Quartic isospin asymmetry energy of nuclear matter from chiral pion-nucleon dynamics

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Based on a chiral approach to nuclear matter, the quartic term in the expansion of the equation of state of isospin-asymmetric nuclear matter is calculated. The contributions to the quartic isospin asymmetry energy $A_4(k_f)$ arising from 1π exchange and chiral 2π exchange in nuclear matter are calculated analytically together with three-body terms involving virtual $\Delta(1232)$ isobars. From these interaction terms one obtains at saturation density $\rho_0 = 0.16\ \text{fm}^{-3}$ the value $A_4(k_{f0}) = 1.5$ MeV, more than three times as large as the kinetic energy part. Moreover, iterated 1π exchange exhibits components for which the fourth derivative with the respect to the isospin asymmetry parameter δ becomes singular at $\delta = 0$. The genuine presence of a nonanalytical term $\delta^4 \ln|\delta|$ in the expansion of the energy per particle of isospin-asymmetric nuclear matter is demonstrated by evaluating an s-wave contact interaction at second order.

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I. INTRODUCTION AND SUMMARY

The determination of the equation of state of isospin-asymmetric nuclear matter has been a longstanding goal shared by both nuclear physics and astrophysics [1]. Usually one assumes a parabolic form for the energy per nucleon at zero temperature, $\bar{E}_{\text{as}}(\rho_p,\rho_n) = \bar{E}(\rho) + A_2(\rho)\,\delta^2 + \mathcal{O}(\delta^4)$, where $\rho = \rho_p + \rho_n$ is the total nucleon density and $\delta = (\rho_n - \rho_p)/\rho$ the isospin asymmetry related to unequal proton and neutron densities $\rho_p \neq \rho_n$. The validity of the quadratic approximation has been verified with good numerical accuracy from isospin-symmetric nuclear matter ($\delta = 0$) up to pure neutron matter ($\delta = 1$) by most of the existing nuclear many-body theories using various interactions [2,3]. Nonetheless, it has been shown consistently in numerous studies [4] that for some properties of neutron stars, such as the proton fraction at beta equilibrium, the core-crust transition density and the critical density for the direct Urca process to occur, even a very small quartic isospin asymmetry energy $A_4(\rho)$ (multiplied with δ^4 in the expansion of the energy per nucleon) can make a big difference.

Given the fact that all the available numerical solutions of the nuclear many-body problem confirm the validity of the quadratic approximation, the quartic isospin asymmetry $A_4(\rho)$ should be rather small. However, in the recent work by Cai and Li [5], which employs an empirically constrained isospin-dependent single-nucleon momentum distribution and the equation of state of pure neutron matter near the unitary limit, a significant quartic term of $A_4(\rho_0)^{(\text{kin})} = (7.2 \pm 2.5)$ MeV has been found from the kinetic energy of interacting nucleons. This value amounts to about 16 times the free Fermi gas prediction; see Eq. (2). On the other hand, recent calculations of isospin-asymmetric nuclear matter based on chiral low-momentum interactions and many-body perturbation theory [6] lead to a small value of $A_4(\rho_0) \simeq 1$ MeV.

The purpose of the present paper it to give a prediction for the density-dependent quartic isospin asymmetry energy $A_4(k_f)$ in the chiral approach to nuclear matter developed in Refs. [7,8]. In this approach the long- and medium-range nucleon-nucleon (NN) interactions arising from multipion exchange are treated explicitly and a few parameters encoding the relevant short-distance dynamics are adjusted to bulk properties of nuclear matter. A systematic expansion in small momenta is performed up to three-loop order. Single-particle potentials [8], quasiparticle interactions [9], the thermodynamic behavior of nuclear matter at finite temperatures [10], and the density dependence of the in-medium quark condensate [11] follow then as predictions in that framework (see also the recent review article [12]).

The present paper is organized as follows. In Sec. II, analytical expressions are given for the contributions to the quartic isospin asymmetry energy $A_4(k_f)$ as they arise from 1π exchange and chiral 2π exchange. The three-nucleon interaction generated by 2π exchange and excitation of a virtual $\Delta(1232)$ isobar is considered as well. These interaction contributions lead at saturation density $\rho_0 = 0.16\ \text{fm}^{-3}$ (or $k_{f0} = 263$ MeV) to the (small) value $A_4(k_{f0}) = 1.5$ MeV, which amounts to about three times the kinetic energy part. Moreover, in the course of the calculation one encounters components of the second-order 1π exchange whose representation of the fourth derivative with the respect to δ at $\delta = 0$ is singular. In Sec. III, the generic presence of a non-analytical term $\delta^4 \ln|\delta|$ in the expansion of the energy per particle of isospin-asymmetric nuclear matter is demonstrated by calculating in closed form the second-order contribution from an s-wave contact interaction. Clearly, after having established its existence, the nonanalytical term $\delta^4 \ln|\delta|$ should be included in future fits of the equation of state of (zero-temperature) isospin-asymmetric nuclear matter.

II. ONE-PION AND TWO-PION EXCHANGE CONTRIBUTIONS

In this section the expressions for the quartic isospin asymmetry $A_4(k_f)$ are given as they arise from one-pion and two-pion exchange diagrams following Refs. [7,8]. Isospin-asymmetric (spin-saturated) nuclear matter is characterized by

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different proton and neutron Fermi momenta, $k_{p,n} = k_f(1 \mp \delta)^{1/3}$. Expanding the energy per particle at fixed nucleon density $\rho = 2k_f^3/3\pi^2$ in the isospin asymmetry parameter δ up to fourth order gives

$$\bar{E}_{\rm as}(k_p,k_n) = \bar{E}(k_f) + \delta^2 A_2(k_f) + \delta^4 A_4(k_f) + \mathcal{O}(\delta^6), \quad (1)$$

with $A_2(k_f)$ the (usual) quadratic isospin asymmetry energy. The density-dependent expansion coefficients $\bar{E}(k_f), A_2(k_f)$, and $A_4(k_f)$ are viewed as functions of the Fermi momentum k_f, since in this form they emerge directly from the calculation. The first contribution to $A_4(k_f)$ comes from the relativistically improved kinetic energy $T_{\rm kin}(p) = p^2/2M - p^4/8M^3$, and it reads

$$A_4(k_f)^{(\rm kin)} = \frac{k_f^2}{162M}\left(1 + \frac{k_f^2}{4M^2}\right), \quad (2)$$

with $M = 939$ MeV the average nucleon mass. The corresponding value at nuclear matter saturation density $\rho_0 = 0.16$ fm^{-3} (or at Fermi momentum $k_{f0} = 263$ MeV) is $A_4(k_{f0})^{(\rm kin)} = 0.464$ MeV. The (positive) relativistic $1/M^3$ correction in Eq. (2) amounts to about 2%.

For the treatment of two-body interactions that depend on the momentum transfer $|\vec{p}_1 - \vec{p}_2|$, the following expansion formulas for six-dimensional integrals over two Fermi spheres are most helpful:

$$\int \frac{d^3p_1 d^3p_2}{(2\pi)^6} F(|\vec{p}_1 - \vec{p}_2|)[\theta(k_p - |\vec{p}_1|)\theta(k_p - |\vec{p}_2|) + \theta(k_n - |\vec{p}_1|)\theta(k_n - |\vec{p}_2|)]$$
$$= \frac{2k_f^6}{3\pi^4}\int_0^1 dz\left\{\left[z^2(1-z)^2(2+z) + \frac{\delta^2 z^3}{3}\right]F(2zk_f) + \frac{\delta^4 k_f}{162}[F'(2k_f) - 7z^4 F'(2zk_f)]\right\}, \quad (3)$$

$$\int \frac{d^3p_1 d^3p_2}{(2\pi)^6} F(|\vec{p}_1 - \vec{p}_2|)\theta(k_p - |\vec{p}_1|)\theta(k_n - |\vec{p}_2|)$$
$$= \frac{k_f^6}{3\pi^4}\int_0^1 dz\left\{\left[z^2(1-z)^2(2+z) + \frac{\delta^2 z}{3}(z^2-1)\right]F(2zk_f) + \frac{\delta^4 k_f}{162}(8z^2 - 1 - 7z^4)F'(2zk_f)\right\}. \quad (4)$$

The z-dependent weighting functions at order δ^2 and δ^4 have been obtained by applying several partial integrations. The contribution of the 1π-exchange Fock diagram to the quartic isospin asymmetry energy reads

$$A_4(k_f)^{(1\pi)} = \frac{g_A^2 m_\pi^3}{(36\pi f_\pi)^2}\left\{\left(4u + \frac{21}{8u}\right)\ln(1+4u^2) - 2u^3 - \frac{33u}{4} - \frac{u(9+44u^2)}{4(1+4u^2)^2}\right.$$
$$\left. + \frac{m_\pi^2}{M^2}\left[2u^5 + 2u^3 + \frac{3u}{8} - u^3\ln(1+4u^2) - \frac{u(3+16u^2)}{8(1+4u^2)^2} - \frac{3u^2}{2}\arctan 2u\right]\right\}, \quad (5)$$

with the dimensionless variable $u = k_f/m_\pi$. The occurring physical parameters are: nucleon axial-vector coupling constant $g_A = 1.3$, (neutral) pion mass $m_\pi = 135$ MeV, and pion decay constant $f_\pi = 92.4$ MeV. The second line in Eq. (5) gives the relativistic $1/M^2$ correction. It amounts at density $\rho_0 = 0.16$ fm^{-3} to a reduction of the static 1π-exchange contribution by about 16%.

Next in the chiral expansion comes the iterated (second-order) 1π exchange. With two medium insertions $\frac{1}{2}(1+\tau_3)\theta(k_p - |\vec{p}_i|) + \frac{1}{2}(1-\tau_3)\theta(k_n - |\vec{p}_i|)$ one gets a Hartree contribution of the form

$$A_4(k_f)^{(\rm H2)} = \frac{g_A^4 M m_\pi^4}{(24\pi)^3 f_\pi^4}\left\{10u^3 - \frac{61u}{2} + \frac{200u^2+49}{6u}\ln(1+4u^2) - \frac{u(13+60u^2)}{6(1+4u^2)^2} - \frac{128u^2}{3}\arctan 2u\right\}, \quad (6)$$

and the corresponding Fock exchange-term reads

$$A_4(k_f)^{(\rm F2)} = \frac{g_A^4 M m_\pi^4}{(12\pi)^3 f_\pi^4}\left\{\frac{u}{8} - \frac{u^3}{3} - \frac{u}{12(1+2u^2)} - \frac{u}{24(1+u^2)} + u^4\arctan u + \frac{u^2(2+11u^2+16u^4)}{6(1+2u^2)^2}[\arctan u - \arctan 2u]\right.$$
$$\left. + \int_0^u dx\,\frac{21x^2 - 16u^2}{6u(1+2x^2)}[(1+8x^2+8x^4)\arctan x - (1+4x^2)\arctan 2x]\right\}. \quad (7)$$

Pauli-blocking effects at second order are included through diagrams with three (isospin-asymmetric) medium insertions [7]. Here only the factorizable Fock contribution is considered for which the energy denominator gets canceled by factors from the momentum-dependent πN vertices (see Eqs. (11) and (26) in Ref. [7]). Its contribution to the quartic isospin asymmetry energy can be represented as a one-parameter integral, $A_4(k_f)^{(\rm fac)} = g_A^4 M m_\pi^4 (12\pi f_\pi)^{-4}\int_0^u dx\, I(x,u)$, where the lengthy integrand $I(x,u)$ involves the function $\ln[1+(u+x)^2] - \ln[1+(u-x)^2]$ and its square. The corresponding value at saturation density is $A_4(k_{f0})^{(\rm fac)} = -1.35$ MeV, thus counterbalancing most of the Fock term $A_4(k_{f0})^{(\rm F2)} = 1.70$ MeV without Pauli-blocking written in Eq. (7). For the nonfactorizable pieces the representation of the fourth derivative with respect to δ at $\delta = 0$ includes singularities of the form $(u-x)^{-\nu}$, $\nu = 1,2$. When subtracting these singular terms from the integrand only very small numerical values are obtained for the nonfactorizable Hartree contribution. In the case of the quadratic isospin asymmetry energy $A_2(k_{f0})$ one finds that the nonfactorizable pieces (see Eqs. (24) and (26) in Ref. [7]) tend to cancel each other almost completely, as $(-11.6+12.0)$ MeV. Therefore, one can expect that the omission of the nonfactorizable pieces does not change much the final result for the quartic isospin asymmetry energy $A_4(k_f)$. However, the observation that the iterated 1π exchange has components with a singular representation of their fourth derivative with respect to δ at $\delta = 0$, indicates that the expansion in Eq. (1) becomes nonanalytic beyond the quadratic order δ^2. This feature is demonstrated in Sec. III by calculating in closed form the second-order contribution from an s-wave contact interaction.

One continues with the contribution of the irreducible 2π exchange to the quartic isospin asymmetry energy. Using a twice-subtracted dispersion relation for the 2π-exchange NN potential in momentum-space and the master formulas in Eqs. (3) and (4), one obtains

$$A_4(k_f)^{(2\pi)} = \frac{1}{81\pi^3}\int_{2m_\pi}^{\infty} d\mu \bigg\{ \mathrm{Im}(V_C + 2\mu^2 V_T)\bigg[\frac{7\mu k_f}{4} - \frac{2k_f^5}{3\mu^3} - \frac{\mu k_f^3(7\mu^2+36k_f^2)}{2(\mu^2+4k_f^2)^2} - \frac{7\mu^3}{16k_f}\ln\left(1+\frac{4k_f^2}{\mu^2}\right)\bigg] + \mathrm{Im}(W_C + 2\mu^2 W_T)\bigg[\frac{2k_f^5}{\mu^3} + \frac{k_f^3}{\mu} + \frac{21\mu k_f}{4} - \frac{\mu k_f^3(7\mu^2+36k_f^2)}{2(\mu^2+4k_f^2)^2} - \frac{\mu}{16k_f}(21\mu^2+32k_f^2)\ln\left(1+\frac{4k_f^2}{\mu^2}\right)\bigg]\bigg\}, \quad (8)$$

where Im $V_{C,T}$ and Im $W_{C,T}$ are the spectral functions of the isoscalar and isovector central and tensor NN amplitudes, respectively. These imaginary parts are composed of the functions $\sqrt{\mu^2-4m_\pi^2}$, $\sqrt{\mu^2-4m_\pi^2}/(\mu^2-4m_\pi^2+4\Delta^2)$ and $\arctan(\sqrt{\mu^2-4m_\pi^2}/2\Delta)$, with $\Delta = 293$ MeV the delta-nucleon mass splitting. Note that due to the implemented subtractions the k_f expansion of $A_4(k_f)^{(2\pi)}$ in Eq. (8) starts with the power k_f^7. A short-distance contribution proportional to k_f^5 is supplemented by the subtraction constants

$$A_4(k_f)^{(\mathrm{sc})} = \frac{10k_f^5}{(3M)^4}\left(\frac{2B_5}{3} - B_{n,5}\right), \quad (9)$$

with the parameters $B_5 = 0$ and $B_{n,5} = -3.58$ adjusted in Ref. [8] to the empirical nuclear matter saturation point and quadratic isospin asymmetry energy $A_2(k_{f0})^{(\mathrm{emp})} = 34$ MeV.

Finally, in order to complete the small-momentum expansion in the $\Delta(1232)$-full chiral effective field theory up to three-loop order [8], one considers the long-range three-nucleon interaction generated by 2π exchange and virtual excitation of a $\Delta(1232)$ isobar. The corresponding three-body Hartree contribution reads

$$A_4(k_f)^{(\Delta)} = \frac{g_A^4 m_\pi^6 u^2}{\Delta(6\pi f_\pi)^4}\left\{\left(\frac{16u^2}{3}+\frac{21}{4}\right)\ln(1+4u^2) - \frac{4u^4}{3} - \frac{41u^2}{3} - \frac{2u^2(11+99u^2+236u^4)}{3(1+4u^2)^3}\right\}, \quad (10)$$

while the associated three-body Fock term can be represented as $g_A^4 m_\pi^6 (12\pi f_\pi)^{-4}\Delta^{-1}\int_0^u dx\, J(x,u)$, where the lengthy integrand $J(x,u)$ involves the functions $\arctan(u+x) + \arctan(u-x)$ and $\ln[1+(u+x)^2] - \ln[1+(u-x)^2]$. Note that the three-body contact-term proportional to ζ introduced additionally in Ref. [8] does not contribute to the quartic isospin asymmetry energy $A_4(k_f)$.

Summing up all the calculated contributions, one obtains the result for the density-dependent quartic isospin asymmetry energy $A_4(k_f)$ of nuclear matter as shown in Fig. 1 in the density region $0 < \rho < 2\rho_0 = 0.32\ \mathrm{fm}^{-3}$. The predicted value at saturation density $\rho_0 = 0.16\ \mathrm{fm}^{-3}$ is $A_4(k_{f0}) = 1.49$ MeV and this amounts to 3.2 times the free Fermi-gas part $A_4(k_{f0})^{(\mathrm{kin})} = 0.464$ MeV. For orientation, the density dependence of the kinetic part $A_4(k_f)^{(\mathrm{kin})}$ is shown separately by the dashed line in Fig. 1. It is worth mentioning that interaction contributions to $A_4(k_f)$ start (at least) with the power k_f^5. The density dependence of the full line in Fig. 1 is to a good approximation $\rho^{5/4}$. For comparison, the variety of phenomenological Skyrme forces give a quartic isospin asymmetry energy $A_4(k_f)^{(\mathrm{Sk})} = k_f^2/162M + k_f^5[3t_1(1+x_1) + t_2(1-x_2)]/972\pi^2$ with values typically smaller than 1 MeV at saturation density [13]. Moreover, one can study the sensitivity of the outcome for $A_4(k_{f0})$ on the fitting procedure of the short-range parameter $B_{n,5}$. According to Eq. (46) in Ref. [8] a variation of the quadratic isospin asymmetry energy $A_2(k_f)$ gets scaled down (via $B_{n,5}$) by a factor -27 for the quartic isospin asymmetry energy $A_4(k_f)$. Therefore, taking the lower value $A_2(k_{f0})^{(\mathrm{emp})} = 28$ MeV as a benchmark would lead to a somewhat larger (predicted) value of $A_4(k_{f0}) = 1.7$ MeV. The estimated uncertainty of 0.2 MeV is presumably conservative, since a variation of $A_2(k_{f0})^{(\mathrm{emp})}$ can also be linked to the short-range parameter $B_{n,3}$, which does not at all affect the quartic isospin asymmetry energy $A_4(k_f)$.

III. s-WAVE CONTACT INTERACTION TO SECOND ORDER

The analysis of the Pauli-blocking corrections to the second-order (iterated) 1π exchange has indicated that

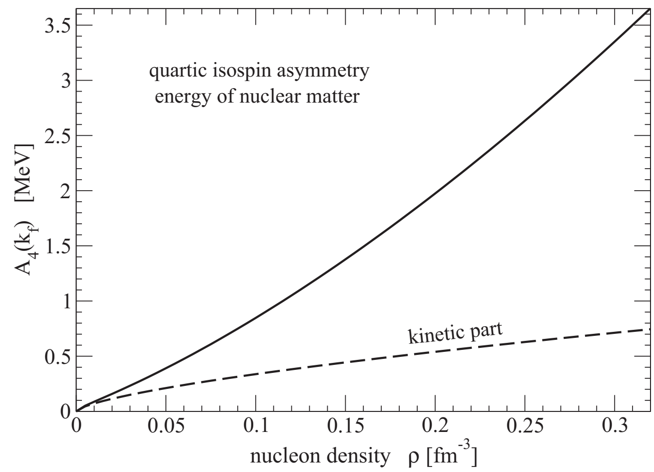

FIG. 1. Quartic isospin asymmetry $A_4(k_f)$ as a function of the nucleon density $\rho = 2k_f^3/3\pi^2$.

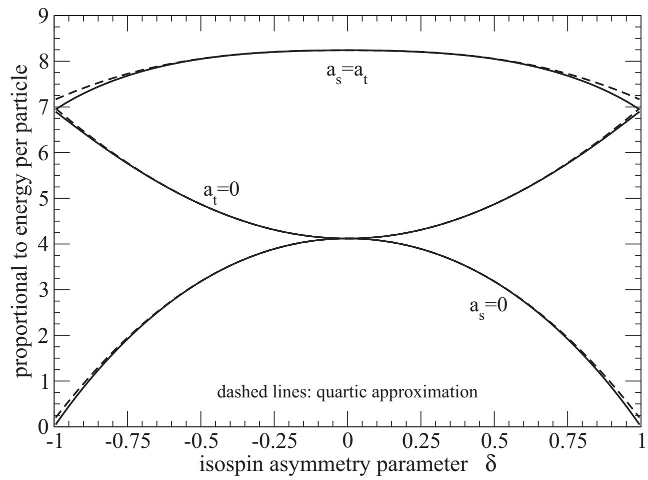

FIG. 2. Dependence of the second-order energy per particle $\bar{E}_{\rm as}(k_p,k_n)^{(2{\rm nd})}$ on the isospin asymmetry δ. Three different choices for the scattering lengths, $a_s = a_t$, $a_t = 0$, and $a_s = 0$, are considered.

nonanalytical terms may occur in the δ expansion of the energy per particle of isospin-asymmetric nuclear matter beyond the quadratic order. In the extreme case there could be a cubic term $|\delta|^3$, which after all is even under the exchange of protons and neutrons: $\delta \to -\delta$. In order to clarify the situation, one considers an s-wave contact interaction,

$$V_{\rm ct} = \frac{\pi}{M}[a_s + 3a_t + (a_t - a_s)\vec{\sigma}_1 \cdot \vec{\sigma}_2], \tag{11}$$

and examines it in second-order many-body perturbation theory. For this simple interaction, the occurring nine-dimensional principal-value integrals over three Fermi spheres with (at most) two different radii, k_p or k_n, can be solved in closed analytical form. The pertinent function to express the result in the isospin-asymmetric configuration of interest is

$$35\int_0^1 dz\,(z - z^4)\left\{2xz + (x^2 - z^2)\ln\frac{x+z}{|x-z|}\right\} = \frac{x}{2}(15 + 33x^2 - 4x^4) + \frac{1}{4}(42x^2 - 15 - 35x^4)\ln\frac{x+1}{|x-1|} + 2x^7\ln\frac{x^2}{|x^2-1|}, \tag{12}$$

where the variable $x > 0$ is set to a ratio of Fermi momenta, $[(1+\delta)/(1-\delta)]^{\pm 1/3}$ or 1. Note that the function defined in Eq. (12) has at $x = 1$ the value $22 - 4\ln 2$. Combining the second-order Hartree and Fock diagrams generated by $V_{\rm ct}$ according to their spin- and isospin-factors and performing the expansion in powers of δ, one obtains the following result for the energy per particle:

$$\bar{E}_{\rm as}(k_p,k_n)^{(2{\rm nd})} = \frac{k_f^4}{5\pi^2 M}\left\{\frac{3}{7}(a_s^2 + a_t^2)(11 - 2\ln 2) + \frac{4\delta^2}{3}\left[a_s^2(3 - \ln 2) - a_t^2(2 + \ln 2)\right] + \frac{\delta^4}{81}\left[a_s^2\left(10\ln\frac{|\delta|}{3} + 2\ln 2 - \frac{41}{6}\right) + a_t^2\left(30\ln\frac{|\delta|}{3} + 2\ln 2 + \frac{3}{2}\right)\right] + \mathcal{O}(\delta^6)\right\}. \tag{13}$$

The crucial and novel feature which becomes evident from this expression is the presence of the nonanalytical logarithmic term $\delta^4\ln(|\delta|/3)$. Interestingly, the corresponding coefficient is three times as large in the spin-triplet channel as in the spin-singlet channel. For comparison the first-order contribution of the s-wave contact interaction $V_{\rm ct}$ reads $\bar{E}_{\rm as}(k_p,k_n)^{(1{\rm st})} = k_f^3[-a_s - a_t + \delta^2(a_t - a_s/3)]/2\pi M$, without any higher powers of δ. Note that the sign convention for the scattering lengths $a_{s,t}$ has been chosen here such that positive values correspond to attraction. As a check the same results at first and second order have been derived by using the alternative (and equivalent) form of the s-wave contact interaction, $V'_{\rm ct} = \pi[3a_s + a_t + (a_s - a_t)\vec{\tau}_1 \cdot \vec{\tau}_2]/M$.

In Fig. 2 the dependence of the second-order energy per particle $\bar{E}_{\rm as}(k_p,k_n)^{(2{\rm nd})}$ on the isospin asymmetry parameter δ is shown for three different choices of the s-wave scattering lengths: $a_s = a_t$, $a_t = 0$, and $a_s = 0$. In each case the full line shows the exact result and the (nearby) dashed line gives the expansion in powers of δ truncated at fourth order according to Eq. (13). One observes that these expansions [including the nonanalytical logarithmic term $\delta^4\ln(|\delta|/3)$] reproduce the full δ dependence very well over the whole range $-1 \leqslant \delta \leqslant 1$. Note also that the prefactor $k_f^4 a_{s,t}^2/5\pi^2 M$ of dimension energy has been scaled out in Fig. 2.

If one performs for the second-order energy density the fourth derivative with respect to δ at $\delta = 0$ under the integral, then one encounters integrands with singularities of the form $(1-z)^{-\nu}$, $\nu = 1,2$. The origin of these singularities, or in the proper treatment the nonanalytical term $\delta^4\ln(|\delta|/3)$, lies in the energy denominator of second-order diagrams. For an infinite (normal) many-fermion system the energy spectrum has a vanishing gap between bound states in the Fermi sea and excited states in the continuum. Such a gapless energy spectrum causes a singularity, respectively a nonanalyticity, if small asymmetries of the Fermi levels of two components are analyzed with too high resolution.

In summary, it has been demonstrated that the nonanalytical term $\delta^4 \ln(|\delta|/3)$ will be generically present in calculations of isospin-asymmetric nuclear matter when going beyond the mean-field approximation. Therefore, a term $\delta^4 \ln|\delta|$ should be included in future fits of the equation of state of (zero-temperature) isospin-asymmetric nuclear matter and its role should be further examined.

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