# Energies and widths of $T = 1$ single-particle states in $^{14}$O and $^{14}$N

H. T. Fortune

*Department of Physics and Astronomy, University of Pennsylvania, Philadelphia, Pennsylvania 19104, USA*

(Received 11 April 2015; revised manuscript received 19 May 2015; published 11 June 2015)

I used a simple potential model to compute energies and widths of single-particle states in $^{14}$O and their corresponding $T = 1$ states in $^{14}$N, using information for the parent states in $^{14}$C as input. Agreement is reasonable, but some discrepancies exist.

DOI: 10.1103/PhysRevC.91.064306 PACS number(s): 21.10.Dr, 21.10.Jx, 21.10.Sf, 27.20.+n

## I. INTRODUCTION

Long ago, a variety of shell-model calculations were performed for $^{14}$C. Cohen and Kurath [1] did the normal-parity states for most $p$-shell nuclei. True [2] took $^{12}$C to be an inert core and allowed two nucleons to occupy the $p_{1/2}$ orbital and the $sd$ shell. Hsieh and Horie [3] computed the non-normal parity states. Lie [4] combined weak coupling (in the sense of Ellis and Engeland [5]) and shell models and calculated states of both parities, allowing up to six nucleons in the $sd$ shell. More recently, an ambitious $p$-$sd$ calculation [6] was performed for many nuclei from $^{10}$B to $^{23}$O. They presented energy-level diagrams, but no numerical values that are useful in the present context. Of course, all the states of $^{14}$C have analogs in $^{14}$N, which also contains a set of $T = 0$ states. Most of the calculations included these $T = 0$ states, but they are not of interest here, and I will not mention them further.

Of special interest in $^{14}$C are the four lowest negative-parity states [7], which are well described [8,9] as the states expected from coupling $s$ and $d$ neutrons to the $1/2^-$ ground state (g.s.) of $^{13}$C. [Throughout, I use *s, d*, and *d'* to denote the $2s_{1/2}$, $1d_{5/2}$, and $1d_{3/2}$ orbitals, respectively.] Their spectroscopic factors in the reaction $^{13}$C$(d,p)$ range from 0.7 to 1.0 [10], reinforcing the idea of a dominant single-particle structure. They were treated as single-particle states in a recent work on $s$ and $d$ neutron states in light nuclei [11]. The mirrors of these states in $^{14}$O should then consist of $s$ and $d$ protons coupled to the g.s. of $^{13}$N. These states in $^{14}$C and $^{14}$O could have quite different energies because of the different behavior of the Coulomb energy for $s$ and $d$ nucleons. The aim here is to use the available information concerning these four negative-parity states in $^{14}$C as input to a potential model that allows calculation of the energies and widths of their mirror states in $^{14}$O. I will also compute the properties of the analogs in $^{14}$N.

## II. MODEL AND CALCULATIONS

I use a single-particle potential model that has been successful in accounting for the properties of mirror states in many light nuclei. Whenever detailed wave functions are available, the model correctly reproduces the mirror energy differences to within about 30 keV. In the model, a nucleon interacts with the nucleus through a Woods–Saxon potential, having geometrical parameters $r_0$, $a = 1.26$, 0.60 fm. For a proton, the interaction also includes the Coulomb potential of a uniformly charged sphere with $r_{0c} = 1.40$ fm. For a given state in $^{14}$C, the first step of the procedure is to vary the potential-well depth to reproduce the neutron separation energy of that state as $^{14}\mathrm{C} = {}^{13}\mathrm{C} + n$. For most of the calculations reported here, I use only the g.s. of $^{13}$C, but I will present some results obtained with inclusion of multiple cores. The second step is then to use this well depth (with the Coulomb potential added) to compute the proton energy for the mirror state as $^{14}\mathrm{O} = {}^{13}\mathrm{N} + p$. These proton energies are then converted to excitation energies in $^{14}$O and compared with experimental values.

### A. $^{14}$O

Relevant states in $^{14}$O are listed in Table I, where I include energies and widths from the latest compilation [7] and from two later experiments [12–14]. For later use I have computed the weighted average of the excitation energies, but I will consider the various width values separately. The compilation does not list a $0^-$ or $2^-$ state in the relevant region, but one state listed as tentative turned out later to be the $2^-$ state. Ball and Cerny [15] first suggested the $2^-$ assignment, later confirmed by Refs. [13,14]. The ($^3$He, $t$) experiment did not report a $0^-$ state, but a broad bump near 5.6 MeV is evident in one of their spectra, and it is probably $0^-$ because of its large width and because that energy is close to the one found in the $^{13}\mathrm{N} + p$ elastic experiments. Reference [14] confirmed the $0^-$ assignment and width of Ref. [13] and reported a smaller uncertainty in the $0^-$ width (45 keV, compared to 100). That 45 keV uncertainty [14] may be an underestimate. The resonance is very weak, the fit is relatively poor, and part of the resonance is obscured by a detector dead layer. Also, they reported a much smaller width for the $3^-$ state—25(3) compared with 42(2). However, visual inspection of the resonance fits in Ref. [14] suggests that a larger width might provide a better fit.

The first four negative-parity states in $^{14}$C and results for their mirrors in $^{14}$O are listed in Table II. The larger shifts of the $0^-$ and $1^-$ states between $^{14}$C and $^{14}$O exhibit the well-known Thomas–Ehrman effect for $s$-wave nucleons [16]. It is encouraging that the computed single-particle $1^-$ energy is below the actual one, because (as noted below) any neglected components in its wave function will have smaller shifts.

For the $1^-$ state, I have performed calculations for several core + nucleon combinations. Three of them are listed in Table III. All four of these negative-parity states in $^{14}$C are

 

TABLE I. Experimental energies (MeV) and widths (keV) of $s$ and $d$ single-particle states in $^{14}$O from various sources.

| $J^\pi$ | Compilation[a] | | $^{14}$N($^3$He,$t$)[b] | | $^{13}$N + $p$ elastic[c] | | Wt. ave. |
|---|---|---|---|---|---|---|---|
| | $E_x$ | Γ | $E_x$ | Γ | $E_x$ | Γ | $E_x$ |
| $1^-$ | 5.173(10) | 38.1(18) | 5.178(10) | 37(14) | 5.159(10) | 42(4) | 5.170(6) |
| $3^-$ | 6.272(10) | 103(6) | 6.284(9) | 50(6) | 6.285(12) | 42(2)[d] | 6.280(6) |
| $0^-$ | | | 5.6? | Broad | 5.71(2) | 400(100)[e] | 5.71(2) |
| $2^-$ | (6.79(30)) | | (6.762(30)) | 107(40) | 6.767(11) | 90(5) | 6.769(10) |

[a]Reference [7].
[b]Reference [12].
[c]Reference [13], unless otherwise noted.
[d]Reference [14] reports 25(3) keV.
[e]Reference [14] reports 400(45) keV.

TABLE II. Energies (MeV) of $s$ and $d$ single-particle states in $^{14}$C and $^{14}$O.

| | $^{14}$C | | | $^{14}$O | | |
|---|---|---|---|---|---|---|
| $J^\pi$ | $E_x$ | $\ell$ | $E_n$ | $E_p$ (calc.) | $E_x$ (calc.) | $E_x$ (expt.)[a] |
| $1^-$ | 6.094 | 0 | −2.083 | 0.422 | 5.050 | 5.170(6) |
| $3^-$ | 6.728 | 2 | −1.449 | 1.556 | 6.184 | 6.280(6) |
| $0^-$ | 6.903 | 0 | −1.274 | 1.050 | 5.678 | 5.71(2) |
| $2^-$ | 7.341 | 2 | −0.836 | 2.096 | 6.724 | 6.769(10) |

[a]Weighted averages from Table I.

TABLE III. Computed energies (MeV) of $1^-$ state of $^{14}$O for various cores.

| $E_x$ ($^{14}$C) | Core | $E_x$ (core) | $\ell$ | $E_n$ | $E_p$ (calc.) | $E_x$ (core) | $E_x$ ($^{14}$O) | $S$ (Lie) |
|---|---|---|---|---|---|---|---|---|
| 6.094 | g.s. | 0 | 0 | −2.083 | 0.422 | 0 | 5.050 | 0.846 |
| 6.094 | $3/2^-$ | 3.685 | 0 | −5.768 | −2.691 | 3.511 | 5.448 | 0.137 |
| 6.094 | g.s. | 0 | 2 | −2.083 | 0.992 | 0 | 5.620 | <0.02 |

TABLE IV. Energies (MeV), widths (keV) (from Table I), and spectroscopic factors of $s$ and $d$ single-particle states in $^{14}$O.

| $J^\pi$ | $E_x$ (expt.) | $E_p$ (expt.) | $\Gamma_{sp}$ | $\Gamma_{expt}$ | $S = \Gamma_{expt}/\Gamma_{sp}$ | $S$ (mirror)[a] |
|---|---|---|---|---|---|---|
| $1^-$ | 5.170 | 0.542 | 54 | 38.1(18) | 0.71(3) | 0.75 |
| $3^-$ | 6.280 | 1.652 | 53 | 103(6) | 1.94(11) | 0.65 |
| | | | | 50(6) | 0.94(11) | |
| | | | | 42(2) | 0.79(4) | |
| | | | | 25(3) | 0.47(6) | |
| $0^-$ | 5.71 | 1.16 | 650 | 400(100)[b] | 0.62(16) | 1.02 |
| $2^-$ | 6.769 | 2.141 | 130 | 107(40) | 0.82(31) | 0.72 |
| | | | | 90(5) | 0.69(4) | |

[a]Reference [10].
[b]The width of 400(45) keV [14] produces $S = 0.62(7)$, about 2.3$\sigma$ below $S(d,p)$.

TABLE V. Calculated and experimental energies (MeV) for $1^-$ and $0^-$ states in $^{14}$O.

| $J^\pi$ | Calculated | MED fit[a] | | |
|---|---|---|---|---|
| | Potential model | Linear | Quadratic | Experimental |
| $1^-$ | 5.050 | 5.140 | 5.155 | 5.170(6) |
| $0^-$ | 5.678 | 5.69 | 5.70 | 5.71(2) |

[a]Using fit parameters from Ref. [18] for mirror energy differences (MED).

TABLE VI. Calculated and experimental energies (MeV) for selected $T = 1$ negative-parity states in $^{14}$N.

| $J^\pi$ | $^{14}$C | | | $^{13}$C$+p$ | | $^{13}$N$+n$ | $^{14}$N ($T = 1$) | |
|---|---|---|---|---|---|---|---|---|
| | $E_x$[a] | $\ell$ | $E_n$ | $E_p$ | $E_x$ | $E_x$ | $E_x$ (calc.) | $E_x$ (expt.)[a] |
| $1^-$ | 6.094 | 0 | −2.083 | 0.057 | 7.608 | 8.471 | 8.039 | 8.062 |
| $3^-$ | 6.728 | 2 | −1.449 | 1.133 | 8.684 | 9.105 | 8.894 | 8.907(3) |
| $0^-$ | 6.903 | 0 | −1.274 | 0.642 | 8.193 | 9.280 | 8.737 | 8.776(7) |
| $2^-$ | 7.341 | 2 | −0.836 | 1.677 | 9.228 | 9.718 | 9.473 | 9.509(3) |

[a]Reference [7].

primarily of the structure $^{13}$C(g.s.) $+ n$, with $\ell = 0$ or 2 for the neutron. For $1^-$, the largest impurity configuration should be an $s_{1/2}$ neutron coupled to the $3/2^-$ state at $E_x = 3.685$ MeV in $^{13}$C. Another possible admixture for $1^-$ is $^{13}$C(g.s.) $\times d'$. Lie estimated the primary component to have a strength of 85%, with 14% for $3/2^- \times s$, and less than 2% for $1/2^- \times d'$. The weakness of the latter is understandable because the $d_{3/2}$ strength should lie 6 to 7 MeV higher in $^{14}$C. With my computed energies and Lie's $1^-$ wave function, the predicted excitation energy is 5.11 MeV, to be compared with the experimental value of 5.17 MeV. With only the two largest components, I would reproduce the experimental energy with 69% $1/2^- \times s$ and 31% $3/2^- \times s$.

For the $3^-$ state, calculations that included components with excited cores would also increase slightly the predicted energy in $^{14}$O. Such configuration mixing is known for this $J^\pi$, because a higher $3^-$ state at 9.8 MeV has some single-particle strength [10]. The fact that $S(d,p)$ is smallest for $3^-$ is consistent with such mixing.

Table IV lists the experimental widths from various sources for the four relevant states in $^{14}$O. Also listed there are the single-particle widths computed in the present potential model. For these calculations I used the experimental energies. In the present case, the spectroscopic factor $S$ is related to the width via the relationship $S = \Gamma_{\rm expt}/\Gamma_{\rm sp}$. If mirror symmetry is valid, these should be equal to the values of $S$ extracted in the reaction $^{13}$C$(d,p)$ to the mirror states (listed in the last column). For the sake of comparisons, I assign an uncertainty of 10% to the latter. Uncertainties in the single-particle widths arising from uncertainties in energy are much smaller. For example, for the $1^-$ state, an uncertainty of 6 keV in energy results in less than 5% change in single-particle width. Percentage uncertainties for other states are even smaller. Because of its importance for astrophysical considerations, the $1^-$ width has been carefully measured [17]. We note that the $S$ obtained from the width and that from $(d,p)$ agree very well for this state.

The first three experimental widths for the $3^-$ state produce values of $S$ that are larger than the $S$ from $(d,p)$. It would appear that the width value of 103(6) keV can be ruled out on these grounds. The next two widths agree within uncertainty, but both appear to be too large if the $(d,p)$ $S$ is correct. The most recent width [14] of 25(3) keV produces a spectroscopic factor of 0.47(6)—smaller than $S(d,p)$, but almost in agreement within uncertainties.

On the other hand, the only experimental widths for the broad $0^-$ state appear to be too low. This might be due to its small cross section and very large width. Furthermore, in the elastic experiments [13,14], the $0^-$ energy is in a region that is partially obscured by detector dead layers. Of the two widths for the $2^-$ state, one has a much larger uncertainty than the other, but both are in good agreement with $S(d,p)$.

Earlier, I investigated a simple fit to the mirror energy differences (MEDs) for $2s_{1/2}$ single-particle states in several light nuclei [18]. The fit used a linear or quadratic function of the neutron separation energy $S_n$ $(= -E_n)$ times a factor of $Z_{\rm core}/A^{1/3}$, where $Z_{\rm core}$ is $Z - 1$ for the proton-rich member of the mirror pair. The nuclei included in the fit had $Z_{\rm core} = 6$ or 8, whereas here $Z_{\rm core}$ is 7. I have computed the $^{14}$O $1^-$ and $0^-$ energies from the earlier fit parameters. Results are listed in Table V. Agreement is good.

### B. $^{14}$N ($T = 1$)

The procedure for the $T = 1$ states of $^{14}$N is the same, except for the fact that the $^{14}$N states are 50% $^{13}$C$+p$ and 50% $^{13}$N$+n$. Computed energies are compared with experimental ones in Table VI. Here, again (as expected), the calculated energies are all less than the experimental energies; but in $^{14}$N,

TABLE VII. Energies (MeV), widths (keV) (from Table V), and spectroscopic factors for $s$ and $d$ single-particle $T = 1$ states in $^{14}$N, compared with $S$ values from $^{14}$C and $^{14}$O.

| $J^\pi$ | $E_x$ (expt.) | $E_p$ (expt.) | $\Gamma_{\rm sp}$ | $\Gamma_{\rm expt}$ | $S = 2\,\Gamma_{\rm expt}/\Gamma_{\rm sp}$ | $S$ ($^{14}$C)[a] | $S$ ($^{14}$O)[b] |
|---|---|---|---|---|---|---|---|
| $1^-$ | 8.062 | 0.511 | 88 | 30(1) | 0.68(2) | 0.75 | 0.71(3) |
| $3^-$ | 8.907 | 1.356 | 35 | 16(2) | 0.91(11) | 0.65 | 0.94(11), 0.79(4), 0.47(6) |
| $0^-$ | 8.776 | 1.225 | ∼1100 | 410(20) | ∼0.75(4) | 1.02 | 0.62(16) |
| $2^-$ | 9.509 | 1.958 | 110 | 41(2) | 0.75(4) | 0.72 | 0.82(31), 0.69(4) |

[a]Reference [10].
[b]Table IV.

the average deviation between experimental and computed energies is only 28 keV.

Widths and spectroscopic factors are listed in Table VII. Because these states are only 50% $^{13}C+p$, the relationship between $S$ and width is $S = 2\Gamma_{expt}/\Gamma_{sp}$. As for $^{14}O$, the results for the $1^-$ and $2^-$ states are in good agreement with expectations from the assumption of isospin conservation. And, as in $^{14}O$, the $S$ obtained from the width for $0^-$ is too small, and that for $3^-$ is too large. The agreement between results for $^{14}N$ and $^{14}O$ can be seen by comparing the last column with the third column from the end. This pattern of consistent differences might profit from further study.

## III. SUMMARY

A simple potential model has been used to compute energies and widths for $T = 1$ single-particle states in $^{14}O$ and $^{14}N$, using information from $^{14}C$ as input and assuming isospin conservation. Results in all cases are reasonable. For the spectroscopic factors computed from the widths, results for $^{14}N$ and $^{14}O$ agree, but $S$ $(0^-)$ is too small and three of four values of $S(3^-)$ are too large compared with $S$ values in $^{14}C$ obtained from the reaction $^{13}C(d,p)$. The most recent $3^-$ width [14] provides a spectroscopic factor smaller than $S(d,p)$. Repeating the measurements of the widths for the $0^-$ and $3^-$ states is desirable.


[1] S. Cohen and D. Kurath, Nucl. Phys. **73**, 1 (1965); **101**, 1 (1967).
[2] W. W. True, Phys. Rev. **130**, 1530 (1963).
[3] S. T. Hsieh and H. Horie, Nucl. Phys. A **151**, 243 (1970).
[4] S. Lie, Nucl. Phys. A **181**, 517 (1972).
[5] P. J. Ellis and T. Engeland, Nucl. Phys. A **144**, 161 (1970).
[6] Cenxi Yuan, Toshio Suzuki, Takaharu Otsuka, Furong Xu, and Naofumi Tsunoda, Phys. Rev. C **85**, 064324 (2012).
[7] F. Ajzenberg-Selove, Nucl. Phys. A **523**, 1 (1991).
[8] P. C. Sood and Y. R. Waghmare, Nucl. Phys. **46**, 181 (1963).
[9] H. T. Fortune, Phys. Rev. C **89**, 017302 (2014).
[10] R. J. Peterson, H. C. Bhang, J. J. Hamill, and T. G. Masterson, Nucl. Phys. A **425**, 469 (1984).
[11] C. R. Hoffman, B. P. Kay, and J. P. Schiffer, Phys. Rev. C **89**, 061305(R) (2014).
[12] A. Negret *et al.*, Phys. Rev. C **71**, 047303 (2005).
[13] T. Teranishi *et al.*, Phys. Lett. B **650**, 129 (2007).
[14] Y. B. Wang, B. X. Wang, X. Qin, X. X. Bai, B. Guo, C. Jiang, Y. J. Li, Z. H. Li, G. Lian, J. Su, S. Zeng, and W. P. Liu, Phys. Rev. C **77**, 044304 (2008).
[15] G. C. Ball and J. Cerny, Phys. Rev. **155**, 1170 (1967).
[16] R. G. Thomas, Phys. Rev. **88**, 1109 (1952); J. B. Ehrman, *ibid.* **81**, 412 (1951).
[17] T. E. Chupp, R. T. Kouzes, A. B. McDonald, P. D. Parker, T. F. Wang, and A. Howard, Phys. Rev. C **31**, 1023 (1985).
[18] H. T. Fortune, Phys. Rev. C **88**, 024309 (2013).