

**Double- $\beta$  decay within a consistent deformed approach**

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(Received 29 January 2015; revised manuscript received 28 April 2015; published 28 May 2015)

In this paper we present a timely application of the proton-neutron deformed quasiparticle random-phase approximation ( $pn$ -dQRPA), designed to describe in a consistent way the  $1^+$  Gamow-Teller states in odd-odd deformed nuclei. For this purpose we apply a projection before variation procedure by using a single-particle basis with projected angular momentum, provided by the diagonalization of a spherical mean field plus quadrupole-quadrupole interaction. The residual Hamiltonian contains pairing plus proton-neutron dipole terms in particle-hole and particle-particle channels, with constant strengths. As an example we describe the two-neutrino double-beta ( $2\nu\beta\beta$ ) decay of  $^{150}\text{Nd}$  to the ground state of  $^{150}\text{Sm}$ . The experimental ( $p, n$ ) type of strength in  $^{150}\text{Nd}$  and the ( $n, p$ ) type of strength in  $^{150}\text{Sm}$  are reasonably reproduced and the  $2\nu\beta\beta$  decay matrix element depicts a strong dependence upon the particle-particle strength  $g_{pp}$ . The experimental half-life is reproduced for  $g_{pp} = 0.05$ . It turns out that the measured half-lives for  $2\nu\beta\beta$  transitions between other deformed superfluid partners with mass numbers  $A = 82, 96, 100, 128, 130, 238$  are reproduced with fairly good accuracy by using this value of  $g_{pp}$ .

DOI: [10.1103/PhysRevC.91.054329](https://doi.org/10.1103/PhysRevC.91.054329)

PACS number(s): 23.40.Bw, 23.40.Hc, 21.60.Jz, 27.70.+q

**I. INTRODUCTION**

One of the important topics in both nuclear physics and particle physics is the investigation of nuclear double- $\beta$  decays [1,2]. The neutrinoless mode,  $0\nu\beta\beta$  decay, is especially interesting due to its potential to explore physics beyond the standard model, in particular to discover the fundamental nature of the neutrino and describing in a reasonable way the  $1^+$  Gamow-Teller states its absolute mass scale. The major problem here is to relate quantitatively the potential experimental discoveries to the neutrino properties since this has to be done through the nuclear matrix elements (NMEs) which depend on detailed many-body features of nuclei [3]. At present there are many models that are able to tackle the problem of double- $\beta$  decay in medium-heavy and heavy nuclei. All these models have their deficiencies and strong points concerning the model space, configurations, deformation, shell closures, etc. For recent reviews and analyses of these models see [4–7]. The traditionally used microscopic model for double- $\beta$  calculations is the proton-neutron quasiparticle random-phase approximation ( $pn$ -QRPA) [8]. Mostly the  $pn$ -QRPA based on a spherical mean field has been used in the calculations. However, many  $\beta$  and double- $\beta$  decaying nuclei are more or less deformed and therefore it is very important to extend the description to a deformed mean field. This is the starting point of the deformed  $pn$ -QRPA ( $pn$ -dQRPA). Most earlier approaches describe Gamow-Teller  $\beta$  decays by using a  $pn$ -dQRPA phonon in the intrinsic system of coordinates, i.e., in terms of pairs of Nilsson quasiparticles coupled to a  $K = 1$  spin projection. The physical observables, like  $\beta$ -decay transition probabilities, are then estimated by rotating the intrinsic phonon to the laboratory system of coordinates [9,10]. This formalism was applied in order to

describe the  $1^+$  Gamow-Teller states and  $2\nu\beta\beta$  decay in several papers [11–14].

Let us mention that this projection after variation procedure restores only the symmetry of the phonon, by leaving the  $pn$ -dQRPA ground state deformed. A more consistent approach is to use a single-particle (sp) basis with good angular momentum, i.e., a projection before variation procedure. One way to obtain this basis consists in projecting good angular momentum from the product between a coherent state, describing the deformed core, and a spherical sp state [15]. The  $pn$ -dQRPA phonon, describing Gamow-Teller  $\beta$  decays, is built by using pairs of these quasiparticles that are “dressed by deformation”, coupled to the spin  $J = 1$  [16,17]. In Ref. [18] this approach was generalized, by considering all allowed spherical sp states in order to build a sp state “dressed by deformation”. A particular case is the adiabatic limit, which coincides with the usual Nilsson wave function rotated to the laboratory frame. We successfully described the available experimental  $B(E2)$  values for collective states in the range  $50 \leq Z \leq 100$  in even-even nuclei, by using the adiabatic version of this formalism [18].

**II. THEORETICAL BACKGROUND**

In order to describe the  $1^+$  Gamow-Teller states in odd-odd deformed nuclei, we will generalize this formalism to the  $pn$ -dQRPA case. To this purpose we will perform the following steps:

- (i) we built a deformed sp basis with good angular momentum, by diagonalizing a deformed mean field;
- (ii) we then transform dipole  $\beta$ -decay operators in this deformed sp representation;

- (iii) we introduce quasiparticle representation separately for protons and neutrons and then we diagonalize dipole-dipole interaction within the  $pn$ -dQRPA;
- (iv) we finally compute Gamow-Teller transitions.

### A. Deformed single-particle basis

We use a deformed sp basis with good angular momentum [18]

$$a_{\tau\epsilon jm}^\dagger(\Omega) = \sum_{Jk} \mathcal{X}_{\tau\epsilon j}^{Jk} [D_{.0}^{J*}(\Omega) c_{\tau k}^\dagger]_{jm}, \quad (1)$$

in terms of normalized Wigner functions  $D_{M0}^J$  (the dot denotes that the  $M$  projection is used for angular momentum coupling) and spherical creation operators  $c_{\tau k}^\dagger$ , describing the eigenstates of a spherical nuclear plus proton Coulomb mean field. The expansion coefficients  $\mathcal{X}$ , together with eigenvalues  $\epsilon$ , are found by diagonalizing a quadrupole-quadrupole interaction. In the adiabatic approach, where the expansion coefficients are proportional to standard Nilsson coefficients, one has  $j = K$ , where  $K$  is the spin projection on the intrinsic symmetry axis.

### B. Dipole proton-neutron $\beta$ -decay operators

The dipole operators describing Gamow-Teller  $\beta$  decays are given by

$$\begin{aligned} D_{1\mu}^- &= \frac{1}{\sqrt{3}} \sum_{pn} (p||\sigma||n) [a_p^\dagger \tilde{a}_n]_{1\mu}, \\ P_{1\mu}^- &= \frac{1}{\sqrt{3}} \sum_{pn} (p||\sigma||n) [a_p^\dagger a_n^\dagger]_{1\mu}. \end{aligned} \quad (2)$$

Here,  $\sigma_\mu$  is the Pauli operator and the reduced matrix element in the deformed basis (1) is given in terms of the standard spherical matrix element by [18]

$$\begin{aligned} (p||\sigma||n) &= \hat{j}_p \hat{j}_n \sum_{J=\text{even}} \sum_{k_p k_n} \mathcal{X}_{\tau\epsilon_p j_p}^{Jk_p} \mathcal{X}_{\tau\epsilon_n j_n}^{Jk_n} \\ &\times (-)^{k_p - j_n} W(j_p k_p j_n k_n; J 1) \langle k_p ||\sigma||k_n \rangle, \end{aligned} \quad (3)$$

where  $\hat{j} = \sqrt{2j+1}$  and  $W$  is the Racah symbol. In the spherical limit with  $J = 0$  it becomes the standard reduced matrix element of the Pauli operator.

We use an ordinary monopole pairing plus a separable dipole-dipole proton-neutron interaction, with constant strengths, in both the particle-hole (ph) and particle-particle (pp) channels, i.e.,

$$\begin{aligned} H &= \sum_p (\epsilon_p - \lambda^{\text{prot}}) N_p - \frac{G_{\text{pair}}^{\text{prot}}}{4} \sum_{pp'} P_p^\dagger P_{p'} \\ &+ \sum_n (\epsilon_n - \lambda^{\text{neut}}) N_n - \frac{G_{\text{pair}}^{\text{neut}}}{4} \sum_{nn'} P_n^\dagger P_{n'} \\ &+ g_{\text{ph}} \sum_\mu D_{1\mu}^- (D_{1\mu}^-)^\dagger - g_{\text{pp}} \sum_\mu P_{1\mu}^- (P_{1\mu}^-)^\dagger, \end{aligned} \quad (4)$$

where the meaning of the short-hand notation is  $\tau \equiv (\tau\epsilon j)$ ,  $\tau = p, n$ . Here, the chemical potential for protons (neutrons)

is denoted by  $\lambda^{\text{prot}}$  ( $\lambda^{\text{neut}}$ ). The deformed particle-number and pairing operators are respectively given by [18]

$$\begin{aligned} N_\tau &= \frac{2}{2j_\tau + 1} \sum_m a_{\tau m}^\dagger a_{\tau m}, \\ P_\tau^\dagger &= \frac{2}{2j_\tau + 1} \sum_m a_{\tau m}^\dagger a_{\tau -m}^\dagger (-)^{j_\tau - m}. \end{aligned} \quad (5)$$

Let us mention that any interaction can be expanded in the multipole-multipole separable form, and therefore the deformed representation of the one-body operator can be used to build a general interaction.

### C. Quasiparticle representation

We use the quasiparticle representation

$$a_{\tau m}^\dagger = u_\tau \alpha_{\tau m}^\dagger + v_\tau \alpha_{\tau -m} (-)^{j_\tau - m}, \quad (6)$$

where  $u$  and  $v$  are the BCS vacancy and occupation amplitudes, respectively, in order to obtain the  $\beta$ -decay operators entering the Hamiltonian (4). By using the dipole phonon

$$\Gamma_{1\mu}^\dagger(\omega) = \sum_{pn} \{ X_{pn}^\omega [\alpha_p^\dagger \alpha_n^\dagger]_{1\mu} - Y_{pn}^\omega [\alpha_p^\dagger \alpha_n^\dagger]_{1\mu}^\dagger \}, \quad (7)$$

one obtains in a standard  $pn$ -dQRPA equations of motion determining the eigenvalues  $\omega$  and amplitudes  $X^\omega, Y^\omega$  [18,19]. They formally coincide with the spherical  $pn$ -QRPA equations, but the pair basis in the phonon (7) couple proton and neutron states with deformed sp spectra. Thus, in the present approach the QRPA vacuum is spherical, in contrast to the approximations adopted earlier where the spherical symmetry of the phonon was restored after variation, still leaving the vacuum itself deformed.

### D. Gamow-Teller transitions

Gamow-Teller  $\beta$  decay transition matrix elements [19,20] are given by

$$\begin{aligned} \langle 0 || \beta^- || 0 \rangle &= \sum_{pn} (p||\sigma||n) [u_p v_n X_{pn}^\omega + v_p u_n Y_{pn}^\omega], \\ \langle 0 || \beta^+ || 0 \rangle &= \sum_{pn} (p||\sigma||n) [v_p u_n X_{pn}^\omega + u_p v_n Y_{pn}^\omega]. \end{aligned} \quad (8)$$

We write the  $2\nu\beta\beta$  Gamow-Teller matrix element as follows [8]:

$$M_{\text{GT}} = \sum_{mm} \frac{\langle 0 || \beta^- || \omega_m^f \rangle \langle \omega_m^f | \omega_n^i \rangle \langle \omega_n^i || \beta^- || 0 \rangle}{D_m}, \quad (9)$$

where the energy denominator is given by

$$D_m = \frac{\frac{1}{2}(\Delta_{\text{exp}} + \tilde{\omega}_m^i + \tilde{\omega}_m^f) + E_{\text{ex}}(1_1^+) + \Delta M_i^{\text{exp}}}{m_e c^2}. \quad (10)$$

Here,  $\tilde{\omega}_m = \omega_m - \omega_1$ ,  $\Delta_{\text{exp}}$  is the nuclear mass difference between initial and final states,  $E_{\text{ex}}(1_1^+)$  is the experimental energy of the first  $1^+$  state in the intermediate odd-odd nucleus,  $\Delta M_i^{\text{exp}}$  is the measured difference of the mass energies of the intermediate and initial nuclei, and  $m_e c^2$  the electron rest mass. Here we use as much as possible experimental information in

constructing the energy denominator (10) in order to avoid additional uncertainties rising from the description of nuclear mass differences by the  $pn$ -dQRPA formalism. The overlap between the initial  $1_n^+$  and final  $1_m^+$  states in Eq. (9),  $\langle \omega_m^f | \omega_n^i \rangle$ , was estimated according to a relation similar to Eq. (29) of Ref. [11], where we used  $pn$ -dQRPA amplitudes. This permits the use of a different deformation in the initial and final nucleus of double- $\beta$  decay.

### III. NUMERICAL APPLICATION

We now analyze the  $2\nu\beta\beta$  decay process  $^{150}\text{Nd} \rightarrow ^{150}\text{Sm}$  by using our  $pn$ -dQRPA formalism. To this purpose we describe the  $1^+$  states in the intermediate odd-odd nucleus  $^{150}\text{Pm}$  by using  $pn$ -dQRPA eigenstates for both the initial and final nuclei. We use as spherical  $sp$  states  $c_\tau^\dagger$  the eigenstates of the Woods-Saxon plus proton Coulomb mean field with the universal parametrization of [21]. The deformed eigenstates  $a_{\tau m}^\dagger$ , given by Eq. (1), are obtained by diagonalizing the quadrupole-quadrupole interaction in the adiabatic limit for both the initial and final nuclei. The deformation parameters  $\beta_2^i = 0.24$  and  $\beta_2^f = 0.21$  were taken from Ref. [22]. The  $u$  and  $v$  amplitudes were determined by solving the BCS equations with monopole interaction and by reproducing the experimental pairing gaps in the initial and final nuclei.

We then estimated the Gamow-Teller strength

$$B(\text{GT}^\pm) = [g_A(\omega || \beta^\pm || 0)]^2, \quad (11)$$

as a function of the energy relative to the ground state of the intermediate nucleus  $^{150}\text{Pm}$ . Here,  $g_A$  is the effective axial-vector strength. The dipole strength  $g_{ph} = 0.12$  was chosen to reproduce the experimental centroid of the Gamow-Teller ( $p,n$ )-type strength  $B(\text{GT}^-)$  in  $^{150}\text{Nd}$  [23], as can be seen in Fig. 1(a). Here, the experimental data are given by filled circles. The position of the centroid is insensitive to the value of the  $g_{pp}$  strength. Thus, in this figure we considered  $g_{pp} = 0$ . Here we choose the value of the effective axial-vector constant  $g_A = 0.8$ , which is consistent with the cumulative Gamow-Teller strength, as can be seen from Fig. 1(b). In Fig. 1(c) we plot the ( $n,p$ )-type  $B(\text{GT}^+)$  strength in the final nucleus  $^{150}\text{Sm}$  versus the excitation energy in  $^{150}\text{Pm}$ , and in Fig. 1(d) the corresponding cumulative strength. As can be seen, a reasonable agreement with the available experimental data is achieved.

It is worth noting that the  $B(\text{GT}^-)$  strength reproduces the order of magnitude of experimental data in the low part of the spectrum, as can be seen in Fig. 2.

We then estimated the magnitude of the  $2\nu\beta\beta$  matrix element, given by Eq. (9). Here, an important role is played by the overlap between the initial and final BCS wave functions with different deformations. In order to illustrate this point, in Fig. 3 we plot the dependence of the BCS overlap versus the deformation parameter of the final nucleus, by considering for the initial nucleus a fixed value  $\beta_2^i = 0.3$ . By dashed/dot-dashed lines are given proton/neutron BCS overlaps. One notices a very strong decrease of the overlap by increasing the difference between the two deformations. Even for the same deformation the resulting overlap is around 0.85.

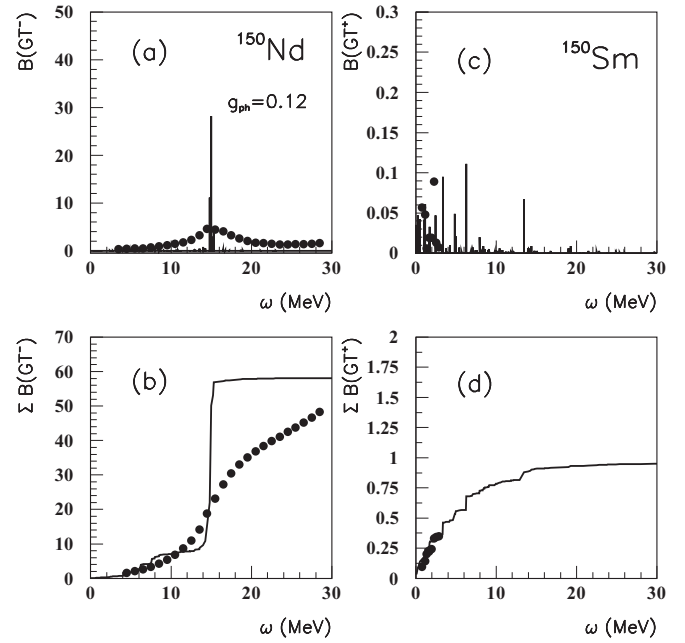


FIG. 1.  $B(\text{GT})$  strength versus the excitation energy in the intermediate nucleus  $^{150}\text{Pm}$ . The experimental data [23] are plotted by filled circles. (a)  $B(\text{GT}^-)$  strength in  $^{150}\text{Nd}$ , (b) Cumulative  $B(\text{GT}^-)$  strength in  $^{150}\text{Nd}$ , (c)  $B(\text{GT}^+)$  strength in  $^{150}\text{Sm}$ , and (d) Cumulative  $B(\text{GT}^+)$  strength in  $^{150}\text{Sm}$ .

In Fig. 4 we plot the  $2\nu\beta\beta$  matrix element (9) as a function of the particle-particle strength. By horizontal lines we indicated the experimental area allowed by experimental errors and by the range  $g_A = 0.8$ – $1.27$  of the effective axial-vector coupling

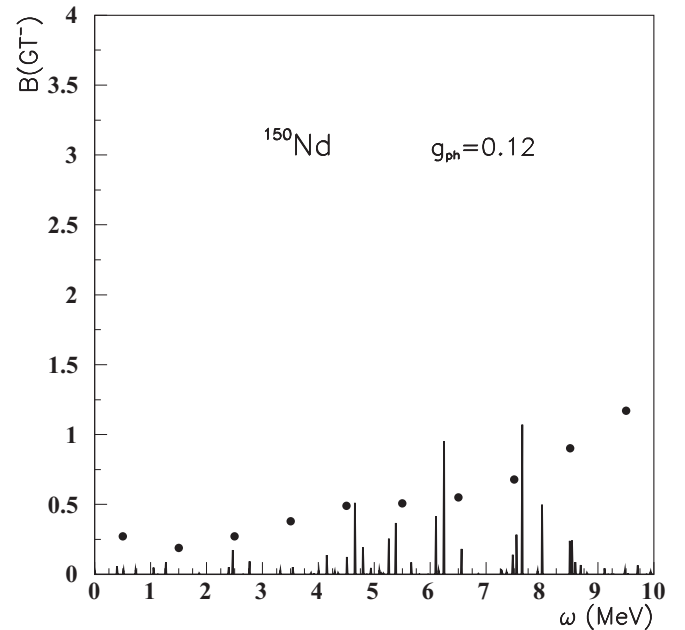


FIG. 2. Low-energy  $B(\text{GT}^-)$  strength in  $^{150}\text{Nd}$  versus the excitation energy in the intermediate nucleus  $^{150}\text{Pm}$ . The experimental data [23] are plotted by filled circles.

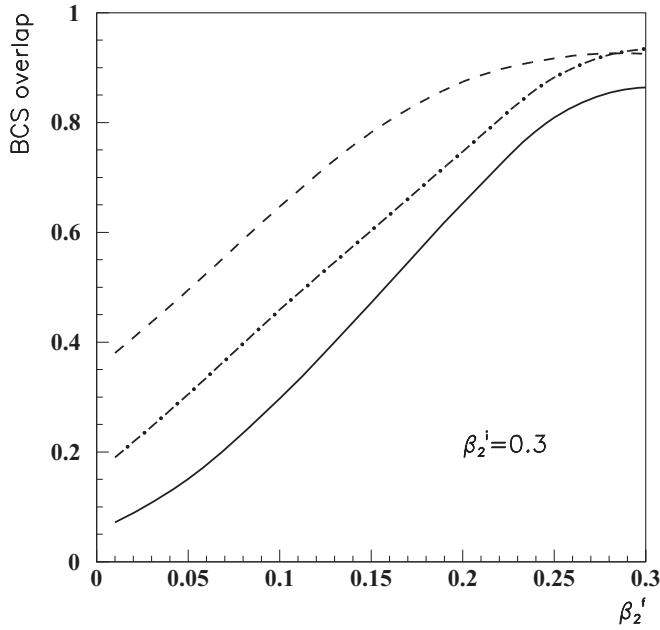


FIG. 3. Overlap between the initial and final BCS wave functions as a function of the quadrupole deformation of the final nucleus  $^{150}\text{Sm}$  (solid line). The quadrupole deformation of the initial nucleus  $^{150}\text{Nd}$  is taken to be  $\beta_2^i = 0.3$ . Proton/neutron overlaps are plotted by a dashed/dot-dashed line.

coefficient. The value  $g_A = 1.27$  corresponds to the bare value of  $g_A$ . In order to point out to the importance of the excitation energy  $E_{\text{ex}}$  in Eq. (9), which is usually neglected in

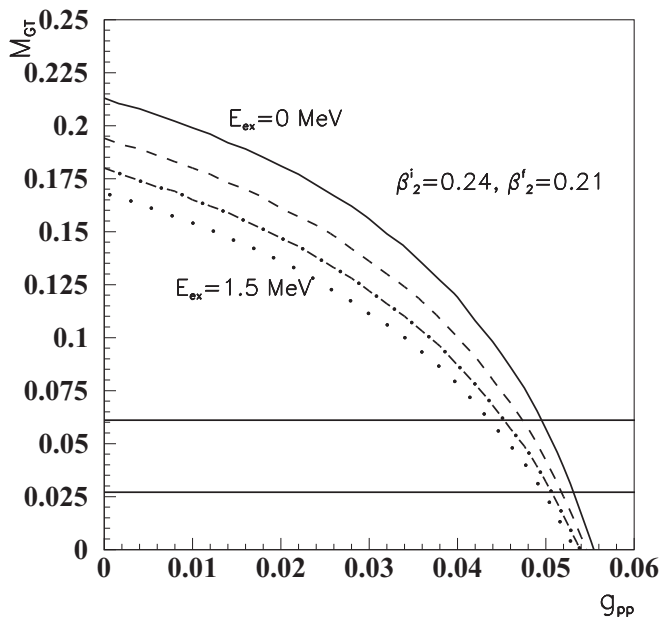


FIG. 4. Gamow-Teller  $2\nu\beta\beta$  matrix element versus  $g_{\text{pp}}$  for different values of the excitation energy of the first  $1^+$  state:  $E_{\text{ex}} = 0$  MeV (solid line),  $E_{\text{ex}} = 0.5$  MeV (dashed line),  $E_{\text{ex}} = 1$  MeV (dot-dashed line),  $E_{\text{ex}} = 1.5$  MeV (dotted line). The two horizontal lines denote the allowed experimental area for the interval  $g_A = 0.8-1.27$ .

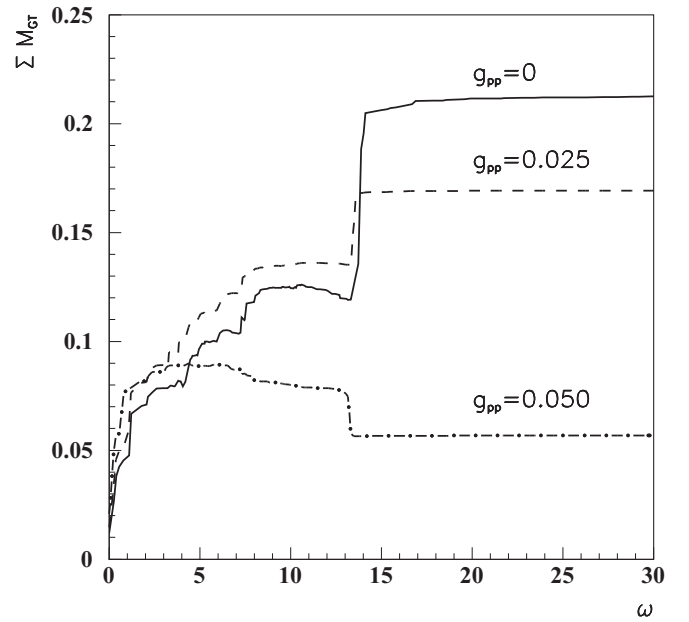


FIG. 5. Cumulative Gamow-Teller strength (12) versus the excitation energy in  $^{150}\text{Pm}$  for  $g_{\text{pp}} = 0$  (solid line),  $g_{\text{pp}} = 0.025$  (dashed line), and  $g_{\text{pp}} = 0.05$  (dot-dashed line).

most papers, we plotted this dependence for  $E_{\text{ex}} = 0$  MeV (solid line),  $E_{\text{ex}} = 0.5$  MeV (dashed line),  $E_{\text{ex}} = 1$  MeV (dot-dashed line), and  $E_{\text{ex}} = 1.5$  MeV (dotted line). Available experimental data of Ref. [23], shown in Fig. 2, indicate that  $E_{\text{ex}} = (0-0.5)$  MeV. Thus, the value of the particle-particle strength which reproduces the experimental value of the double- $\beta$ -decay strength is  $g_{\text{pp}} \approx 0.05$ . In order to point out the importance of the pp strength for the double- $\beta$ -decay matrix element  $M_{\text{GT}}$ , we plot in Fig. 5 the cumulative  $M_{\text{GT}}$  strength

$$\sum M_{\text{GT}} = \sum_{E_{\text{ex}}(1^+) \leq \omega} M_{\text{GT}}(1^+) \quad (12)$$

versus excitation energy for  $g_{\text{pp}} = 0$  (solid line),  $g_{\text{pp}} = 0.025$  (dashed line), and  $g_{\text{pp}} = 0.05$  (dotted line). One notices a strong dependence upon  $g_{\text{pp}}$ , especially for the region beyond the  $B(\text{GT}^-)$  maximum in Fig. 1(a).

To further test our model, we computed the half-lives for several superfluid  $2\nu\beta\beta$  partners, with known experimental half-life values. The results are given in the last column of the Table I. In the fifth column of this table there are given theoretical values estimated by using spherical approach, while in the sixth column we used the deformed method. For both we use  $g_{\text{pp}} = 0.05$  overall. We notice a significant improvement by the deformed approach compared to the spherical one, especially for nuclei with different deformations.

Finally, to put the presently introduced theory framework in context we perform here a brief comparison with other recent models that take into account the deformation in  $\beta$ -decay and/or double- $\beta$ -decay calculations. A very popular nuclear-structure model is the proton-neutron interacting boson model, IBA2. In Ref. [24] it has been used to compute  $0\nu\beta\beta$  decay rates of many cases of interest for experimental investigation. In these calculations the closure approximation has been

TABLE I.  $2\nu\beta\beta$  emitters with charge and mass numbers given in the first and second columns. Mother/daughter quadrupole deformation parameter [22] is given in the third/fourth column, theoretical spherical/deformed half-life in fifth/sixth column and experimental value in the last column.

$Z$	$A$	$\beta_L$	$\beta_R$	$\log_{10} T_{\text{th}}^{(\text{sph})}$	$\log_{10} T_{\text{th}}^{(\text{def})}$	$\log_{10} T_{\text{exp}}$
34	82	0.150	0.070	18.83	19.05	19.96
40	96	0.220	0.080	17.71	18.95	19.36
42	100	0.240	0.160	17.70	18.63	18.85
52	128	0.000	0.140	24.99	24.70	24.30
52	130	0.000	-0.110	22.31	21.23	20.84
60	150	0.240	0.210	18.55	18.93	18.91
92	238	0.210	0.210	20.93	21.54	21.30

exploited since the IBA2 model cannot calculate the wave functions of the intermediate odd-odd nucleus. While the  $0\nu\beta\beta$  decay rates are calculable in the closure approximation the  $2\nu\beta\beta$  decay rates are not [3]. This is why the IBA2 model is not suited for calculation of NMEs for  $2\nu\beta\beta$  decays. Another problem with IBA2 model is that it can only exploit a model space of one major shell, thus leaving out the spin-orbit partner orbitals in the adjacent major shells, known to be very relevant for Gamow-Teller type of transitions occurring in  $2\nu\beta\beta$  decays. The projected Hartree-Fock-Bogoliubov model (PHFB) of Ref. [25] has been used to compute both  $2\nu\beta\beta$  and  $0\nu\beta\beta$  decay rates for quite some time now. The model is based on a deformed Hartree-Fock-Bogoliubov mean field complemented by a ‘summation method’ to take into account, in an effective way, the correlations beyond the mean field. However, this model is still basically just a mean-field model that is unable to calculate the wave functions of the intermediate odd-odd nucleus. A third interesting model is the mean-field model [26] based on a Gogny energy-density functional. The sore point of this approach is the same as for IBA2: both these models have to use closure approximation thus excluding calculations of  $2\nu\beta\beta$  decay rates. A further model, designed to calculate Gamow-Teller and other nuclear transitions is the projected shell model (PSM) [27]. In fact,

the basic philosophy of PSM is very close to the  $\beta$ -dQRPA in that it also starts from a deformation dressed sp basis with good spherical quantum numbers. Since the PSM is applied to (axially) deformed nuclei the effectively deformed sp wave functions lead to an efficient handling of nuclear structure and small dimensions of the many-body model space. It is also a multishell model suited to a description of, e.g., parity-changing decay operators. As far as we know, its feasibility for calculations of  $2\nu\beta\beta$  decay properties has not been tested yet.

#### IV. CONCLUSIONS

Concluding, we described the  $1^+$  Gamow-Teller states in odd-odd deformed nuclei within a consistent  $pn$ -dQRPA framework, by using a sp basis with good angular momentum. This particle-core basis is provided by the diagonalization of a spherical mean field plus quadrupole-quadrupole interaction. The main features of  $\beta$ -decay strengths are reasonably described within a schematic pairing plus proton-neutron dipole residual interaction in particle-hole and particle-particle channels. We have confirmed the well known fact that the  $2\nu\beta\beta$  matrix element for  $^{150}\text{Nd}$  strongly depends upon the particle-particle strength. The value  $g_{\text{pp}} \approx 0.05$  reproduces the experimental half-life. By using this value of  $g_{\text{pp}}$  the experimental half-lives for superfluid emitters are rather well reproduced by the deformed approach, clearly better than the spherical one. The present projection-before-variation approach is able to give a consistent description of a deformed nucleus in the laboratory system of coordinates. It seems a very promising procedure to describe, in a relative simple way, deformed many-body systems.

#### ACKNOWLEDGMENTS

This work has been partially supported by the Academy of Finland under the Finnish Centre of Excellence Programme 2012-2017 (Nuclear and Accelerator Based Programme at JYFL) and grant nos. PN-II-ID-PCE-2011-3-0092 and PN-09370102 of the Romanian National Authority for Scientific Research.

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