Probing quark charge correlations by identified hadrons in ultrarelativistic AA collisions

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We propose a new kind of two-particle correlation of identified hadrons in longitudinal rapidity space which can reflect directly the conserved charge correlations of a hot quark system produced in AA collisions at Large Hadron Collider (LHC) energies. It is derived from the basic scenario of quark combination mechanism of hadron production. Like the elliptic flow of identified hadrons at intermediate transverse momentum, this correlation is independent of the absolute hadronic yields but depends only on the flavor compositions of hadrons and thus exhibits interesting properties for different kinds of hadrons. We suggest the measurement of this correlation function in AA collisions at the LHC to gain more insight into the charge-correlation properties of the produced hot quark matter.

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I. INTRODUCTION

Correlations between different identified hadrons in momentum space were sensitive probes of prompt hadron production dynamics in different high-energy reactions already in 1980s and 1990s [1–7]. The experimental data of baryon-baryon and baryon-antibaryon correlations in rapidity space in e^+e^- annihilations [4–6] provided important tests of existing phenomenological models of nonperturbative hadronization process. In ultrarelativistic heavy-ion collisions, correlation measurements and studies continue to serve as an indispensable means for exploring the properties of the strong-interacting quark gluon plasma (QGP) produced in collisions [8–17]. In particular, recent studies of the charge balance functions in low-p_T domain provided deep understanding of the properties of conserved charge correlations of QGP produced in collisions [13–27]. We note that these studies are mainly of charged particles while those of two-particle correlations of identified hadrons are relatively less concerned. Definitely, the correlations of conserved charges generated in the QGP stage, after hadronization, should not only impose the bulk constraints on the production of all hadrons from QGP but also visualize themselves in the correlation between different identified hadrons, e.g., p-$\bar{p}$, p-$\bar{\Lambda}$, and p-K correlations, etc. The latter relates to the physics of how the charge correlation properties of QGP, after hadronization, present themselves in the identified two-hadron correlations, or, in the converse philosophy, the physics of what QGP charge correlation properties can be extracted from the correlations between specific hadrons. The purpose of this paper is to find a direct and clear connection between the identified two-hadron correlations and the conserved charge correlations of deconfined quark systems in the framework of the quark combination mechanism of hadron production. Conserved charges specifically refer to three conserved quantum numbers, i.e., baryon number, strangeness and electric charge. To this end, a new properly designed two-hadron correlation function in longitudinal rapidity space is proposed. ALICE experiments at the Large Hadron Collider (LHC) have the ability of high-precision measurement of identified hadrons and the ability of correcting the weak decays of strange hyperons and thus can provide a good experimental platform for studying the identified two-hadron correlations in momentum space.

The paper is organized as follows: We present in Sec. II the new hadronic correlation function $G_{\alpha\beta}(\Delta y)$ and illustrate its interesting properties. Then we study in Sec. III the observation of conserved charge correlations of quark system by measuring hadronic $G_{\alpha\beta}$. Influences of resonance decay on $G_{\alpha\beta}(\Delta y)$ are studied in Sec. IV. A summary and discussion are given in Sec. V.

II. A NEW HADRONIC CORRELATION FUNCTION $G_{\alpha\beta}(\Delta y)$

We propose the following two-hadron correlation function in the longitudinal rapidity space as a new observable in AA collisions at LHC energies:

$$G_{\alpha\beta}(y_1,y_2) = \frac{\langle [n_\alpha(y_1) - n_{\bar{\alpha}}(y_1)][n_\beta(y_2) - n_{\bar{\beta}}(y_2)]\rangle}{\langle n_\alpha(y_1)\rangle\langle n_\beta(y_2)\rangle}, \quad (1)$$

which measures directly the correlation between two hadronic species by a symmetrical combination of $\alpha\beta$, $\bar{\alpha}\bar{\beta}$, $\alpha\bar{\beta}$, and $\bar{\alpha}\beta$ correlations but can reflect in a quite clear manner the properties of conserved charge correlations of the quark system produced in collisions. Here, angle brackets denote event average and $n_\alpha(y_1)$ denote the number of hadrons α at y_1. We confine ourselves to the situation of zero baryon number density, e.g., the central plateau region of collisions as a good approximation, otherwise one would subtract the term $\langle n_\alpha(y_1) - n_{\bar{\alpha}}(y_1)\rangle\langle n_\beta(y_2) - n_{\bar{\beta}}(y_2)\rangle$ in the numerator. In the denominator, we put the product of two-hadron yields rather than either one, which is the major difference from the previous balance functions [3,13,27]. At first sight, this correlation is hadronic-yield dependent since it seems to be of the magnitude of $1/n_\alpha$ or $1/n_\beta$. In fact, however, this is not the case; rather, it presents a clean quark-level insight as we interpret Eq. (1) in

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the framework of the quark combination mechanism (QCM) of hadron production.

The QCM describes the production of hadrons as the quark system hadronizes by the combination of neighboring quarks and antiquarks in phase space. Two highlights in the early years of the BNL Relativistic Heavy Ion Collider (RHIC) experiments, i.e., high-p/π ratio and quark number scaling of hadronic elliptic flow at intermediate transverse momentum (p_T), are the foremost suggestion of QCM as an important hadronization candidate for the hot quark system produced in relativistic heavy-ion collisions [28–33]. Applying QCM to the low-p_T region of higher quark number density is natural and the related issues, such as entropy conservation or pion production, have been properly addressed [31,32,34–37]. Various phenomenological models based on QCM [38,39] have successfully explained lots of low-p_T data from CERN Super Proton Synchrotron (SPS), RHIC, and recent LHC experiments; in particular those of yields and longitudinal rapidity spectra of identified hadrons [34,40–43]. There are also many successful applications of QCM on correlation studies, e.g., multihadron yield correlations [43–45], baryon-meson correlated emission [46,47], as well as the charge balance function [15,20].

Applying QCM to one-dimensional longitudinal rapidity space and considering that the averaged rapidity interval between neighboring quarks at LHC energies is of the order of 10^{-3}, i.e., a thousand quarks per unit rapidity, the combination of neighboring quarks into a hadron can be treated as an ideal equal-y combination to make the study illustrative and insightful and meanwhile keep a good numerical approximation. Then, we have the single-hadron rapidity distribution, e.g., for the produced meson $M_i(q_1\bar{q}_2)$,

$$\mathcal{F}_{M_i}(y) = \mathcal{P}_{q_1\bar{q}_2,M_i}(y)\, f_{q_1\bar{q}_2}(y), \tag{2}$$

and similarly for baryonic $\mathcal{F}_{B_i}(y)$. Here, $f_{q_1\bar{q}_2}(y)$ is the density of $q_1\bar{q}_2$ pairs at rapidity y in the system just before hadronization. $\mathcal{P}_{q_1\bar{q}_2,M_i}$ denotes the probability of a $q_1\bar{q}_2$ pair combining into a meson. We also consider the two-hadron joint distributions $\mathcal{F}_{\alpha\beta}(y_1,y_2)$, since $G_{\alpha\beta}$ essentially measures $(\mathcal{F}_{\alpha\beta}+\mathcal{F}_{\bar{\alpha}\bar{\beta}}-\mathcal{F}_{\bar{\alpha}\beta}-\mathcal{F}_{\alpha\bar{\beta}})/\mathcal{F}_\alpha\mathcal{F}_\beta$, and have

$$\mathcal{F}_{M_iM_j}(y_1,y_2) = \mathcal{P}_{(q_1\bar{q}_2)(q_3\bar{q}_4),M_iM_j}(y_1,y_2)\, f_{(q_1\bar{q}_2)(q_3\bar{q}_4)}(y_1,y_2), \tag{3}$$

and similarly for $\mathcal{F}_{M_iB_j}$ and $\mathcal{F}_{B_iB_j}$. $f_{(q_1\bar{q}_2)(q_3\bar{q}_4)}(y_1,y_2)$ is the joint distribution of a $q_1\bar{q}_2$ pair at y_1 and a $q_3\bar{q}_4$ pair at y_2. $\mathcal{P}_{(q_1\bar{q}_2)(q_3\bar{q}_4),M_iM_j}(y_1,y_2)$ is the joint production probability of a M_iM_j pair given a $q_1\bar{q}_2$ pair at y_1 and a $q_3\bar{q}_4$ pair at y_2, which has the factorization form $\mathcal{P}_{q_1\bar{q}_2,M_i}(y_1)\,\mathcal{P}_{q_3\bar{q}_4,M_j}(y_2)$ because of the locality of hadronization. In the system without net charges we expect $\mathcal{P}_{q_1\bar{q}_2,M_i}=\mathcal{P}_{\bar{q}_1q_2,\bar{M}_i}$ and $\mathcal{P}_{q_1q_2q_3,B_j}=\mathcal{P}_{\bar{q}_1\bar{q}_2\bar{q}_3,\bar{B}_j}$ in charge conjugation symmetry.

The kernel $\mathcal{P}$ involves the complex nonperturbative hadronization dynamics and is far from being solved strictly from first principles. Fortunately, by adopting Eq. (1) we do not need to know the precise form of $\mathcal{P}$. By using Eqs. (2) and (3) we obtain that $G_{\alpha\beta}$ actually measures the $(f^{(q)}_{\alpha\beta}+f^{(q)}_{\bar{\alpha}\bar{\beta}}-f^{(q)}_{\bar{\alpha}\beta}-f^{(q)}_{\alpha\bar{\beta}})/f^{(q)}_\alpha f^{(q)}_\beta$, where $f^{(q)}_\alpha$ and $f^{(q)}_{\alpha\beta}$ denote the corresponding multiquark distributions in single α and joint $\alpha\beta$ productions in Eqs. (2) and (3), respectively. Then we immediately get

$$G_{\alpha\beta}(y_1,y_2) = \frac{\big\langle\big[n^{(q)}_\alpha(y_1)-n^{(q)}_{\bar{\alpha}}(y_1)\big]\big[n^{(q)}_\beta(y_2)-n^{(q)}_{\bar{\beta}}(y_2)\big]\big\rangle}{\big\langle n^{(q)}_\alpha(y_1)\big\rangle\big\langle n^{(q)}_\beta(y_2)\big\rangle}, \tag{4}$$

where $n^{(q)}_\alpha(y)$ is the number of multiquark clusters in an event for α production. In the meson $\alpha(q_1\bar{q}_2)$ case, $n^{(q)}_\alpha(y)=n_{q_1\bar{q}_2}(y)=n_{q_1}(y)n_{\bar{q}_2}(y)$ is the number of $q_1\bar{q}_2$ pairs at rapidity y. In the baryon $\alpha(q_1q_2q_3)$ case, $n^{(q)}_\alpha(y)=n_{q_1q_2q_3}(y)$ is the number of $q_1q_2q_3$ combinations which satisfies $n_{q_1q_2q_3}(y)\approx n_{q_1}(y)n_{q_2}(y)n_{q_3}(y)$ in the large-quark-number limit. $n_q(y)$ is the number of q-flavor quarks at y. To second order in the fluctuations of quark numbers, we have

$$G_{\alpha\beta}(y_1,y_2) = \sum_{f_1,f_2} A_{f_1f_2}\frac{C_{f_1f_2}(y_1,y_2)}{\langle n_{f_1}(y_1)\rangle\langle n_{f_2}(y_2)\rangle}, \tag{5}$$

where the detailed procedure of its derivation from Eq. (4) is in the appendix. Here, indices f_1 and f_2 run over all flavors of quarks and antiquarks. The coefficient $A_{f_1f_2}=(n_{\alpha,f_1}-n_{\bar{\alpha},f_1})(n_{\beta,f_2}-n_{\bar{\beta},f_2})$. n_{α,f_1} is the number of quark f_1 contained in hadron α. $C_{f_1f_2}(y_1,y_2)=\langle n_{f_1}(y_1)n_{f_2}(y_2)\rangle-\langle n_{f_1}(y_1)\rangle\langle n_{f_2}(y_2)\rangle$ is the ordinary two-point correlation function. Clearly, $G_{\alpha\beta}(y_1,y_2)$ depends only on the quark-level correlations as well as the flavor compositions of hadrons α and β, independent of the absolute yields of two hadrons. We emphasize that this result is independent of the precise form of the kernel $\mathcal{P}$, which does not mean the absence of hadronization dynamics in our formula. We incorporated the most basic dynamics of QCM in our formula, i.e., the combination of neighboring quarks and antiquarks in phase space into hadrons. This independence of kernel $\mathcal{P}$ just means that the result does not depend on the sophisticated hadronization details. Therefore, Eq. (5) is a general result of QCM. We also verify that the analytical result of Eq. (5) derived from the equal-y-combination approximation totally agrees with the numerical results of the combination model developed by the Shandong group (SDQCM) [39], which applies more practical quark combinations in longitudinal hadron production.

As applying the approximation of zero baryon number density to the central-plateau region of AA collisions at LHC energies, we further assume a Bjorken longitudinally boost invariance [48] for the system, and then the two-point correlation functions depend only on $\Delta y = y_2 - y_1$ rather than on y_1 and y_2 individually. Since the rapidity density of particles is uniform in this case, we use n_α and n_f to denote the rapidity densities of hadron α and quark f, respectively.

One of the advantages of $G_{\alpha\beta}$ is that it can conveniently relate to the correlation of conserved charges in the quark system. Here we consider the system made up mainly of the three quark flavors, i.e., up (u), down (d), and strange (s) quarks. There are three conserved charges in the system, i.e., baryon number (B), electric charge (C), and strangeness (S). Alternatively, we use the net number of up ($n_u-n_{\bar{u}}$), down ($n_d-n_{\bar{d}}$), and strange quarks ($n_s-n_{\bar{s}}$) instead of B, C, and S charges because quark numbers are more convenient

TABLE I. $G_{\alpha\beta}(\Delta y)$ of directly produced hadrons after hadronization in terms of $G_{ab}(\Delta y)$ of the quark system before hadronization.

$G_{\alpha\beta}$	π^+	p	K^+	Λ	Ξ^0	Ω^-
π^-	$-2G_{uu}+2G_{ud}$					
$\bar{p}$	$-G_{uu}+G_{ud}$	$-5G_{uu}-4G_{ud}$			Transpose symmetric	
K^-	$-G_{uu}+G_{ud}$	$-2G_{uu}-G_{ud}+3G_{us}$	$-G_{uu}+2G_{us}-G_{ss}$			
$\bar{\Lambda}$	0	$-3G_{uu}-3G_{ud}-3G_{us}$	$-G_{uu}-G_{ud}+G_{us}+G_{ss}$	$-2G_{uu}-2G_{ud}-4G_{us}-G_{ss}$		
$\bar{\Xi}^0$	$-G_{uu}+G_{ud}$	$-2G_{uu}-G_{ud}-6G_{us}$	$-G_{uu}-G_{us}+2G_{ss}$	$-G_{uu}-G_{ud}-5G_{us}-2G_{ss}$	$-G_{uu}-4G_{us}-4G_{ss}$	
$\bar{\Omega}^+$	0	$-9G_{us}$	$-3G_{us}+3G_{ss}$	$-6G_{us}-3G_{ss}$	$-3G_{us}-6G_{ss}$	$-9G_{ss}$

in hadronic correlations. Furthermore, charge correlations of quark system $G_{ab}(\Delta y)$ can be also described by Eq. (1). Under the isospin symmetry and electric charge conjugation symmetry, we have four kinds of quark number correlation functions, i.e.,

$$\begin{aligned} G_{uu}(\Delta y) &= 2\left[C_{uu}(\Delta y) - C_{u\bar{u}}(\Delta y)\right]/\langle n_u\rangle^2, \\ G_{ss}(\Delta y) &= 2\left[C_{ss}(\Delta y) - C_{s\bar{s}}(\Delta y)\right]/\langle n_s\rangle^2, \\ G_{ud}(\Delta y) &= 2[C_{ud}(\Delta y) - C_{u\bar{d}}(\Delta y)]/\langle n_u\rangle^2, \\ G_{us}(\Delta y) &= 2\left[C_{us}(\Delta y) - C_{u\bar{s}}(\Delta y)\right]/(\langle n_u\rangle\langle n_s\rangle), \end{aligned} \tag{6}$$

for the quark system just before hadronization.

Substituting Eq. (6) into Eq. (5), we finally get

$$G_{\alpha\beta}(\Delta y) = \sum_{f_1,f_2=u,d,s} Q_{\alpha,f_1} Q_{\beta,f_2} G_{f_1 f_2}(\Delta y), \tag{7}$$

where $Q_{\alpha,f_1} = n_{\alpha,f_1} - n_{\alpha,\bar{f}_1}$ denotes the net number of f_1 in hadron α. The equation shows a direct and simple connection between the hadronic correlation and the charge correlation of the quark system before hadronization. There is no such concision if one adopts $C_{\alpha\beta}(\Delta y)$ and existing balance functions. In Table I, we show $G_{\alpha\beta}(\Delta y)$ of various identified hadrons in terms of the charge correlations $G_{ab}(\Delta y)$ of the quark system.

There are several interesting results in Table I. First, we see that signs before G_{ab} in G_{MM} and G_{MB} have both the positive and negative parts while in $G_{B\bar{B}}$ their signs are the same. It arises from the flavor composition nature of $M(q\bar{q})$ and $B(qqq)$. Second, we find $G_{\pi^+\bar{\Lambda}}(\Delta y) = G_{\pi^+\bar{\Omega}^+}(\Delta y) = 0$ which means their productions are independent of each other. This is because π^+ is composed of u and $\bar{d}$ and their correlations with quarks in Λ and Ω^- are canceled out under the isospin symmetry. Third, we see $G_{\pi^+\bar{p}}(\Delta y) = G_{\pi^+K^-}(\Delta y) = G_{\pi^+\bar{\Xi}^0}(\Delta y) = -G_{uu}(\Delta y) + G_{ud}(\Delta y)$ in which strangeness correlations disappear. This is also because the correlation between u in π^+ and strange quarks in K^-, $\bar{\Xi}^0$ cancel that part of $\bar{d}$ in the pion and thus only light-flavor correlations are left. These results are independent of quark's G_{ab} and thus can be regarded as the characteristic properties of QCM. For $G_{\alpha\beta}(\Delta y)$ of other hadrons, they are also correlated with each other by four-quark correlation functions in Eq. (6), e.g.,

$$\begin{aligned} & G_{p\bar{\Omega}^+}(\Delta y) - 3G_{K^+\bar{\Omega}^+}(\Delta y) \\ &\quad = 2G_{\Xi^0\bar{\Omega}^+}(\Delta y) - G_{\Lambda\bar{\Omega}^+}(\Delta y) = G_{\Omega^-\bar{\Omega}^+}(\Delta y), \\ & G_{K^+K^-}(\Delta y) - G_{\Omega^-\bar{\Omega}^+}(\Delta y) + G_{\Xi^0\bar{\Xi}^0}(\Delta y) \\ &\quad = 2G_{K^+\bar{\Xi}^0}(\Delta y). \end{aligned} \tag{8}$$

The applicability of the QCM can be tested by these intrinsic relationships between different $G_{\alpha\beta}(\Delta y)$.

Besides, the quark combination also predicts that K^+ is associated in production with Λ, Ξ^0, and Ω^- rather than their antiparticles for the regular quark G_{ab} such as those discussed later. This is because the $+G_{ss}$ item always overwhelms numerically other parts in their correlation decompositions; see Table I. It is a natural result since K^+ production consumes a $\bar{s}$ while the remaining s enters into a hyperon, thereby passing the $s\bar{s}$ quark correlation to these strange hadrons.

The fact of $G_{\alpha\beta}$ being the linear combination of a few G_{ab}, as shown in Table I, also suggests that not only the correlation width measured in the past but also the correlation magnitude should be regarded as significant observations at LHC experiments. On the other hand, these simple relations provide the possibility of extracting the charge correlation properties of quark system from the measurable correlations of identified hadrons. By the measurement of $G_{\alpha\beta}(\Delta y)$ in Table I, we can extract directly $G_{uu}(\Delta y)$, $G_{ud}(\Delta y)$, $G_{us}(\Delta y)$, and $G_{ss}(\Delta y)$. Their combinations give the correlations among the usual B, C, and S charges. Relating net quark numbers to B, C, and S charges, i.e.,

$$\begin{aligned} B &= \tfrac{1}{3}(n_u - n_{\bar{u}}) + \tfrac{1}{3}(n_d - n_{\bar{d}}) + \tfrac{1}{3}(n_s - n_{\bar{s}}), \\ C &= \tfrac{2}{3}(n_u - n_{\bar{u}}) - \tfrac{1}{3}(n_d - n_{\bar{d}}) - \tfrac{1}{3}(n_s - n_{\bar{s}}), \\ S &= n_{\bar{s}} - n_s, \end{aligned} \tag{9}$$

we have, for example,

$$\begin{aligned} C_{BB}(\Delta y) &= \tfrac{1}{9}\big\{2\langle n_u\rangle^2\left[G_{uu}(\Delta y) + G_{ud}(\Delta y)\right] \\ &\qquad + \langle n_s\rangle^2 G_{ss}(\Delta y) + 4\langle n_u\rangle\langle n_s\rangle G_{us}(\Delta y)\big\}, \\ C_{BS}(\Delta y) &= -\tfrac{1}{3}\left[\langle n_s\rangle^2 G_{ss}(\Delta y) + 2\langle n_u\rangle\langle n_s\rangle G_{us}(\Delta y)\right]. \end{aligned} \tag{10}$$

Quark number densities $\langle n_u\rangle$ and $\langle n_s\rangle$ can be obtained by fitting the experimental data of yields of identified hadrons in a quark combination model (see, e.g., SDQCM [39]).

III. EXTRACTION OF QUARK CHARGE CORRELATIONS BY $G_{\alpha\beta}$ MEASUREMENTS

The results in Table I enable us to systematically extract the properties of charge correlations of quark system produced in collisions from the identified two-hadron correlations. In this section, we demonstrate an application of the above results.

By the measurement of only a few $G_{\alpha\beta}(\Delta y)$ shown in Table I, e.g., $G_{p\bar{\Lambda}}(\Delta y)$, $G_{p\bar{\Xi}^0}(\Delta y)$, and $G_{p\bar{\Omega}^+}(\Delta y)$, we can probe some basic or qualitative properties of quark systems produced in collisions. We also discuss two different methods of measuring $G_{\alpha\beta}(\Delta y)$ in experiments.

First, we study the qualitative properties of charge correlations for quark system. By the definition in Eq. (6), $G_{ab}(\Delta y)$ for quark systems has a two-component feature, which is given by the competition between $C_{ab}(\Delta y)$ and $C_{a\bar{b}}(\Delta y)$. Here, $C_{ab}(\Delta y)$ describes the distribution of the correlated ab quark pairs while $C_{a\bar{b}}(\Delta y)$ describes that of the correlated $a\bar{b}$ pairs. Note that the a and/or b appeared in the subscript of G variable denote the charge a and/or b (i.e., net-u, net-d, and net-s numbers) while those in C variables denote the specific flavors. In this paper, we focus on the properties of short-range correlation (SRC) between charge a and charge b, generated by interactions of thermal partons during the late stage of QGP evolution, which is our most interesting part relating to QGP properties. One of important sources of such SRC correlations comes from gluons near hadronization. In QCM, a gluon hadronizes by splitting first into a quark-antiquark pair which then combines with other quarks and antiquarks into hadrons. This contributes a tight diagonal correlation in small-rapidity distance on the quark system at hadronization via a narrow-distributed $C_{u\bar{u}}(\Delta y)$, $C_{d\bar{d}}(\Delta y)$, and $C_{s\bar{s}}(\Delta y)$. On the other hand, $G_{ab}(\Delta y)$ variable suffers also the constraint of global charge conservation (GCC) imposed on charges a and b, respectively. In the case of zero charges for the system, one should have $\int G_{ab}(\Delta y)d\Delta y = 0$. GCC has a long-range characteristic since it is mainly induced by the charge separation during the first fm/c of the collisions. Definitely, GCC will contaminate the intrinsic correlation between charge a and charge b. In a naive way, as the short-range correlation is dominated by the $a\bar{b}$ pair correlations [by item $C_{a\bar{b}}(\Delta y)$], GCC would manifest itself by the long-range feature of ab correlation [by item $C_{ab}(\Delta y)$], and vice versa.

Based on the above discussions of two necessary ingredients of quark correlations, we can build a naive parametrization for the $G_{ab}(\Delta y)$ of quark system. We take a Gaussian approximation for the shapes of GCC and SRC correlations in rapidity, respectively, in light of the observed shape of the charge balance function in relativistic heavy-ion collisions [23]. Their distributions are denoted by $\mathcal{N}(0,\sigma_g)$ and $\mathcal{N}(0,\sigma_s)$, respectively. As the ab pair correlation dominates the SRC, we have

$$G_{ab}(\Delta y) = \frac{2n_{ab}^{(c)}}{\langle n_a\rangle\langle n_b\rangle}[\mathcal{N}(0,\sigma_s) - \mathcal{N}(0,\sigma_g)], \qquad (11)$$

where $n_{ab}^{(c)}$ denotes the number of correlated ab quark pairs. The coefficient 2 comes from the definition of Eq. (6) which incorporates the conjugation contribution of the $\bar{a}\bar{b}$ pair correlation. On the contrary, as the $a\bar{b}$ pair correlation dominates the SRC, we have

$$G_{ab}(\Delta y) = -\frac{2n_{a\bar{b}}^{(c)}}{\langle n_a\rangle\langle n_b\rangle}[\mathcal{N}(0,\sigma_s) - \mathcal{N}(0,\sigma_g)], \qquad (12)$$

where $n_{a\bar{b}}^{(c)}$ denotes the number of correlated $a\bar{b}$ quark pairs. The minus sign comes directly from the definition of Eq. (6). Summarizing the two above cases of the coefficient into a universal charge correlation matrix

$$\chi_{ab} = \begin{cases} 2n_{ab}^{(c)} & \text{when } ab \text{ flavor correlation dominates} \\ -2n_{a\bar{b}}^{(c)} & \text{when } a\bar{b} \text{ flavor correlation dominates,} \end{cases} \qquad (13)$$

we have

$$G_{ab}(\Delta y) = \frac{\chi_{ab}}{\langle n_a\rangle\langle n_b\rangle}[\mathcal{N}(0,\sigma_s) - \mathcal{N}(0,\sigma_g)]. \qquad (14)$$

Here, we consider three different charge correlation scenarios that are possible for the quark system at hadronization: (1) The system consists of only quasifree individual constituent quarks and antiquarks. The only correlation comes from the pair production of $u\bar{u}$, $d\bar{d}$, and $s\bar{s}$ in collisions. The χ_{ab} matrix is diagonal, i.e., $\chi_{uu} = -2n_{u\bar{u}}^{(c)} = -2\langle n_u\rangle$, $\chi_{ss} = -2n_{s\bar{s}}^{(c)} = -2\langle n_s\rangle$, and $\chi_{ud} = \chi_{us} = 0$. (2) Among quarks and antiquarks in the system there exists some tight correlations between quarks and antiquarks with different flavors in rapidity space. χ_{ab} has negative off-diagonal matrix elements which will generally decrease G_{MM} and G_{BM} but increase $G_{B\bar{B}}$ in magnitude. As an illustration, we consider an extreme case of strong quark-antiquark correlations with different flavors [27]. Every $\bar{s}$ quark is also correlated with a u or d quark, besides its intrinsic pair correlation with a s quark and we have $\chi_{us} = -2n_{u\bar{s}}^{(c)} \approx -\langle n_s\rangle$. Similarly, every u quark is also associated with a $\bar{d}$ or $\bar{s}$ antiquark and we have $\chi_{ud} = -2n_{u\bar{d}}^{(c)} \approx -2(\langle n_u\rangle - n_{u\bar{s}}^{(c)}) \approx -2(\langle n_u\rangle - \frac{1}{2}\langle n_s\rangle)$. (3) Different from the former, there exist a tight correlation between two (anti-)quarks. χ_{ab} has positive off-diagonal matrix elements which generally decrease $G_{B\bar{B}}$ but increase G_{MM} and G_{BM}. As an example of this scenario, we take a thermal ansatz under the Boltzmann distribution. The number of correlated two quarks in rapidity is assumed to be thermal distributed. The mass of two correlated quarks is the sum of individual quark masses. Fixing the hadronization temperature $T = 165$ MeV and the mass of individual quarks $m_u = 330$ MeV and $m_s = 500$ MeV, we estimate the magnitude of off-diagonal elements to be $\chi_{ud} \approx 0.34\langle n_u\rangle$ and $\chi_{us} \approx 0.52\langle n_s\rangle$, respectively.

Three different charge correlation scenarios can be identified by the measurement of few hadronic $G_{\alpha\beta}$. In Fig. 1, we show calculations of $G_{p\bar{\Lambda}}(\Delta y)$, $G_{p\bar{\Xi}^0}(\Delta y)$, and $G_{p\bar{\Omega}^+}(\Delta y)$ at three scenarios as their effective discrimination. In calculations, the quark rapidity densities are taken to be $\langle n_u\rangle = \langle n_d\rangle = 710$ and $\langle n_s\rangle = 290$, respectively, by using SDQCM to fit the experimental data of rapidity density of pions and kaons in central Pb + Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV [49]. For widths σ_g and σ_s we temporarily assume a flavor-blind value for the purpose of qualitative analysis only. The GCC distribution width σ_g is taken to be 3.8, fixed roughly by the data of pseudorapidity distribution of charged particles [50]. The SRC width σ_s is fixed to be 0.45 by the data of charge balance function [26] in the collisions.

It can be seen from Fig. 1 that, in the diagonal case of χ_{ab}, the production between p and $\bar{\Lambda}$, $\bar{\Xi}^0$ is associated. This is quite natural due to the local neutrality of net-u charge

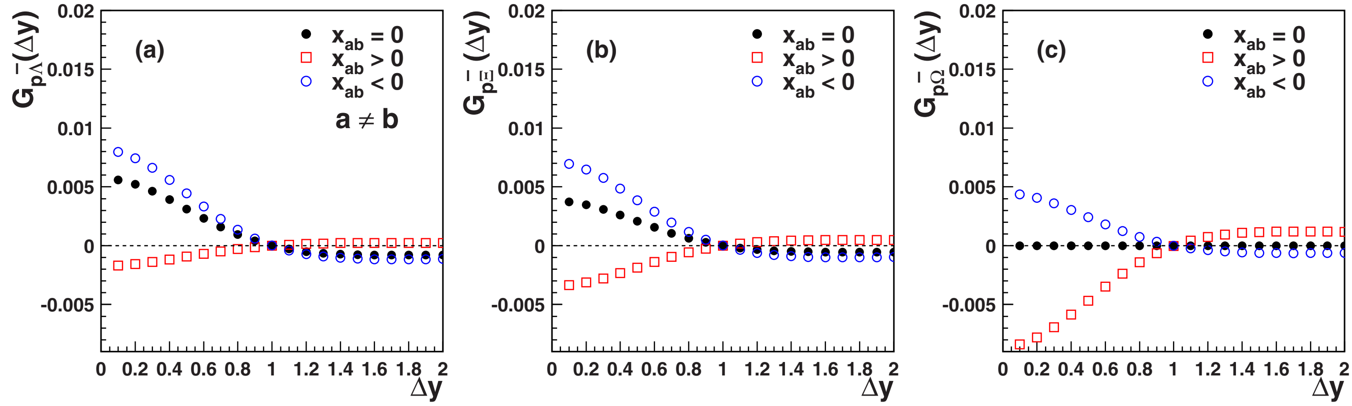

FIG. 1. (Color online) $G_{p\bar{\Lambda}}(\Delta y)$, $G_{p\bar{\Xi}^0}(\Delta y)$, and $G_{p\bar{\Omega}^+}(\Delta y)$ as a function of Δy, as χ_{ab} matrix is diagonal (filled circles), has positive off-diagonal elements (open squares) and has negative off-diagonal elements (open circles).

in the system or, in other words, the light-quark and light-antiquark production association. The production of p and of Ω^- are independent of each other in this case because of the vanishing G_{us} component. The negative off-diagonal χ_{ab} elements increase their production association magnitudes, in particular for p and $\bar{\Omega}^+$. On the contrary, the large positive values of the off-diagonal χ_{ab} elements may change the sign of the $G_{p\bar{\Lambda}}$, $G_{p\bar{\Xi}^0}$, and even $G_{p\bar{\Omega}^+}$, which means the production between p and these antihyperons is no longer concomitant but repulsive. We can see that $G_{p\bar{\Lambda}}(\Delta y)$ and $G_{p\bar{\Xi}^0}(\Delta y)$ can effectively discriminate the scenario (3) from others while $G_{p\bar{\Omega}^+}(\Delta y)$ is all powerful in three cases.

There are two available methods measuring the hadron $G_{\alpha\beta}(\Delta y)$ in experiments. The first is that adopted in e^+e^- and $p\bar{p}$ reactions in the early years, i.e., choosing hadrons α and $\bar{\alpha}$ at a specific rapidity, e.g., at $y = 0$, as the test particles and then recording rapidity distances between every hadron β ($\bar{\beta}$) and test particles event by event. The second is that used recently in balance function measurements in AA collisions. Considering that the detectors have a finite acceptance rapidity window y_w, statistics of all hadrons $\alpha, \bar{\alpha}, \beta$, and $\bar{\beta}$ in this window generates the partial correlation function $G_{\alpha\beta}(\Delta y|y_w)$, and then divide it by the scale factor $1 - \Delta y/y_w$ proposed in Ref. [14] to remove the finite-window effects and restore the theoretical definition.

IV. RESONANCE DECAY EFFECTS

A significant contribution to the correlation function of final hadrons measured experimentally comes from the decay of short-life hadrons. Let us consider first the decay contribution to the single-hadron rapidity distribution:

$$n_i^{(f)}(y) = n_i(y) + \sum_j \int dy_0 n_j(y_0)\mathcal{D}(j, y_0 \to i, y), \quad (15)$$

where superscript (f) denotes the final-state hadrons including resonance decays. The decay function $\mathcal{D}(j, y_0 \to i, y)$ denotes the probability of finding an i-type hadron of rapidity y from the decay products of a j-type hadron of rapidity y_0. It is determined by the branch ratio of the decay channel $Br(j \to i)$ and the corresponding decay kinematics and by the transverse motion of the mother particle. In order to focus on longitudinal dynamics, we use an event-averaged transverse momentum (p_T) distribution for mother particles in decay.

Using Eq. (15) we obtain the correlation function of final hadrons:

$$\begin{aligned} G_{\alpha\beta}^{(f)}(y_1, y_2) = {} & \frac{\langle n_\alpha(y_1)\rangle\langle n_\beta(y_2)\rangle}{\langle n_\alpha^{(f)}(y_1)\rangle\langle n_\beta^{(f)}(y_2)\rangle} G_{\alpha\beta}(y_1, y_2) + \sum_k \int dy_0 \mathcal{D}(k, y_0 \to \beta, y_2) \frac{\langle n_\alpha(y_1)\rangle\langle n_k(y_0)\rangle}{\langle n_\alpha^{(f)}(y_1)\rangle\langle n_\beta^{(f)}(y_2)\rangle} G_{\alpha k}(y_1, y_0) \\ & + \sum_k \int dy_0 \mathcal{D}(k, y_0 \to \alpha, y_1) \frac{\langle n_k(y_0)\rangle\langle n_\beta(y_2)\rangle}{\langle n_\alpha^{(f)}(y_1)\rangle\langle n_\beta^{(f)}(y_2)\rangle} G_{k\beta}(y_0, y_2) \\ & + \sum_{j,k} \iint dy_0 dy_0' \mathcal{D}(j, y_0 \to \alpha, y_1)\mathcal{D}\left(k, y_0' \to \beta, y_2\right) \times \frac{\langle n_j(y_0)\rangle\langle n_k\left(y_0'\right)\rangle}{\langle n_\alpha^{(f)}(y_1)\rangle\langle n_\beta^{(f)}(y_2)\rangle} G_{jk}\left(y_0, y_0'\right). \end{aligned} \quad (16)$$

Here we have utilized the symmetry of the summation index, e.g., $\sum_k \mathcal{D}(k, y_0 \to \beta, y_2) C_{\alpha k}(y_1, y_0) = \sum_k \mathcal{D}(\bar{k}, y_0 \to \beta, y_2) C_{\alpha\bar{k}}(y_1, y_0)$, and charge conjugation symmetry $\mathcal{D}(k, y_0 \to \beta, y_2) = \mathcal{D}(\bar{k}, y_0 \to \bar{\beta}, y_2)$ for the decay function.

Applying the broken longitudinal boost invariance [48] to heavy-ion collisions at LHC energies, we expect a rapidity-independent yield density $\langle n_\alpha(y)\rangle = \langle n_\alpha\rangle$ in the central rapidity region. The double hadron ratio in Eq. (16) can be abbreviated as

$$\frac{\langle n_{\alpha'}(y_1)\rangle\langle n_{\beta'}(y_2)\rangle}{\langle n_\alpha^{(f)}(y_1)\rangle\langle n_\beta^{(f)}(y_2)\rangle} = \frac{\langle n_{\alpha'}\rangle\langle n_{\beta'}\rangle}{\langle n_\alpha^{(f)}\rangle\langle n_\beta^{(f)}\rangle} \equiv R_{\alpha\beta}^{\alpha'\beta'}.$$

The correlation function $G_{\alpha\beta}(\Delta y)$ depends only on the relative rapidity interval $\Delta y = y_2 - y_1$. After abbreviating the decay function $\mathcal{D}(k, y_0 \to \beta, y_2) = \mathcal{D}_{k\beta}(y_2 - y_0)$, we get finally

$$G_{\alpha\beta}^{(f)}(\Delta y) = R_{\alpha\beta}^{\alpha\beta} G_{\alpha\beta}(\Delta y) + \sum_k R_{\alpha\beta}^{\alpha k} \int d\Delta \mathcal{D}_{k\beta}(\Delta) G_{\alpha k}(\Delta y - \Delta) + \sum_k R_{\alpha\beta}^{k\beta} \int d\Delta \mathcal{D}_{k\alpha}(\Delta) G_{k\beta}(\Delta y + \Delta)$$
$$+ \sum_{j,k} R_{\alpha\beta}^{jk} \iint d\Delta_1 d\Delta_2 \mathcal{D}_{j\alpha}(\Delta_1) \mathcal{D}_{k\beta}(\Delta_2) G_{jk}(\Delta y + \Delta_1 - \Delta_2). \tag{17}$$

We see that $G_{\alpha\beta}^{(f)}(\Delta y)$ receives three parts of decay contributions. The first and second are single-decay contributions from αk and $k\beta$ correlations, respectively. Here k denotes the resonance that can decay into α and/or β. The third is the double decay contribution from jk correlations where j and k are resonances that can decay into α and β, respectively. Note that neutral particles such as ϕ and ω do not contribute to final state $G_{\alpha\beta}^{(f)}(\Delta y)$ because $G_{\phi h}(\Delta y) = G_{\omega h}(\Delta y) = 0$.

Considering that ALICE experiments at LHC have the ability of correcting the weak decays of strange hyperons, we present only the effects of the strong and electromagnetic (S&EM) decays on the correlation function. In calculations, only $J^P = 0^-$ and 1^- mesons and $J^P = (1/2)^+$ and $(3/2)^+$ baryons in the flavor SU(3) ground state are included. The decay function $\mathcal{D}(j, y_0 \to i, y)$ in Eqs. (15) and (17) is evaluated using the SDQCM, in which the decay branch ratios are taken from the Particle Data Group [51] and the influence of transverse motion of the mother particle in decay is also included based on our recent work of p_T distribution of identified hadrons at LHC [45]. Double hadron ratios $R_{\alpha\beta}^{\alpha'\beta'}$ are calculated according to previous results in Refs. [43,45].

Figure 2 shows $G_{\alpha\beta}(\Delta y)$ of initial hadrons produced at hadronization (open circles) and those $G_{\alpha\beta}^{(f)}(\Delta y)$ including S&EM decays (open squares), as the quark system that existed previously has the vanishing off-diagonal correlation matrix elements. We find that the effects of S&EM decays are varied with hadron species. Calculations of resonance decay effects in other two scenarios of quark system discussed in Sec. III are found to be similar.

Correlations involving pion $G_{\pi h}^{(f)}(\Delta y)$ are mostly decay influenced because final-state pions contain lots of resonance decay contributions. For example, the number of decay contribution channels (including single and double decay contributions) in $G_{\pi^+\pi^-}^{(f)}(\Delta y)$ exceeds 170, and those in $G_{\pi^+K^+}^{(f)}(\Delta y)$ and others are also up to dozens. This leads to the complex resonance decay effects. We see that $G_{\pi^+\pi^-}^{(f)}(\Delta y)$, $G_{\pi^+K^-}^{(f)}(\Delta y)$, $G_{\pi^+\bar{p}}^{(f)}(\Delta y)$, and $G_{\pi^-\bar{\Xi}^0}^{(f)}(\Delta y)$ are significantly suppressed by S&EM decays while $G_{\pi^+\bar{\Lambda}}^{(f)}(\Delta y)$ and $G_{\pi^+\bar{\Omega}^+}^{(f)}(\Delta y)$ are almost unchanged. In addition, the rapidity shift $\Delta = y_\pi - y_R$ in resonance decay $R \to \pi$ is large, usually on average $\Delta \gtrsim 0.5$. This causes a nontrivial smearing effect to the correlation function by a wide distribution of decay function in Eq. (17).

We find that final state $G_{K^+K^-}^{(f)}(\Delta y)$ and $G_{pK^-}^{(f)}(\Delta y)$ are decreased by about 20% in magnitude, compared with the initial ones. This is mainly because the channel $K^{*0} \to K^+ + \pi^-$ contributes to the final K^+, which introduces contribution terms $G_{K^{*0}h}(\Delta y)$ in $G_{K^+h}^{(f)}(\Delta y)$. In addition, the averaged rapidity shift $\Delta = y_K - y_{K^*} \gtrsim 0.3$ is also relatively large, causing the certain smearing effect on the final-state correlation function. Due to similar but small-in-magnitude reasons, $G_{K^+\Lambda}^{(f)}(\Delta y)$ and $G_{K^+\Omega^-}^{(f)}(\Delta y)$ are slightly decreased. $G_{p\bar{p}}^{(f)}(\Delta y)$ and $G_{p\bar{\Xi}^+}^{(f)}(\Delta y)$ are also slightly changed due to similar reasons in resonance decaying into proton process.

We see that $G_{p\bar{\Lambda}}^{(f)}(\Delta y)$, $G_{K^+\Xi^0}^{(f)}(\Delta y)$ as well as two-hyperon correlations $G_{\Lambda\bar{\Lambda}}^{(f)}(\Delta y)$, $G_{\Lambda\bar{\Xi}^0}^{(f)}(\Delta y)$, $G_{\Lambda\bar{\Omega}^+}^{(f)}(\Delta y)$, $G_{\Xi^0\bar{\Xi}^0}^{(f)}(\Delta y)$, and $G_{\Xi^0\bar{\Omega}^+}^{(f)}(\Delta y)$ are almost unchanged by S&EM decays. This is mainly because of the quite narrow rapidity shift ($\Delta \sim 0.1$) and better flavor inheritance in $B^* \to B$ decays. These slightly changed and almost unchanged correlation functions are good probes in heavy-ion collisions at LHC energies.

V. SUMMARY AND DISCUSSION

In summary, the study of two-hadron production correlations is one of the important means to understand the charge correlation properties of hot quark system produced in ultrarelativistic heavy-ion collisions. By introducing a new correlation function $G_{\alpha\beta}(\Delta y)$, we presented a direct connection between identified two-hadron correlations and charge correlations of the quark system that existed previously in the framework of QCM of hadron production. The correlation between hadron species α and β via the function $G_{\alpha\beta}(\Delta y)$ is just expressed as the linear combination of several charge correlations of quarks before hadronization. This conciseness is very beneficial for the extraction of charge correlation of quark system in experiments by hadronic observation. As an instance, we discussed three possible charge correlation scenarios of the quark system, i.e., the system dominated by quasifree quarks and antiquarks, by correlated quark-antiquark pairs with different flavors, and by two correlated (anti-)quarks with different flavors, and we clarified their hadronic signals by the correlations $G_{p\bar{\Lambda}}(\Delta y)$, $G_{p\bar{\Xi}^0}(\Delta y)$, and $G_{p\bar{\Omega}^+}(\Delta y)$. This new kind of the correlation function is an important supplement to available experimental measurements mainly by balance functions and is useful to gain more insights into the charge correlation properties of the produced hot quark matter in heavy-ion collisions at LHC.

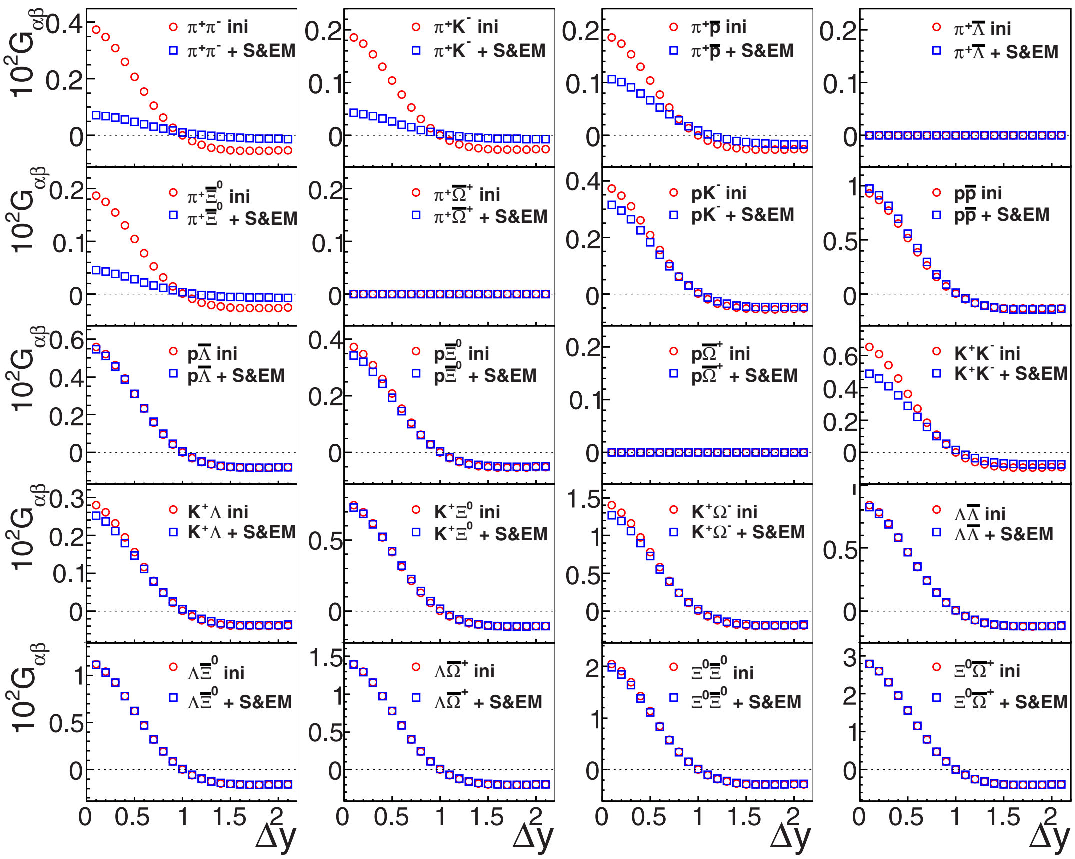

FIG. 2. (Color online) $G_{\alpha\beta}(\Delta y)$ of initial hadrons produced at hadronization (open circles) and those including strong and electromagnetic decays (open squares), as the quark system existed previously has vanishing off-diagonal correlation-matrix elements.

One point needed to address at last is the hadronization effects on the charge correlation properties of the system. One may puzzle that Eqs. (5) and (7) seem to just only translate quark charge correlations into hadrons. In fact, hadronization implicitly plays its role in two-hadron correlations which is manifested by calculating the absolute correlations $\langle n_\alpha\rangle\langle n_\beta\rangle G_{\alpha\beta}(\Delta y)$. Also, if we directly calculate the correlations among B, C, and S, we can find the explicitly nontrivial effects of the hadronization. For example, the baryon-strangeness correlation $(\langle BS\rangle - \langle B\rangle\langle S\rangle)/\langle S^2\rangle$ will decrease obviously after hadronization. Here, we emphasize that the direct and clear connection between quark charge correlations and identified hadron correlations in Eqs. (5) and (7) result from the adoption of the proper charge transmitters (i.e., net quark numbers) as well as the proper correlation function we proposed. In addition, in the QCM description of QGP hadronization, the gluon in the system at hadronization is usually replaced by a quark-antiquark pair and then these quark-antiquark pairs together with those original quarks and antiquarks in QGP combine into various hadrons. These newly produced quarks and antiquarks by gluon splitting contribute extra charge correlations in small-Δy region on the system via $u\bar{u}$, $d\bar{d}$, $s\bar{s}$ production association, which is also a significant effect of the hadronization. In the above formulas we do not distinguish these different origins of quarks and antiquarks since we directly start from the system of quarks and antiquarks. Separating gluon effects from $G_{\alpha\beta}$ measurements is not an easy task, which needs a careful nonperturbative phenomenological analysis. However, even though the quantitative calculation is difficult, some qualitative properties are easily obtained. For example, the production of quark antiquark pairs at hadronization will increase χ_{uu} and χ_{ss} of the system and therefore strengthen mostly the correlations between particle pairs with opposite quantum numbers. This conclusion is in agreement with the expectation of Ref. [13] but is different from Ref. [27] in which the hadronization might weaken the production association between proton and antiproton. Finally, we argue that through the measurements of $G_{\alpha\beta}(\Delta y)$ we can observe the correlation properties between different flavors of quarks just before hadronization. This point is similar to the case of hadronic elliptic flow studies in which, as the constituent quark number scaling is observed, we say that we observe the flow of quarks.

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APPENDIX: DERIVATION OF EQ. (5)

Regarding the $n_\alpha^{(q)}(y_1)$ and $n_\alpha^{(q)}(y_1)n_\beta^{(q)}(y_2)$ as functions of quark numbers at y_1 and/or y_2, we expand the $[n_\alpha^{(q)}(y_1)-n_{\bar\alpha}^{(q)}(y_1)][n_\beta^{(q)}(y_2)-n_{\bar\beta}^{(q)}(y_2)]$ around the event-averaged values of quark numbers:

$$\begin{aligned}
&\big[n_\alpha^{(q)}(y_1)-n_{\bar\alpha}^{(q)}(y_1)\big]\big[n_\beta^{(q)}(y_2)-n_{\bar\beta}^{(q)}(y_2)\big]\\
&\quad=\big[n_\alpha^{(q)}(y_1)-n_{\bar\alpha}^{(q)}(y_1)\big]\big[n_\beta^{(q)}(y_2)-n_{\bar\beta}^{(q)}(y_2)\big]\Big|_{\substack{\langle n_q(y_1)\rangle\\ \langle n_q(y_2)\rangle}}\\
&\qquad+\sum_{f_1}\frac{\partial\big[n_\alpha^{(q)}(y_1)-n_{\bar\alpha}^{(q)}(y_1)\big]\big[n_\beta^{(q)}(y_2)-n_{\bar\beta}^{(q)}(y_2)\big]}{\partial n_{f_1}(y_1)}\Bigg|_{\substack{\langle n_q(y_1)\rangle\\ \langle n_q(y_2)\rangle}}\delta n_{f_1}(y_1)\\
&\qquad+\sum_{f_2}\frac{\partial\big[n_\alpha^{(q)}(y_1)-n_{\bar\alpha}^{(q)}(y_1)\big]\big[n_\beta^{(q)}(y_2)-n_{\bar\beta}^{(q)}(y_2)\big]}{\partial n_{f_2}(y_2)}\Bigg|_{\substack{\langle n_q(y_1)\rangle\\ \langle n_q(y_2)\rangle}}\delta n_{f_2}(y_2)\\
&\qquad+\frac{1}{2}\sum_{f_1,f_2}\frac{\partial^2\big[n_\alpha^{(q)}(y_1)-n_{\bar\alpha}^{(q)}(y_1)\big]\big[n_\beta^{(q)}(y_2)-n_{\bar\beta}^{(q)}(y_2)\big]}{\partial n_{f_1}(y_1)\,\partial n_{f_2}(y_1)}\Bigg|_{\substack{\langle n_q(y_1)\rangle\\ \langle n_q(y_2)\rangle}}\delta n_{f_1}(y_1)\,\delta n_{f_2}(y_1)\\
&\qquad+\frac{1}{2}\sum_{f_1,f_2}\frac{\partial^2\big[n_\alpha^{(q)}(y_1)-n_{\bar\alpha}^{(q)}(y_1)\big]\big[n_\beta^{(q)}(y_2)-n_{\bar\beta}^{(q)}(y_2)\big]}{\partial n_{f_1}(y_2)\,\partial n_{f_2}(y_2)}\Bigg|_{\substack{\langle n_q(y_1)\rangle\\ \langle n_q(y_2)\rangle}}\delta n_{f_1}(y_2)\,\delta n_{f_2}(y_2)\\
&\qquad+\sum_{f_1,f_2}\frac{\partial^2\big[n_\alpha^{(q)}(y_1)-n_{\bar\alpha}^{(q)}(y_1)\big]\big[n_\beta^{(q)}(y_2)-n_{\bar\beta}^{(q)}(y_2)\big]}{\partial n_{f_1}(y_1)\,\partial n_{f_2}(y_2)}\Bigg|_{\substack{\langle n_q(y_1)\rangle\\ \langle n_q(y_2)\rangle}}\delta n_{f_1}(y_1)\,\delta n_{f_2}(y_2)\\
&\qquad+O(\delta^3),
\end{aligned}\tag{A1}$$

where $\delta n_{f_1}(y_1)=n_{f_1}(y_1)-\langle n_{f_1}(y_1)\rangle$. Here, f_1 and f_2 run over all kinds of the flavors of quarks and antiquarks contained in system. Note that subscript $\langle n_q(y_1)\rangle$ and $\langle n_q(y_2)\rangle$ denote the evaluations of these items at event averaged numbers of all quark flavors at y_1 and y_2. In the case of zero baryon number density, the first five items on the right-hand side of equation are zero and only the last item has the nontrivial contributions, up to second order in the fluctuations of quark numbers. After event average, we have

$$\begin{aligned}
&\big\langle\big[n_\alpha^{(q)}(y_1)-n_{\bar\alpha}^{(q)}(y_1)\big]\big[n_\beta^{(q)}(y_2)-n_{\bar\beta}^{(q)}(y_2)\big]\big\rangle\\
&\quad=\sum_{f_1,f_2}\frac{\partial^2\big[n_\alpha^{(q)}(y_1)-n_{\bar\alpha}^{(q)}(y_1)\big]\big[n_\beta^{(q)}(y_2)-n_{\bar\beta}^{(q)}(y_2)\big]}{\partial n_{f_1}(y_1)\,\partial n_{f_2}(y_2)}\Bigg|_{\substack{\langle n_q(y_1)\rangle\\ \langle n_q(y_2)\rangle}}\langle\delta n_{f_1}(y_1)\,\delta n_{f_2}(y_2)\rangle+O(\langle\delta^3\rangle).
\end{aligned}\tag{A2}$$

For $\langle n_\alpha^{(q)}(y)\rangle$, we have similarly

$$\big\langle n_\alpha^{(q)}(y)\big\rangle=n_\alpha^{(q)}(y)\big|_{\langle n_q(y)\rangle}+\sum_{f_1,f_2}\frac{\partial^2 n_\alpha^{(q)}(y)}{\partial n_{f_1}(y)\,\partial n_{f_2}(y)}\Bigg|_{\langle n_q(y)\rangle}\langle\delta n_{f_1}(y)\,\delta n_{f_2}(y)\rangle+O(\langle\delta^3\rangle).\tag{A3}$$

Substituting them into Eq. (4), we have

$$\begin{aligned}
G_{\alpha\beta}(y_1,y_2)&=\sum_{f_1,f_2}\left\{\frac{1}{n_\alpha^{(q)}(y_1)\,n_\beta^{(q)}(y_2)}\frac{\partial^2\big[n_\alpha^{(q)}(y_1)-n_{\bar\alpha}^{(q)}(y_1)\big]\big[n_\beta^{(q)}(y_2)-n_{\bar\beta}^{(q)}(y_2)\big]}{\partial n_{f_1}(y_1)\,\partial n_{f_2}(y_2)}\right\}\Bigg|_{\substack{\langle n_q(y_1)\rangle\\ \langle n_q(y_2)\rangle}}\\
&\quad\times\langle\delta n_{f_1}(y_1)\,\delta n_{f_2}(y_2)\rangle+O(\langle\delta^3\rangle)\\
&=\sum_{f_1,f_2}\frac{A_{f_1f_2}}{\langle n_{f_1}(y_1)\rangle\langle n_{f_2}(y_2)\rangle}\langle\delta n_{f_1}(y_1)\,\delta n_{f_2}(y_2)\rangle+O(\langle\delta^3\rangle).
\end{aligned}\tag{A4}$$

Since higher-order fluctuations are usually suppressed by the factor of $1/\sqrt{n_q}$, we neglect them and get, to second order in fluctuations of quark numbers,

$$G_{\alpha\beta}(y_1,y_2) = \sum_{f_1,f_2} \frac{A_{f_1 f_2}}{\langle n_{f_1}(y_1)\rangle\langle n_{f_2}(y_2)\rangle}\langle \delta n_{f_1}(y_1)\,\delta n_{f_2}(y_2)\rangle = \sum_{f_1,f_2} A_{f_1 f_2}\frac{C_{f_1 f_2}(y_1,y_2)}{\langle n_{f_1}(y_1)\rangle\langle n_{f_2}(y_2)\rangle}, \tag{A5}$$

where $A_{f_1 f_2} = (n_{\alpha,f_1} - n_{\bar{\alpha},f_1})(n_{\beta,f_2} - n_{\bar{\beta},f_2})$. n_{α,f_1} is the number of quarks f_1 contained in hadron α. f_1 and f_2 run over all types of quark and antiquark flavors contained in system. $C_{f_1 f_2}(y_1,y_2) = \langle \delta n_{f_1}(y_1)\delta n_{f_2}(y_2)\rangle$ is the ordinary two-point correlation function.

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