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Spallation reaction versus heavy-ion fusion: Fission excitation functions as a fundamental probe of presaddle nuclear dissipation

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Due to nuclear dissipation, fission cross sections drop with respect to standard statistical model predictions. Using the stochastic dynamical model of fission, we calculate the drop of fission cross sections, σ_f^{drop}, as a function of presaddle dissipation strength (β) under two contrasting initial conditions for produced ^{224}Th compound nuclei: (i) high excitation energy but low angular momentum (available in spallation reactions) and (ii) low excitation energy but high angular momentum (available in heavy-ion fusion). We find that the former type of conditions not only significantly increases the influence of friction on fission cross sections but also substantially enhances the sensitivity of σ_f^{drop} to β. Our findings suggest that on the experimental side, to accurately obtain information of presaddle friction strength with fission excitation function, it is optimal to choose the spallation reaction approach induced by energetic protons as a way to populate excited nuclear systems.

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I. INTRODUCTION

Experimental explorations of the decay properties of hot nuclei have been performed using heavy-ion fusion reactions. It has been found in numerous experiments [1–9] that when excitation energy deposited into the compound system is increased, the measured prescission particle multiplicity exceeds that estimated by standard statistical models (SMs). This discrepancy has been demonstrated [10–21] to arise from dissipation effects that are not accounted for in theoretical calculations. Nuclear dissipation retards the fission process that affects the competition of fission with evaporation, two principle decay modes of highly excited nuclei. While intense experimental and theoretical investigations on nuclear dissipation have been made, the nature and magnitude of nuclear dissipation are still uncertain and currently under vigorous debate. In particular, how to accurately determine the strength of presaddle dissipation (β) is the focus of controversy [22].

While prescission particles have been widely used to survey nuclear dissipation, they are a less direct signature of presaddle dissipation, because they depend on both pre- and postsaddle dissipation. To extract reliable and precise information about β, it is important to explore those experimental signals sensitive to presaddle dissipation effects only. While several observables (e.g., evaporation residue cross section [23,24] and its spin distribution [25,26]) were proposed, these evaporation-channel-related quantities are an indirect indicator of presaddle dissipation as compared to those provided via fission channels. As the most direct consequence of dissipation, fission is hindered; that is, fission probability (or fission cross section) is reduced. Therefore, the fission cross section is identified as the most sensitive and fundamental probe of presaddle dissipation [22,27–29].

On the experimental side, besides conventional heavy-ion fusion, two new approaches, i.e., energetic proton- and antiproton-induced spallation reactions [30–32] and peripheral relativistic heavy-ion collisions [33,34], have recently been applied to produce excited nuclei, which can have a high excitation energy E^* (up to 1 GeV) but a low angular momentum (ℓ_c), in contrast with that in the heavy-ion fusion approach where excited nuclei produced have a low E^* ($\sim$120 MeV) but a large ℓ_c ($>50\hbar$). They thus generate widespread interest in the potential of exploring fission properties with particle multiplicity and fission cross section [35–37].

In this context, the present work is devoted to a comparative study of two different types of initial conditions for the formed hot nuclear system, namely (high E^*, low ℓ_c) and (high ℓ_c, low E^*), in order to survey which condition can provide a stronger constraint on the determination of β with fission cross sections. Towards that goal, we compare the sensitivity of fission cross sections to β for different E^* and ℓ_c in the framework of Langevin models. The stochastic approach [10,12–14,16–18,21,38,39] has been utilized to successfully reproduce a great number of experimental data on fission excitation functions and prescission particle multiplicities for a lot of fissioning systems over a wide range of excitation energy, angular momentum, and fissility.

II. THEORETICAL MODEL

An account of the combination of the dynamical Langevin equations with a statistical decay model (CDSM) is given. We refer the reader to Refs. [10,11] for more details. As pointed out in Refs. [11,40], the driving force of a hot system is not simply the negative gradient of the conservative force, but should also contain a thermodynamic correction; therefore, the dynamic part of the CDSM is described by the Langevin equation that is expressed by entropy. We employ the following one-dimensional overdamped Langevin equation [11] to perform the trajectory calculations:

$$\frac{dq}{dt} = \frac{T}{M\beta}\frac{dS}{dq} + \sqrt{\frac{T}{M\beta}}\Gamma(t). \tag{1}$$

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Here q is the dimensionless fission coordinate and is defined as half the distance between the center of mass of the future fission fragments divided by the radius of the compound nucleus, M is the inertia parameter [41], and β is the dissipation strength. The temperature in Eq. (1) is denoted by T and $\Gamma(t)$ is a fluctuating force with $\langle\Gamma(t)\rangle = 0$ and $\langle\Gamma(t)\Gamma(t')\rangle = 2\delta(t-t')$. The driving force of the Langevin equation is calculated from the entropy:

$$S(q,E^*) = 2\sqrt{a(q)[E^* - V(q)]}, \tag{2}$$

where E^* is the excitation energy of the system. Equation (2) is constructed from the Fermi-gas expression with a finite-range liquid-drop potential [42] $V(q)$ that includes q-dependent surface, Coulomb, and rotation energy terms. In our dynamical calculations we use $\{c,h,\alpha\}$ [43] parametrization of the compound nucleus shape. Since only symmetrical fission is considered, the parameter describing the asymmetry of the shape is set to $\alpha = 0$ [11,24]. The deformation coordinate q is obtained by the relation $q(c,h) = (3c/8)\{1 + \frac{2}{15}[2h + (c-1)/2]c^3\}$ [10,44], where c and h correspond to the elongation and neck degrees of the freedom of the nucleus, respectively.

In constructing the entropy, the deformation-dependent level density parameter is used:

$$a(q) = a_1 A + a_2 A^{2/3} B_s(q), \tag{3}$$

where A is the mass number, and $a_1 = 0.073$ and $a_2 = 0.095$ are taken from Ignatyuk *et al.* [45]. B_s is the dimensionless surface area (for a sphere $B_s = 1$) which can be parametrized by the analytical expression [46]

$$B_s(q) = \begin{cases} 1 + 2.844(q-0.375)^2, & \text{if} \quad q < 0.452, \\ 0.983 + 0.439(q-0.375), & \text{if} \quad q \geqslant 0.452. \end{cases} \tag{4}$$

In the CDSM, evaporation of prescission light particles along Langevin fission trajectories from their ground state to their scission point has been taken into account. The emission width of a particle, Γ_ν $(\nu = n,p,\alpha)$, is calculated with the formula given in Ref. [47]. After each emission act of a particle, the intrinsic energy $[E^*_{\text{intr}} = E^* - V(q)]$, the entropy, and the temperature in the Langevin equation are recalculated and the dynamics is continued.

Light-particle evaporation is coupled to the fission mode by a Monte Carlo method. The present simulation allows for the discrete emission of light particles. The procedure is the following [41]: We calculate the decay widths for light particles at each Langevin time step τ. Then the emission of particle is allowed by asking along the trajectory at each time step τ whether a random number ζ is less than the ratio of the Langevin time step τ to the decay time $\tau_{\text{dec}} = \hbar/\Gamma_{\text{tot}}$: $\zeta < \tau/\tau_{\text{dec}}$ $(0 \leqslant \zeta \leqslant 1)$, where Γ_{tot} is the sum of light particles decay widths. If this is the case, a particle is emitted and we ask for the kind of particle ν $(\nu = n,p,\alpha)$ by a Monte Carlo selection with the weights $\Gamma_\nu/\Gamma_{\text{tot}}$. This procedure simulates the law of radioactive decay for the different particles.

The CDSM describes the competition between fission and evaporation as follows: a dynamical trajectory will either reach the scission point, in this case it is counted as a fission event, or if the intrinsic excitation energy E^*_{intr} for a trajectory still inside the saddle $(q < q_{sd})$ reaches a value $E^*_{\text{intr}} < \min(B_f,\ B_\nu)$ (B_f is the height of the fission barrier and B_ν is the binding energy of the particle ν) the event is counted as an evaporation residue event. We do not follow the subsequent cooling of the evaporation residues which proceeds exclusively by γ-ray emission. After the fission probability flow over the fission barrier attains its quasistationary value, the decay of the compound system is described by the statistical part of the CDSM. When entering the statistical branch, we calculate the particle decay widths Γ_ν again [47] and the fission width [10] and use a standard Monte Carlo cascade procedure, which allows for multiple emissions of light particles and higher-chance fission. In case fission is decided there, one switches again to the Langevin equation for computing the evolution from saddle to scission. After each emission act we again recalculate the intrinsic energy, and continue the cascade until the intrinsic energy is $E^*_{\text{intr}} < \min(B_f,\ B_\nu)$. In this case we count the event as evaporation residue and do not follow the de-excitation process further. Fission and evaporation residue probability as well as prescission particle emission probability are calculated by countering the number of corresponding fission, evaporation residue, and emitted particle events registered in the dynamical and statistical branch of the CDSM. To accumulate sufficient statistics, 10^7 Langevin trajectories are simulated.

For starting a trajectory an orbit angular momentum value is sampled from the fusion spin distribution, which reads

$$\frac{d\sigma(\ell)}{d\ell} = \frac{2\pi}{k^2}\frac{2\ell+1}{1+\exp[(\ell-\ell_c)/\delta\ell]}. \tag{5}$$

The parameters ℓ_c and $\delta\ell$ are the critical angular momenta for fusion and diffuseness, respectively.

III. RESULTS AND DISCUSSION

Besides the nuclear dissipation strength, which has been found to have a strong effect on fission process, including fission time scale, angular momentum (ℓ_c), and the excitation energy (E^*) of the fissioning nucleus play a role in fission and affect the time scale for fission. Since these three factors influence the fission process, it means that employing a suitable condition of ℓ_c and E^* will favor a more accurate probe of β with fission cross section. Thus, it is necessary to investigate the evolution of the sensitivity of fission cross section to presaddle dissipation strength β for different ℓ_c and E^*, which is the purpose of the present work.

The essential difference between standard statistical model and diffusion model is that the latter accounts for the dissipation effects in fission. So a new element, i.e., nuclear dissipation, enters into the description for the competition between evaporation and fission channels as a hot nucleus decays. Moreover, ℓ_c, E^*, and some other factors (that are not related to nuclear dissipation) affect fission both in the absence and in the presence of nuclear dissipation. Hence, calculating the deviation (due to nuclear dissipation) of fission cross section with respect to the standard statistical model prediction is a sensitive method to reveal presaddle dissipation effects.

Dissipation delays fission by about 10^{-20}s in which light particles could be emitted. This causes a deviation of the measured fission cross section (σ_f) from that predicted by

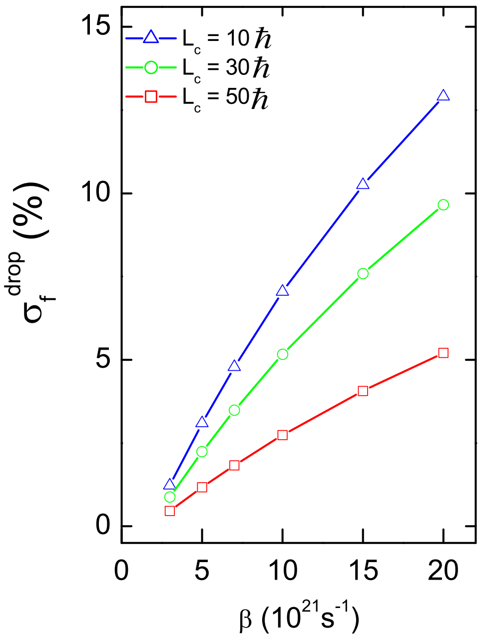

FIG. 1. (Color online) Dynamical drop of the fission cross section of ^{224}Th relative to that predicted by SMs as a function of the presaddle dissipation strength β at excitation energy $E^* = 150$ MeV and at three angular momenta $\ell_c = 10\hbar$, $30\hbar$, and $50\hbar$.

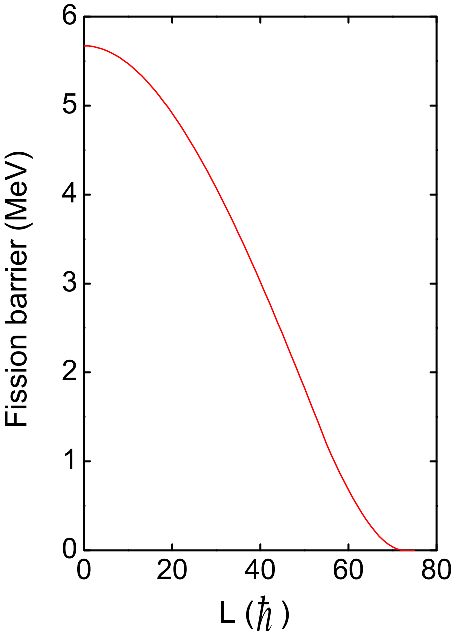

FIG. 2. (Color online) Fission barrier of the system ^{224}Th as a function of angular momentum.

SMs, and the amplitude of the deviation depends strongly on β. A study for the deviation thus provides a sensitive method of determining β. To that end, we adopt a definition similar to that suggested by Lazarev *et al.* [48], and define the relative drop of σ_f calculated by SMs over the value by taking into account the dissipation and fluctuations of collective nuclear motion

$$\sigma_f^{\text{drop}} = \frac{\langle \sigma_f^{\text{SM}} \rangle - \langle \sigma_f^{\text{dyn}} \rangle}{\langle \sigma_f^{\text{SM}} \rangle}. \tag{6}$$

Figure 1 shows the angular momentum effect on the fission cross section as a probe of β. Two prominent features are observed. First, the symbol △ is always above the symbol □ for any β, meaning that a low nuclear spin can increase the dissipation effects on fission cross sections. Another feature is that the slope of curve σ_f^{drop} vs β, which reflects the sensitivity of fission cross sections to friction, becomes steeper with decreasing ℓ_c. The physical understanding for the enhanced sensitivity of fission cross sections to friction at low ℓ_c is as follows: Both fission barrier and friction can affect fission cross sections. At high angular momentum, fission barriers drop (Fig. 2), favoring fission. Thus, while friction effects modify fission cross sections, the fission cross section estimated by SMs, σ_f^{SM}, becomes larger with an increase of angular momentum, which dominates the amplitude of fission cross section at high ℓ_c. Consequently, high spins lead to a smaller σ_f^{drop} [see Eq. (6)]. A picture like Fig. 1 is seen at other excitation energies and, hence, it is not displayed here.

Previous works concerning β [10,14,16,28] employed fission excitation functions from heavy-ion reactions, where the formed compound nuclei (CNs) have a high spin ($>50\hbar$). But Fig. 1 illustrates that a low spin can significantly increase the sensitivity of fission cross sections to friction. The finding suggests that experimentally, when using fission cross sections to probe presaddle dissipation, it is desirable to produce CNs with a low spin. In addition, in heavy-ion fusion, due to a high spin of the populated CNs, the shape of CNs at equilibrium ground state is distorted; that is, it deviates apparently from a spherical shape. However, the decaying systems populated by light ions have a shape that is close to a spherical one because of the low spin involved, a prominent advantage for describing subsequent de-excitation, as indicated in Ref. [49]. Thus, producing and employing fission excitation function data induced by light ions such as protons can put more severe constraints on β. This could provide more reliable values of the friction parameter.

Figure 3 presents the role of excitation energy in pinning down friction with fission cross sections. It can be seen that dissipation effects on fission cross sections are amplified with increasing E^*. Apart from that, the slope of curve σ_f^{drop} vs β is larger with a rise in E^*. Specifically speaking, as β varies from 3 zs^{-1} (1 zs $= 10^{-21}$ s) to 20 zs^{-1}, σ_f^{drop} at $E^* = 100$ MeV changes by 9.8%, which is far below that at $E^* = 350$ MeV where the change arrives at 36.1%, indicating an enhanced sensitivity of fission cross sections to friction at high E^*. The reason for the influence of excitation energy is that the entropy, a crucial quantity in the Langevin model, is a function of excitation energy [see Eq. (2)]. As a result, a variation in E^* can alter the location of saddle point, which affects the magnitude of the ratio of level-density parameter at saddle to that at equilibrium ground state, a_f/a_n, an important parameter controlling the competition between fission and particle emission. Friction and a_f/a_n have opposite effects on

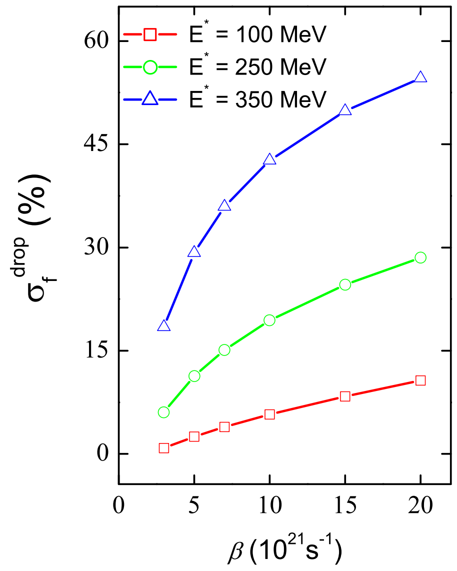

FIG. 3. (Color online) Dynamical drop of the fission cross section of ^{224}Th relative to that predicted by SMs as a function of the presaddle dissipation strength β at angular momentum $\ell_c = 10\hbar$ and at three excitation energies $E^* = 100$, 250, and 350 MeV.

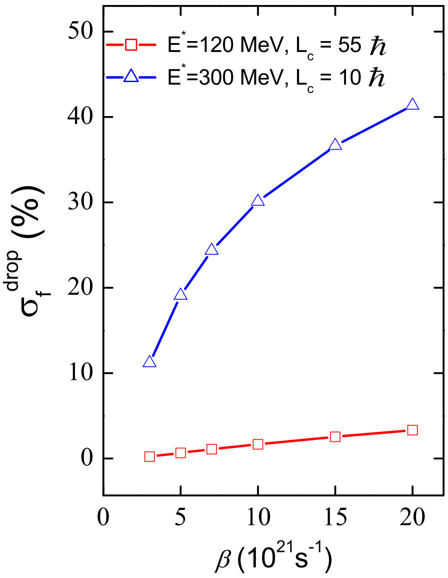

FIG. 4. (Color online) Comparison of dynamical drop of the fission cross section of ^{224}Th relative to statistical-model values vs presaddle dissipation strength β for ^{224}Th nuclei between case (i) $E^* = 120$ MeV and $\ell_c = 55\hbar$ and case (ii) $E^* = 300$ MeV and $\ell_c = 10\hbar$.

fission cross section. A greater a_f/a_n can increase fission rates. It has been noted [10,50] that the distance between saddle point and equilibrium ground state gets closer with E^*, meaning a smaller a_f [see Eq. (3)] and correspondingly, a smaller a_f/a_n at a high E^*. The excitation-energy dependence of the level-density parameters has also been indicated recently [51]. A low a_f/a_n weakens its influence on fission channels; in other words, it amplifies the effect of friction on fission cross section.

We carry out calculations at different angular momenta and find that the conclusion remains the same as that drawn from Fig. 3.

Figure 3 exhibits that making use of a highly excited system can significantly raise the sensitive dependence of fission cross section on friction. As shown in a number of works [35,37], spallation reactions can yield a decaying system with a high energy, indicating that it is a feasible experimental way for investigating the characteristics of fission processes.

Nuclear systems populated in both spallation reactions and heavy-ion fusion have a marked difference in angular momentum and excitation energy. The two types of reactions thus offer an opportunity for examining the role of these two important parameters in exploiting decay features of hot nuclei, particularly concerning dissipation properties in fission. We calculate σ_f^{drop} as a function of β for two cases: (i) $E^* = 120$ MeV and $\ell_c = 55\hbar$ and (ii) $E^* = 300$ MeV and $\ell_c = 10\hbar$. It is easily noted that case (ii) contains low ℓ_c and high E^* as compared to case (i). So, based on the observation made in Figs. 1 and 3, a greater sensitivity of fission cross section to friction can be expected for case (ii). The expectation is confirmed in Fig. 4.

In heavy-ion fusion approach, it is quite difficult to simultaneously obtain the conditions of a low ℓ_c and a very high E^* (>250 MeV) for the populated CNs. This is because with increasing bombarding energy of the projectile, on the one hand, the spin of the formed CNs rises rapidly (up to $75\hbar$ [52–56]) and, on the other hand, as a consequence of high incident energy (which can trigger other reaction channels such as multifragmentation [57]), the excitation energy deposited into CNs via fusion mechanism is not very high. But the results of Fig. 4 demonstrate that a combination of these two conditions can place a stronger constraint on presaddle friction. Moreover, energetic proton-induced spallation reaction can generate decaying systems [30–32,37] with a small angular momentum and a rather large excitation energy, suggesting that it constitutes a powerful tool for exploring nuclear dissipation with fission cross sections.

Due to pre-equilibrium emission, spallation reactions lead to an excitation energy distribution in the struck nucleus. Results presented in Fig. 3 show that high E^* favors a determination of β, thus as far as the present research aim is concerned, one only needs to select those events that satisfy requirements for E^* (e.g., $E^* > 250$ MeV) from this excitation energy distribution for fission studies; one does not need to characterize the distribution. Specifically speaking, experimentally, by detecting both fission fragments and various light particles at the same time one can select those events with a larger excitation energy for fission studies. Theoretically, intranuclear cascade (INCL) model [58] can predict the excitation energy distribution in the struck nucleus caused by the pre-equilibrium particle emission. In this model, all output information including the residue hot nuclei generated is recorded in an event-by-event format. The case is similar for the quantum molecular dynamics (QMD) model [59], which is also widely employed in the study of spallation reactions. It treats pA collisions in a dynamical way and also stores the resulting E^* and other related information for the generated residue hot nucleus with an event-by-event format. By imposing conditions on E^* for all output events provided by INCL and QMD calculations by means of data analysis software called ROOT [60], it is convenient to select those events that satisfy requirements for E^* (e.g., $E^* > 250$ MeV) for fission studies. Thus, spallation reactions could be used in investigating fission.

Spallation reactions are considered as a two-step processes, i.e., collision process between protons and target nuclei and decay process of residue nuclei generated in the former process, which can be treated by INCL (or QMD) model and decay model, respectively. While our prediction, that is, decaying systems having high E^* and low ℓ_c populated in spallation reactions are more preferable conditions for probing β with fission cross section, is solely dependent on Langevin model calculation and does not rely on INCL (or QMD) model, a meta-analysis of fission cross section can strengthen the prediction. In previous works, INCL-GEMINI [35,37,58] and QMD-GEMIN [59] models were applied to make such a meta-analysis for spallation reaction. However, to more precisely explore dissipation effects via the spallation reaction approach, it is necessary to develop a new framework based on INCL-Langevin model, whose difference with INCL-GEMINI model is in the choice of decay model used to deal with de-excitation of the residue hot nuclei. As far as our present research purpose is concerned, the difference is critical. The reason is as follows: While statistical decay model (e.g., GEMINI) uses a fission width that is modified to include Kramers' correction in order to consider dissipation effects, in comparison with the description of the Langevin model, it only partially describes the dissipation effects on fission. In addition, the Langevin model considers a large number of dynamical features, such as the time dependence of the fission width that is neglected in statistical models. Thus, for extracting a precise β value from experimental data, the use of Langevin fission width in calculation is preferable to that of Kramers' fission width. Moreover, it also indicates the necessity of formulating the INCL-Langevin approach.

We note that the way that the INCL model couples with the Langevin model is analogous to that with the statistical model GEMINI; that is, the INCL model provides relevant information on the generated residue nuclei that will be used in subsequent decay calculation. But unlike the statistical model GEMINI, which performs a Monte Carlo–type calculation, the Langevin model makes a dynamical trajectory calculation. As a result, the computation time required in the INCL-Langevin approach is greater than that required in the INCL-GEMINI approach. So, to better perform a large-scale calculation for spallation reaction with the new approach, developing more effective numerical computation methods is necessary. Furthermore, given that spallation reactions can provide more favorable conditions to stringently constrain β with fission cross section and that a meta-analysis can strengthen the present results that are obtained by the Langevin model, theoretical efforts towards developing the INCL-Langevin approach are therefore urgently needed. Work along this direction is under way.

For a spallation reaction, it is currently described by INCL model followed by de-excitation model. Similar to the INCL-GEMINI framework, in the framework of the INCL-Langevin model, the INCL model calculation can give information of the related parameters (i.e., E^*, A, Z, etc.) characterizing the generated residue nucleus, which is recorded as an event. Using the data analysis software ROOT [60], one can get the event number of the generated residue nucleus having the specified values of E^*, A, Z, etc. and its production cross section is calculated by the countered event number corresponding to it times a numerical factor that is given by geometrical cross section for the pA collision divided by the total number of runs; see Refs. [35,37,58,61] for more details. The Langevin model then calculates the fission probability of the generated residue hot nucleus. So, using the information of the production cross section and fission probability obtained, one can get fission cross section for this struck nucleus with the specified E^*.

In previous analyses of fission cross section induced in spallation reactions performed with the INCL-GEMINI model [35], all events are used to analyze presaddle dissipation effects. However, it is shown in Fig. 3 that high-energy conditions (e.g., $E^* > 250$ MeV) can significantly enhance the sensitivity of fission cross section to β. It suggests that when one uses the INCL-GEMINI model to carry out a meta-analysis for fission cross section, selecting and analyzing those events having high E^* will favor a more accurate determination of β.

IV. SUMMARY AND CONCLUSIONS

In the framework of stochastic models we have investigated the drop of fission cross sections with respect to SM values caused by friction effects, $\sigma_f^{\rm drop}$, with presaddle friction strength β for different angular momenta and excitation energies. It has been found that the sensitivity of $\sigma_f^{\rm drop}$ to β is significantly increased at low spin and high excitation energy. Furthermore, our Langevin calculations have illustrated that $\sigma_f^{\rm drop}$ shows a greater sensitivity to β under the conditions of (high E^*, low ℓ_c) than under the conditions of (low E^*, high ℓ_c), two contrasting conditions that are respectively provided in spallation reactions and heavy-ion fusion. The conclusions are helpful for the choice of the decaying system to be investigated. Thus, they provide a good orientation for future experiments and theoretical efforts. Specifically, the conclusions suggest that experimentally, to accurately probe information of presaddle dissipation by measuring fission excitation functions, it is optimal to choose energetic proton-induced spallation reaction approach to populate a decaying system with low spin and high energy.

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