Charge symmetry breaking in electromagnetic nucleon form factors in elastic parity-violating electron-nucleus scattering

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The effects of charge symmetry breaking in nucleon electromagnetic form factors on parity-violating elastic electron- ^{12}C scattering is studied and found to be much smaller than other known effects. The analysis of a planned experiment is discussed. Nuclear isospin violation is likely to provide the largest correction term.

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I. INTRODUCTION

The standard model can be tested in low-energy electron-nucleus scattering [1–4]. For nuclei with $J^\pi = 0^+$ the parity violating (PV) asymmetry acquires a very simple, model-independent expression in terms of the weak nuclear charge, with nuclear structure effects canceling out if the nuclear ground state is purely isospin 0 and if effects of strangeness and charge symmetry breaking in the nucleon electromagnetic form factors can be ignored.

Indeed, plans are underway to measure the weak charge of the ^{12}C nucleus as part of the P2 experiment at Mainz [5]. This is a low-momentum transfer PV elastic electron-nucleus scattering experiment with the aim of reaching a relative precision of 0.3%. Much work has already been done on the effects of nuclear isospin mixing [6] as well as nucleon strangeness [4,7,8]. The purpose here is to asses the effects of charge symmetry breaking (CSB) of nucleon electromagnetic form factors on nuclear parity-vioalting electron scattering.

In the following, the general expression for the PV asymmetry are first discussed, including all correction terms expected to be relevant. My focus is on comparing the computed size of the nucleon CSB effects with the strangeness and nuclear isospin violation effects that are already in the literature for the kinematics $0 \leqslant Q^2 \leqslant 0.063$ GeV^2 of the planned P2 experiment [5].

II. PARITY-VIOLATION ASYMMETRY

Polarized electron elastic scattering from unpolarized nuclei has been used to study parity violation, because both electromagnetic (EM) and weak interactions contribute to the process via γ and Z^0 exchange. The PV asymmetry is given by [4]

$$\mathcal{A} = \frac{d\sigma^+ - d\sigma^-}{d\sigma^+ + d\sigma^-}\,, \tag{1}$$

where $d\sigma^+(d\sigma^-)$ is the cross section for electrons longitudinally polarized parallel (antiparallel) to their momentum. The asymmetry $\mathcal{A}$ for a target state of $J^\pi = 0^+$ predicted by the standard model can be written as

$$\mathcal{A} = \frac{G_F}{2\pi\alpha\sqrt{2}} Q^2 a_A \frac{\tilde{F}_{C0}(q)}{F_{C0}(q)}, \tag{2}$$

where G_F and α are the Fermi and fine-structure coupling constants, Q^2 is the negative of the square of the four-momentum transfer in the scattering process, $a_A = -1$, and the terms F_{C0} and $\tilde{F}_{C0}$ are the electromagnetic and weak neutral current nuclear form factors. This result is obtained in the plane wave Born approximation by keeping only the square of the photon-exchange amplitude for the spin-averaged EM cross section and using the interference between the γ and Z^0 exchange amplitudes in the cross section difference.

For $N = Z$ nuclear ground states that are pure isospin zero, only isoscalar matrix elements contribute and the weak and EM form factors obey the proportionality relation

$$\tilde{F}_{C0}(q) = \beta_V^{(0)} F_{C0}(q), \tag{3}$$

so that the resulting PV asymmetry $\mathcal{A}^0$ depends only on fundamental constants

$$\mathcal{A}^0 \equiv \left[\frac{G_F\, Q^2}{2\pi\alpha\sqrt{2}}\right] a_A \beta_V^{(0)} \cong 3.22 \times 10^{-6} \frac{Q^2}{\text{fm}^{-2}}, \tag{4}$$

where, within the standard model, $a_A\beta_V^{(0)} = 2\sin^2\theta_W$, with θ_W as the weak mixing angle. This proportionality with $\sin^2\theta_W$, provides an ability to test the standard model, which has intrigued many. But one must handle corrections which occur as the result of the effects of nuclear isospin mixing, strangeness content, and charge symmetry breaking in nucleon electromagnetic form factors.

One can begin to assess these different effects, starting by taking matrix elements of the basic weak interaction. In the standard model the weak neutral vector coupling between a Z boson and a quark is given by $\frac{1}{2}(\tau^3 - 4s_W Q_q)$, where $s_W \equiv \sin^2\theta_W$ and Q_q is the quark charge in units of the proton charge. For the numerical work $s_W = 0.234$ shall be used. Then the nucleon (N) weak form factors are given in terms of the quark and electromagnetic current form factors as

$$F_{1,2}^{Z,N} = \tfrac{1}{2}\big(F_{1,2}^{u,N} - F_{1,2}^{d,N} - F_{1,2}^{s,N} - 4s_W F_{1,2}^{em,N}\big), \tag{5}$$

where $F_{1,2}^q$ is the contribution of the quark (q) to the nucleon Dirac or Pauli form factor.

The CSB form factors $F^{\not{s}}$ and $F^{\not{v}}$ are related to matrix elements of an isoscalar current $j_s^\mu = \frac{1}{6}(\overline{u}\gamma^\mu u + \overline{d}\gamma^\mu d)$ and isovector current $j_v^{3,\mu} = \frac{1}{2}(\overline{u}\gamma^\mu u - \overline{d}\gamma^\mu d)\tau^3$ by

$$\begin{aligned}
&\overline{u}_N(P+q)\left[F_1^{\not s}(Q^2)\gamma^\mu + F_2^{\not s}(Q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2m_N}\right]u_N(P)\\
&\quad = \langle p|j_s^\mu|p\rangle - \langle n|j_s^\mu|n\rangle,\\
&\overline{u}_N(P+q)\left[F_1^{\not v}(Q^2)\gamma^\mu + F_2^{\not v}(Q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2m_N}\right]u_N(P)\\
&\quad = \langle p|j_v^\mu|p\rangle + \langle n|j_v^\mu|n\rangle,
\end{aligned} \tag{6}$$

with m_N as the average nucleon mass. One can then express isoscalar and isovector combinations as

$$F_{1,2}^{Z,p} + F_{1,2}^{Z,n} = F_{1,2}^{\not v} - F_{1,2}^{s} - 2s_W\left(F_{1,2}^{em,p} + F_{1,2}^{em,n}\right), \tag{7}$$

where

$$F_{1,2}^{\not v} \equiv \tfrac{1}{2}\left(F_{1,2}^{u,p} - F_{1,2}^{d,p} + F_{1,2}^{u,n} - F_{1,2}^{d,n}\right), \tag{8}$$

and

$$F_{1,2}^{Z,p} - F_{1,2}^{Z,n} = (1-2s_W)\left(F_{1,2}^{em,p} - F_{1,2}^{em,n}\right) - F_{1,2}^{\not s}, \tag{9}$$

where

$$F_{1,2}^{\not s} \equiv \tfrac{1}{6}\left(F_{1,2}^{u,p} + F_{1,2}^{d,p} - F_{1,2}^{u,n} - F_{1,2}^{d,n}\right). \tag{10}$$

These form factors are multiplied by the point-nucleon form factors $F_{p,n}(Q^2)$ of the nucleus to obtain the form factors F_{C0} and $\widetilde{F}_{C0}$. This assumes that all of the nuclear strangeness lies within individual nucleons. Any other nuclear strangeness would arise from an s quark confined to one baryon and an $\overline{s}$ confined to another nucleon. The existence of such exotic components is highly suppressed by large energy denominators and is ignored here. Meson exchange currents are neglected, as these are expected to be very small [6].

The relevant ratio $\frac{\widetilde{F}_{C0}(Q^2)}{F_{C0}(Q^2)}$ is given by

$$\frac{\widetilde{F}_{C0}(Q^2)}{F_{C0}(Q^2)} = \frac{G_E^{Z,p}(Q^2)F_p(Q^2) + G_E^{Z,n}(Q^2)F_n(Q^2)}{G_E^{em,p}(Q^2)F_p(Q^2) + G_E^{em,n}(Q^2)F_n(Q^2)}, \tag{11}$$

where $G_E^{Z,N}, G_E^{em,N}$ are the Sach's electric form factors computed using the average value of the nucleon mass. The above expression is obtained by neglecting the leading relativistic correction term in the nucleon current, a term of the order of the nucleon momentum divided by the nucleon mass. The equations in Ref. [6] show that for a C^{12} nucleus, such terms are at most approximately $Q^2/(12m_N^2) \approx 5\times10^{-3}$ of the small correction terms kept at the low values of momentum transfer of interest to the experiment [5].

Next Eq. (11) is simplified by defining

$$F_p(Q^2) \equiv \overline{F}(Q^2) + \tfrac{1}{2}\Delta F(Q^2), \tag{12}$$

$$F_n(Q^2) \equiv \overline{F}(Q^2) - \tfrac{1}{2}\Delta F(Q^2), \tag{13}$$

$$G_\pm^Z(Q^2) \equiv G_E^{Z,p}(Q^2) \pm G_E^{Z,n}(Q^2), \tag{14}$$

$$G_\pm^{em}(Q^2) \equiv G_E^{em,p}(Q^2) \pm G_E^{em,n}(Q^2). \tag{15}$$

Using this notation and keeping the leading term and those of first-order in the corrections $G_E^s, G_E^{\not v}$, and ΔF gives

$$\frac{\widetilde{F}_{C0}(Q^2)}{F_{C0}(Q^2)} = -2s_W + \frac{G_E^{\not v} - G_E^s}{G_+^{em}} + \frac{(1-2s_W)^2 G_-^{em}}{G_+^{em}}\frac{\Delta F}{2\overline{F}}. \tag{16}$$

The net result is that

$$\mathcal{A} = \left[\frac{G_F\, Q^2}{2\pi\alpha\sqrt{2}}\right]\left(2s_W - \frac{G_E^{\not v} - G_E^s}{G_+^{em}} - \frac{(1-2s_W)^2 G_-^{em}}{G_+^{em}}\frac{\Delta F}{2\overline{F}}\right). \tag{17}$$

The nucleon electromagnetic form factors of Kelly [9] are used in the calculations.

One may define the correction to the $2s_W$ term as $C(Q^2) \equiv -\frac{G_E^{\not v} - G_E^s}{G_+^{em}} - \frac{(1-2s_W)^2 G_-^{em}}{G_+^{em}}\frac{\Delta F}{2\overline{F}}$ so that

$$\mathcal{A} = \left[\frac{G_F\, Q^2}{2\pi\alpha\sqrt{2}}\right](2s_W + C(Q^2)). \tag{18}$$

One way to analyze an experiment is to make an extrapolation linear in Q^2 to determine the value of s_W, so we shall be concerned with the linearity of $C(Q^2)$.

III. THE CORRECTION TERM $C(Q^2)$

The three contributions to $C(Q_2)$ are considered.

A. Charge symmetry breaking (CSB) of the electromagnetic form factors

We have previously evaluated [10] the leading-order CSB effects of the pion cloud of the nucleon and of vector mesons which contribute to the leading low energy constant [11]. Our previous work did not obtain the separate terms $F_{1,2}^{\not v,\not s}$. This is done here. The pionic terms are given by

$$F_1^{\not s} = -\left(\frac{g_A m_N}{f_\pi}\right)^2[\widetilde{I}_1(Q^2,m_p,m_n) - \widetilde{I}_1(Q^2,m_n,m_p)], \tag{19}$$

$$F_2^{\not s} = 2\left(\frac{g_A m_N}{f_\pi}\right)^2[I_2(Q^2,m_p,m_n) - I_2(Q^2,m_n,m_p)], \tag{20}$$

$$\begin{aligned}
F_1^{\not v} = &\left(\frac{g_A m_N}{f_\pi}\right)^2[\widetilde{I}_1(Q^2,m_p,m_n) - \widetilde{I}_1(Q^2,m_n,m_p)\\
&- \widetilde{J}_1(Q^2,m_p,m_n) + \widetilde{J}_1(Q^2,m_n,m_p)],
\end{aligned} \tag{21}$$

$$\begin{aligned}
F_2^{\not v} = &\left(\frac{g_A m_N}{f_\pi}\right)^2[-2I_2(Q^2,m_p,m_n) + 2I_2(Q^2,m_n,m_p)\\
&- 2J_2(Q^2,m_p,m_n) + 2J_2(Q^2,m_n,m_p)].
\end{aligned} \tag{22}$$

The values of the axial vector coupling constant, g_A, the pion decay constant f_π, and the average nucleon mass are presented in Ref. [10]. The terms $\widetilde{I}_1(Q^2,m_p,m_n)$, $I_2(Q^2,m_p,m_n)$, $\widetilde{J}_1(Q^2,m_n,m_p)$, and $J_2(Q^2,m_p,m_n)$ are obtained from the relevant Feynman diagrams and are specified in Eqs. (9) and (10) of Ref. [10].

I also need to include the resonance saturation assumptions for the phenomenologically unconstrained contact terms $\kappa^{\not s}$ and $\kappa^{\not v}$ discussed in Ref. [10]. These terms dominate the CSB contribution to G_E of the proton [11]. The ω couples to isoscalar currents, and so the diagram $\omega \to \rho$ where the ω couples to a current and then mixes with a ρ that couples to a nucleon as an isovector contributes to $F^{\not s}$. Conversely the ρ couples to isovector currents, so the diagram with $\rho \to \omega$

contributes to $F^{\not{p}}$. This gives

$$
\begin{aligned}
F_1^{VM,\not{s}} &= g_\rho F_\omega \Theta_{\rho\omega} \frac{Q^2}{m_V\left(m_V^2+Q^2\right)^2},\\
F_2^{VM,\not{s}} &= -g_\rho \kappa_\rho F_\omega \Theta_{\rho\omega} \frac{m_V}{\left(m_V^2+Q^2\right)^2},\\
F_1^{VM,\not{p}} &= g_\omega F_\rho \Theta_{\rho\omega} \frac{Q^2}{m_V\left(m_V^2+Q^2\right)^2},\\
F_2^{VM,\not{p}} &= -g_\omega \kappa_\omega F_\rho \Theta_{\rho\omega} \frac{m_V}{\left(m_V^2+Q^2\right)^2}.
\end{aligned}
\tag{23}
$$

The effects of CSB are to be compared with those of strangeness in the nucleon.

B. Strangeness

The effects of strangeness on nucleon electromagnetic form factors has been parameterized [6] as

$$G_E^{(s)} = \rho_s \tau G_D^V \xi_E^{(s)}, \quad G_M^{(s)} = \mu_s G_D^V, \tag{24}$$

with (for instance, Ref. [4])

$$G_D^V = (1+4.97\tau)^{-2}, \quad \xi_E^{(s)} = (1+5.6\tau)^{-1}. \tag{25}$$

The parameter ρ_s and μ_s are constrained by PV electron scattering measurements on hydrogen, deuterium, and helium-4. Reference [6] used the range $-1.5 < \rho_s < 1.5$. Later work [8] made a statistical analysis of the full set of parity-violating asymmetry data for elastic electron scattering. This found $\rho_s = 0.92 \pm 0.58$. This range of values is used in the numerical work. However, experiments on deep inelastic scattering restrict the s and $\bar{s}$ parton distribution functions to very small values [12] and reality may correspond to an order of magnitude smaller values of ρ_s [13].

C. Nuclear isospin violation

Reference [6] used a Skyrme-type density-dependent interaction to generate the ground-state wave function in the Hartree-Fock plus BCS approximation. This procedure yields ground-state densities for ^{12}C, ^{24}Mg, ^{28}Si, and ^{32}S nuclei, which give computed nuclear charge form factors in excellent agreement with electron scattering data. The difference in proton and neutron charge densities is generated mainly by the Coulomb interaction. The result of a different formalism are used here: a nuclear density functional of Bulgac *et al.* [14]. This calculation produces nuclear densities constrained by nuclear binding energies and charge densities for the entire periodic table. To study the model dependence I compare the effects of this model with those of Ref. [6]. Figure 1 shows the quantity

$$\frac{\Delta\Gamma}{\Gamma} \equiv \frac{(1-2s_W)^2 G_-^{em}}{2s_W G_+^{em}} \frac{\Delta F}{2\overline{F}} \tag{26}$$

for the two models. For ^{12}C the effects of the calculation of Ref. [14] have the same Q^2 dependence as the one of Ref. [6]. This lends credence to the idea that the many-body nuclear theory is under control. Its uncertainties would not impact experimental extractions of the weak mixing angle or strangeness content, because these use a linear extrapolation in Q^2 [15]. However, the effects of Ref. [15] are 30% larger than those of Ref. [14]. This difference is not surprising because the isospin-violating effect is the difference between two large quantities. Fortunately, the experiment will not rely on knowledge of the magnitude of these effects, but rather on the Q^2 dependence, which is the same. Note in passing that the effects of isospin-violating strong forces (absent in both calculations) are much smaller than those of the Coulomb interaction for all nuclei [16–18].

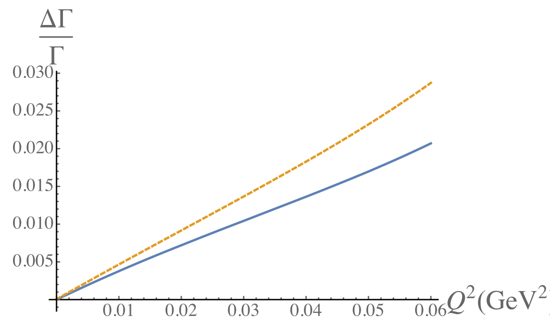

FIG. 1. (Color online) Comparison of $\frac{\Delta\Gamma}{\Gamma}$ of Ref. [14] (solid) with that of Ref. [6] (dashed).

IV. RESULTS AND CONCLUSIONS

My aim is to present calculations relevant for the planned experiment [5]. Therefore the momentum transfer range is restricted to $0 \leqslant Q^2 \leqslant 0.0625$ GeV^2.

I begin by comparing the effects of charge symmetry breaking (CSB) in nucleon electromagnetic form factors with the effects of nuclear isospin violation; see Fig. 2. As expected, the nuclear effects are far larger than those of the nucleon. The range of curves for the CSB terms is obtained from using the compilations of Refs. [19] and [20]. If the value of Q^2 were increased by about 15%, the effects of nuclear isospin would become very large and nonlinear in the variable Q^2. This feature is in agreement with the results of Ref. [6]. However, the restriction of the value of Q^2 to an upper limit of 0.0625 GeV^2 is sufficient to ensure a linear behavior. Note also that any effects of the uncertainty in the nuclear isospin

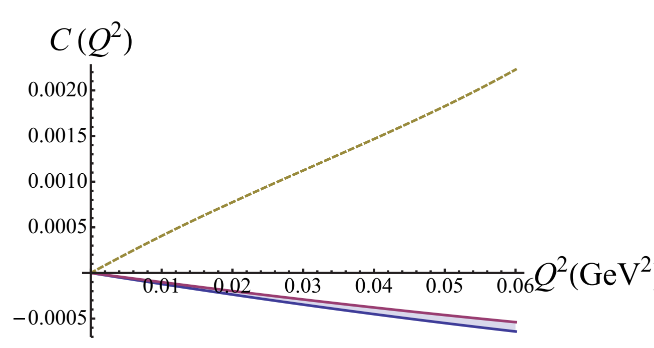

FIG. 2. (Color online) Contributions to the correction $C(Q^2)$ due to nuclear isospin violation (dashed) and CSB in the nucleon form factors (solid) for two sets of meson-nucleon coupling constants; see text.

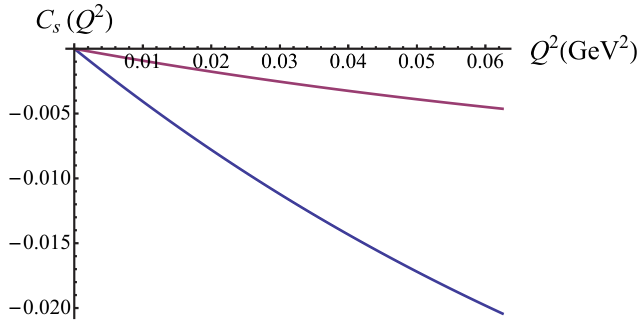

FIG. 3. (Color online) Contributions to the correction $C(Q^2)$ due to strangeness. The two curves are obtained using the upper (+0.15) and lower (0.34) limits on ρ_s from Ref. [8].

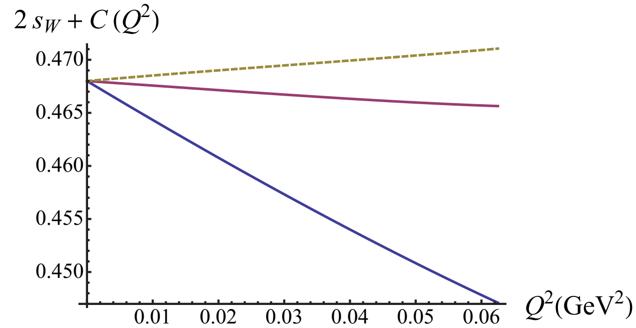

FIG. 4. (Color online) $2s_W + C(Q^2)$. The two solid curves include the effects of nuclear isospin violation, the average of nucleon CSB, and the upper and lower limits of the strangeness contribution. The dashed curve is obtained by setting the strangeness contribution to 0. The use of each of the parameter sets discussed in this paper would lead to a straight line on this figure.

violation terms (expected to be no more than 5%) are expected to be far smaller than the uncertainty goal of the planned experiment [5].

Next I assess the effects of nucleon strangeness using the range of values from Ref. [8], $\rho_s = 0.92 \pm 0.58$; see Fig. 3. A comparison of Figs. 2 and 3 shows that the CSB effects are generally more than an order of magnitude smaller than those of nucleon strangeness obtained from these limits. This statement is consistent with that of Ref. [10], which compared proton CSB effects with experimental uncertainties.

Finally I plot the quantity $2s_W + C(Q^2)$ which gives via Eq. (18) the PV asymmetry in units of $\frac{G_F Q^2}{2\pi\alpha\sqrt{2}}$; see Fig. 4. The two solid curves result from using the previously stated [8] upper and lower limits on ρ_s. A third dashed curve sets the strangeness contribution to zero ($\rho_s = 0$). Recent work relates the strangeness contribution to deep inelastic scattering to that of proton electromagnetic form factors [13] through the use of light-front models, and ρ_s is limited to values about 10 times smaller than in Ref. [8]. If these models are valid, the dashed curve (with dominant contribution arising from nuclear isospin violation) would be the best prediction.

To summarize: The parity-violating elastic- ^{12}C scattering asymmetry $\mathcal{A}$ at very low values of Q^2 is dominated by the size of the weak mixing angle, s_W. All of the corrections to that value are linear in Q^2, for the relevant range of $0 \leqslant Q^2 \leqslant 0.0625$ GeV^2. The CSB effects on nucleon electromagnetic form factors are at least an order of magnitude smaller than the contributions expected from nuclear isospin breaking, which themselves are about 10^{-3} of the weak nuclear charge at $Q^2 = 0.01$ GeV^2. The effects of nucleon strangeness are uncertain, but are linear with Q^2 in the relevant kinematic range. A measurement of the weak mixing angle to the desired relative accuracy of 0.3% in the weak charge of ^{12}C would require the ability to determine the slope of $C(Q^2)$ to that accuracy to distinguish a deviation from the standard model from an effect of the correction term. This requires a measurement at more than one value of Q^2.

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