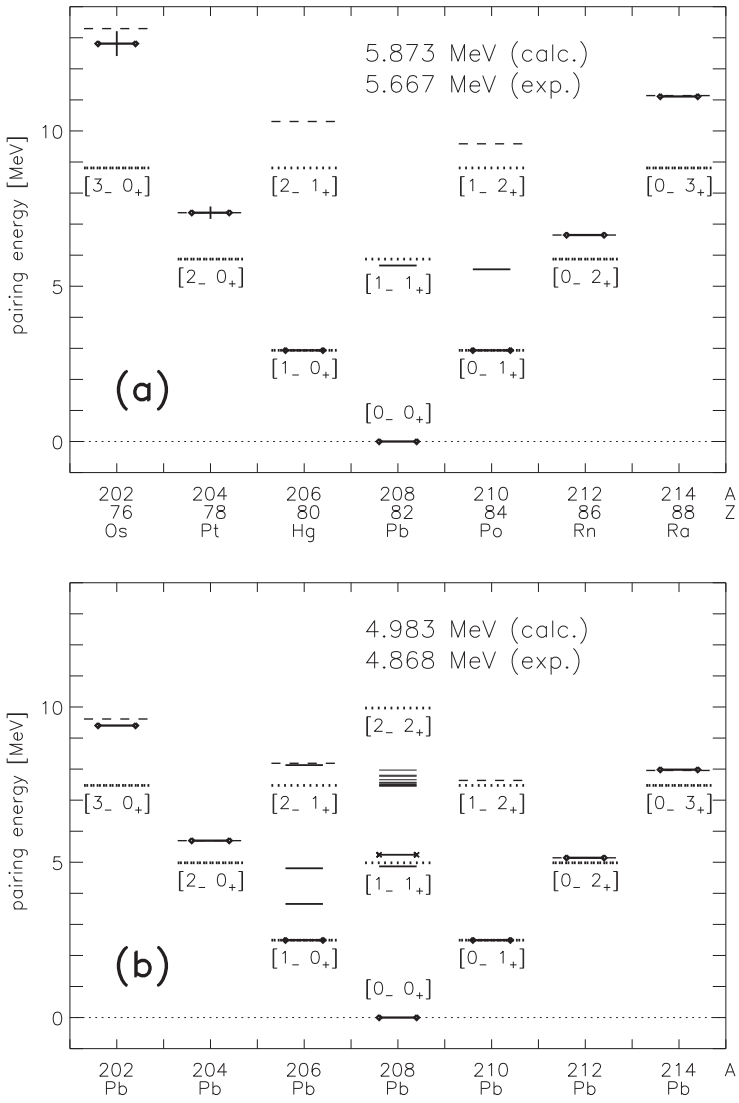

FIG. 2. Pairing vibration states determined by experiment (solid lines with diamonds) and calculated in the harmonic approximation [Eq. (6) with vanishing anharmonic terms V_{+-}, V_{++}, and V_{--}, dotted lines] (a) for ^{208}Pb isotones and (b) for lead isotopes (adapted from Fig. 6-62 in [14]). The dashed lines show the prediction in the anharmonic approximation [Eq. (6)]. The parameter V_{+-} [Eqs. (6)–(8)] derives from the energies of the pairing state $[1_-1_+]$ shown at top.

The ratio of the shell gaps in ^{208}Pb for protons and neutrons between the filled orbits and the empty orbits (Fig. 3-3 in [20]) agrees with the ratio of the neutron and proton pairing vibrational energies $\hbar\omega$ [Eqs. (7) and (8)] almost exactly:

$$E^{\rm gap}(\pi, ^{208}{\rm Pb}) = 4214\ {\rm keV},$$
$$E^{\rm gap}(\nu, ^{208}{\rm Pb}) = 3431\ {\rm keV},$$
$$\frac{E^{\rm gap}(\pi, ^{208}{\rm Pb})}{E^{\rm gap}(\nu, ^{208}{\rm Pb})} \Big/ \frac{\hbar\omega(\pi)}{\hbar\omega(\nu)} = 1.02. \qquad (9)$$

As Fig. 1 shows, there is a cusp at $A = 208$ in the dependence of the relative binding energies $E^{\rm rel}$ [Eq. (3)] on the atomic weight number A. The nonlinearity of the function $E^{\rm rel}(A)$ manifests the anharmonicity with the parameters V_{+-}, V_{++}, and V_{--} [Eq. (6)].

Some other 0^+ states in the nuclei around ^{208}Pb with $76 \leqslant Z \leqslant 88$ or $120 \leqslant N \leqslant 132$ [21] and in ^{208}Pb [1] are known. They are shown by solid lines in Fig. 2. Among four known 0^+ states in ^{206}Pb, the state at $E_x = 5637$ keV is well known as the three-phonon state, the $[2_-1_+]$ state [22]. The 0^+ state at $E_x = 2609$ keV in ^{210}Po consists mainly of the configuration $f^2_{7/2}$ [23].

The double octupole 0^+ state in ^{208}Pb at $E_x = 5241$ keV [5,6] is shown in Fig. 2(b) by a solid line marked with crosses. Near the predicted $[2_-1_+]$ and $[1_-2_+]$ states, a bundle of nine states with the assigned spin of 0^+ is reported in ^{208}Pb (and, in addition, several more states with tentative assignment of spin 0^+), just above the neutron threshold $S(n) = 7368$ keV [1]; they are shown in Fig. 2(b) by thin lines. The isoscalar giant monopole resonance in ^{208}Pb is reported at $E_x = 13.9$ MeV [1,24] but not shown in Fig. 2.

Quite generally, we conclude that in heavy nuclei the forces between protons and neutrons responsible for the pairing are qualitatively similar.

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